Causality of Effective Field Theories in Gravitational Spacetimes

A Thesis

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by

Madhukar Deb

Indian Institute of Science Education and Research, Pune
Dr. Homi Bhabha Road,
Pashan, Pune 411008, INDIA

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Supervisor: Dr. Diptimoy Ghosh

Certificate

This is to certify that this dissertation entitled 'Causality of Effective Field Theories in Gravitational Spacetimes' towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by Madhukar Deb at Indian Institute of Science Education and Research under the supervision of Dr Diptimoy Ghosh, Associate Professor, Department of Physics, during the academic year 2025-2026.

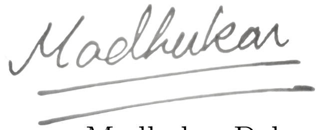
Dr Diptimoy Ghosh

Committee:
Dr Diptimoy Ghosh
Dr Sachin Jain

Declaration

I hereby declare that the matter embodied in the report entitled ’Causality of Effective Field Theories in Gravitational Spacetimes’ are the results of the work carried out by me at the Department of Physics, Indian Institute of Science Education & Research (IISER) Pune, under the supervision of Dr Diptimoy Ghosh, and the same has not been submitted elsewhere for any other degree. Wherever others contribute, every effort is made to indicate this clearly, with due reference to the literature and acknowledgement of collaborative research and discussions.

I declare that the use of Generative AI in this thesis did not extend beyond what is described in Section 4.6.1 of the ‘Guidelines for Generative AI usage at IISER Pune’, and accordingly, no attribution is required.



Madhukar Deb
Roll No: 20211036

This thesis is dedicated to my mentor, teachers, family and friends

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Abstract

Causality, along with unitarity, can be used to bound EFT coefficients of various EFTs. Here, we use microcausality and the Paley-Wiener theorem to bound the EFT coefficients of the scalar Goldstone boson as well as the Euler-Heisenberg effective action. These bounds on the coefficients match the bounds derived independently by imposing subluminality of the group velocity. We then show when microcausality-related bounds are equivalent to the subluminality of group velocity. In general, subluminal group velocity is a stronger condition when compared to microcausality.

We would also like to extend causality analysis on EFTs on flat spacetime to EFTs on curved spacetime when they show superluminality. Here, we specifically look at the Drummond-Hathrell (DH) effective action. Local superluminal photon propagation arises at $\mathcal{O}(\alpha/m_e^2)$ in the DH effective action obtained by integrating out the electron in QED coupled to gravity. Whether such superluminality implies a genuine violation of causality in curved spacetime is subtle and remains conceptually nontrivial. In this work we revisit this question using two complementary and largely symmetry-independent diagnostics.

First, we analyse the global causal structure of the effective (optical) metric governing DH photon propagation and identify conditions under which it remains stably causal, thereby excluding the formation of closed causal curves. Second, we treat the gravitational background to be a non-dynamical Lorentz-breaking field with a non-zero vacuum expectation value, and apply the microcausality bounds in flat spacetime on the photon propagator in the EFT-geometrical optics regime.

For two representative examples, a circular photon orbit in Schwarzschild and a linear trajectory in a two-black-hole geometry, we find that, within the regime of validity of the DH effective theory, both diagnostics indicate that the superluminal photon propagation is causally benign. These checks do not constitute a general definition of microcausality in curved spacetime, but provide a controlled and instructive check of causal consistency for EFT superluminality in gravitational backgrounds.

Contents

Chapter 1

Introduction

We use an effective field theory (EFTs) to describe some physical theory at low energy scales if we do not have access to its high-energy behaviour. An EFT is usually defined by some energy scale Λ, called the cutoff energy scale of the EFT. This EFT is only valid at energy scales $E \ll \Lambda$. The heavy degrees of freedom ($m \gtrsim \Lambda$) of the initial theory are captured in the coefficients of the EFT, which depend on the energy cutoff, as well as other parameters of the full theory. Given that we do not know the form of the full theory and only have access to the low-energy behaviour of the theory, an important question arises: *What methods can we use to constrain the EFT coefficient parameter space?* In this thesis, we will use causality as a constraint on the parameter space of various EFTs in flat and curved spacetime.

Relativistic causality places sharp restrictions on the propagation of signals in flat spacetime: no information can travel outside the Minkowski light cone, and in a quantum field theory, this requirement is encoded in microcausality, namely the vanishing of operator commutators at spacelike separation. These principles lead to powerful constraints on EFTs, particularly in settings where Lorentz invariance and an S-matrix description are available (see for example, [1]–[14]).

Restricting ourselves to flat spacetime, we see that the relativistic causality condition imposes non-trivial conditions on the coefficients of the EFT [1]. Causality was used in [15] by assuming the subluminality of the signal's group velocity to derive bounds on the coefficients of various EFTs, which are compatible with bounds derived using other methods, such as positivity and analyticity.

An alternate formalism of causality is microcausality (the vanishing of the commutator between spacelike-separated operators). Since microcausality is an operator statement and holds for all states of the theory, it holds even when Lorentz invariance is spontaneously broken by the background. Considering the commutator between two bosonic operators $G_c(t,\vec{x})$, [16] derived bounds on its mixed Fourier transform $\tilde{G}_c(t,\vec{k})$ by using the Paley-Wiener theorem [17]. According to the Paley-Wiener theorem, the commutator $G_c(t,\vec{x})$ is consistent with microcausality if and only if:

1. The mixed Fourier transform $\tilde{G}_c(t,\vec{k})$ is analytic for all $\vec{k} \in \mathbb{C}$
2. The function $\tilde{G}_c(t,\vec{k})$ is bounded as follows.

$$|\tilde{G}_c(t,\vec{k})| < C(A + ||\vec{k}||)^N e^{||\mathrm{Im}(\vec{k})||t} \quad \forall \;\; \vec{k} \in \mathbb{C}$$

   for some choice of positive constants of C, A and N.

As we will see, the above bound on $\tilde{G}_c(t,\vec{k})$ can be used to find constraints on the coefficients of various EFTs. In particular, we will apply this bound to constrain the parameters of the massless and massive Goldstone boson and the low-energy Euler-Heisenberg Lagrangian. As we will see, for these theories, the constraints match the ones derived the results derived before, particularly the results derived in [15].

In curved spacetime, however, the relation between signal propagation and causality is more subtle. Even in a UV-complete and causal theory such as QED coupled to gravity, integrating out heavy fields generates higher-derivative operators that can deform the effective causal cone relative to the background metric. A classic example is the Drummond–Hathrell (DH) effective action [18], in which $\mathcal{O}(\alpha/m_e^2)$ curvature couplings modify the photon dispersion relation. In various spacetimes, these corrections permit photon trajectories that are spacelike with respect to the background metric [19]–[22]. We refer to this phenomenon as "superluminality".

A natural question therefore arises:

> *Does local superluminal propagation in the DH EFT imply a breakdown of causality?*

In flat spacetime, the answer would immediately be no because of the equivalence of reference frames, which leads to changing ordering of events. In curved spacetime, the issue is far more intricate. Lorentz invariance is generically broken by the background geometry, reference frames are not equivalent, and familiar flat-space arguments no longer apply. As a result, superluminality does not automatically entail causality violation, a point that has been demonstrated in a variety of curved backgrounds [23]–[26].

Several works have addressed the causality of DH photons using time-delay arguments [27]–[29]. In situations where an S-matrix exists and the geometry enjoys suitable symmetries, DH corrections produce either a small positive time delay across a shockwave [27] or a parametrically negligible negative time delay in the Schwarzschild background [28], [29]. While these results are reassuring, time-delay-based reasoning has important limitations. Such arguments rely on asymptotic flatness, the existence of an S-matrix, and often strong symmetries such as spherical symmetry, making them difficult to generalise to multi-centred or non-symmetric geometries. Moreover, as emphasised by Penrose [30], even the operational meaning of "time delay" in asymptotically flat spacetimes is subtle, since curved-spacetime null cones can lie outside their flat-space counterparts. Time delay, while useful, is therefore not a universal diagnostic of causality.

These limitations motivate a more geometric and state-independent approach. In this work we adopt two complementary perspectives:

(1) **Stable causality:** We analyse the global causal structure of the effective (optical) metric governing DH photon propagation. Stably causal spacetime, which is defined to have no closed causal curves even under small metric perturbations, is equivalent to the existence of a global time function as seen in [31], [32]. This approach was previously applied to DH photons in FLRW backgrounds [33]. Here we extend the analysis to a circular photon orbit in the Schwarzschild geometry and to a straight-line trajectory in a two-centre, extremal Reissner–Nordström background.

(2) **Quantum microcausality:** Microcausality is a consequence of the causal structure of a theory rather than of Lorentz invariance itself [34]. Using the analyticity framework developed in [16], [35], we examine whether the modified DH dispersion relations are compatible with flat-spacetime microcausality bounds when the gravitational background is treated as a fixed Lorentz-breaking field. We further employ the formalism of [36] to analyse the structure of Green's functions in $(\omega, \vec{k})$ space.

We apply these diagnostics to two explicit examples exhibiting DH superluminality: (i) a circular photon orbit in Schwarzschild, and (ii) a straight-line trajectory between two extremal Reissner–Nordström black holes. In both cases we find that, within the geometric-optics window of the EFT,

> *the DH superluminality appears compatible with stable causality and with flat-spacetime microcausality bounds.*

In the two–black–hole geometry, the stable-causality analysis additionally imposes a parametric condition $M \gg m_e^{-1}$, while the microcausality test remains valid independently of this assumption.

This thesis is divided into two main parts. Part I is dedicated to using microcausality-related bounds to find constraints on the EFT coefficients for three different EFTs, namely, massless and massive Goldstone bosons, and the Euler-Heisenberg Lagrangian.
In Part II, we will analyse whether superluminality of the Drummond-Hathrell photon is consistent with causality in curved spacetime, or if this superluminality leads to some pathologies in the spacetime by using various methods. Part II is divided into various

sections. Chapter 5 explicitly shows how superluminality arises for the DH photon in the two spacetimes that we are considering (Schwarzschild and Two Reissner-Nordstrom Black Holes). Chapter 7 will use the concept of stable causality to analyse the causality of the DH photon. Similarly, in chapter 8, we will define a new causality condition that depends on quantum microcausality and use it to check the causality of the DH photon.

Convention and Notation

In this paper, we consider $\hbar = c = 1$. We also consider that $G = 1$. Thus, in the metric, wherever we write M, it is GM. Also, the metric signature is mostly negative $(+, -, -, -)$. Thus, timelike vectors are those which are such that $g_{\mu\nu}u^\mu u^\nu > 0$.

We denote the background metric as $g_{\mu\nu}$. The general *optical metric* (valid for all photon paths) is denoted by $\mathcal{G}'_{\mu\nu}$ and that for a specific photon path is denoted by $\tilde{\mathcal{G}}_{\mu\nu}$.

Note

1. Parts of the introduction, conclusion and most of the results derived in part II are based on the preprint [37], written by Dr Diptimoy Ghosh, Jay Desai and me.
2. I declare that the use of Generative AI in this thesis did not extend beyond what is described in Section 4.6.1 of the 'Guidelines for Generative AI usage at IISER Pune', and accordingly, no attribution is required.

Chapter 2

Prerequisite- Microcausality and Paley-Wiener theorem

Here, we review and compile key insights derived in [16]. These results are derived under the assumption of microcausality, a key assumption of defining a QFT in flat spacetime. The property of microcausality implies that given operators that define some local observables at a given point in spacetime, they will commute if the points are spacelike separated, that is:

$$[\mathcal{O}_1(x), \mathcal{O}_2(y)] = 0 \qquad \text{if} \;\; (x-y)^2 < 0 \tag{2.1}$$

Here, we assume that the metric is the Minkowski (flat spacetime) metric and has the signature $(+,-,-,-)$. Thus, the square of the difference of two spacelike points in this metric will be negative.

Since this is an operator statement, we note that it must hold for all possible states of the theory. If we assume that $\mathcal{O}(x) = \phi(x)$, which is the field operator, then the above statement must hold for all $\phi(x)$, even if they are not Lorentz invariant states.

Now, we consider the following $\mathcal{O}(x) = \phi(x)$, where $\phi(x)$ is the field operator, which is a 3-scalar field operator (invariant under space rotations and translations) and define

$$G_c(\vec{x}, t) = \langle [\phi(\vec{x}, t), \phi(0)] \rangle \tag{2.2}$$

Since $G_c(\vec{x}, t)$ is the expectation of the commutator of the form given in equation (2.1), it will disappear if $x = (\vec{x}, t)$ lies outside the light cone of 0, which is a reference point with respect to which we measure x. At a fixed time t, we can consider the function $G_c(\vec{x}, t)$ to be a complex function of $\vec{x}$. So, for a finite value of t, we get that $G_c(\vec{x}, t)$ is a compact function which has support only for $\vec{x}$ such that $|\vec{x}| < t$. This means that we can use the properties of complex functions to derive some useful results.

We now state the Paley-Wiener theorem [17]: An $L(1)$ function $f(\vec{x})$ has the compact support $\|\vec{x}\| < R$ for some finite R if and only if its Fourier transform $\tilde{f}(\vec{k})$ has the following properties:

1. $\tilde{f}(\vec{k})$ is analytic for $\vec{k} \in \mathbb{C}$
2. $\tilde{f}(\vec{k})$ is exponentially bounded. The exponential bound is given as follows:

$$|\tilde{f}(\vec{k})| < Ce^{||\mathrm{Im}(\vec{k})||R} \tag{2.3}$$

   where C is a constant. If $f(x)$ is a tempered distribution, we see that the bound is modified as follows:

$$|\tilde{f}(\vec{k})| < C(A + ||\vec{k}||)^N e^{||\mathrm{Im}(\vec{k})||R} \tag{2.4}$$

   for some choice of constants C, A and N.

We see that from microcausality, $G_c(t, \vec{x})$ has a compact support of $|\vec{x}| < t$ for a fixed value of t. Thus, from Paley-Wiener theorem, we know that the Fourier transform $\tilde{G}_c(t, \vec{k})$ is analytic for all $\vec{k} \in \mathbb{C}$. We see that the property of Green's functions lead to $G_c(t, \vec{x})$ being tempered. This means that we see the following bound:

$$|\tilde{G}_c(t, \vec{k})| < C(A + ||\vec{k}||)^N e^{||\mathrm{Im}(\vec{k})||t} \tag{2.5}$$

Since we are dealing with rotationally invariant operators $\phi(\vec{x})$, which means that the G_c will only depend on $||\vec{x}||^2$. Thus, $\tilde{G}_c$ will only depend on $||\vec{k}||^2 = k^2$ and will be analytic in k^2. Our bound in equation (2.5) will be modified to:

$$|\tilde{G}_c(t, \vec{k})| < C(A + |k|)^N e^{|\mathrm{Im}(k)|t} \tag{2.6}$$

We will use the bounds in the above equation, as given, in a few examples to derive bounds on the parameters, as we will see in Part I. This bound will also be useful while trying to study causality of superluminal signals in curved space, as we will see in Part 5

Part I

Microcausality Bounds on EFTs in Flat spacetime

Chapter 3

Microcausality bounds on EFT parameters

In this section, we will independently try to find bounds on the parameters of various EFTs by using microcausality analysis as derived in [16]. This serves as a review as well as a consistency check for the bounds derived by imposing causality via a subluminality analysis as seen in [15].

We will first use the microcausality analysis to do a consistency check on a free scalar theory, and then we will extend this analysis to various EFTs

3.1 Free Scalar Field

Let us consider a system described by the Lagrangian:

$$\mathcal{L} = \frac{1}{2}((\partial\phi)^2 - m^2\phi^2) \tag{3.1}$$

For the above given Lagrangian, $\tilde{G}_c$ can be written as:

$$\tilde{G}_c(t, \vec{k}) = -i\frac{\sin(\omega_k t)}{\omega_k} \tag{3.2}$$

where $\omega_k = \sqrt{k^2 + m^2}$. Now, we can check that the analyticity of $\tilde{G}_c$ in the variable k^2. At first glance, it looks like $\tilde{G}_c$ is non analytic as $\omega_k = \sqrt{k^2 + m^2}$ and will lead to branch cuts, but if we write $\sin(\omega_k t)$ as polynomial by Taylor expansion, we get that

$$\sin(\omega_k t) = \sum_{n=0}^{\infty} (-1)^n \frac{(\omega_k t)^{2n+1}}{(2n+1)!} \tag{3.3}$$

and we see that:

$$\frac{\sin(\omega_k t)}{\omega_k} = \sum_{n=0}^{\infty}(-1)^n \frac{(\omega_k)^{2n} t^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty}(-1)^n \frac{(k^2+m^2)^n t^{2n+1}}{(2n+1)!} \tag{3.4}$$

Thus, we see that $\tilde{G}_c$ is an analytic function of k^2.

Next, we would like to check the boundedness of $\tilde{G}_c$ to verify the microcausality result. Assuming that k is complex, we get that ω_k too is complex, which will result in:

$$\tilde{G}_c(t, \vec{k}) \sim e^{\pm \omega_k t} \tag{3.5}$$

if we ignore the non exponential prefactors. We can ignore $1/\omega_k$ because we can find a polynomial of the form $C(A + ||k||)^N$ such that $1/\omega_k < C(A + ||k||)^N$. So, for $\tilde{G}_c(t, \vec{k})$ to satisfy the bound given by equation (2.6), we require that:

$$e^{|\text{Im}(\omega_k)|t} \leq e^{|\text{Im}(k)|t} \tag{3.6}$$

Thus, we get the condition that;

$$|\text{Im}(\omega_k)| \leq |\text{Im}(k)| \tag{3.7}$$

But we can show that this is true for $\omega_k = \sqrt{k^2+m^2}$ as follows. If we assume that $\omega_k = \text{Re}(\omega_k) + i\text{Im}(\omega_k)$. This would imply that
$\omega_k^2 = (\text{Re}(\omega_k))^2 - (\text{Im}(\omega_k))^2 + 2i\text{Re}(\omega_k)\text{Im}(\omega_k)$. Thus, we get that:

$$\begin{aligned}(\text{Re}(\omega_k))^2 - (\text{Im}(\omega_k))^2 + 2i\text{Re}(\omega_k)\text{Im}(\omega_k) = (\text{Re}(k))^2 - (\text{Im}(k))^2 \\ + 2i\text{Re}(k)\text{Im}(k) + m^2\end{aligned} \tag{3.8}$$

If we rewrite $k = a + ib$ and $\omega_k = A + iB$, by equating both real and imaginary parts of equation (3.8), we get that:

$$\begin{aligned} A^2 - B^2 &= a^2 + m^2 - b^2 \\ A^2B^2 &= a^2b^2 \end{aligned} \tag{3.9}$$

The real part will generate the following inequality:

$$A^2 - B^2 \geq a^2 - b^2 \tag{3.10}$$

Now, we see that a^2, b^2, A^2 and B^2 are non-negative numbers such that their products are equal from the equation for the imaginary part and their differences are unequal. Let us make the following parameterization:

$$a^2 = \frac{x+p}{2} \qquad b^2 = \frac{x-p}{2}$$
$$A^2 = \frac{y+q}{2} \qquad B^2 = \frac{y-q}{2}$$

where x, y, p and q are non negative numbers, such that $x^2 - p^2 = y^2 - q^2$ and $x \geq y$. This would imply that $q^2 \geq p^2$ or $q \geq p$. which would mean that $x - p \geq y - q$. This leads to the inequality that:

$$b^2 \geq B^2 \implies \text{Im}(\omega_k) \leq \text{Im}(k) \tag{3.11}$$

Hence, equation (3.11) verifies the bound obtained in equation (3.7).

3.2 Massless Goldstone Boson

We can bound the coefficients for the massless Goldstone boson as follows. We first write the Lagrangian as given in [15]:

$$\mathcal{L} = \frac{1}{2}\left((\partial\phi)^2 + \frac{4a}{3\Lambda^2}\dot{\phi}^3 + \frac{2b}{3\Lambda^4}\dot{\phi}^4\right) \tag{3.12}$$

where, Λ is the energy cutoff for the theory.

We can use the microcausality analysis to derive bounds on a and b. We start by expanding the Lagrangian about a background $\phi = Ct + \pi$, where π is the fluctuation about the background Ct and C is very small. So, we can expand the equation in powers of C. The leading order terms in the Lagrangian will look like:

$$\mathcal{L} = \frac{1}{2}\left(\alpha\dot{\pi}^2 - (\nabla\pi)^2\right) = \frac{1}{2}\alpha(\dot{\pi}^2 - \alpha^{-1}(\nabla\pi)^2) \tag{3.13}$$

Here $\alpha = 1 + \frac{4a}{3\Lambda^2}C + \frac{2b}{3\Lambda^4}C^2$.

The function describing $\tilde{G}_c$ will be of the same form as given in equation (3.2) with $\omega_k = \alpha^{-1/2}k$. For $|\text{Im}(\omega_k)| \leq |\text{Im}(k)|$ to be true, we would need that $\alpha \geq 1$ for any choice of C. If we only keep terms up to first power of C in α, we get that:

$$\alpha \approx 1 + \frac{4a}{3\Lambda^2}C \tag{3.14}$$

Since C can be either positive or negative, we have no non zero choice of a for which $\alpha \geq 1$ for any choice of C. Hence, $a = 0$

Now, if we consider terms of the order C^2, we get that:

$$\alpha = 1 + \frac{2b}{3\Lambda^4}C^2 \tag{3.15}$$

Here, we see that since $C^2 \geq 0$ for any choice of C, for $\alpha \geq 1$, we need that $b \geq 0$. Hence, we have a bound on a and b by using microcausality analysis that is given by:

$$a = 0 \qquad b \geq 0 \tag{3.16}$$

This result is consistent with bounds obtained using subluminality analysis as seen in [15].

3.3 Massive Goldstone boson

Now, we can try to find bounds on the coefficients in the Lagrangian representing a massive Goldstone boson as given in [15]. The Lagrangian is given by:

$$\mathcal{L} = \frac{1}{2}\left((\partial\phi)^2 + \frac{2a}{\Lambda^4}(\partial\phi)^4 - m^2\phi^2 - 2J\phi\right) \tag{3.17}$$

Here, m represents the mass of the boson and J is a source term that sources the background that we will choose. The equation of motion is given by:

$$\partial^2\phi\left(1 + \frac{4a(\partial\phi)^2}{\Lambda^4}\right) + \frac{8a}{\Lambda^4}(\partial_\nu\phi)(\partial^\mu\partial^\nu\phi)(\partial_\nu\phi) + m^2\phi + J = 0 \tag{3.18}$$

Now, we choose a background about which we expand, which is given by $\phi = \pi + \phi_0$, such that $\partial_\mu\phi_0 = C_\mu$, where C_μ is a constant vector and $C_\mu C^\mu \ll \Lambda^4$. Since J sources the background, we get that

$$m^2\phi_0 + J = 0 \tag{3.19}$$

Thus, the equation of motion for π upto linear order and upto order $C^2 = C_\mu C^\mu$ is given by:

$$\partial^2\pi\left(1 + \frac{4aC^2}{\Lambda^4}\right) + \frac{8a}{\Lambda^4}C_\mu C_\nu(\partial^\mu\partial^\nu\pi) + m^2\pi = 0 \tag{3.20}$$

Now, we see that upto lowest order in C the solution to this equation is $\partial^2\pi = -m^2\pi$. So, we can rewrite the above equation as:

$$\partial^2\pi + \frac{8a}{\Lambda^4}C_\mu C_\nu(\partial^\mu\partial^\nu\pi) + m^2\pi\left(1 - \frac{4aC^2}{\Lambda^4}\right) = 0 \tag{3.21}$$

Now, we see that the dispersion relation will be given by:

$$k^2 + \frac{8a}{\Lambda^4} C_\mu C_\nu k^\mu k^\nu - m^2 \left(1 - \frac{4aC^2}{\Lambda^4}\right) = 0 \tag{3.22}$$

Now, we choose $C_\mu = (0, 0, 0, C_3)$ and thus, $C^2 = -C_3^2$ and this leads to equation (3.22) looking like:

$$\begin{aligned} \omega^2 - |\vec{k}|^2 + \frac{8a}{\Lambda^4} C_3^2 k_3^2 - m^2 \left(1 + \frac{4aC_3^2}{\Lambda^4}\right) = 0 \\ \implies \omega^2 = |\vec{k}|^2 - \frac{8a}{\Lambda^4} C_3^2 k_3^2 + m^2 \left(1 + \frac{4aC_3^2}{\Lambda^4}\right) \end{aligned} \tag{3.23}$$

If we assume that the boson only has momentum parallel to the vector C_μ (in the 3 direction), we get the following dispersion relation:

$$\omega_k^2 = \alpha k^2 + \beta m^2 \tag{3.24}$$

where $\alpha = 1 - \frac{8aC_3^2}{\Lambda^4}$ and $\beta = 1 + \frac{4aC_3^2}{\Lambda^4}$

From equation (3.21), we can explicitly show that the mixed commutator is of the form (3.2) with $\omega_k = \alpha k^2 + \beta m^2$. If we require microcausality to hold for this system, we see that $(\text{Im}(\omega_k))^2 \leq (\text{Im}(k))^2$. Now if we start with this condition, we get the following:

$$\begin{aligned} &(\text{Re}(\omega_k))^2 - (\text{Im}(\omega_k))^2 + 2i\text{Re}(\omega_k)\text{Im}(\omega_k) \\ &= \alpha[(\text{Re}(k))^2 - (\text{Im}(k))^2 + 2i\text{Re}(k)\text{Im}(k)] + \beta m^2 \end{aligned} \tag{3.25}$$

Now, if we equate the real and imaginary parts, we get that:

$$\begin{aligned} &(\text{Re}(\omega_k))^2 - (\text{Im}(\omega_k))^2 = \alpha[(\text{Re}(k))^2 - (\text{Im}(k))^2] + \beta m^2 \\ &\text{Re}(\omega_k)\text{Im}(\omega_k) = \alpha\text{Re}(k)\text{Im}(k) \end{aligned}$$

Thus, by applying our microcausality condition to the imaginary part equation, we get that $\text{Re}(\omega_k) \geq \alpha\text{Re}(k)$. Substituting this into the real part equation, we get that:

$$\begin{aligned} \alpha[(\text{Re}(k))^2 - (\text{Im}(k))^2] + \beta m^2 \geq (\alpha\text{Re}(k))^2 - (\text{Im}(k))^2 \\ \implies \alpha(\alpha - 1)(\text{Re}(k))^2 + (\alpha - 1)(\text{Im}(k))^2 \leq \beta m^2 \end{aligned} \tag{3.26}$$

Upto order C^2, the l.h.s can be written as

$$(\alpha - 1)((\text{Re}(k))^2 + (\text{Im}(k))^2) \leq \beta m^2 \tag{3.27}$$

But, we know that $(\text{Re}(k))^2 + (\text{Im}(k))^2 = |k|^2$, which is always positive and we know that

$\alpha - 1 = -\frac{8aC_3^2}{\Lambda^4}$. Thus, we get that:

$$\frac{8aC_3^2}{\Lambda^4} \geq -\frac{\beta m^2}{|k|^2}$$
$$\implies a \geq -\frac{\beta \Lambda^4 m^2}{8C_3^2 |k|^2} \tag{3.28}$$

The issue with this bound is that it explicitly depends on the momentum of the boson and thus, cannot be considered a reliable bound because the momentum may vary over a given range and we do not get a strict bound for a. Also, the above constraint is dependent on $1/C^2$, which is a very large number as we have considered C to be a perturbative constant.

We see that if we try to derive a bound using subluminality, we get a similar bound, which is not helpful. Whereas if we try to bound a using amplitude analysis, we get a similar bound that we got in the equation (3.16), where we get that $a \geq 0$. Hence, this is an example of a system where we cannot get a reliable bound from just using causality, but need a different method to bound the EFT parameter.

3.4 Euler-Heisenberg Lagrangian

The Euler-Heisenberg action [38], [39] is the low-energy effective-action derived by considering the theory of QED at energy scales lower than m_e. This theory is derived by considering upto one loop interactions of QED. The action can be written as:

$$\mathcal{L} = -\frac{1}{4}F^2 + c_1 F^4 + c_2 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2 \tag{3.29}$$

Here, $\tilde{F}^{\mu\nu} = \epsilon^{\rho\sigma\mu\nu}F_{\rho\sigma}$. For our choice of background, $F_{\mu\nu}\tilde{F}^{\mu\nu}$ will be 0, so we will only focus on the F^4 term for the rest of this section. Also, both c_1 and c_2 are of the order α^2/m_e^4.

We can get some constraints on the sign of c_1 just by assuming microcausality. We can vary the above equation with respect to A_μ to get the equation of motion as follows:

$$\partial_\mu F^{\mu\nu} - c_1(16F_{\alpha\beta}F^{\mu\nu}\partial_\mu F^{\alpha\beta} + 8F^2\partial_\mu F^{\mu\nu}) = 0 \tag{3.30}$$

If we assume a background $\bar{A}_\mu = (0, 0, 0, Ct)$, we get the following $\bar{F}_{\mu\nu} = C(\delta^0{}_\mu\delta^3{}_\nu - \delta^3{}_\mu\delta^0{}_\nu)$. Also, we get that $F^2 = -2C^2$. If we assume that $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$, we get the linearized equation of motion for $f_{\mu\nu}$ to be:

$$(1 + 16c_1C^2)\partial_\mu f^{\mu\nu} - 16c_1\bar{F}_{\alpha\beta}\bar{F}^{\mu\nu}\partial_\mu f^{\alpha\beta} = 0 \tag{3.31}$$

Now, going into Fourier space and assuming Lorentz gauge or $\partial_\mu a^\mu = 0$ (where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$), we get that:

$$k_\mu k^\mu a^\nu - \frac{A}{C^2}\bar{F}_{\alpha\beta}\bar{F}^{\mu\nu}k_\mu k^\alpha a^\beta = 0 \tag{3.32}$$

where $A = 32c_1C^2/(1+16\alpha C^2)$. Now, we can define a new vector:

$$\tilde{k}_\nu = \frac{\bar{F}_{\mu\nu}}{C}k^\mu \tag{3.33}$$

So the equation of motion can be written as:

$$k_\mu k^\mu a^\nu - A\tilde{k}^\nu\tilde{k}_\beta a^\beta = 0 \tag{3.34}$$

We see that if $\nu = 1$ or 2, we get the same dispersion relation that we would from the Maxwell Lagragian. Thus, we have to look at $\nu = 3$ and $\nu = 0$. Thus, we see that :

$$\begin{aligned} k_\mu k^\mu a^3 - A(\tilde{k}^3\tilde{k}_3a^3 + \tilde{k}^3\tilde{k}_0a^0) = 0 \\ k_\mu k^\mu a^0 - A(\tilde{k}^0\tilde{k}_3a^3 + \tilde{k}^0\tilde{k}_0a^0) = 0 \end{aligned} \tag{3.35}$$

An alternate method to find the dispersion relation is by finding the the poles of the propagator. To do this, we start with the Lagrangian and we can write the Lagrangian as:

$$\mathcal{L} = -\frac{1}{4}(1-8c_1\bar{F}^2)f^2 + 4c_1\bar{F}^{\alpha\beta}\bar{F}^{\mu\nu}f_{\alpha\beta}f_{\mu\nu} \tag{3.36}$$

where we have written $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ and have ignored the background terms. Now if we go to the Fourier space:

$$\tilde{\mathcal{L}} = \left(\frac{1}{2}(1+16c_1C^2)(k^2\eta^{\mu\nu} - k^\mu k^\nu) - 16c_1C^2\tilde{k}^\mu\tilde{k}^\nu\right)a_\mu a_\nu \tag{3.37}$$

The propagator is the inverse of the operator in the bracket. Hence, we get:

$$G_{\nu\rho}((k^2\eta^{\mu\nu} - k^\mu k^\nu) - A\tilde{k}^\mu\tilde{k}^\nu) = -i\delta^\mu{}_\rho \tag{3.38}$$

If we add a Lagrange multiplier to the Lagrangian of the form $(1/2\xi)(\partial_\mu A_\nu)^2$, the above equation changes as follows:

$$k^2\left(\left(\eta^{\mu\nu} - \left(1-\frac{1}{\xi}\right)\frac{k^\mu k^\nu}{k^2}\right) - A\frac{\tilde{k}^\mu\tilde{k}^\nu}{k^2}\right)G_{\nu\rho} = -i\delta^\mu{}_\rho \tag{3.39}$$

Now, we pick the Feynman gauge, where $\xi = 1$. Thus, we can write the above equation as:

$$k^2\left(\eta^{\mu\nu} - \beta\frac{\tilde{k}^\mu\tilde{k}^\nu}{k^2}\right)G_{\nu\rho} = -i\delta^\mu{}_\rho \tag{3.40}$$

We can now expand $G_{\nu\rho}$ as a power series in β as given:

$$G_{\nu\rho} = G^{(0)}_{\nu\rho} + \beta G^{(1)}_{\nu\rho} + \beta^2 G^{(2)}_{\nu\rho}\dots \tag{3.41}$$

where, $G^{(0)}_{\nu\rho}$ is the propagator for Maxwell Lagrangian with the gauge $\xi = 1$, which is given by:

$$G^{(0)}_{\mu\nu} = \frac{-i\eta_{\mu\nu}}{k^2} \tag{3.42}$$

Now, substituting the value of $G_{\mu\nu}$ into equation (3.40), we get the following relation by matching powers of β:

$$G^{(n)}_{\nu\rho}\eta^{\mu\nu} - G^{(n-1)}_{\nu\rho}\frac{\tilde{k}^\mu\tilde{k}^\nu}{k^2} = 0 \tag{3.43}$$

If we take $n = 1$, we get that:

$$\begin{aligned} G^{(1)}_{\nu\rho}\eta^{\mu\nu} &= \frac{-i\eta_{\nu\rho}}{k^2}\frac{\tilde{k}^\mu\tilde{k}^\nu}{k^2} \\ \implies G^{(1)}_{\sigma\rho} &= \frac{\eta_{\nu\rho}\eta_{\mu\sigma}}{k^2}\frac{\tilde{k}^\mu\tilde{k}^\nu}{k^2} = \frac{-i\tilde{k}_\rho\tilde{k}_\sigma}{k^4} \end{aligned} \tag{3.44}$$

Similarly, if we take $n = 2$, we get

$$\begin{aligned} G^{(2)}_{\nu\rho}\eta^{\mu\nu} &= \frac{\tilde{k}_\rho\tilde{k}_\nu}{k^4}\frac{\tilde{k}^\mu\tilde{k}^\nu}{k^2} \\ \implies G^{(2)}_{\sigma\rho} &= \frac{\tilde{k}_\rho\tilde{k}_\nu\eta_{\mu\sigma}}{k^4}\frac{\tilde{k}^\mu\tilde{k}^\nu}{k^2} = \frac{\tilde{k}^2(\tilde{k}_\rho\tilde{k}_\sigma)}{k^6} = G^{(1)}_{\nu\rho}\frac{\tilde{k}^2}{k^2} \end{aligned} \tag{3.45}$$

Similarly, for higher n, we will see that the following holds:

$$G^{(n)}_{\rho\sigma} = G^{(1)}_{\rho\sigma}\left(\frac{\tilde{k}^2}{k^2}\right)^{(n-1)} \tag{3.46}$$

Thus, our Feynman propagator can be written as:

$$\begin{aligned} G_{\rho\sigma}(k) &= \frac{-i}{k^2}\left(\eta_{\rho\sigma} + \frac{\tilde{k}_\rho\tilde{k}_\sigma}{\tilde{k}^2}\sum_{n=1}^{\infty}\left(A\frac{\tilde{k}^2}{k^2}\right)^n\right) \\ &= \frac{-i}{k^2}\left(\eta_{\rho\sigma} + \frac{A\tilde{k}_\rho\tilde{k}_\sigma}{k^2 - A\tilde{k}^2}\right) \end{aligned} \tag{3.47}$$

We know that the momentum space commutator only has support on shell. This means that, we can write $(G'_c)_{\mu\nu}$ as:

$$(G'_c)_{\mu\nu} \sim \left(\left(\eta_{\mu\nu} - \frac{\tilde{k}_\mu \tilde{k}_\nu}{\tilde{k}^2}\right)\delta(k^2) + \frac{\tilde{k}_\mu \tilde{k}_\nu}{\tilde{k}^2}\delta(k^2 - A\tilde{k}^2)\right) \tag{3.48}$$

Hence, we have a non-trivial dispersion relation given by:

$$\begin{aligned} & k^2 - A\tilde{k}^2 = 0 \\ \implies & \omega^2 - |\vec{k}|^2 - Ak_3^2 + A\omega^2 = 0 \end{aligned} \tag{3.49}$$

If we assume that propagation is in the 1 and 2 direction, we get:

$$(1+A)\omega^2 - |\vec{k}|^2 = 0 \implies \omega^2 = \frac{|k|^2}{1+A} \tag{3.50}$$

Now, if we apply that the group velocity must be subluminal for this propagation, we get that:

$$\frac{1}{1+A} \leq 1 \implies A \geq 0 \implies c_1 \geq 0 \tag{3.51}$$

which we see matches with the fact that in the Euler-Heisenberg Lagrangian, $c_1 > 0$. This matches with the sign of c_1 derived by Euler and Heisenberg in [38]

We also see that if we do an inverse Fourier transform of equation (3.47) with respect to the time coordinate to get $\tilde{G}_{\mu\nu}(\vec{k}, t)$, it is actually analytic for all complex $\vec{k}$. And also:

$$|(\tilde{G}_{\mu\nu})_c(t, \vec{k})| \sim e^{\mathrm{Im}(k)t} + e^{\mathrm{Im}(k)t/(1+A)^{1/2}} \tag{3.52}$$

For the Euler-Heisenberg Lagrangian to obey microcausality, we require that:

$$\frac{\mathrm{Im}(k)}{(1+A)^{1/2}} \leq \mathrm{Im}(k) \implies \frac{1}{1+A} \leq 1 \tag{3.53}$$

otherwise, we see that the mixed commutator will not follow the microcausality-related bound.

The above result is equivalent to using the claiming the subluminality of the group velocity, as seen in [15]. Thus, the microcausality bound that we derived using the formalism in [16] seems to match the bounds that we derived from subluminality.

Chapter 4

Group velocity

All the bounds that we have derived so far have been equivalent to the subluminality bounds, which depend on the group velocity. So, in this section, we would like to see how the group velocity is related to the imaginary part of ω_k and k.

If we assume that $\text{Im}(\omega_k)|_{k\in\mathbb{R}^+} = 0$, we can show that for certain excitations [16]:

$$\text{Im}(\omega_k)|_{k\to\mathbb{R}^+} = \frac{d\omega_k}{dk}\text{Im}(k) \tag{4.1}$$

To prove this, we assume that $\omega_k = f(k)$ such that $\text{Im}(f(k))|_{k\in\mathbb{R}^+} = 0$. Expanding about $k_0 \in \mathbb{R}^+$:

$$\omega_k = f(k_0) + f'(k)|_{k=k_0}(k-k_0) + f''(k_0)|_{k=k_0}\frac{(k-k_0)^2}{2!}\dots \tag{4.2}$$

Now, we know that $\text{Im}(f^{(n)}(k)|_{k=k_0}) = 0$. Thus, taking the imaginary part on both sides of equation (4.2), we get that:

$$\text{Im}(\omega_k) = f'(k)|_{k=k_0}\text{Im}(k-k_0) + f''(k_0)|_{k=k_0}\frac{\text{Im}((k-k_0)^2)}{2!}\dots \tag{4.3}$$

If we assume that $k \to k_0$ along the real axis, we set $k = k_0 + i\epsilon$, where $\epsilon \to 0$. Thus, we get that $\text{Im}(k-k_0) = \text{Im}(k)$, which means that upto lowest order, we get that:

$$\text{Im}(\omega_k) = f'(k)|_{k=k_0}\text{Im}(k) = \left.\frac{d\omega_k}{dk}\right|_{k=k_0}\text{Im}(k) \tag{4.4}$$

Now, from equation (3.7) and (4.4), we get that:

$$v_g \equiv \frac{d\omega_k}{dk} \quad \text{and} \quad |v_g| \leq 1 \tag{4.5}$$

This is the condition for subluminality that we have derived independently by using microcausality. This implies that our microcausality bound naturally requires subluminal group velocity, as used in [15] to similarly bound various EFT parameters.

We also observe that the bound given in (2.5) is a necessary and sufficient condition for the commutator vanishing outside the light cone (obeying microcausality). But, imposing subluminality is actually a necessary but not sufficient condition for microcausality to hold. This is mainly because deriving this relation between microcausality and subluminality requires a non-trivial assumption that $\mathrm{Im}(\omega_k)|_{k\in\mathbb{R}^+} = 0$, which may not always be true. Particularly, if there is a resonance, the $\mathrm{Im}(\omega_k)$ may be negative even for real k. This will lead to the group velocity being superluminal.

For all the examples of EFTs that we have considered, imposing subluminality seems to give an identical bound to the EFT parameters, as if we apply a full microcausality analysis to the EFTs. This implies that in all the theories that we considered in the previous chapter, there seem to be no unstable modes or resonances that would cause the two bounds to differ.

Part II

Causality Analysis of superluminal Drummond-Hathrell Photons

Chapter 5

Superluminality of the Drummond-Hathrell Photon

In their classic analysis [18], Drummond and Hathrell showed that integrating out the electron in QED coupled to gravity generates higher-derivative electromagnetic operators suppressed by m_e^{-2}. The resulting effective action, valid well below the electron mass, admits an expansion in powers of α/m_e^2,

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(R - \frac{1}{4}F^2 + a\,R\,F^2 + b\,R_{\mu\nu}F^{\mu}{}_{\rho}F^{\nu\rho} + c\,R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + d\,\nabla_\mu F^{\mu\lambda}\nabla_\nu F^{\nu}{}_{\lambda}\right). \tag{5.1}$$

The coefficients (a, b, c, d) are all of order α/m_e^2, and their explicit values are derived in [18], [21], [22]. For the purposes of this work, the scaling and sign of these coefficients are more relevant than their precise numerical values.

Varying the action with respect to A_μ yields the equations of motion

$$\begin{aligned}\nabla_\mu F^{\mu\nu} - 4a\nabla_\mu(RF^{\mu\nu}) - 2b\Big(R^{\mu}{}_{\rho}\nabla_\mu F^{\rho\nu} - F^{\rho\mu}\nabla_\mu R^{\nu}{}_{\rho} - R^{\nu}{}_{\rho}\nabla_\mu F^{\rho\mu} + F^{\rho\nu}\nabla_\mu R^{\mu}{}_{\rho}\Big)\\ - 4c\nabla_\mu(R^{\mu\nu\rho\sigma}F_{\rho\sigma}) = 0.\end{aligned} \tag{5.2}$$

The term proportional to d multiplies $\nabla_\mu F^{\mu\nu}$, which itself is $\mathcal{O}(\alpha/m_e^2)$, so keeping the d-term would only contribute at higher order. We therefore consistently drop it in leading-order analyses.

In the backgrounds that we consider, the equation of motion is given by:

$$\nabla_\mu F^{\mu\nu} - (2b\,R^{\mu}{}_{\lambda}\nabla_\mu F^{\lambda\nu} + 4c\,R^{\mu\nu\alpha\beta}\nabla_\mu F_{\alpha\beta}) = 0 \tag{5.3}$$

Thus, relative to minimally coupled Maxwell theory, the DH terms induce a curvature-dependent correction to the photon propagation.

Geometric optics and the modified light cone

Superluminal behaviour arises cleanly in the geometric optics approximation. Consider a DH photon with wavelength λ propagating in a region of spacetime whose curvature radius is much larger than λ,

$$m_e^{-1} \ll \lambda \ll (R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})^{-1/4}. \tag{5.4}$$

This implies a smallness condition

$$\frac{(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})^{1/4}}{m_e} \ll 1, \tag{5.5}$$

ensuring that higher-order EFT corrections remain parametrically suppressed.

Under (5.4), the WKB expansion is valid. We write

$$A_\mu = A\bar{a}_\mu e^{i\Theta}, \tag{5.6}$$

with $A\bar{a}_\mu$ being a slowly varying amplitude on the curvature scale and Θ rapidly varying on the wavelength scale. To leading order,

$$\partial_\mu \Theta = k_\mu, \tag{5.7}$$

where k_μ is the photon wavevector, and $\nabla_\mu A_\nu \simeq A\bar{a}_\nu \partial_\mu e^{i\Theta}$.

Inserting this ansatz into (5.3) and multiplying the equation by $\bar{a}_\nu$ gives us:

$$k^2 - 2b\, R^{\mu\nu} k_\mu k_\nu + 8c\, R^{\mu\alpha\nu\beta} k_\mu k_\nu \bar{a}_\alpha \bar{a}_\beta = 0 \tag{5.8}$$

working in Lorenz gauge $k_\mu \bar{a}^\mu = 0$, with normalization $\bar{a}^\mu \bar{a}_\mu = -1$ (so $\bar{a}_\mu$ is spacelike). Here, we have assumed that the background A_μ field is absent. If we assume that the background field $\bar{A}_\mu$ is non-zero, then the above equation is modified by:

$$k^2 - 2b\, R^{\mu\nu} k_\mu k_\nu + 8c\, R^{\mu\alpha\nu\beta} k_\mu k_\nu \bar{a}_\alpha \bar{a}_\beta + 4c\, R^{\mu\nu\alpha\beta} \nabla_\mu \bar{F}_{\alpha\beta} \bar{a}_\nu = 0 \tag{5.9}$$

The curvature-dependent term suggests defining an *effective* or *optical* inverse metric $\tilde{g}^{\mu\nu}$ such that

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - 2b\, R^{\mu\nu} + 8c\, R^{\mu\rho\nu\sigma} \bar{a}_\rho \bar{a}_\sigma \tag{5.10}$$

for which the propagation condition takes the simple form

$$\tilde{g}^{\mu\nu} k_\mu k_\nu = 4c\, R^{\mu\nu\alpha\beta} \nabla_\mu \bar{F}_{\alpha\beta} \bar{a}_\nu \tag{5.11}$$

Equivalently, the effective metric $\tilde{\mathcal{G}}_{\mu\nu}$ satisfies as seen in [21], [40]

$$\tilde{\mathcal{G}}_{\mu\nu} u^\mu u^\nu = 4c\, R^{\mu\nu\alpha\beta} \nabla_\mu \bar{F}_{\alpha\beta} \bar{a}_\nu \tag{5.12}$$

with $\tilde{\mathcal{G}}_{\mu\alpha} \tilde{g}^{\alpha\nu} = \delta^\nu{}_\mu$. To leading order,

$$\tilde{\mathcal{G}}_{\mu\nu} = g_{\mu\nu} + 2b\, R_{\mu\nu} - 8c\, R^\rho{}_\mu{}^\sigma{}_\nu\, \bar{a}_\rho \bar{a}_\sigma \tag{5.13}$$

with corrections controlled by (5.5). In the spacetimes we will consider, the right hand side of (5.12) is 0. Hence, the DH terms slightly deform the causal cone of $g_{\mu\nu}$, while the photon remains null with respect to $\tilde{\mathcal{G}}_{\mu\nu}$.

This effective metric is obtained for a particular path γ ($\tilde{\mathcal{G}}^\gamma_{\mu\nu}$). For a general path, the effective metric ($\mathcal{G}'_{\mu\nu}$) is defined by

$$\mathcal{G}'_{\mu\nu}|_\gamma = \tilde{\mathcal{G}}^\gamma_{\mu\nu} \tag{5.14}$$

for all paths γ. In pure Maxwell theory the causal structure is governed by $g_{\mu\nu}$, and $g^{\mu\nu} k_\mu k_\nu = 0$ encodes both null propagation and microcausality. The DH modification does *not* alter the fact that the photon moves on a null trajectory relative to some metric; it merely shifts the null cone from that of $g_{\mu\nu}$ to that of $\tilde{\mathcal{G}}_{\mu\nu}$. Thus, "superluminality" means that a photon trajectory may be spacelike with respect to $g_{\mu\nu}$ but remains lightlike with respect to $\tilde{\mathcal{G}}_{\mu\nu}$.

The geometric optics window (5.4) is essential. For $\lambda \lesssim m_e^{-1}$ the DH effective action breaks down and one must revert to the UV theory (QED with gravity), while for $\lambda \gtrsim (R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma})^{-1/4}$ geometric optics fails and wave effects become important. The conclusions about superluminality and optical metrics only apply inside the EFT window.

Example I: circular photon orbit in Schwarzschild

Consider an equatorial circular photon orbit around a Schwarzschild black hole of mass M,

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2. \tag{5.15}$$

Along a circular null trajectory $u^r = u^\theta = 0$, with $u^t, u^\phi \neq 0$. In this background the Riemann tensor components relevant to our polarization choice satisfy

$$R^{\mu\rho}{}_{\nu\sigma} = \begin{cases} R^{\mu\rho}{}_{\mu\rho} & \mu = \nu,\ \rho = \sigma, \\ 0 & \text{otherwise.} \end{cases} \tag{5.16}$$

Considering the propagation of the photon in the ϕ direction in the plane $\theta = \pi/2$, with a radial polarization, we see that:

$$\left(1 - \frac{2M}{r}\right)(1 + 8cR^{tr}{}_{tr})\left(\frac{dt}{d\tau}\right)^2 - r^2(1 + 8cR^{\phi r}{}_{\phi r})\left(\frac{d\phi}{d\tau}\right)^2 = 0 \tag{5.17}$$

Now, we can rewrite the above as:

$$\begin{aligned}\left(1 - \frac{2M}{r}\right)\left(\frac{dt}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2 = 8c\bigg(& r^2 R^{\phi r}{}_{\phi r}\left(\frac{d\phi}{d\tau}\right)^2 \\ & - \left(1 - \frac{2M}{r}\right)R^{tr}{}_{tr}\left(\frac{dt}{d\tau}\right)^2\bigg)\end{aligned} \tag{5.18}$$

Substituting the Riemann tensor values

$$R^{\phi r}{}_{\phi r} = \frac{M}{r^3}, \qquad R^{tr}{}_{tr} = -\frac{2M}{r^3}, \tag{5.19}$$

and with $c < 0$ we obtain

$$g_{\mu\nu}u^\mu u^\nu < 0, \tag{5.20}$$

so the trajectory is spacelike relative to the background metric. Locally, the photon appears "superluminal" with respect to $g_{\mu\nu}$.

Example II: straight line between two extremal black holes

Next consider a multicenter Reissner-Nordström configuration with two extremal black holes at $(0, 0, \pm L)$. The spacetime metric is [41]

$$ds^2 = U^{-2}(\vec{x})\, dt^2 - U^2(\vec{x})\, d\vec{x}^2, \qquad U(\vec{x}) = 1 + \frac{M}{\Delta X_+} + \frac{M}{\Delta X_-}, \tag{5.21}$$

with $\Delta X_\pm = \sqrt{(L \mp z)^2 + x^2 + y^2}$. In this background, equation (5.3) applies. In this case, the EM background $\bar{F}_{\mu\nu}$ will be non-zero. From [41], we find that

$$\bar{A}_0 = U^{-1} - 1, \quad \bar{A}_i = 0 \tag{5.22}$$

With this background, and considering polarization along a single (z) direction, the RHS of (5.12) vanishes due to the fact that $R^{tijk} = 0$. Consider propagation along x with polarization along z. To leading order, one finds

$$U^{-2}(1 + 8c\, R^{tz}{}_{tz} + 2b\, R^{t}{}_{t})\Big(\frac{dt}{d\tau}\Big)^2 - U^2(1 + 8c\, R^{xz}{}_{xz} + 2b\, R^{x}{}_{x})\Big(\frac{dx}{d\tau}\Big)^2 = 0. \tag{5.23}$$

Matching to the background null condition $U^2 dx/d\tau = U^{-2} dt/d\tau$ yields

$$U^{-2}\Big(\frac{dt}{d\tau}\Big)^2 - U^2\Big(\frac{dx}{d\tau}\Big)^2 = (8c\,(R^{xz}{}_{xz} - R^{tz}{}_{tz}) - 2b\,(R^{t}{}_{t} - R^{x}{}_{x}))\, U^{-2}\Big(\frac{dt}{d\tau}\Big)^2. \tag{5.24}$$

But since $R^{t}{}_{t} = R^{x}{}_{x}$ in the X-Y plane, the second term on the right-hand side is 0.

Using the values of the Riemann tensor given in [22], we get

$$R^{xz}{}_{xz} - R^{tz}{}_{tz} = \frac{6U^{-2}L^2M}{(L^2 + x^2)^{5/2}} > 0, \tag{5.25}$$

and because $c < 0$, one again finds

$$g_{\mu\nu}u^{\mu}u^{\nu} < 0, \tag{5.26}$$

so the trajectory is spacelike relative to $g_{\mu\nu}$. Superluminality is therefore also present in this multicenter geometry.

The appearance of local superluminal propagation is familiar from flat spacetime as an immediate signal of acausality: propagation outside the Minkowski light cone conflicts with microcausality and allows signal transmission outside causal order. The DH setup therefore raises an important conceptual question: if a consistent UV theory such as QED coupled to gravity generates small polarization-dependent superluminal effects at low energies, does this necessarily endanger causality in a curved background? In gravitational settings the notion of causal structure is richer, and the "background" light cone need not universally control signal propagation. The central issue investigated in later sections is whether such local superluminality can accumulate into a global causal pathology, or whether it remains entirely benign once the full spacetime structure is taken into account. We now turn to causality-based diagnostics aimed at addressing this question.

Chapter 6

Superluminality and Causality: A Review of Time-Delay Arguments

As shown in the previous section, the Drummond-Hathrell (DH) corrections cause photon trajectories to deviate slightly from the null cone of the background metric $g_{\mu\nu}$. In flat spacetime even an infinitesimal superluminal deviation would immediately signal a breakdown of relativistic causality, since it allows information to propagate outside the Minkowski light cone. In curved spacetime, however, the conceptual landscape is richer: different effective metrics may govern propagation, the background lacks global Lorentz symmetry, and the meaning of "delay", "advance", or "signal speed" depends sensitively on the geometry. This has led to a substantial literature exploring whether DH superluminality is truly dangerous or merely an innocuous EFT artifact.

One widely studied diagnostic is the *time delay*. Intuitively, we can ask whether a photon arrives earlier or later than it would have in some reference geometry (often flat spacetime or minimally coupled Maxwell theory).[1] Positive time delay is traditionally associated with causal propagation, while negative time delay ("time advance") is often regarded as a red flag for causality violation. In this section we review two influential lines of reasoning that employ time delay to argue that DH superluminality does not jeopardize causality, and then explain why time delay is not reliable as a general criterion. This motivates the use of more robust causal diagnostics in the rest of the paper.

[1] Here "time delay" refers to a quantity extracted from a coordinate that plays the role of time in an appropriate asymptotic region. The interpretation is therefore tied to the asymptotic structure of the spacetime.

Shockwave Backgrounds and Local Shifts in Null Coordinates

In [27], the authors study a DH photon in a gravitational shockwave background in D dimensional space described by the Aichelburg-Sexl metric [42],

$$ds^2 = du\, dv - h(u, x_i)\, du^2 - \sum_{i=1}^{D-2} dx_i^2, \tag{6.1}$$

with v a null Killing direction and $r = \sqrt{x_i x_i}$. By arranging two shockwaves localized at $r = \pm b$ and $u = 0$, one can track a photon traveling along the u direction at $r = 0$. The relevant observable is the discontinuity

$$\Delta v = v(u > 0) - v(u < 0), \tag{6.2}$$

interpreted as the time delay across the shock.

The result is

$$\Delta v = K\left(1 + c\frac{(D-2)(D-4)}{b^2}\left(\frac{\vec{a}\cdot\vec{n}}{a^2} - \frac{1}{D-2}\right)\right), \tag{6.3}$$

with $K > 0$, $c \sim \alpha/m_e^2$, $\vec{a}$ the polarization, and $\vec{n} = \vec{b}/b$. The DH correction is suppressed provided the geometric optics condition

$$m_e^{-1} \ll \lambda \ll b, \tag{6.4}$$

holds, ensuring $(m_e b)^{-1} \ll 1$. Thus, the DH term is always a small correction, and Δv remains strictly positive in the EFT regime. So we see that no time advance occurs in this case.

Eisenbud-Wigner Scattering Time Delay

A conceptually different notion of time delay arises in scattering theory and is well defined only in asymptotically flat spacetimes. In [28], [29], the authors consider the Eisenbud-Wigner time delay:

$$\Delta T_\ell = 2\,\frac{\partial \delta_\ell(\omega)}{\partial \omega}, \tag{6.5}$$

where $\delta_\ell(\omega)$ is the phase shift in the ℓ-th partial wave.

For the DH photon,

$$\Delta T_\ell = \Delta T_\ell^g + \Delta T_\ell^{\mathrm{EFT}}, \tag{6.6}$$

where $\Delta T_\ell^g > 0$ is the usual time delay for Schwarzschild. Causality requires

$$|\Delta T_\ell^{\text{EFT}}| \ll \omega^{-1}, \tag{6.7}$$

so that the EFT correction cannot overwhelm the positive gravitational delay. The DH contribution scales as

$$|\Delta T_\ell^{\text{EFT}}| \sim \frac{M}{b^2 m_e^2}, \tag{6.8}$$

and in the geometric optics regime ($b \gg M$, $\omega b \gg 1$),

$$|\Delta T_\ell^{\text{EFT}}| \ll \sqrt{\frac{M}{b}}\,\omega^{-1} < \omega^{-1}. \tag{6.9}$$

Thus the DH photon again exhibits no time advance in this setting.

Why Time Delay Is Not a Universal Causality Diagnostic

While the above examples are reassuring, time delay suffers from important conceptual and geometric limitations.

(1) Issues with asymptotically flat spacetimes: Both the papers [28] and [27] use time delay in asymptotically flat spacetimes to argue that causality holds. While calculating the time delay, both papers use the S-matrix formalism either explicitly (in the case of [28]) or implicitly by using scattering amplitudes to calculate the time delay. But, in asymptotically flat spacetimes, we have to be very careful how we define S-matrices, because, as pointed out in [30], we cannot define an S-matrix without violating the timelike vectors going outside the flat spacetime light cone. So the condition of a positive time delay is too restrictive and may lead to us calling causal spacetimes acausal due to this time delay condition.

Also, as shown by Gao and Wald in [43], under the null energy condition, null geodesics in any asymptotically AdS spacetimes are always delayed relative to null geodesics in pure AdS. This is a rigorous time delay theorem that works only for asymptotically AdS spacetimes. There is no such equivalent theorem for all asymptotically flat spacetimes. Therefore, time delay may not be the most appropriate quantity to understand causality of asymptotically flat spacetimes.

(2) Spacetime Symmetries: The argument made in [28] requires that the spacetime has spherical symmetry so we can define the scattering phase shift for each partial wave ℓ. This may not work for even a general asymptotically flat spacetime, as seen in the case of the two extremal Reissner-Nordstrom black hole case, where spherical symmetry is broken and thus, we cannot use the concept of scattering time delay using the scattering phase shift.

These reasons motivate the approach taken in the remainder of this paper, where we try to assess the causality in a symmetry and background-independent manner. The two particular methods that we will discuss in the next two sections are given below

- **Stable causality** of the optical metric, a global geometric condition ensuring the absence of closed causal curves even under metric perturbations.
- **Microcausality** of the photon field, a quantum condition requiring vanishing of commutators at spacelike separation even in Lorentz-breaking curved backgrounds.

These provide a more robust and general framework for analyzing causality in the Drummond-Hathrell effective theory.

Chapter 7

Stable Causality of the Optical Metric

In the previous section, we reviewed how the Drummond-Hathrell (DH) corrections lead to local superluminal propagation in two explicit geometries: a circular photon orbit in Schwarzschild, and a linear trajectory in the field of two extremal Reissner–Nordström black holes. Local superluminality by itself does not immediately diagnose a breakdown of causality in general curved spacetimes due to the arguments made for flat spacetime not translating to curved spacetime. In curved spacetime, the appropriate criterion for causal well-definedness is inherently global.

A spacetime can be defined as *causal* if it contains no CCCs. However, checking this directly is highly nontrivial. Instead, following [31], [32], one typically employs the stronger condition of *stable causality*. A spacetime $(\mathcal{M}, g_{\mu\nu})$ is stably causal if it does not admit CCCs even after small metric perturbations. Following the proof from [31], a spacetime is stably causal iff there exists a smooth function f whose gradient is everywhere timelike. Operationally, f serves as a global time function: if such an f exists, then the spacetime does not admit CCCs and remains non-pathological under arbitrarily small perturbations of the metric. This analysis was also done for the FLRW metric in [33], where Shore found that a DH photon propagating in the FLRW metric is consistent with stable causality.

In our context, the relevant geometry is not the background metric $g_{\mu\nu}$ but the *effective* (or "optical") metric $\mathcal{G}'_{\mu\nu}$ governing DH photon propagation. For each causal curve of the photon, $\mathcal{G}'_{\mu\nu}$ reduces to the optical metric $\tilde{\mathcal{G}}_{\mu\nu}$ defined in (5.13). Throughout this section, the manifold $\mathcal{M}$ denotes the region exterior to black-hole horizons, where the EFT and geometric optics approximations remain valid.

Our aim is modest: to identify regimes in which $\mathcal{G}'_{\mu\nu}$ is stably causal. This is sufficient to show that, in those regimes, the DH photon—although superluminal relative to $g_{\mu\nu}$—does not generate closed causal curves with respect to the physical propagation metric.

7.1 Schwarzschild Geometry

For Schwarzschild, the effective metric for all DH photons propagating in this spacetime can be brought to the following form as shown by [18]:

$$ds^2 = \left(1 - \beta \frac{\alpha}{m_e^2} \frac{M}{r^3}\right) \left(F(r)\, dt^2 - F^{-1}(r) dr^2\right) - r^2 d\Omega^2, \tag{7.1}$$

where $F(r) = 1 - 2M/r$ and $\beta \sim \mathcal{O}(1)$ (β is a metric parameter which depends on polarization and direction of propagation).

To test stable causality, we choose the natural candidate

$$f(x^\mu) = t. \tag{7.2}$$

The gradient $\nabla_\mu f$ is purely timelike if $g'^{tt} > 0$ everywhere ($g'^{\mu\nu}$ is the inverse of $\mathcal{G}'_{\mu\nu}$). From (7.1),

$$g'^{tt} = 1 + \beta \frac{\alpha}{m_e^2} \frac{M}{r^3}. \tag{7.3}$$

Using the EFT inequality (5.4) and the fact that $R_{\mu\nu\rho\sigma} \propto M/r^3$, we find $g'^{tt} > 0$ for all $r > 2M$ and for all polarizations.

Thus t is a global time function on $(\mathcal{M}, \mathcal{G}'_{\mu\nu})$, establishing stable causality in the EFT regime. In particular:

- the optical metric does not admit closed causal curves,
- and the superluminal DH photon remains globally causal even though its null cone is slightly wider than that of $g_{\mu\nu}$.

This result does not rule out all possible causal issues in more exotic regions of the parameter space, but it confirms that within the geometric-optics EFT window, no causal pathology arises in Schwarzschild due to DH corrections.

7.2 Two Black–Hole Geometry

For the two–center extremal Reissner–Nordström configuration, the explicit effective metric $\mathcal{G}'_{\mu\nu}$ is not known in closed form. Nevertheless, its causal properties can be inferred directly from the structure of the DH correction.

From (5.10), along any causal photon trajectory γ,

$$g'^{\mu\nu}|_\gamma = g^{\mu\nu} - 2b\,R^{\mu\nu} + 8cR^{\mu\alpha\nu\beta}\bar{a}_\alpha\bar{a}_\beta. \tag{7.4}$$

Thus,

$$g'^{tt}|_\gamma = g^{tt} - 2b\,R^{tt} + 8cR^{t\alpha t\beta}\bar{a}_\alpha\bar{a}_\beta. \tag{7.5}$$

Near either black hole, the curvature satisfies $R^{\mu\alpha\nu\beta}\bar{a}_\alpha\bar{a}_\beta \sim 1/M^2$, the maximum scale in the geometry. Suppose we again take $f(x) = t$. Then

$$g'^{tt} \gtrsim U^2(\vec{x})\left(1 + \gamma\,U^{-2}(\vec{x})\,\frac{1}{M^2}\right), \tag{7.6}$$

with $U(\vec{x}) > 1$ in the exterior region and $\gamma \sim \mathcal{O}(\alpha/m_e^2)$ (γ is a parameter depending on the polarization and direction of propagation).

Demanding stable causality, we find the condition $\gamma/M^2 \sim \alpha/(m_e^2 M^2) \ll 1$ which is equivalent to

$$m_e M \gg 1, \tag{7.7}$$

meaning that the BH mass is well above the mass scale $M_{Pl}^2/m_e \sim 10^{-13} M_\odot$. In this regime,

$$\left|\gamma\,U^{-2}(\vec{x})\,\frac{1}{M^2}\right| \ll 1, \tag{7.8}$$

independent of its sign.

Hence $g'^{tt} > 0$ everywhere outside the horizons, and t again defines a global time function. We therefore conclude:

> *In the two–black–hole geometry, whenever $M \gg m_e^{-1}$, the optical metric governing DH photon propagation is stably causal. Local superluminality does not lead to closed causal curves.*

We emphasize that this is not a proof of causality for all parameter values; rather, it identifies a clean parametric window where stable causality is ensured.

Chapter 8

Microcausality Bounds in the EFT Regime

Stable causality of the optical metric provides a classical diagnostic of whether Drummond–Hathrell (DH) superluminality can endanger the global causal structure of a spacetime. A complementary question is whether DH photon propagation is compatible with *microcausality*, the quantum condition that local operators commute at spacelike separation. Unlike stable causality, microcausality probes the structure of local observables and is sensitive to the analytic properties of Green's functions.

In this section we apply the formalism developed in [16], [35] to obtain a modest but informative check of quantum causal consistency within the DH effective field theory. Our analysis is restricted to the geometric-optics regime of the EFT and should be understood as a controlled diagnostic rather than a fully covariant treatment of quantum fields in curved spacetime.

8.1 Microcausality and the Paley–Wiener Theorem

In flat spacetime, microcausality requires that for any two bosonic operators $\mathcal{O}_1(x)$ and $\mathcal{O}_2(y)$,

$$[\mathcal{O}_1(x), \mathcal{O}_2(y)] = 0 \qquad \text{if } (x-y)^2 < 0. \tag{8.1}$$

For a field operator $\hat{\phi}(x)$, this condition implies that the commutator

$$G_c(t_1, \vec{x}_1; t_2, \vec{x}_2) := [\,\hat{\phi}(x_1), \hat{\phi}(x_2)\,] \tag{8.2}$$

vanishes outside the flat-spacetime light cone.

Using the Paley–Wiener theorem [17], the above spacetime statement can be reformulated as a condition on the analyticity and boundedness of the spatial Fourier transform $\tilde{G}_c(\Delta t, \vec{k})$ at fixed Δt. As shown in [16], flat-spacetime microcausality holds if and only if $\tilde{G}_c(\Delta t, \vec{k})$ admits an analytic continuation to all $\vec{k} \in \mathbb{C}^3$ and satisfies

$$|\tilde{G}_c(\Delta t, \vec{k})| \leq C(|\vec{k}|)\, e^{\mathrm{Im}(|\vec{k}|)\,\Delta t} \qquad \forall \vec{k} \in \mathbb{C}^3, \tag{8.3}$$

for some polynomial C. Importantly, this is an operator statement: it must hold for *all* states of the theory, including Lorentz-breaking states.

In what follows, we treat the gravitational background as a fixed Lorentz-breaking state associated with a non-trivial metric configuration. In this sense, the metric is viewed as a classical spin–2 field living on flat spacetime, analogous to a background medium, and we examine whether the modified DH dispersion relations respect the analyticity bound (8.3) within the geometric-optics window of the EFT.

Before proceeding, it is important to clarify the scope of this analysis. In a fully curved spacetime, there is no unique or universally accepted definition of microcausality for effective field theories whose characteristic cones differ from that of the background metric. In particular, for DH photons the relevant propagation cones are governed by the optical metric rather than $g_{\mu\nu}$. Establishing a fully covariant notion of microcausality in such settings remains a subtle and open problem.

Here we therefore adopt a more limited but well-defined perspective. We apply flat-spacetime microcausality criteria—formulated as analyticity and boundedness conditions on momentum-space Green's functions—to the photon field propagating on a fixed Lorentz-breaking gravitational background. This approach follows the framework of [16], [35] and allows us to test whether the DH-modified dispersion relations are compatible with standard microcausality bounds within the regime of validity of the EFT.

Our goal is not to propose a general definition of microcausality in curved spacetime, but rather to provide a controlled and instructive check: whether superluminal DH photon propagation, viewed as a Lorentz-breaking effective excitation on flat spacetime, violates flat-space microcausality bounds in the geometric-optics regime.

8.2 Circular Orbit in the Schwarzschild Geometry

Using the geometric-optics ansatz, in appendix B we derive that the form of the field A_μ must be as follows:

$$A_\mu = \int \bar{a}^k_\mu\, e^{i(\omega t - kr\phi)}\, dk, \tag{8.4}$$

and substituting into the DH equation of motion (5.3), we obtain the modified dispersion relation

$$\omega^2 F^{-1}(1 - 8cR^{tr}{}_{tr}) - k^2(1 - 8cR^{\phi r}{}_{\phi r}) = 0. \tag{8.5}$$

Solving for ω yields

$$\omega = k\,\frac{1 - 4cR^{\phi r}{}_{\phi r}}{1 - 4cR^{tr}{}_{tr}} F^{1/2}(r), \tag{8.6}$$

with $F(r) = 1 - 2M/r$. The commutator in momentum space (Eq (B.12)) behaves as

$$(\tilde{G}_{\mu\nu})_c(t,k) \sim i\frac{\sin(\omega t)}{\omega}. \tag{8.7}$$

Microcausality Eq (8.3) requires $\mathrm{Im}(\omega) \leq \mathrm{Im}(k)$, which gives the condition

$$\frac{1 - 4cR^{\phi r}{}_{\phi r}}{1 - 4cR^{tr}{}_{tr}} F^{1/2}(r) \leq 1. \tag{8.8}$$

In the Schwarzschild background one finds

$$1 \leq \frac{1 - 4cR^{\phi r}{}_{\phi r}}{1 - 4cR^{tr}{}_{tr}} \leq \left(1 - \frac{2M}{r}\right)^{-1/2},$$

while $F^{1/2}(r) \leq 1$.

Thus the inequality (8.8) is satisfied throughout the geometric-optics EFT regime. Although DH photons are locally superluminal relative to the background metric $g_{\mu\nu}$, the modified dispersion relation does not violate the Paley–Wiener analyticity bound (8.3). In this sense, when treated as a Lorentz-breaking effective excitation on a fixed background, the photon commutator remains compatible with flat-spacetime microcausality within the regime of validity of the EFT.

8.3 Linear Trajectory in the Two Black–Hole Geometry

For the two-center extremal Reissner-Nordström configuration, the geometric-optics ansatz takes the form (shown in appendix B)

$$A_\mu = \int \bar{a}^k_\mu\, e^{i(\omega t - kx^*)}\, dk, \tag{8.9}$$

with the effective X coordinate

$$x^* = \int^x dx' \, \frac{1 - 4c\, R^{tz}{}_{tz} - b\, R^t{}_t}{1 - 4c\, R^{xz}{}_{xz} - b\, R^x{}_x} U^2(x') \tag{8.10}$$

where $U(x) = U((x, 0, 0))$ is defined in (5.21). In this case the dispersion relation simplifies to

$$k = \omega. \tag{8.11}$$

Consequently,

$$(\tilde{G}_{\mu\nu})_c(t, k) \sim i\frac{\sin(kt)}{k}, \tag{8.12}$$

which is easily bounded by a function satisfying (8.3). The commutator has compact support within the flat-space light cone, and the analyticity requirements of microcausality are met inside the EFT window.

Remarks on the Regime of Validity

The above conclusions apply only within the geometric-optics EFT regime,

$$l^{-1} \ll |\vec{k}| \ll m_e, \tag{8.13}$$

where $l^{-1} \sim (R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma})^{1/4}$ is the curvature scale. For $|\vec{k}| \lesssim l^{-1}$ the geometric-optics approximation fails, while for $|\vec{k}| \gtrsim m_e$ the DH EFT is no longer valid and the UV completion (QED coupled to gravity) must be used. Since the UV theory is known to be causal [44], [45], one does not expect any fundamental causal inconsistency to arise even outside the EFT regime, although the effective description employed here is no longer applicable.

Our results should therefore be viewed as a limited but constructive check:

> *Within the window of validity of the Drummond–Hathrell effective theory, the modified dispersion relation appears compatible with the microcausality bounds derived from the Paley–Wiener theorem, at least for the explicit trajectories examined here.*

This does not constitute a general proof of microcausality for DH photons in arbitrary backgrounds, but it provides additional evidence that local superluminality in the DH EFT need not entail a causal pathology.

Chapter 9

Microcausality from the Green's function in $(\omega, \vec{k})$ space

Now, that we have discussed the causality condition derived from the mixed space $(t, \vec{k})$ Green's function for superluminality that is derived from the DH action, we can use the Fourier transformed Green's function in $(\omega, \vec{k})$ space to derive the causality conditions. To do this, we will use the formalism developed in the recently published paper [36] to see if our Green's function is consistent with causality.

Considering $G_{\mathrm{R}}(\omega, \vec{k})$ is the retarded Green's function in the $(\omega, \vec{k})$ space, then according to [36], $\mathrm{Im}G_{\mathrm{R}}(\omega, \vec{k})$ will be microcausal if

$$\int d\zeta \ \mathrm{Im}G_{\mathrm{R}}(\zeta, k + \xi\zeta) = 0 \tag{9.1}$$

which is shown to be true for the Lorentz invariant case. For the Lorentz invariant case, we see that $\mathrm{Im}G_{\mathrm{R}}(\omega, \vec{k})$ has the following form:

$$\mathrm{Im}G_{\mathrm{R}}(\omega, \vec{k}) = \mathrm{Sign}(\omega)\theta(\omega^2 - k^2)\rho(\omega^2 - k^2) \tag{9.2}$$

where $\rho(\omega^2 - k^2)$ is the spectral density and $\theta(\omega^2 - k^2)$ is the Heaviside step function.

In [36], the authors show that if the Green's function is given by (9.2), then it follows equation (9.1) and thus, is consistent with causality. They also show that if the spectral density is of the form $\rho(\omega^2 - c_s^2 k^2)$ (spectral density for fields in a Lorentz broken background), then this Green's function is also consistent with causality as long as $c_s^2 \leq 1$.

The spectral density in the EFT regime for the DH action is of the form $\rho(\omega^2 - c_s^2 k^2)$ as the background breaks Lorentz invariance. We will show that $c_s^2 \leq 1$ for each of the cases, which will imply that the superluminality seen in the DH action is consistent with causality. For the single DH photon in curved backgrounds, the spectral density is of the form of a delta function.

Circular Path in Schwarzschild

In this section, we will find the value of c_s^2 for a superluminal DH photon propagating in a circular path in the Schwarzschild metric. The value of c_s is given in equation (8.6). Thus, we see that:

$$c_s^2 = \frac{1 - 8cR^{\phi r}{}_{\phi r}}{1 - 8cR^{tr}{}_{tr}} F(r) \tag{9.3}$$

Thus, the spcetral density ρ is given by:

$$\rho(\omega^2 - c_s^2 k^2) = 2\pi\delta(\omega^2 - c_s^2 k^2) \tag{9.4}$$

We have shown in section 8 that the above value of $c_s^2 \leq 1$ and thus, we see that according to the required conditions for causality in $(\omega, \vec{k})$ space, superluminal propagation of a DH photon is consistent with causality in the EFT regime.

Linear Path in Two Black Hole case

For the superluminal photon propagating in the linear path in the two black hole case, we see that if our Green's function in position space is of the form $G_{\mathrm{R}}(t, x^*)$, where x^* is given in equation (8.10), then the imaginary part of the retarded Green's function in $(\omega, \vec{k})$ space is given by the equation (9.2), as can be seen from the previous section. Thus, superluminality in this case too will be consistent with the microcausality in the EFT regime.

Chapter 10

Conclusion and Remarks

In this thesis, we sought to apply microcausality to constrain various EFT parameters and to examine the relationship between the subluminality of the group velocity and microcausality. We also revisited the question of whether the superluminal photon propagation predicted by the Drummond–Hathrell (DH) effective action signals a breakdown of causality in curved spacetime.

Using the formalism derived and derivation from [16], we were able to find bounds on the EFT coefficients for the massless Goldstone boson, massive Goldstone boson, as well as Euler-Heisenberg theories. The bounds that were derived for these theories seem to match the bounds that were already derived by imposing subluminality on the group velocity, as seen in [15] for the first two theories, and the bound for the Euler-Heisenberg action is consistent with the coefficient derived explicitly by taking the low-energy limit [38], [39]. Thus, we see that microcausality bounds are equivalent to the bounds derived by a subluminality analysis.

Seeing that the EFT bounds derived from the microcausality seem to be equivalent to the bounds derived by the subluminality analysis, we tried to see when we could impose subluminality of group velocity as a valid statement of causality by using the microcausality bound as our guiding principle. As we derived and as shown in [16], subluminality of group velocity is not a universal causality condition, but depends on the theory that we are considering. We see that if there are no resonances (i.e. $\mathrm{Im}(\omega_k)|_{k\in\mathbb{R}^+} = 0$), then the statement of subluminality of group velocity is equivalent to microcausality being valid for the theory.

Having used causality to bound the coefficients of various EFTs in flat spacetime, we wanted to study the causality of EFTs in curved spacetime. Specifically, we wanted to analyse the Drummond-Hathrell (DH) effective action, which is the EFT that is derived by considering QED minimally coupled to gravity and integrating out the electron. Thus, this EFT is valid at energy scales below the mass of the electron (m_e). Applying a geometric optics approximation, we find that, for certain gravitational backgrounds, the DH photon can propagate outside the background light cone (superluminality). In flat spacetime, we clearly observe that superluminal propagation violates causality, but it is not clear if this idea of causality can be generalised to curved spacetime. This led us to investigate whether superluminality in curved spacetimes necessarily led to causality pathologies, or whether it was completely benign and didn't cause any causality violations.

Previous literature has also addressed this question using various time-delay arguments [27], [28]. The time delay arguments are limited by the fact that they are dependent on not only being able to define an S-matrix, but also the spherical symmetry of the background spacetime. These limitations have led us to define and focus on two complementary and largely symmetry-independent diagnostics (one classical and one quantum) that probe different aspects of causal consistency.

On the classical side, we analysed the global causal structure of the effective optical metric governing DH photon propagation. For both examples studied—the circular orbit in Schwarzschild and the straight trajectory in a two–black–hole geometry—we identified parametric regimes in which the optical metric is stably causal. In these regimes, a global time function exists, implying the absence of closed causal curves and ruling out global causal paradoxes.

On the quantum side, we examined microcausality using the analyticity criteria developed in [16], [36]. Treating the gravitational background as a fixed Lorentz-breaking field, we tested whether the DH-modified dispersion relations satisfy certain analyticity and boundedness conditions within the geometric-optics regime of the EFT. For the trajectories considered, these conditions are satisfied, indicating that the photon commutator remains compatible with flat-spacetime microcausality bounds. In this restricted but well-defined sense, the DH corrections do not permit acausal signalling, despite the local widening of the effective light cone with respect to the background light cone.

Taken together, these analyses suggest that DH superluminality is causally benign within the geometric-optics EFT window,

$$m_e^{-1} \ll \lambda \ll (R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})^{-1/4}. \tag{10.1}$$

This does not rule out the possibility of causal pathologies in more extreme regimes or for other effective field theories. Rather, the results derived here highlight that superluminality in gravitational EFTs does not *automatically* imply causality violation, and that global geometric and local quantum diagnostics can jointly provide a sharper picture of when genuine problems may arise.

Appendix A

Calculation of mixed commutator $G_C(t, \vec{k})$

In this appendix, we will explicitly compute the mixed commutator $G_c(t, \vec{k})$ for theories where the equation of motion can be written in the following general form:

$$\ddot{\pi} - \zeta \nabla^2 \pi + m^2 \pi = 0 \tag{A.1}$$

where, ζ is some constant and m is the mass of the scalar.

We can parameterize the field π by considering the Fourier transform of the field with respect to the position variable:

$$\pi(t, \vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3} f_k(t) e^{i\vec{k}.\vec{x}} \tag{A.2}$$

Substituting it back into (A.1), we get the equation of motion to look like:

$$\ddot{f}_k(t) = -(\zeta \vec{k}^2 + m^2) \tag{A.3}$$

The solution to this equation is given by:

$$f_k(t) = e^{\pm i\omega_k t} \qquad \text{where} \ \ \omega_k = \zeta \vec{k}^2 + m^2$$

Considering the positive energy mode, we will take the solution to be:

$$f_k(t) = Ce^{-i\omega_k t}$$

By applying normalization conditions, we get that:

$$C = \frac{1}{\sqrt{2\omega_k}}$$

As derived in [16], the mixed commutator by taking a Fourier transform with respect to $(\vec{x}_1 - \vec{x}_2)$ can be written as:

$$\tilde{G}_c(t_1, t_2; \vec{k}) = 2i\text{Im}(f_k(t_1) f_k^*(t_2)) \tag{A.4}$$

Now substituting the value of $f_k(t)$ for this equation of motion, we get that.

$$\tilde{G}_c(t_1, t_2; \vec{k}) = i\frac{\text{Im}(e^{-i\omega_k(t_1 - t_2)})}{\omega_k} = -i\frac{\sin(\omega_k(t_1 - t_2))}{\omega_k} \tag{A.5}$$

If $(t_1, \vec{x}_1) = (t, \vec{x})$ and $(t_1, \vec{x}_2) = 0$, we get the general form of the mixed propagator, $\tilde{G}_c(t, \vec{k})$, which is given by:

$$\tilde{G}_c(t_1, t_2; \vec{k}) = -i\frac{\sin(\omega_k t)}{\omega_k} \tag{A.6}$$

Thus, for any theory with the equation of motion given by equation (A.1), the mixed commutator will be given by equation (A.6)

Appendix B

Calculation of commutators photon fields in curved spacetime

In this appendix, we will derive A_μ in the geometrical optics regime, which will lead us to explicitly find the commutator for both cases. We will follow the procedure given in [16] for this. The first thing that we will observe is that the equation of motion is given by:

$$\nabla_\mu F^{\mu\nu} - (2b\, R^\mu{}_\lambda \nabla_\mu F^{\lambda\nu} + 4c\, R^{\mu\nu\alpha\beta}\nabla_\mu F_{\alpha\beta}) = 0. \tag{B.1}$$

In the Lorenz gauge, we see that $\nabla_\mu A^\mu = 0$. This leads to the equation of motion being:

$$\begin{aligned}&\nabla^\mu\nabla_\mu A_\nu(1 - 2b\, R^\lambda{}_\lambda - 8c\, R^{\mu\nu}{}_{\mu\nu}) - 2b(R^\mu{}_\lambda \nabla_\mu F^\lambda{}_\nu - R^\lambda{}_\lambda \nabla^\mu\nabla_\mu A_\nu)\\ &-8c(\, R^\mu{}_\nu{}^{\alpha\beta}\nabla_\mu F_{\alpha\beta} - \, R^{\mu\nu}{}_{\mu\nu}\nabla^\mu\nabla_\mu A_\nu) = 0\end{aligned} \tag{B.2}$$

Now, using the geometric optics ansatz, we can substitute:

$$A_\mu = A a_\mu e^{i\Theta}$$

and:

$$k_\mu = \partial_\mu \Theta$$

So that equation (B.2) looks like:

$$\begin{aligned}&a_\nu \nabla^\mu\nabla_\mu e^{i\Theta}(1 - 2b\, R^\lambda{}_\lambda - 8c\, R^{\mu\nu}{}_{\mu\nu}) - 2b(R^\mu{}_\lambda \nabla_\mu f^\lambda{}_\nu - R^\lambda{}_\lambda a_\nu \nabla^\mu\nabla_\mu e^{i\Theta})\\ &-8c(\, R^\mu{}_\nu{}^{\alpha\beta}\nabla_\mu f_{\alpha\beta} - \, R^{\mu\nu}{}_{\mu\nu} a_\nu \nabla^\mu\nabla_\mu e^{i\Theta}) = 0\end{aligned} \tag{B.3}$$

where, $f_{\mu\nu} = k_\mu a_\nu - k_\nu a_\mu$. Now, we can analyze each of the cases as given below

Circular path in Schwarzschild

For the Schwarzschild metric, only the first term in equation (B.3) survives. So, the equation of motion simplifies to:

$$a_\nu \nabla^\mu \nabla_\mu e^{i\Theta}(1 - 8c\, R^{\mu\nu}{}_{\mu\nu}) = 0 \tag{B.4}$$

Also note that the operator $\nabla^\mu \nabla_\mu$ acting on a scalar s has the following property:

$$\nabla^\mu \nabla_\mu s = \frac{1}{\sqrt{-g}} \partial_\mu(\sqrt{-g} g^{\mu\nu} \partial_\nu s) \tag{B.5}$$

Since $e^{i\Theta}$ is a scalar, we will use the above formula to simplify equation (B.4).

Now, for the circular path, we see that $k_r = k_\theta = 0$ in Schwarzschild coordinates. Assuming the ansatz for Θ is given by:

$$\Theta = \omega t - rk\phi$$

This ansatz will lead to equation (B.4) looking like:

$$a_\nu e^{i\Theta}((1 - 8c\, R^{t\nu}{}_{t\nu})\omega^2 - (1 - 8c\, R^{\phi\nu}{}_{\phi\nu})k^2) = 0 \tag{B.6}$$

leading to the relation between ω and k seen in equation (8.6).

Now, we would like to find the commutator $G_{\mu\nu}$. To do this, we will have to quantize the field A_μ as follows. We define $\hat{A}_\mu$ as:

$$\hat{A}_\mu = \int \frac{dk}{2\pi} \sum_{i=1}^{2} (a_\mu^{i,k}(x) e^{i\Theta} \hat{b}_{i,k} + a_\mu^{i,k}(x)^* e^{-i\Theta} \hat{b}_{i,k}^\dagger) \tag{B.7}$$

where $\hat{b}_{i,k}$ and $\hat{b}_{i,k}^\dagger$ are the annihilation and creation operators for a photon with polarization vector $a_\mu^i(k)$ and momentum k in the circular path. Now, we require the following normalization condition on $\hat{A}_\mu$ so that the quantization condition for fields in [46] is satisfied:

$$-ir^2 F^{-1}(r) |a_\mu^{i,k}(x)|^2 (e^{-i\omega t} \partial_t e^{i\omega t} - e^{i\omega t} \partial_t e^{-i\omega t}) = 1 \tag{B.8}$$

for a non-zero $a_\mu^{i,k}(x)$. This leads to the value of a_μ^i when it is non-zero:

$$a_\mu^{i,k}(x) = \frac{F^{1/2}(r)}{r\sqrt{2\omega}} e^{i\alpha} \tag{B.9}$$

where α is a phase factor. Now, if we require that equal time commutation relations hold, we automatically get the condition that:

$$[\hat{b}_{i,k}, \hat{b}^{\dagger}_{j,k'}] = \delta_{ij}\delta(k-k') \tag{B.10}$$

This leads to the commutator $G_{\mu\nu}$, being:

$$G_{\mu\nu}(x,x') = [A_\mu(x), A_\nu(x')] = 2i\sum_{i=1}^{2}\int a_\mu^{i,k} a_\nu^{i,k}\ \mathrm{Im}(e^{i\omega(t-t')})e^{-ikr(\phi-\phi')}dk \tag{B.11}$$

This is the form of the propagator in the EFT regime when geometric optics is valid. So taking a spatial Fourier transform, we get that $\tilde{G}_{\mu\nu}(k;t,t')$ is given by

$$\tilde{G}_{\mu\nu}(k;t,t') \sim iF(r)\frac{\sin(\omega(t-t'))}{\omega} \tag{B.12}$$

This gives us the structure of the commutator in the (k,t) space, which we need for the microcausality bounds.

Linear path in the Two-Black-Hole case

For this case too, we will see that on the linear path from $(x,0,0)$ to $(x',0,0)$, the second term in equation (B.3) is also 0. This means that, similar to the case of the circular path in the Schwarzschild metric, the equation of motion is again given by (B.4). Using equation (B.5), we see that the equation of motion looks like:

$$a_\nu[U^2(x)(1-8c\,R^{t\nu}{}_{t\nu}-2b\,R^t{}_t)\partial_t^2 e^{i\Theta} - U^{-2}(x)(1-8c\,R^{x\nu}{}_{x\nu}-2b\,R^x{}_x)\partial_x^2 e^{i\Theta}] = 0 \tag{B.13}$$

Considering an ansatz of the form:

$$e^{i\Theta} = e^{i\omega t}f(x)$$

Substituting this into the equation, we get that

$$(1-8c\,R^{t\nu}{}_{t\nu}-2b\,R^t{}_t)f(x)\omega^2 - U^{-4}(x)(1-8c\,R^{x\nu}{}_{x\nu}-2b\,R^x{}_x)f''(x) = 0 \tag{B.14}$$

Now, using the WKB approximation [47], we find that:

$$f(x) \approx e^{ig(x)} \tag{B.15}$$

where,

$$g(x) = \omega\int^x dx'\frac{1-4c\,R^{t\nu}{}_{t\nu}-b\,R^t{}_t}{1-4c\,R^{x\nu}{}_{x\nu}-b\,R^x{}_x}U^2(x') \tag{B.16}$$

Similar to how we quantized the photon field in the Schwarzschild background, we can quantize this field. This leads to us getting that

$$a_\mu^{i,k}(\vec{x}) = U^2(x)\frac{1}{\sqrt{2k}}e^{i\alpha} \tag{B.17}$$

This means that the commutator $G_{\mu\nu}(x, x')$ is given by:

$$G_{\mu\nu}(x, x') = [A_\mu(x), A_\nu(x')] = 2i\sum_{i=1}^{2}\int a_\mu^{i,k}a_\nu^{i,k}\ \mathrm{Im}(e^{i\omega(t-t')})e^{-ik(x^*-x'^*)}dk \tag{B.18}$$

And we thus, get that:

$$\tilde{G}_{\mu\nu}(k; t, t') \sim iU^{-2}(x^*)U^{-2}(x'^*)\frac{\sin(k(t-t'))}{k} \tag{B.19}$$

where, $U^{-2}(x^*)$ and $U^{-2}(x'^*)$ are constants that only depend on the starting and ending position vector of the photon observed, and since both of these are less than 1, we see that

$$|\tilde{G}_{\mu\nu}(k; t, t')| \leq \left|\frac{\sin(k(t-t'))}{k}\right| \tag{B.20}$$

which is the required form of the commutator $\tilde{G}_{\mu\nu}(k; t, t')$.

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