$\mathcal{N}$-Extended Super-BMS$_3$ Algebras and Generalized Gravity Solutions:

A THESIS

submitted by

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for the award of the degree

of

DOCTOR OF PHILOSOPHY

DEPARTMENT OF PHYSICS
IISER Pune

AUGUST 2018

Declaration:

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CERTIFICATE

Certified that the work incorporated in the thesis entitled "$\mathcal{N}$-Extended Super-BMS_3 Algebras and Generalized Gravity Solutions", submitted by Turmoli Neogi, was carried out by the candidate, under my supervision. The work presented here or any part of it has not been included in any other thesis submitted previously for the award of any degree or diploma from any other University or institution.

Prof. Sunil Mukhi
(Supervisor)

Date: 08.08.2018

ACKNOWLEDGEMENTS

It is a pleasure to thank the following people whose help and support have been indispensable for carrying out my research work and writing this thesis.

First I would like to thank my advisor Sunil Mukhi for his guidance and for the many enriching discussions I had with him. I have learnt a lot of physics from my interactions with him.

Working with Nabamita Banerjee has been an absolute delight! I am indebted to her, not only for all the valuable lessons in physics that I have learnt from her, but also for the constant support and motivation that she has provided me, especially when I faced hard times.

I want to sincerely thank Dileep Jatkar, who is a collaborator in some of my papers. Discussions with him on various topics in physics have always been very enlightening, and have provided me with great insights. I would also like to take this opportunity to thank my MSc project supervisor, Debashis Ghoshal. Attending his excellent courses and working on a project with him during MSc got me keenly interested in this wonderful field of theoretical high energy physics.

Attending some regular courses at IISER Pune has been extremely beneficial for my understanding of the subject. I am grateful to Arjun Bagchi, Avinash Khare, Nabamita Banerjee, Suneeta Vardarajan for taking some of these courses. I have also learnt a great deal from my discussions with them, as well as with some other past/current members of the high energy group. Some of them deserve special mention like Sachin Jain and Rudranil Basu.

I am indebted to a very good friend, Ramit Dey, for teaching me some invaluable lessons in LaTeX*, that I have used to write this thesis. Special thanks to Sucheta Majumder for being a very nice and helpful friend. I am thankful to all other friends for their support and encouragement, especially Arindam Bhattacharjee, Bappa Ghosh and Taniya Mandal. Thanks to the staff at IISER Pune for always being supportive and making* *life easier for me. Some of them have been exceptionally helpful like Anita Mishra, Dhanashree Sheth, Prabhakar Anagare and Tushar Kurulkar. I am indebted to the Indian Strings community for selection and support at various schools and conferences.*

I would like to thank my thesis referees for very constructive reviews of my thesis, and pointing out several interesting subtleties as well as important future directions along this line of work.

Last but not the least, I want to sincerely thank my parents and grandparents for their support, motivation and unwavering faith in me throughout the long journey, especially at unfavourable times.

ABSTRACT

In this thesis we analyze the properties of asymptotically flat three dimensional spacetimes with extended supersymmetry. We then give the construction of the asymptotic algebra for the extended supersymmetric cases by two different methods: first by Inönü-Wigner contraction of two copies of superconformal algebra, and then by an asymptotic symmetry analysis. We thereafter go on to explore important physical properties like energy bounds and generic gravity solutions. Finally, we construct the free field realizations of the (super) BMS$_3$ *systems, which may be an interesting step towards understanding flat holography.*

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CHAPTER 1

Introduction

1.1 Introduction to Three-Dimensional Gravity:

(Super)gravity in three dimensions has many interestesting aspects. These arise from the unique fact that there are no propagating degrees of freedom for gravity in $2+1$-d. This can be understood from the following consideration:

The curvature tensor, better known as the Riemann tensor, is one of the most important objects in general relativity. In case of a pseudo-Riemannian manifold, it measures the extent to which the metric tensor deviates from the Minkowski metric of flat spacetime. Using the standard Weyl decomposition, it can be shown that the Riemann tensor can be split into parts consisting of the Ricci scalar, the traceless part of the Ricci tensor and the Weyl tensor, which is tracefree, that is, contraction of any pair of indices gives zero. By virtue of the Einstein equations, the Ricci tensor (including its trace) is determined fully by the stress energy tensor, ie controlled by the matter distribution. The gravitational degrees of freedom reside entirely in the Weyl tensor.

Now in three dimensions, it so happens that the Weyl tensor vanishes identically. Hence the Riemann tensor is fully determined by the Ricci tensor. Therefore in 3-d, a space where the Ricci tensor $R_{\mu\nu}$ is zero makes the Riemann tensor zero as well. As a result, solutions with zero cosmological constant are always locally Minkowski spacetime: the asymptotic flat solutions of Einstein equations do not possess any local degrees of freedom. This implies that there is no gravitational radiation (in classical theory) and no propagating gravitons (in quantum theory).

However, although the solutions are locally Minkowski, globally these may differ from one another. In fact, a large class of gravitational solutions exist, depending on the holonomy of the manifold. If the holonomy is trivial, then a single coordinate patch, which parameterizes the neighbourhood of a point with the Minkowski metric $\eta_{\mu\nu}$, can be globally extended throughout the spacetime. However, if the holonomy is non-trivial, that is, non-contractible cycles are present in the manifold, then a single coordinate patch is not sufficient to cover the entire spacetime. Thus in this case, the global solution differs from $\eta_{\mu\nu}$. This is why solutions of three dimensional gravity can be classified by their holonomy structure. [1]

1.2 Asymptotic Symmetry:

Asymptotically flat spacetimes were analyzed by Bondi, van der Burg, Metzner and Sachs in the early 60's [2, 3]. They studied it for four spacetime dimensions in the context of gravitational radiation. It was found that the symmetry group for such spacetimes at null infinity was not merely the Poincare group (which is the symmetry group of flat spacetime, consisting of translations and Lorentz transformations), but an infinite dimensional extension of it, which later came to be known as the BMS$_4$ group. It consists of the usual Lorentz transformations, however, the familiar translations of the Poincare group now get extended to arbitrary angle-dependent translations, called supertranslations. Later it was shown by Barnich and Troessaert that one can consider further extension of the BMS group if one considers the Killing vectors at asymptotic infinity to be meromorphic functions [4–6]. Under this generalization of allowing local singularities in the asymptotic Killing vectors, the Lorentz part of the algebra gets enhanced to two copies of the Virasoro algebra, and these enhanced generators are now called superrotations. A natural question that arises is whether this algebra encodes information about the bulk gravity with flat space asymptotics. Relevant references in this context include [4, 7–12].

In this thesis, we are interested in a simpler case, which is the asymptotically flat space in three spacetime dimensions. The corresponding algebra is known as the BMS$_3$ algebra. To derive this, one starts with a set of Bondi coordinates u, r, ϕ where r and ϕ are the usual radial and angular coordinates respectively, while $u = t - r$, where t is the familiar time coordinate. In this specific choice of coordinate system, the flat Minkowski metric is given by

$$ds^2 = -du^2 - 2du\, dr + r^2 d\phi^2$$

It is then obvious that here u acts as the timelike coordinate and r as the null coordinate.

We are however interested in asymptotically flat spacetimes (and not particularly in Minkowski spaetime), hence we start with a more generic choice of metric with arbitrary coeffiients, having specific fall-off conditions in r. A systematic analysis of the transformations that leave the form of the metric invariant finally leads to the symmetry algebra of the system with most generic central extension as [4]:

$$\begin{aligned}
[\mathfrak{J}_n, \mathfrak{J}_m] &= (n-m)\mathfrak{J}_{n+m} + \frac{c_1}{12}n(n^2-1)\delta_{n+m,0} \\
[\mathfrak{J}_n, \mathcal{M}_m] &= (n-m)\mathcal{M}_{n+m} + \frac{c_2}{12}n(n^2-1)\delta_{n+m,0} \\
[\mathcal{M}_n, \mathcal{M}_m] &= 0
\end{aligned}$$

Thus we see that the algebra is infinite dimensional, made out of generators $\mathcal{M}_n$ and $\mathfrak{J}_n$, which we can identify as the supertranslation and superrotation generators respectively.

In the next chapter, we will show that alternatively, the BMS_3 algebra can also be derived as a In$\ddot{o}n\ddot{u}$-Wigner contraction of two copies of conformal algebras.

1.3 Thesis at a glance

Here we present a brief outline of the thesis. In the present chapter, we start with a brief introduction of three dimensional gravity, especially the features which make it interesting. Then we give a brief discussion of asymptotic symmetries, particularly emphasizing the same for three dimensional flat space.

In chapter two, we will first review how BMS_3 algebra has been derived in the literature from contraction of conformal algebras. Then we will systematically extend the procedure to derive super-BMS_3 and extended super BMS_3 algebras by contracting two copies of corresponding super-conformal algebras. The important results in this chapter are the correct methods of contraction of different generators to reproduce the expected flat algebra, based on various physical requirements.

In chapter three, we will derive the $\mathcal{N} = 4$ super BMS_3 algebra by direct asymptotic symmetry analysis. For this, we will utilise the important fact that three-dimensional gravity has a Chern-Simons formulation. A crucial fact that we will show here is that the algebra obtained by this method will match exactly with that obtained by contraction in the previous chapter. We shall then also perform a similar asymptotic symmetry analysis for the corresponding super-AdS_3 case, and obtain the correct superconformal algebra. Then we will show that the flat case, including the gauge field, can be reproduced as suitable combination from the AdS case. Finally, we shall also perform some related analysis, for example, that of energy bound and Killing spinor solutions.

One subtlety that we will encounter in chapter three is the appearance of various non-linear terms at intermediate stages of deriving the final algebra by asymptotic symmetry analysis. However, we have shown that upon suitable modification, the justification for which we have explained in details, all the non-linear terms are cancelled or absorbed, and the final algebra is linear. However, it turns out that this nice vanishing of non-linear terms is due to the choice of a very specific transformation of the fermions under R-symmetry. In chapter four, we will allow more generic transformations, and as a consequence, it will turn out that there are explicit non-linear terms in the final algebra! This will have non-trivial physical effects, like the shift of the energy bound, which we will compute in details. Finally, we shall also discuss a class of purely bosonic topological solutions, and analyze their thermodynamics.

In chapter five, we shall discuss the free field realisations of BMS_3 as well as (extended) super-BMS_3 algebras. We shall then also give the free field realisations of a few other related cases.

CHAPTER 2

Extended Supersymmetric BMS$_3$ Algebras:

In this chapter we shall derive $\mathcal{N} = 2, 4, 8$ super-BMS$_3$ algebras by contracting two copies of extended super conformal algebras.

2.1 Introduction:

One simple way to derive the BMS algebra is by direct contraction of Virasoro algebra, which is the asymptotic symmetry algebra of asymptotically AdS spacetimes [13]. The Virasoro algebra is given by:

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}$$
$$[\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{n+m} + \frac{\bar{c}}{12}n(n^2-1)\delta_{n+m,0}$$

where in general, the central charges c and $\bar{c}$ are independent. However, for Einstein gravity, $c = \bar{c} = \frac{3l}{2G}$, where l is the AdS$_3$ radius. Now one can obtain flat space by taking the AdS$_3$ radius l to infinity, however, the generators of the Virasoro algebra have to be scaled correctly while taking this limit. We take the linear combinations:

$$\mathfrak{J}_n = L_n - \bar{L}_{-n}, \qquad \mathcal{M}_n = \epsilon\left(L_n + \bar{L}_{-n}\right)$$

in the limit $\epsilon \to 0$, where $\epsilon = \frac{1}{l}$. In addition, we also scale the central charges as $c_1 = c - \bar{c}$ and $c_2 = \epsilon(c + \bar{c})$.

This results in the following algebra, which is called the BMS_3 algebra:

$$
\begin{aligned}
[\mathfrak{J}_n, \mathfrak{J}_m] &= (n-m)\mathfrak{J}_{n+m} + \frac{c_1}{12}n(n^2-1)\delta_{n+m,0} \\
[\mathfrak{J}_n, \mathcal{M}_m] &= (n-m)\mathcal{M}_{n+m} + \frac{c_2}{12}n(n^2-1)\delta_{n+m,0} \\
[\mathcal{M}_n, \mathcal{M}_m] &= 0
\end{aligned}
$$

2.1.1 $\mathcal{N} = 1$ Super-BMS_3 Algebra:

Three-dimensional $\mathcal{N} = 1$ super-BMS algebra is also known in literature [14, 15]. Here we start with 2 copies of Virasoro algebra, one of which is augemented by supersymmetry. The algebras is:

$$
\begin{aligned}
&[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0} \\
&[\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{n+m} + \frac{\bar{c}}{12}n(n^2-1)\delta_{n+m,0} \\
&[L_n, Q_r] = \left(\frac{n}{2} - r\right)Q_{n+r}, \qquad \{Q_r, Q_s\} = L_{r+s} + \frac{c}{6}r^2\delta_{r+s,0}
\end{aligned}
$$

Here Q_r are the fermionic generators corresponding to the 'unbarred' sector of the Virasoro algebra. Note that there are no $\bar{Q}_r$ generators, hence this corresponds to the case of $(1,0)$ supersymmetry.

Now suitable contraction of this will give us the minimal supersymmetrization of the BMS_3 algebra, which we can identify as the asymptotic symmetry algebra of $\mathcal{N} = 1$ supergravity of asymptotically flat spacetime. As far as the bosonic generators and central charges are concerned, the contraction is exactly the same as in the pure BMS case. The new input here is the contraction of fermionic generators, which was shown to be $\Psi_r = \sqrt{\epsilon}Q_r$, where Ψ_r is defined as the fermionic generator of the super-BMS algebra.

Taking the limit $\epsilon \to 0$, one obtains the following algebra:

$$
\begin{aligned}
[\mathfrak{J}_n, \mathfrak{J}_m] &= (n-m)\mathfrak{J}_{n+m} + \frac{c_1}{12} n(n^2-1)\delta_{n+m,0} \\
[\mathfrak{J}_n, \mathcal{M}_m] &= (n-m)\mathcal{M}_{n+m} + \frac{c_2}{12} n(n^2-1)\delta_{n+m,0} \\
\{\Psi_r, \Psi_s\} &= \mathcal{M}_{r+s} + \frac{c_2}{2} r^2 \delta_{r+s,0} \qquad\qquad [\mathcal{M}_n, \mathcal{M}_m] = 0 \\
[\mathfrak{J}_n, \Psi_r] &= \left(\frac{n}{2} - r\right)\Psi_{n+r} \qquad\qquad\quad [\mathcal{M}_n, \Psi_r] = 0
\end{aligned}
$$

We have considered the generic case $c \neq \bar{c}$. This algebra is the $\mathcal{N} = 1$ BMS$_3$ algebra.

2.2 Extended Super-BMS$_3$ Algebras:

We shall now write down the extended super-BMS$_3$ algebras that we have derived in our paper [16].

2.2.1 $\mathcal{N} = 2$ Super-BMS$_3$:

Here the starting point will be the $(1,1)$ superconformal algebra:

$$
\begin{aligned}
&[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12} n(n^2-1)\delta_{n+m,0} \\
&[\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{n+m} + \frac{\bar{c}}{12} n(n^2-1)\delta_{n+m,0} \\
&[L_n, Q_r] = \left(\frac{n}{2} - r\right) Q_{n+r}, \qquad [\bar{L}_n, \bar{Q}_r] = \left(\frac{n}{2} - r\right) \bar{Q}_{n+r} \\
&\{Q_r, Q_s\} = L_{r+s} + \frac{c}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0} \qquad \{\bar{Q}_r, \bar{Q}_s\} = \bar{L}_{r+s} + \frac{\bar{c}}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}
\end{aligned}
\tag{2.2.1}
$$

We will scale the Virasoro generators as before, that is asymmetrically and with mixing of modes. However, now the scaling of the supercharges has more than one choice: Either one can combine both of these symmetrically, or one can construct linear combinations out of these and scale them asymmetrically, as we have done for the bosonic generators. Let us consider both these cases separately:

For the first option, let us define the scaled generators as:

$$\Psi^1_r = \sqrt{\epsilon} Q_r, \qquad\qquad \Psi^2_r = \sqrt{\epsilon} \bar{Q}_{-r}$$

This scaling is called symmetric scaling because both the holomorphic and the anti-holomorphic generators of the original suoerconformal algebra are scaled with the same power of ϵ. However this contraction is not completely symmetric as the mode number is preserved on the holomorphic side but is flipped on the anti-holomorphic side.
Now taking the limit $\epsilon \to 0$, the algebra we get is:

$$\begin{aligned}
[\mathfrak{J}_n, \mathfrak{J}_m] &= (n-m)\mathfrak{J}_{n+m} + \frac{c_1}{12} n(n^2-1)\delta_{n+m,0} \\
[\mathfrak{J}_n, \mathcal{M}_m] &= (n-m)\mathcal{M}_{n+m} + \frac{c_2}{12} n(n^2-1)\delta_{n+m,0} \\
\{\Psi_r, \Psi_s\} &= \frac{1}{2}\left[\mathcal{M}_{r+s} + \frac{c_2}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\right] \qquad\qquad [\mathcal{M}_n, \mathcal{M}_m] = 0 \\
[\mathfrak{J}_n, \Psi^a_r] &= \left(\frac{n}{2} - r\right)\Psi^a_{n+r} \qquad\qquad [\mathcal{M}_n, \Psi^a_r] = 0
\end{aligned} \tag{2.2.2}$$

where the bosonic modes n, m are integral whereas the fermionic modes r, s are half-integral, while $a = \pm$. We can recognize this as the most generically extended three dimensional $\mathcal{N} = 2$ BMS_3 algebra. As expected in supergravity, the translation subgroup appears on the right hand side of the fermion anticommutators. In fact, we can identify the whole super-Poincare algebra with the following set of generators:

$$\mathfrak{J}_\pm, \mathfrak{J}_0, \mathcal{M}_\pm, \mathcal{M}_0, \Psi^\pm_{\frac{1}{2}}$$

The other choice is the asymmetric scaling that we have mentioned before. In that case we do not get a consistent algebra, because the anticommutator of the supercharges turns out to be divergent. This is shown in the appendix.

Here let us recall that for the pure bosonic case, the BMS algebra was found to be isomorphic to the Galilean Conformal Algebra (GCA), although the two are obtained by different contractions of the conformal generators. For the GCA one has to perform asymmetric scaling of the combinations $L_n \pm \bar{L}_n$ (hence no mixing of modes), whereas for the BMS, one needs to contract the linear combinations with mixing of modes. The two ways of contraction give rise to the same final algebra, which has been called the

BMS-GCA correspondence. It is now natural to ask the question whether this correspondence holds in the supersymmetric case as well. The supersymmetric GCA has been constructed in [17]. However as we have shown above, the asymmetric scaling of the combinations $Q_r + \bar{Q}_r$ does not lead to a consistent algebra in the BMS case. Hence it follows that the BMS GCA correspondence of [8] does not hold at the supersymmetric level. The correspondence seems to be accidental and only limited to the bosonic case.

2.2.2 $\mathcal{N} = 4$ Super-BMS$_3$:

Here we start from a theory of gauged supergravity admitting $(2, 2)$ supersymmetry. This means that there are two supercharges in both the holomorphic and the anti-holomorphic sector. As there is more than one charge in each sector, this is the first time that the R-symmetry generator appears in our analysis. Let us denote the supercharges in the holomorphic sector as Q^+ and Q^-. Then under the R-symmetry, these two have charges ± 1 respectively. The anti-holomorphic part is exactly analogous to this.

The holomorphic part reads:

$$\begin{aligned}
&[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}, && [R_n, R_m] = \frac{c}{3}n\delta_{n+m,0}\\
&[L_n, Q_r^{\pm}] = \left(\frac{n}{2} - r\right) Q^{\pm}_{n+r}, \qquad [L_n, R_m] = -mR_{n+m}, && [R_n, Q_r^{\pm}] = \pm Q^{\pm}_{r+n}\\
&\{Q_r^+, Q_s^-\} = L_{r+s} + \frac{1}{2}(r-s)R_{r+s} + \frac{c}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}, && \{Q_r^{\pm}, Q_s^{\pm}\} = 0
\end{aligned} \tag{2.2.3}$$

The anti-holomorphic sector is exactly same, albeit with barred generators and central charge. Note that $(Q_r^+)^\dagger = Q^-_{-r}$, and similarly for $\bar{Q}$.

By now we already know the scalings for the Virasoro and the fermionic generators. So we only need to find out the correct contraction for the R-symmetry part. Let us analyse both the symmetric and the antisymmetric cases.

$\mathcal{N}=4$ **Super-BMS$_3$ with Asymmetric Scaling for the R-currents:**

We use the contractions for Virasoro and fermionic generators as before, and scale the R-currents asymmetrically:

$$
\begin{aligned}
\mathfrak{J}_n &= \lim_{\epsilon\to 0}\left(L_m - \bar{L}_{-m}\right), & \mathcal{M}_n &= \lim_{\epsilon\to 0}\epsilon\left(L_m - \bar{L}_{-m}\right)\\
\Psi_r^{1,\pm} &= \lim_{\epsilon\to 0}\sqrt{\epsilon}Q_r^{\pm}, & \Psi_r^{2,\pm} &= \lim_{\epsilon\to 0}\sqrt{\epsilon}\bar{Q}_{-r}^{\pm}\\
c_1 &= \lim_{\epsilon\to 0}(c-\bar{c}), & c_2 &= \lim_{\epsilon\to 0}\epsilon(c+\bar{c})\\
\mathcal{R}_m &= \lim_{\epsilon\to 0}(R_m - \bar{R}_{-m}), & \mathcal{S}_m &= \lim_{\epsilon\to 0}\epsilon(R_m + \bar{R}_{-m})
\end{aligned}
\tag{2.2.4}
$$

The scaled supercharges satisfy: $(\Psi_r^{a,\pm})^\dagger = \Psi_{-r}^{a,\mp}$ with $a=1,2$.

This gives the algebra:

$$
\begin{aligned}
[\mathfrak{J}_n,\mathfrak{J}_m] &= (n-m)\mathfrak{J}_{n+m} + \frac{c_1}{12}n(n^2-1)\,\delta_{n+m,0}\\
[\mathfrak{J}_n,\mathcal{M}_m] &= (n-m)\mathcal{M}_{n+m} + \frac{c_2}{12}\,n(n^2-1)\,\delta_{n+m,0}\\
[\mathcal{M}_n,\mathcal{M}_m] &= 0, \quad [\mathcal{M}_n,\mathcal{S}_m] = 0, \quad [\mathcal{M}_n,\mathcal{R}_m] = -m\,\mathcal{S}_{n+m}\\
[\mathfrak{J}_n,\mathcal{R}_m] &= -m\mathcal{R}_{n+m}, \qquad [\mathfrak{J}_n,\mathcal{S}_m] = -m\mathcal{S}_{n+m}\\
[\mathcal{R}_n,\mathcal{R}_m] &= \frac{c_1}{3}n\delta_{n+m,0}, \quad [\mathcal{S}_n,\mathcal{S}_m] = 0, \quad [\mathcal{R}_n,\mathcal{S}_m] = \frac{c_2}{3}n\delta_{n+m,0}\\
[\mathcal{M}_n,\Psi_r^{a,\pm}] &= 0 \qquad [\mathfrak{J}_n,\Psi_r^{a,\pm}] = \left(\frac{n}{2}-r\right)\Psi_{r+n}^{a,\pm}\\
[\mathcal{R}_n,\Psi_r^{1,\pm}] &= \pm\Psi_{n+r}^{1,\pm}, \quad [\mathcal{R}_n,\Psi_r^{2,\pm}] = \mp\Psi_{n+r}^{2,\pm}, \quad [\mathcal{S}_n,\Psi_r^{a,\pm}] = 0\\
\{\Psi_r^{1,\pm},\Psi_s^{1,\mp}\} &= \frac{1}{2}\left[\mathcal{M}_{r+s} + \frac{1}{2}(r-s)\mathcal{S}_{r+s} + \frac{c_2}{6}\left(r^2-\frac{1}{4}\right)\delta_{r+s,0}\right]\\
\{\Psi_r^{2,\pm},\Psi_s^{2,\mp}\} &= \frac{1}{2}\left[\mathcal{M}_{r+s} - \frac{1}{2}(r-s)\mathcal{S}_{r+s} + \frac{c_2}{6}\left(r^2-\frac{1}{4}\right)\delta_{r+s,0}\right]\\
\{\Psi_r^{1,\pm},\Psi_s^{2,\mp}\} &= 0
\end{aligned}
\tag{2.2.5}
$$

where the index $a,b=1,2$. The anticommutator of each supercharge with its Hermitian conjugate closes into a linear combination of P and $\mathcal{S}$ plus a central term, with the coefficient of $\mathcal{S}$ taking opposite signs for $a=1,2$. The anticommutator of each supercharge with itself vanishes – as expected, given that the result has R-charge 2. The super-Poincaré algebra sits inside this algebra and the corresponding generators are :

$$\mathfrak{J}_{\pm1}, \mathfrak{J}_0, \mathcal{M}_{\pm1}, \mathcal{M}_0, \mathcal{R}_0, \Psi^{a,\pm}_{\pm,\frac{1}{2}}$$

$\mathcal{N}=4$ Super-BMS$_3$ with Symmetric Scaling for the R-currents:

Let us now consider the alternative scenario, that is, if the R-charges are scaled symmetrically, and see if that leads to an algebra that we can identify as a valid super BMS$_3$ algebra. After contraction, the algebra that we get is as follows:

$$\begin{aligned}
[\mathfrak{J}_n, \mathfrak{J}_m] &= (n-m)\mathfrak{J}_{n+m} + \frac{c_1}{12} n(n^2-1)\,\delta_{n+m,0} \\
[\mathfrak{J}_n, \mathcal{M}_m] &= (n-m)\mathcal{M}_{n+m} + \frac{c_2}{12}\, n(n^2-1)\,\delta_{n+m,0} \\
[\mathcal{M}_n, \mathcal{M}_m] &= 0, \quad [\mathcal{M}_n, \mathcal{S}_m] = 0, \quad [\mathcal{M}_n, \mathcal{R}_m] = 0 \\
[\mathfrak{J}_n, \mathcal{R}_m] &= -m\mathcal{R}_{n+m}, \qquad\quad [\mathfrak{J}_n, \mathcal{S}_m] = -m\mathcal{S}_{n+m} \\
[\mathcal{R}_n, \mathcal{R}_m] &= \frac{c_2}{6} n\delta_{n+m,0}, \quad [\mathcal{S}_n, \mathcal{S}_m] = -\frac{c_2}{6} n\delta_{n+m,0}, \quad [\mathcal{R}_n, \mathcal{S}_m] = 0 \\
[\mathcal{M}_n, \Psi^{a,\pm}_r] &= 0 \qquad\quad [\mathfrak{J}_n, \Psi^{a,\pm}_r] = \left(\frac{n}{2} - r\right)\Psi^{a,\pm}_{r+n} \\
[\mathcal{R}_n, \Psi^{a,\pm}_r] &= 0, \quad [\mathcal{S}_n, \Psi^{a,\pm}_r] = 0, \quad \{\Psi^{a,\pm}_r, \Psi^{b,\pm}_s\} = 0, \;\; a \neq b \\
\{\Psi^{1,\pm}_r, \Psi^{1,\mp}_s\} &= \frac{1}{2}\left[\mathcal{M}_{r+s} + \frac{c_2}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\right] \\
\{\Psi^{2,\pm}_r, \Psi^{2,\mp}_s\} &= \frac{1}{2}\left[\mathcal{M}_{r+s} + \frac{c_2}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\right]
\end{aligned} \tag{2.2.6}$$

We see that that this algebra contains no R-symmetry. Indeed, all bosonic operators commute with the $\Psi^{a,\pm}$ except $\mathfrak{J}_m$, which just measures its spin. In particular, the $\mathcal{R}$ and $\mathcal{S}$ operators commute with the fermions, so that the latter are not charged under $\mathcal{R}$ and $\mathcal{S}$. This algebra is therefore trivial and not the correct one to describe the asymptotic symmetry of flat-space extended supergravity. Thus we conclude that the correct scaling is the asymmetric one discussed in the last section and the corresponding algebra is the correct algebra. For the case of $N=8$ super-BMS$_3$ that we consider in the next section, we shall only study the asymmetric scaling for R-currents.

2.2.3 Generic $N = 4$ super-BMS$_3$ algebra

The algebra in eq.(2.2.5) that we have derived via contraction of the $N = 4$ superconformal algebra has specific relations among the central charges appearing in the different commutators. It turns out that more generic central extension of this algebra is possible consistent with all the Jacobi identities among the generators.

It is natural to ponder why there is more freedom in choosing the central charges in this algebra than what we got by naive contraction! This is because the original superconformal algebra had to satisfy the Jacobi identities as well, which related some of its central charges. However, after contraction, several terms drop out when we take the limit $\epsilon \to 0$, and the terms via which the central charges were related by Jacobi identities sometimes vanish. So there is greater freedom in the central charges in this algebra.

In particular, in the superconformal algebra, the central charge appearing in the $[L_n, L_m]$ is related to the central charge appearing in the $[R_n, R_m]$ commutator. However, after contraction, we find that the central term appearing the $[\mathcal{R}_m, \mathcal{R}_n]$ commutator need not be related to that in the $[\mathfrak{J}_m, \mathfrak{J}_n]$ commutator. This is because these two were originally related to each other through the central charge of the supersymmetry algebra. But after contraction, the supersymmetry algebra produces the $\mathcal{S}$ generator on the right-hand-side instead of the R-symmetry generator $\mathcal{R}$. Thus the Virasoro and R-symmetry central charges are no longer related, and the general $N = 4$ super-BMS algebra has independent central terms in the $[\mathcal{R}_m, \mathcal{R}_n]$ and $[\mathfrak{J}_n, \mathfrak{J}_m]$ commutators.

From now on we will consider this maximally centrally extended version of the $N = 4$ super-BMS algebra. In a later section we shall see that the free field realization naturally produces different central charges for these two commutators. One may vary the central extension in the $[\mathcal{R}_m, \mathcal{R}_n]$ commutator can by adding more free fields to the base system.

2.3 $\mathcal{N} = 8$ Super-BMS$_3$:

Now we look at systems containing 8 supercharges. We shall obtain this by contracting two copies of the small $\mathcal{N} = 4$ superconformal algebra which is generated by the bosonic currents T, R^i with $(i = 1, 2, 3)$ and fermionic currents $Q^{a,\alpha}$ with $(a, \alpha = 1, 2)$. The central charge is related to the level of the $SU(2)$ currents. In terms of modes, the holomorphic part of the algebra is: [18]

$$
\begin{aligned}
[L_n, L_m] &= (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0} \qquad & [L_n, R^i_m] &= -mR^i_{n+m} \\
[L_n, Q^{a,\alpha}_r] &= \left(\frac{n}{2} - r\right) Q^{a,\alpha}_{n+r} & \left[R^i_n, R^j_m\right] &= i\epsilon^{ijk}R^k_{n+m} + \frac{c}{12}n\delta^{ij}\delta_{n+m,0} \\
\left[R^i_n, Q^{a,1}_r\right] &= -\frac{1}{2}(\sigma^i)^a_b Q^{b,1}_{n+r} & \left[R^i_n, Q^{a,2}_r\right] &= -\frac{1}{2}(\bar{\sigma}^i)^a_b Q^{b,2}_{n+r}
\end{aligned}
$$

$$
\{Q^{a,+}_r, Q^{b,-}_s\} = \left[\delta_{ab}L_{r+s} - (r-s)(\sigma^i)_{ab}R^i_{r+s} + \frac{c}{6}\left(r^2 - \frac{1}{4}\right)\delta^{ab}\delta_{r+s,0}\right] \tag{2.3.1}
$$

The anti-holomorphic part is similar, with an independent central charge $\bar{c}$. Here $\bar{\sigma}^i_{ab} = \sigma^i_{ba}$ and σ^i are the three Pauli matrices.

As discussed before, we now contract this algebra using the asymmetric scaling for the R-currents.

$$
\begin{aligned}
\mathfrak{J}_n &= \lim_{\epsilon\to 0}\left(L_m - \bar{L}_{-m}\right), \qquad & \mathcal{M}_n &= \lim_{\epsilon\to 0}\epsilon\left(L_m + \bar{L}_{-m}\right), \\
\Psi^{1,a,\alpha}_r &= \lim_{\epsilon\to 0}\sqrt{\epsilon}Q^{a,\alpha}_r, & \Psi^{2,a,\alpha}_r &= \lim_{\epsilon\to 0}\sqrt{\epsilon}\bar{Q}^{a,\alpha}_{-r}, \\
c_1 &= \lim_{\epsilon\to 0}(c - \bar{c}), & c_2 &= \lim_{\epsilon\to 0}\epsilon(c + \bar{c}) \\
\mathcal{R}^i_m &= \lim_{\epsilon\to 0}(R^i_m + \bar{R}^i_{-m}), & \mathcal{S}_m &= \lim_{\epsilon\to 0}\epsilon(R^i_m - \bar{R}^i_{-m})
\end{aligned}
\tag{2.3.2}
$$

where $a = 1, 2$ and $\alpha = \pm$. This gives the algebra:

$$
\begin{aligned}
[\mathfrak{J}_n, \mathfrak{J}_m] &= (n-m)\mathfrak{J}_{n+m} + \frac{c_1}{12}\, n(n^2-1)\,\delta_{n+m,0}, \qquad [\mathcal{S}^i_n, \mathcal{S}^j_m] = 0 \\
[\mathfrak{J}_n, \mathcal{M}_m] &= (n-m)\mathcal{M}_{n+m} + \frac{c_2}{12}\, n(n^2-1)\,\delta_{n+m,0} \qquad [\mathcal{M}_n, \mathcal{M}_m] = 0 \\
[\mathcal{M}_n, \mathcal{R}^i_m] &= -n\mathcal{S}^i_{n+m}, \quad [\mathcal{M}_n, \mathcal{S}^i_m] = 0, \quad [\mathfrak{J}_n, \mathcal{R}^i_m] = -\mathcal{R}^i_{n+m}, \quad [\mathfrak{J}_n, \mathcal{S}^i_m] = -\mathcal{S}^i_{n+m} \\
[\mathcal{R}^i_n, \mathcal{R}^j_m] &= i\epsilon^{ijk}\mathcal{R}^k_{n+m} + \frac{c_1}{12} n\delta^{ij}\delta_{n+m,0}, \quad [\mathcal{R}^i_n, \mathcal{S}^j_m] = i\epsilon^{ijk}\mathcal{S}^k_{n+m} + \frac{c_2}{12} n\delta^{ij}\delta_{n+m,0} \\
[\mathcal{M}_n, \Psi^{A,a,\alpha}_r] &= 0, \qquad [\mathfrak{J}_n, \Psi^{A,a,\alpha}_r] = \left(\frac{n}{2} - r\right)\Psi^{A,a,\alpha}_{r+n}, \qquad [\mathcal{S}^i_n, \Psi^{A,a,\alpha}_r] = 0 \\
[\mathcal{R}^i_n, \Psi^{A,a,1}_r] &= -\frac{1}{2}(\sigma^i)^a_b \Psi^{A,b,1}_{n+r}, \qquad [\mathcal{R}^i_n, \Psi^{A,a,2}_r] = \frac{1}{2}(\bar{\sigma}^i)^a_b \Psi^{A,b,2}_{n+r}, \\
\{\Psi^{1,\pm}_r, \Psi^{1,\mp}_s\} &= \frac{1}{4}[1+(-1)^{A+B}]\left[\delta^{ab}\mathcal{M}_{r+s} - (r-s)(\sigma^i)_{ab}\mathcal{S}^i_{r+s} + \frac{c_2}{6}\left(r^2 - \frac{1}{4}\right)\delta^{ab}\delta_{r+s,0}\right]
\end{aligned}
\tag{2.3.3}
$$

where $A, B = 1, 2$. This is the $N = 8$ super-BMS$_3$ algebra. As in the $N = 4$ case, here also the central charge is too restrictive. Using the Jacobi identity, we can find a more generic version with an independent central term c_3 in the $\mathcal{R} - \mathcal{R}$ commutator. In this case, we identify the super-Poincare algebra as consisiting of the following generators:

$$
\mathfrak{J}_{\pm 1}, \mathfrak{J}_0, \mathcal{M}_{\pm} 1, \mathcal{M}_0, \mathcal{R}^i_0, \Psi^{A,a,\pm}_{\pm\frac{1}{2}}
$$

CHAPTER 3

$\mathcal{N} = 4$ Supersymmetric BMS$_3$ Algebra from Asymptotic Symmetry Analysis:

3.1 Introduction:

In the last chapter, we have shown how to obtain super BMS$_3$ algebras by a method of contraction from the corresponding super-conformal algebras. In this chapter, we shall emphasize on a particular case, namely the $\mathcal{N} = 4$ super-BMS$_3$ algebra, and derive it by a different method. We find the $\mathcal{N} = 4$ super BMS$_3$ algebra, which is the algebra of three dimensional $\mathcal{N} = 4$ supergravity theory at null infinity, by a direct asymptotic analysis following [14] ie by finding appropriate boundary conditions to impose on the fields. If the algebra that we get by this method agrees with our previously obtained algebra via method of contraction, then this will also validate our prescription of scaling the various generators. In fact, as we will see, the algebra obtained naively by asymptotic analysis contains pathological terms, and does not match with what we derived in the previous chapter! However, upon careful analysis, it is realised that the presence of internal R-symmetry calls for certain modifications of some generators in the form of Sugawara shifts, and this finally leads to exactly the algebra found by contraction.

3.1.1 Chern-Simons Formulation for 3 dimensional gravity

The Chern-Simons (CS) action on a three dimensional manifold M, invariant under the action of a compact Lie group G, is given by:

$$I[\mathcal{A}] = \frac{k}{4\pi}\int_M \langle \mathcal{A}, d\mathcal{A} + \frac{2}{3}\mathcal{A}^2\rangle \ . \qquad (3.1.1)$$

Here the gauge field $\mathcal{A}$ is regarded as a Lie-algebra-valued one form, and $\langle , \rangle$ represents a non-degenerate invariant bilinear form taking values on the Lie algebra space and acting as a metric and k is level for the theory. Thus in a particular basis $\{T_a\}$ of the Lie-algebra, we can express $\mathcal{A} = \mathcal{A}^a_\mu \, T_a \, \mathrm{d}x^\mu$. The equation of motion is simply

$$F \equiv dA + A \wedge A = 0.$$

The general solution of the equation of motion is topological, i.e. pure gauge. Consider for instance the Poincaré group $G = \mathrm{ISO}(2,1)$ and a manifold M with a *boundary*. The non-zero commutation relations of the Lie-algebra are:

$$[\mathcal{J}_a, \mathcal{J}_b] = \epsilon_{abc}\mathcal{J}^c \,, \qquad [\mathcal{J}_a, P_b] = \epsilon_{abc}P^c, \tag{3.1.2}$$

where $a = 1, 2, 3$ and ϵ_{abc} is the antisymmetric 3-form. The explicit form of the gauge field is given in this basis by $\mathcal{A}_\mu = e^a_\mu P_a + \omega^a_\mu J_a$, where e^a_μ acts as the vierbein and ω^a_μ is the corresponding spin connection. The above action (3.1.1) then corresponds to the 3D Einstein-Hilbert action

$$S = \frac{1}{16\pi G} \int 2e^a \, R_a \ , \quad R^a = \mathrm{d}\omega^a + \tfrac{1}{2} \, \varepsilon^a{}_{bc} \, \omega^b \, \omega^c \ ,$$

up to identifying the level $k = \frac{1}{4G}$. Thus 3-dimensional gravity invariant under the local $\mathrm{ISO}(2,1)$ Poincaré group, with zero (or non-zero) cosmological constant can be cast as a 3-dimensional CS gauge theory with the same gauge group. Indeed one can show that a generic $\mathrm{ISO}(2,1)$ gauge transformation parametrized by the element $U = E^a P_a + \omega^a J_a$, acts on the gauge field as

$$\delta \mathcal{A}_\mu = -D_\mu U = -(\partial_\mu U + [\mathcal{A}_\mu, U]). \tag{3.1.3}$$

In terms of the gravity fields (e^a_μ, ω^a_μ) the gauge transformation reads:

$$\delta e^a_\mu = -\partial_\mu E^a - \epsilon^{abc} e_{\mu b} \omega_c - \epsilon^{abc} \omega_{\mu b} E_c \tag{3.1.4}$$

$$\delta \omega^a_\mu = -\partial_\mu \omega^a - \epsilon^{abc} \omega_{\mu b} \omega_c \tag{3.1.5}$$

which are the expected local Lorentz transformations generated by ω^a and local diffeomorphism transformations generated by E^a. Recall that under a generic diffeomorphism transformation $x^\mu \to x^\mu + V^\mu$, the fields (e^a_μ, ω^a_μ) transforms as

$$\tilde{\delta} e^a_\mu = V^\nu(\partial_\nu e^a_\mu - \partial_\mu e^a_\nu) + \partial_\mu(V^\nu e^a_\nu), \quad \tilde{\delta}^a_\mu = V^\nu(\partial_\nu \omega^a_\mu - \partial_\mu \omega^a_\nu) + \partial_\mu(V^{\nu a}_\nu). \quad (3.1.6)$$

Thus for $E^a = e^a_\mu V^\mu$ and ignoring the local Lorentz transformation, we can show that the difference between (3.1.4) and (3.1.6) is:

$$\tilde{\delta} e^a_\mu - \delta e^a_\mu = V^\nu(D_\nu e^a_\mu - D_\mu e^a_\nu) - \epsilon^{abc} V^\nu \omega_{\nu b} e_{\mu c} \,. \quad (3.1.7)$$

The 1st term of the RHS of the above equation, the torsion, vanishes on-shell, while the 2nd term can be identified with a local Lorentz transformation with parameter $\omega^a = \omega^a_\mu V^\mu$ [19]. Thus we see that, on-shell, a gauge transformation of Chern Simons theory is identical to a local Lorentz and diffeomorphism transformation of 3D Gravity.

We end this subsection by recalling how to find a nontrivial classical solution in this theory. Since (3.1.1) is a gauge theory, we first need to fix a gauge. In (u, r, ϕ) coordinates, for an arbitrary single valued group element U, the general solution takes the form $A_\mu = U^{-1}\partial_\mu U$. Imposing the gauge-fixing condition $\partial_\phi A_r = 0$, the connection will have following form [20], [21]:

$$A_r(r) = b(r)^{-1}\partial_r b(r), \quad A_\phi(r, \phi, u) = b(r)^{-1} A(\phi, u) b(r), \quad (3.1.8)$$

where $b(r)$ and $A(\phi, u)$ are arbitrary functions. To find A_u, we recall that the gauge fixing condition $\partial_\phi A_r = 0$ must remain invariant under a new gauge transformation, for instance a time (u) evolution, i.e. $\partial_u \partial_\phi \mathcal{A}_u = 0$. Using the equation of motion, this implies $\partial_r \partial_\phi A_u = 0$ which is solved generically by:

$$A_u(r, \phi, u) = b(r)^{-1} B(\phi, u) b(r), \quad (3.1.9)$$

with $B(\phi, u)$ is an arbitrary function of ϕ and time representing the residual gauge freedom of the system. Similarly $A(\phi, u)$ represents the residual part of the gauge field that can not be fixed. Instead, as we shall see in the next subsection, they will give the global conserved charges and centrally extended symmetry algebra at the boundary.

Thus we see that, in a partial gauge fixed CS theory the solution will have the form

$\mathcal{A} = b(r)^{-1}(a + d)b(r)$, with $a = a_u du + a_\phi d\phi$ is a function of ϕ and time. In the following, we choose $b(r) = e^{\alpha r}$, α a Lie-algebra valued constant, as a proper boundary condition on the field. In section 3.3, we have shown the choice of α which reproduces the correct asymptotic gauge field.

3.1.2 Construction of Asymptotic symmetry algebra

Once a solution of CS theory is obtained, one can follow the canonical Hamiltonian approach of [22] to construct the conserved charges that correspond to the residual global part of the gauge symmetry. Here, we shall only outline the procedure detailed in the original paper and [20]. Note that in the reference provided, the analysis is done for AdS space, where asymptotic symmetry is considered at spacelike infinity. For this reason they have constructed spatial slices at $t =$ constant, where t is the usual Minkowski time. In this thesis, however, we are interested in flat space asymptotics, where the asymptotic symmetry algebra is constructed at null infinity. Thus we work in 'Bondi coordinates', as discussed in Section 1.2, and in this case the timelike coordinate is u.

Consider a Chern-Simons theory on a manifold $\Sigma \times \mathbb{R}$, where Σ is a compact two manifold and time is along $\mathbb{R}$. In this gauge theory, one defines global charges by demanding the differentials of the generators of gauge transformations to be regular for a certain choice of boundary conditions. Thus for some arbitrary gauge transformation parameters λ_a (matrix valued function) the charge needs to satisfy:

$$\delta Q(\lambda) = -\frac{k}{2\pi} \int_{\partial\Sigma} \lambda_a \delta A^a_\mu dx^\mu. \tag{3.1.10}$$

Further assuming the parameter function λ to be independent of the gauge field that is varied at the boundary, we readily get the charge $Q(\lambda)$ as,

$$Q(\lambda) = -\frac{k}{2\pi} \int_{\partial\Sigma} \lambda_a A^a_\mu dx^\mu \ , \tag{3.1.11}$$

where the integration constant is set to zero. It is clear from the above expression that $Q(\lambda)$ is defined via the boundary value of the gauge field. Considering the example of ordinary 3 dimensional gravity that we studied in the last section in (u, r, ϕ) coordinates, the boundary consists of the ϕ direction. Thus for this case, we get

$$Q(\lambda) = -\frac{k}{2\pi} \int_{\partial\Sigma} \lambda_a(\phi) A^a(\phi) d\phi \ . \tag{3.1.12}$$

As we have seen in the last section, $A^a(\phi)$ is the residual part of the gauge field that remains unfixed after gauge fixing. Similarly $\lambda_a(\phi)$ corresponds to the residual part of the gauge transformation parameters. Thus, we have constructed global charges that corresponds to the residual gauge symmetry . Expanding the boundary fields and the parameters in modes, one can find find the centrally extended algebra realized by this symmetry.

3.2 Construction of the Action:

In this chapter we want to construct the asymptotic symmetry algebra of $3d$ $\mathcal{N} = 4$ supergravity theory. The global symmetry algebra consists of the bosonic generators $\mathcal{J}_a, P_a, (a = 0,1,2), \mathcal{R}, \mathcal{S}$ and Majorana fermionic generators $\mathcal{Q}^{1\pm}_\alpha, \mathcal{Q}^{2,\pm}_\alpha, \left(\alpha = \pm\frac{1}{2}\right)$. They satisfy the super-Poincare algebra:

$$\begin{aligned}
&[\mathcal{J}_a, \mathcal{J}_b] = \epsilon_{abc}\mathcal{J}^c, \qquad [\mathcal{J}_a, P_b] = \epsilon_{abc}P^c, \qquad [\mathcal{J}_a, \mathcal{Q}^{1,2,\pm}_\alpha] = \frac{1}{2}(\Gamma_a)^\beta_\alpha \mathcal{Q}^{1,2\pm}_\beta \\
&\{\mathcal{Q}^{1\pm}_\alpha, \mathcal{Q}^{1\mp}_\beta\} = -\frac{1}{2}(C\Gamma^a)_{\alpha\beta}P_a \mp \frac{1}{2}C_{\alpha\beta}\mathcal{S}, \qquad [\mathcal{R}^a, \mathcal{Q}^{1\pm}_\alpha] = \pm\frac{1}{2}\mathcal{Q}^{1\pm}_\alpha \\
&\{\mathcal{Q}^{2\pm}_\alpha, \mathcal{Q}^{2\mp}_\beta\} = -\frac{1}{2}(C\Gamma^a)_{\alpha\beta}P_a \pm \frac{1}{2}C_{\alpha\beta}\mathcal{S}, \qquad [\mathcal{R}^a, \mathcal{Q}^{2\pm}_\alpha] = \mp\frac{1}{2}\mathcal{Q}^{2\pm}_\alpha
\end{aligned} \tag{3.2.1}$$

Here $\mathcal{S}$ acts as a possible central extension of the algebra while $\mathcal{R}$ is the proper R-symmetry, as the fermions transform under it. Using the invariant bilinear form for this algebra (shown in Appendix), we get the supertrace elements as

$$< \mathcal{J}_a, P_b >= \eta_{ab}, \qquad < \mathcal{Q}^{1,2\pm}_\alpha, \mathcal{Q}^{1,2\mp}_\beta >= C_{\alpha\beta}, \qquad < \mathcal{R}, \mathcal{S} >= -1 \tag{3.2.2}$$

Now we expand the action of the theory in terms of the basis generators:

$$\mathcal{A} = e^a P_a + \omega^a \mathcal{J}_a + \sum_{\alpha=\pm} \psi^{1\alpha}_\pm \mathcal{Q}^{1\pm}_\alpha + \sum_{\alpha=\pm} \psi^{2\alpha}_\pm \mathcal{Q}^{2\pm}_\alpha + \nu\mathcal{R} + \sigma\mathcal{S}, \tag{3.2.3}$$

Here e^a is the vielbein field, ω^a the corresponding dual spin connection, $\psi^{1\alpha}_{\pm}, \psi^{2\alpha}_{\pm}$ are the Majorana gravitini and ν, σ are the internal gauge fields. Now we can write down the action for $N = 4$ asymptotically flat Supergravity theory:

$$S = \frac{k}{4\pi} \int \left[2e^a \, R_a - \sigma d\nu - \nu d\sigma + \sum_{a=\pm} \bar{\psi}^1_a D\psi^1_{-a} + \sum_{a=\pm} \bar{\psi}^2_a D\psi^2_{-a} \right] \tag{3.2.4}$$

where the covariant derivatives are given by:

$$\begin{aligned} D\psi^1_{\pm} &= \mathrm{d}\psi^1_{\pm} + \tfrac{1}{2}\omega^a \, \Gamma_a \psi^1_{\pm} \pm \tfrac{1}{2}\nu\psi^1_{\pm} \; , \\ D\psi^2_{\pm} &= \mathrm{d}\psi^2_{\pm} + \tfrac{1}{2}\omega^a \, \Gamma_a \psi^2_{\pm} \mp \tfrac{1}{2}\nu\psi^2_{\pm} \; , \\ R^a &= \mathrm{d}\omega^a + \tfrac{1}{2} \, \varepsilon^a{}_{bc} \, \omega^b \, \omega^c \end{aligned} \tag{3.2.5}$$

Now we can check the invariance of the action S under the supersymmetry by using the transformations,

$$\delta\mathcal{A} = \mathrm{d}\lambda^{susy} + [\mathcal{A}, \lambda^{susy}], \quad \lambda^{susy} = \theta^{1\alpha}_{\pm} \mathcal{Q}^{1\pm}_{\alpha} + \theta^{2\alpha}_{\pm} \mathcal{Q}^{2\pm}_{\alpha}.$$

which explicitly read:

$$\begin{aligned} \delta e^a &= \tfrac{1}{2}(\bar{\theta}^1_+ \Gamma^a \psi^1_- + \bar{\theta}^1_- \Gamma^a \psi^1_+ + \bar{\theta}^2_+ \Gamma^a \psi^2_- + \bar{\theta}^2_- \Gamma^a \psi^2_+), \qquad \delta\omega^a = 0 \\ \delta\psi^{1\alpha}_{\pm} &= \mathrm{d}\theta^{1\alpha}_{\pm} + \tfrac{1}{2} \, \omega^a \, \Gamma_a \, \theta^{1\alpha}_{\pm} \pm \tfrac{1}{2}\nu\theta^{1\alpha}_{\pm} = D\theta^{1\alpha}_{\pm} \\ \delta\psi^{2\alpha}_{\pm} &= \mathrm{d}\theta^{2\alpha}_{\pm} + \tfrac{1}{2} \, \omega^a \, \Gamma_a \, \theta^{2\alpha}_{\pm} \mp \tfrac{1}{2}\nu\theta^{2\alpha}_{\pm} = D\theta^{2\alpha}_{\pm} \\ \delta\sigma &= \mp\tfrac{1}{2}(\bar{\psi}^1_{\pm}\theta^1_{\mp} - \bar{\psi}^2_{\pm}\theta^2_{\mp}), \quad \delta\nu = 0. \end{aligned}$$

The supersymmetry algebra closes on-shell into a general coordinate transformation, a Lorentz transformation (with dual parameter $\lambda^a = \epsilon^{abc}\Lambda_{bc}$) and a supersymmetry transformation with parameters $\varepsilon_{\pm} = -\xi^{\nu} \, \psi^1_{\nu\,\pm}$ and $\vartheta_{\pm} = -\xi^{\nu} \, \psi^2_{\nu\,\pm}$:

$$\begin{aligned} [\delta(\varepsilon^1_+, \varepsilon^1_-, \vartheta^1_+, \vartheta^1_-), \delta(\varepsilon^2_+, \varepsilon^2_-, \vartheta^2_+, \vartheta^2_-)] &= \delta_{\mathrm{Lor}}(\lambda^a = -\xi^{\nu}\omega_{\nu}{}^a) + \delta_{\mathrm{susy}}(\varepsilon_+, \varepsilon_-, \vartheta_+, \vartheta_-) \\ &+ \delta_{\mathrm{g.c.}}(\xi^{\nu} = -\tfrac{1}{2}(\bar{\varepsilon}^2_- \Gamma^{\nu} \varepsilon^1_+ + \bar{\varepsilon}^2_+ \Gamma^{\nu} \varepsilon^1_- + \bar{\vartheta}^2_- \Gamma^{\nu} \vartheta^1_+ + \bar{\vartheta}^2_+ \Gamma^{\nu} \vartheta^1_-)). \end{aligned} \tag{3.2.6}$$

The dynamical equations are:

$$
\begin{aligned}
&T^a = -\tfrac{1}{2}\,(\bar\psi^1_+\,\Gamma^a\psi^1_- + \bar\psi^2_+\,\Gamma^a\psi^2_-), \qquad D\psi^{1,2}_\pm = D\bar\psi^{1,2}_\pm = 0\ , \\
&\mathrm{d}\nu = F_\nu = 0, \qquad\qquad 2\,F_\sigma + (\bar\psi^1_-\,\psi^1_+ - \bar\psi^2_+\,\psi^2_-) = 0
\end{aligned}
$$

where T^a is the torsion tensor $T^a = de^a + \epsilon^a_{bc}\omega^b\omega^c$.

Now we make a change of frame to the set of generators $\{M_n, \mathcal{L}_n, q^{1,2\pm}_\alpha, \mathcal{R}, \mathcal{S}\}$ which are related to the old ones by the following relations as was done in [23] [1].

$$
M_n = P_a U^a_n\ , \quad \mathcal{L}_n = \mathcal{J}_a U^a_n\ , \quad q^{1\pm}_\alpha = \sqrt{2}\mathcal{Q}^{1\pm}_\alpha\ , \quad q^{2\pm}_\alpha = \sqrt{2}\mathcal{Q}^{2\pm}_\alpha\ ,
$$

with $(\mathcal{R}, \mathcal{S})$ remaining unchanged. In terms of these generators, the super Poincare algebra is:

$$
\begin{aligned}
&[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{n+m}, \qquad [\mathcal{L}_n, M_m] = (n-m)M_{n+m}, \qquad [M_n, M_m] = 0, \\
&[\mathcal{L}_n, q^{1,2\,\pm}_\alpha] = \Big(\frac{n}{2} - \alpha\Big)\, q^{1,2\,\pm}_{n+\alpha}, \qquad [M_n, q^{1\pm}_\alpha] = 0, \qquad [M_n, q^{2\pm}_\alpha] = 0, \\
&[\mathcal{R}, q^{1\pm}_\alpha] = \pm\tfrac{1}{2}\, q^{1\pm}_\alpha, \qquad [\mathcal{R}, q^{2\pm}_\alpha] = \mp\tfrac{1}{2}\, q^{2\pm}_\alpha, \qquad [\mathcal{S}, q^{1\pm}_\alpha] = 0, \qquad [\mathcal{S}, q^{2\pm}_\alpha] = 0, \\
&\{q^{1\pm}_\alpha, q^{1\mp}_\beta\} = M_{\alpha+\beta} \pm (\alpha-\beta)\mathcal{S}, \qquad \{q^{2\pm}_\alpha, q^{2\mp}_\beta\} = M_{\alpha+\beta} \pm (\alpha-\beta)\mathcal{S}. \qquad (3.2.7)
\end{aligned}
$$

3.3 $\mathcal{N} = 4$ BMS$_3$ asymptotic algebra

To derive the asymptotic symmetry algebra for the above theory, we first need to provide a suitable set of fall-off conditions for the gauge fields at null infinity. The conditions are: (i) it must extend the one of the purely gravitational sector so as to include the bosonic solutions of interest and (ii) is relaxed enough so as to enlarge the set of asymptotic symmetries from BMS$_3$ to its $N = 4$ supersymmetric extension. In order to satisfy these requirements, the behaviour of the gauge fields at the boundary is taken to be of the form,

$$
\mathcal{A} = b^{-1}(a+d)b, \qquad b = exp\left(\frac{r}{2}M_{-1}\right) \qquad (3.3.1)
$$

[1] Our conventions are summarized in appendix A.3

where, the radial dependence is encoded in b. For our purpose, we start with a gauge field

$$
\begin{aligned}
\mathcal{A} =& \Big(M_1 - \tfrac{1}{4}\,\mathcal{M}\,M_{-1} - \frac{i\rho}{2}\,\mathcal{S}\Big)\mathrm{d}u + \frac{\mathrm{d}r}{2}M_{-1} \\
&+ \Big(\mathcal{L}_1 + r\,M_0 \; - \tfrac{1}{4}\,\mathcal{M}\,\mathcal{L}_{-1} - \tfrac{1}{4}\mathcal{N}\,M_{-1} - \frac{i\phi}{2}\,\mathcal{S} - \frac{i\rho}{2}\,\mathcal{R} \\
&- \frac{1}{4}\big(\Psi^1_+\,q^{1+}_- - \Psi^1_-\,q^{1-}_-\big) + \frac{1}{4}\big(\Psi^2_+\,q^{2+}_- - \Psi^2_-\,q^{2-}_-\big)\Big)\mathrm{d}\varphi
\end{aligned}
\tag{3.3.2}
$$

Various fields appearing in the above expression asymptotically will only have u and φ dependence. The above gauge field is so chosen that it correctly reproduces the asymptotically flat metric:

$$ds^2 = \eta_{ab}e^a e^b = \mathcal{M}du^2 - 2dudr + \mathcal{N}dud\varphi + r^2 d\varphi^2$$

The asymptotic symmetries are the set of gauge transformations that preserves this behaviour. The equation of motion and the gauge transformation due to a Lie-algebra-valued parameter Λ must be of the form :

$$d\mathcal{A} + \frac{1}{2}[\mathcal{A},\mathcal{A}] = 0, \qquad \delta\mathcal{A} = \mathrm{d}\Lambda + [\mathcal{A},\Lambda], \tag{3.3.3}$$

where, the parameter Λ depends on various fields that are functions of u and φ and is given as,

$$\Lambda = \Upsilon^n\,\mathcal{L}_n + \xi^n\,M_n + \zeta^{1\,\alpha}_+\,q^{1\,+}_\alpha + \zeta^{1\,\alpha}_-\,q^{1\,-}_\alpha + \zeta^{2\,\alpha}_+\,q^{2\,+}_\alpha + \zeta^{2\,\alpha}_-\,q^{2\,-}_\alpha + \lambda_{\mathcal{R}}\,\mathcal{R} + \lambda_{\mathcal{S}}\,\mathcal{S}. \tag{3.3.4}$$

Before proceeding to compute the variations at infinity, we note down some identities satisfied by various component fields in 3.3.2 due to equations of motion:

$$\partial_\varphi\mathcal{M} = \partial_u\mathcal{N}, \quad \partial_u\mathcal{M} = 0 \tag{3.3.5}$$

Analogously, we can show :

$$\partial_u\Psi^1_\pm = 0\;, \qquad \partial_u\Psi^2_\pm = 0\;, \qquad \partial_u\,\rho = 0\;, \qquad \partial_\varphi\rho = \partial_u\phi. \tag{3.3.6}$$

Similar identities exist for the parameters as well:

$$\partial_u \xi^n = \partial_\varphi \Upsilon^n \ , \quad \partial_u \Upsilon^n = 0, \quad \partial_u \zeta^{1\alpha}_{\pm} = 0, \qquad \partial_u \zeta^{2\alpha}_{\pm} = 0,$$
$$\partial_u \lambda_{\mathcal{S}} = \partial_\varphi \lambda_{\mathcal{R}} \ , \quad \partial_u \, \lambda_{\mathcal{R}} = 0. \tag{3.3.7}$$

Thus the fields and parameters are not independent of each other. We shall come back to this point later.

Next we analyse the gauge variation condition. The conditions that we get on various fields and their variations are given below. First we present the dependent bosonic fields followed by fermionic fields. For bosonic fields we get,

$$\begin{aligned}
&\xi^0 = -\partial_\varphi \xi^+ + r\,\Upsilon^+, \qquad \Upsilon^0 = -\partial_\varphi \Upsilon^+, \quad -\partial_\varphi^2 \Upsilon^+ + 2\Upsilon^- + \tfrac{1}{2}\,\mathcal{M}\,\Upsilon^+ = 0 \\
&-\partial_\varphi^2 \xi^+ + 2\,\xi^- + r\,\partial_\phi \Upsilon^+ + \tfrac{1}{2}\left(\mathcal{M}\,\xi^+ + \mathcal{N}\Upsilon^+\right) + \tfrac{1}{4}\left(\Psi^1_+ \zeta^{1+}_- - \Psi^1_- \zeta^{1+}_+\right) \\
&-\tfrac{1}{4}\left(\Psi^2_+ \zeta^{2+}_- - \Psi^2_- \zeta^{2+}_+\right) = 0
\end{aligned}$$

where we made multiple use of the above identities (3.3.7). The constraints on the dependent fermionic parameters go as follow:

$$\begin{aligned}
\partial_\varphi \zeta^{1+}_{\pm} + \zeta^{1-}_{\pm} \pm \frac{1}{4}\Psi^1_{\pm}\Upsilon^+ \mp \frac{i}{4}\rho\zeta^{1+}_{\pm} = 0 \\
\partial_\varphi \zeta^{2+}_{\pm} + \zeta^{2-}_{\pm} \mp \frac{1}{4}\Psi^2_{\pm}\Upsilon^+ \mp \frac{i}{4}\rho\zeta^{2+}_{\pm} = 0
\end{aligned}$$

Finally we write down the variation of the fields. For bosonic fields, we get,

$$\begin{aligned}
\delta\mathcal{M} &= -2\partial_\varphi^3 \Upsilon^+ + 2\mathcal{M}\partial_\varphi \Upsilon^+ \partial_\varphi \mathcal{M}\Upsilon^+ \\
\delta\mathcal{N} &= -2\partial_\varphi^3 \xi^+ + 2\mathcal{M}\partial_\varphi \xi^+ + 2\mathcal{N}\partial_\varphi \Upsilon^+ + \partial_\varphi \mathcal{M}\xi^+ + \partial_\varphi \mathcal{N}\Upsilon^+ \\
&\quad + \tfrac{1}{2}\big(\partial_\varphi \Psi^1_+ \zeta^{1+}_- + 3\,\Psi^1_+ \partial_\varphi \zeta^{1+}_- - \partial_\varphi \Psi^1_- \zeta^{1+}_+ - 3\,\Psi^1_- \partial_\varphi \zeta^{1+}_+ + \frac{i}{2}\Psi^1_+ \zeta^{1+}_- \rho + \frac{i}{2}\Psi^1_- \zeta^{1+}_+ \rho\big) \\
&\quad - \tfrac{1}{2}\big(\partial_\varphi \Psi^2_+ \zeta^{2+}_- + 3\,\Psi^2_+ \partial_\varphi \zeta^{2+}_- - \partial_\varphi \Psi^2_- \zeta^{2+}_+ - 3\,\Psi^2_- \partial_\varphi \zeta^{2+}_+ + \frac{i}{2}\Psi^2_+ \zeta^{2+}_- \rho + \frac{i}{2}\Psi^2_- \zeta^{2+}_+ \rho\big), \\
\delta\rho &= 2i\partial_\varphi \lambda_{\mathcal{R}}, \quad \delta\phi = 2i\partial_\varphi \lambda_{\mathcal{S}} - \frac{1}{2}(\Psi^1_+ \, \zeta^{1+}_- + \Psi^1_- \, \zeta^{1+}_+) + \frac{1}{2}(\Psi^2_+ \, \zeta^{2+}_- + \Psi^2_- \, \zeta^{2+}_+)
\end{aligned} \tag{3.3.8}$$

For the fermionic fields we get:

$$
\begin{aligned}
\delta\Psi^1_\pm &= \pm 4\partial^2_\varphi\zeta^{1+}_\pm + \Big(\partial_\varphi\Psi^1_\pm\Upsilon^+ + \tfrac{3}{2}\Psi^1_\pm\partial_\varphi\Upsilon^+\Big) \mp \mathrm{i}\Big(\partial_\varphi\rho\zeta^{1+}_\pm + 2\rho\partial_\varphi\zeta^{1+}_\pm\Big) \mp \mathcal{M}\zeta^{1+}_\pm \\
&\quad \mp \frac{\mathrm{i}}{4}\Psi^1_\pm\rho\Upsilon^+ \mp \frac{1}{2}\mathcal{R}\Psi^1_\pm \mp \frac{1}{4}\rho^2\zeta^{1+}_\pm, \\
\delta\Psi^2_\pm &= \mp 4\partial^2_\varphi\zeta^{2+}_\pm + \Big(\partial_\varphi\Psi^2_\pm\Upsilon^+ + \tfrac{3}{2}\Psi^2_\pm\partial_\varphi\Upsilon^+\Big) \mp \mathrm{i}\Big(\partial_\varphi\rho\zeta^{2+}_\pm + 2\rho\partial_\varphi\zeta^{2+}_\pm\Big) \pm \mathcal{M}\zeta^{2+}_\pm \\
&\quad \pm \frac{\mathrm{i}}{4}\Psi^2_\pm\rho\Upsilon^+ \pm \frac{1}{2}\lambda_\mathcal{R}\Psi^2_\pm \pm \frac{1}{4}\rho^2\zeta^{2+}_\pm
\end{aligned}
\tag{3.3.9}
$$

We then obtain the variation of the canonical generators corresponding to the asymptotic symmetries of this theory.

$$
\delta\mathcal{C} = -\frac{k}{2\pi}\int\langle\Lambda, \delta\mathcal{A}_\varphi\rangle d\varphi \tag{3.3.10}
$$

For the asymptotic behaviour described here, it is straightforward to verify that this expression becomes linear in the deviation of the fields with respect to the reference background and is given as,

$$
\begin{aligned}
\delta\mathcal{C} = -\frac{k}{4\pi}\int \big[\Upsilon^+\delta\mathcal{N} + \xi^+\delta\mathcal{M} + (\delta\Psi^1_+\zeta^{1+}_- - \delta\Psi^1_-\zeta^{1+}_+) \\
- (\delta\Psi^2_+\zeta^{2+}_- - \delta\Psi^2_-\zeta^{2+}_+) + i(\lambda_\mathcal{R}\delta\phi + \lambda_\mathcal{S}\delta\rho)\big] d\varphi
\end{aligned}
\tag{3.3.11}
$$

where we have used the supertraces suitable for the current basis:

$$
< \mathcal{L}_n, M_m >= \gamma_{nm}, \qquad < q^{1,2\pm}_\alpha, q^{1,2\mp}_\beta >= 2C_{\alpha\beta}, \qquad < \mathcal{R}, \mathcal{S} >= -1 \tag{3.3.12}
$$

We have already noticed that the fields $\mathcal{N}$ and $\mathcal{M}$ are not independent. Similarly, the gauge transformation parameters ξ^+ and Υ^+ are also not independent. The independent components can be written as,

$$
\mathcal{N} = \mathfrak{J}(\varphi) + u\partial_\varphi\mathcal{M}, \quad \xi^+ = T(\varphi) + u\partial_\varphi\Upsilon^+. \tag{3.3.13}
$$

so the above equation 3.3.11 turns into:

$$
\delta\mathcal{C} = -\frac{k}{4\pi}\int \big[\Upsilon^+\delta\mathfrak{J} + T\d{M} + (\delta\Psi^1_+\zeta^{1+}_- - \delta\Psi^1_-\zeta^{1+}_+) - (\delta\Psi^2_+\zeta^{2+}_- - \delta\Psi^2_-\zeta^{2+}_+) + i\lambda_\mathcal{R}\delta\phi + i\lambda_\mathcal{S}\delta\rho\big] d\varphi. \tag{3.3.14}
$$

We can readily read off the charge from the above variation formula as,

$$\mathcal{C} = -\frac{k}{4\pi}\int \big[\Upsilon^+\mathfrak{J}+T\mathcal{M}+(\Psi^1_+\zeta^{1+}_- - \Psi^1_-\zeta^{1+}_+)-(\Psi^2_+\zeta^{2+}_- - \Psi^2_-\zeta^{2+}_+)+i\lambda_{\mathcal{R}}\phi+i\lambda_{\mathcal{S}}\rho\big]d\varphi. \tag{3.3.15}$$

Then we obtain the Poisson brackets among various modes of the fields as:

$$\{\mathcal{C}[_1],\mathcal{C}[_2]\}_{PB} = \delta_{_1}\mathcal{C}[_2], \tag{3.3.16}$$

and the non-zero Poisson brackets are,

$$\begin{aligned}
&\mathrm{i}\{\mathfrak{J}_n,\mathfrak{J}_m\} = (n-m)\mathfrak{J}_{n+m}, \quad \mathrm{i}\{\mathfrak{J}_n,\mathcal{M}_m\} = (n-m)\mathcal{M}_{n+m} + \frac{c_M}{12}n^3\delta_{n+m,0}\\
&\mathrm{i}\{\mathfrak{J}_n,\Psi^{1,2\pm}_r\} = \big(\frac{n}{2}-r\big)\Psi^{1,2\pm}_{r+n} \mp \tfrac{1}{4}[\Psi^{1,2\pm}\mathcal{S}]_{n+r}\\
&\mathrm{i}\{\mathcal{R}_n,\Psi^{1\pm}_r\} = \pm\tfrac{1}{2}\Psi^{1\pm}_{r+n}, \quad \mathrm{i}\{\mathcal{R}_n,\Psi^{2\pm}_r\} = \mp\tfrac{1}{2}\Psi^{2\pm}_{r+n}, \quad \mathrm{i}\{\mathcal{R}_n,\mathcal{S}_m\} = \frac{c_M}{12}n\delta_{n+m,0}\\
&\{\Psi^{1+},\Psi^{1-}\} = \mathcal{M}_{r+s} + (r-s)\mathcal{S}_{r+n} + \tfrac{1}{4}[\mathcal{S}\mathcal{S}]_n + \frac{c_M}{6}r^2\delta_{r+s,0}\\
&\{\Psi^{2+},\Psi^{2-}\} = \mathcal{M}_{r+s} - (r-s)\mathcal{S}_{r+n} + \tfrac{1}{4}[\mathcal{S}\mathcal{S}]_n + \frac{c_M}{6}r^2\delta_{r+s,0},
\end{aligned} \tag{3.3.17}$$

where $c_M = 12k$.

Here the modes are defined as follows:

$$\begin{aligned}
\mathfrak{J}_n &= \frac{k}{4\pi}\int d\varphi e^{in\varphi}\mathfrak{J}, \qquad \mathcal{M}_n = \frac{k}{4\pi}\int d\varphi e^{in\varphi}\mathcal{M},\\
\mathcal{R}_n &= \frac{k}{4\pi}\int d\varphi e^{in\varphi}\phi, \qquad \mathcal{S}_n = \frac{k}{4\pi}\int d\varphi e^{in\varphi}\rho,\\
\psi^{1,2\pm}_r &= \frac{k}{4\pi}\int d\varphi e^{ir\varphi}\psi^{1,2\pm}, \qquad [\psi^{1,2\pm}\mathcal{S}]_r = \frac{k}{4\pi}\int d\varphi e^{ir\varphi}\psi^{1,2\pm}\phi,\\
[\mathcal{S}\mathcal{S}]_\alpha &= \frac{k}{4\pi}\int d\varphi e^{i\alpha\varphi}\phi\phi, \qquad \delta_{n,0} = \frac{1}{2\pi}\int d\varphi e^{in\varphi}.
\end{aligned} \tag{3.3.18}$$

To get the Poisson brackets, we need the inverse relations among the fields and the modes as well. For example, for fields $\mathfrak{J}(\varphi)$ and $\mathcal{M}(\varphi)$, the inverse relations are given by:

$$\mathfrak{J}(\varphi) = \frac{2}{k}\sum_n e^{-in\varphi}\mathfrak{J}_n, \qquad \mathcal{M}(\varphi) = \frac{2}{k}\sum_n e^{-in\varphi}\mathcal{M}_n. \tag{3.3.19}$$

Here we see that in the Poisson bracket $\{\mathfrak{J}_n,\psi^{1,2,\pm}_r\}$, the last term should not have been present, for $\psi^{1,2,\pm}_r$ to transform as a primary field under $\mathfrak{J}_n$. Also, the Poisson bracket $i\{\mathfrak{J}_n, \mathcal{R}_m\}$ is zero. Furthermore, there is a non linear term of the $\mathcal{S}$ generator in the Poisson bracket $\{\Psi^{1,2+}, \Psi^{1,2-}\}$. Hence, at this point the algebra is not quite as expected [16]. The resolution of this problem is based on a simple argument: as we are dealing with a theory with one internal $U(1)$ symmetry, the physical energy-momentum tensor should have a contribution from the corresponding $U(1)$ current. Thus it is necessary to add a Sugawara-like term to $\mathfrak{J}_n$ as follows:

$$\hat{\mathfrak{J}}_n = \mathfrak{J}_n + \tfrac{1}{2}(\mathcal{R}\mathcal{S})_n \, ; \tag{3.3.20}$$

With this modification, all the spurious terms get cancelled or absorbed and the new Poisson brackets read

$$\begin{gathered} i\{\hat{\mathfrak{J}}_n, \psi_r^{1,2\pm}\} = \left(\frac{n}{2} - r\right)\psi_{r+n}^{1,2\pm}, \quad i\{\hat{\mathfrak{J}}_n, \mathcal{R}_m\} = -m\mathcal{R}_{m+n} \\ i\{\hat{\mathfrak{J}}_n, \mathcal{S}_m\} = -m\mathcal{S}_{m+n} \end{gathered} \tag{3.3.21}$$

Finally, we also perform a shift on $\mathcal{M}_n$ as, $\hat{\mathcal{M}}_n = \mathcal{M}_n + \frac{1}{4}[\mathcal{S}\mathcal{S}]_n$. The justification for this shift will be clearlater. It is easy to check that, under this shift, the non-linear term of the $\{\Psi^{1,2+}, \Psi^{1,2-}\}$ Poisson bracket disappears and we also get ,

$$i\{\hat{\mathcal{M}}_n, \mathcal{R}_m\} = -m\,\mathcal{S}_{n+m}.$$

In the next subsection, we shall present the final BMS algebra and as it turns out, we get exact agreement with the one presented in [16].

Finally, we quantize the algebra as follows:

$$i\{,\}_{PB} \to [,] \quad \text{and} \quad \{,\}_{PB} \to \{,\}.$$

The final quantum algebra is:

$$\begin{aligned}
[\hat{\mathfrak{J}}_n, \hat{\mathfrak{J}}_m] &= (n-m)\hat{\mathfrak{J}}_{n+m} + \frac{c_J}{12}\, n^3 \delta_{n+m,0}, \quad [\mathcal{R}_n, \mathcal{R}_m] = \frac{c_J}{12}\, n\, \delta_{n+m,0} \\
[\hat{\mathfrak{J}}_n, \hat{\mathcal{M}}_m] &= (n-m)\hat{\mathcal{M}}_{n+m} + \frac{c_M}{12}\, n^3 \delta_{n+m,0}, \quad [\mathcal{R}_n, \mathcal{S}_m] = \frac{c_M}{12}\, n\, \delta_{n+m,0} \\
[\hat{\mathfrak{J}}_n, \mathcal{R}_m] &= -m\, \mathcal{R}_{n+m}\ , \quad [\hat{\mathfrak{J}}_n, \mathcal{S}_m] = -m\, \mathcal{S}_{n+m}\ , \quad [\hat{\mathcal{M}}_n, \mathcal{R}_m] = -m\, \mathcal{S}_{n+m} \\
[\hat{\mathfrak{J}}_n, \Psi^{1,2\pm}_r] &= \big(\frac{n}{2} - r\big)\Psi^{1,2\pm}_{r+n}, \quad [\mathcal{R}_n, \Psi^{1\pm}_r] = \pm\tfrac{1}{2}\Psi^{1\pm}_{r+n}, \quad [\mathcal{R}_n, \Psi^{2\pm}_r] = \mp\tfrac{1}{2}\Psi^{2\pm}_{r+n}, \\
\{\Psi^{1+}, \Psi^{1-}\} &= \hat{\mathcal{M}}_{r+s} + (r-s)\mathcal{S}_{r+n} + \frac{c_M}{6}\, r^2 \delta_{r+s,0} \\
\{\Psi^{2+}, \Psi^{2-}\} &= \hat{\mathcal{M}}_{r+s} - (r-s)\mathcal{S}_{r+n} + \frac{c_M}{6}\, r^2 \delta_{r+s,0} \qquad (3.3.22)
\end{aligned}$$

This is the most generic quantum extension of the algebra by allowing possible central extension to the $[\hat{\mathfrak{J}}_n, \hat{\mathfrak{J}}_m]$ and $[\mathcal{R}_n, \mathcal{R}_m]$ commutator, fixed by Bianchi identity. We also notice that after the Sugawara shifts, we get exactly the same algebra as presented in [16].

3.4 Energy bound and Killing spinors

We now look for the energy bounds for 3-d $\mathcal{N} = 4$ asymptotically flat supergravity theories.

3.4.1 Susy Energy bound

It is well-known that supersymmetry imposes constraints on the energy of the supersymmetric states. It can be found from the super algebra. In particular, for our case, considering anti-periodic boundary conditions on the fermions[2], we find that the global part of the algebra consists of $(\mathfrak{J}_m, \hat{\mathcal{M}}_m, \Psi^{1,2\pm}_r, \mathcal{R})$, where $m = -1, 0, 1$ and $r = \pm\frac{1}{2}$. Following [24–26], we consider all possible positive-definite combinations of the fermions $\Psi^{1,2\pm}_{\pm 1/2}$:

$$\hat{\mathcal{M}}_0 = \tfrac{1}{4} \sum_{\substack{i=1,2 \\ \alpha=\pm 1/2}} \Psi^{i+}_\alpha\, \Psi^{i-}_{-\alpha} + \Psi^{i-}_{-\alpha}\, \Psi^{i+}_\alpha - \frac{k}{2} \geq -\frac{k}{2} = -\frac{1}{8G} \qquad (3.4.1)$$

[2] we have not studied the periodic boundary conditions on the fermions

Here it is crucial to note that the above bound is satisfied by the shifted generator $\hat{\mathcal{M}}_0$ and not by $\mathcal{M}_0$. This implies that for extended supersymmetric cases, the right physical charge at null infinity corresponds to $\hat{\mathcal{M}}$ [3]. For the Minkowski vacuum, $\hat{\mathcal{M}}_0 = \mathcal{M}_0 = -\frac{1}{8G}$ as all the other fields, including the R-and S-symmetry gauge fields are vanishing, and the bound is saturated. Hence, Minkowski space is obviously a ground state for this theory.

3.4.2 Asymptotic Killing Spinors

We now want to study the asymptotic supersymmetries that preserve the asymptotically flat backgrounds. For this, we impose that both the gravitinos and their generic variations are zero at infinity. This gives the "asymptotic Killing spinor equation". We thus need to solve a simplified version of the equations (3.3.9), i.e.:

$$\partial_\varphi^2 \zeta_\pm^i \mp \frac{\mathrm{i}}{2}\rho\partial_\varphi\zeta_\pm^i - \tfrac{1}{4}(\mathcal{M} + \tfrac{1}{4}\rho^2)\zeta^i = 0 \tag{3.4.2}$$

where $i = 1, 2$ and we assumed $\partial_\varphi \rho = 0$ and $\mathcal{M}$ constant. The general solutions are:

$$\begin{aligned}\zeta_+^i &= e^{-\mathrm{i}\frac{\rho}{4}\varphi}\big(c_1^i\, e^{\frac{\sqrt{\mathcal{M}}}{2}\varphi} + c_2^i\, e^{-\frac{\sqrt{\mathcal{M}}}{2}\varphi}\big)\\ \zeta_-^i &= e^{\mathrm{i}\frac{\rho}{4}\varphi}\big(d_1^i\, e^{\frac{\sqrt{\mathcal{M}}}{2}\varphi} + d_2^i\, e^{-\frac{\sqrt{\mathcal{M}}}{2}\varphi}\big)\end{aligned} \tag{3.4.3}$$

The solutions are well-defined, given the periodicity of φ only when $\mathcal{M} = -n^2$ and $n > 0$, a strictly positive integer without loss of generality.
For $n = 1, \rho = 0$ we find the Killing spinors for the Minkowski vacuum, $\mathcal{M} = -1$. For $n > 1$, the energy bound is violated and we get angular defect solutions [27].

3.4.3 Global Killing vectors

Global killing vectors are the globally defined supersymmetry transformations that leave the pure bosonic solution in the asymptotic region invariant. Depending on the range of the mass parameter, the pure bosonic zero mode solutions include cosmological solutions [28, 29], stationary conical defects solutions [27], the Minkowski spacetime

[3] $\mathfrak{J}$ and $\mathcal{M}$ belongs to the same super multiplet, hence, if one is shifted, so must be the other.

and angular excess solutions of [30, 31]. The global Killing spinor equation is given as,

$$D\zeta^1_\pm = (\mathrm{d} + \pm\tfrac{1}{2}\nu)\zeta^1_\pm = 0 \tag{3.4.4}$$

From the gauge field (3.3.2), we can obtain the values of the spin connection and the R-gauge field:

$$= \tfrac{1}{2}{}^n\,\tilde{\Gamma}_n = \Lambda^{-1}\mathrm{d}\Lambda\,, \qquad \Lambda = \exp\left(\frac{1}{2}\left(\tilde{\Gamma}_{+1} - \frac{\mathcal{M}}{4}\tilde{\Gamma}_{-1}\right)\varphi\right)\,,$$

The general solution of this equation is obtained from the solution of the homogeneous equation ($\nu = 0$):

$$\zeta^1_{\text{hom}} = \Lambda^{-1}\zeta^1_0 = \begin{pmatrix} \cosh\left(\frac{\sqrt{\mathcal{M}}}{2}\varphi\right) & -\frac{\sqrt{\mathcal{M}}}{2}\sinh\left(\frac{\sqrt{\mathcal{M}}}{2}\varphi\right) \\ -\frac{2}{\sqrt{\mathcal{M}}}\sinh\left(\frac{\sqrt{\mathcal{M}}}{2}\varphi\right) & \cosh\left(\frac{\sqrt{\mathcal{M}}}{2}\varphi\right) \end{pmatrix}\zeta^1_0 \tag{3.4.5}$$

with ζ^1_0 constant spinor (here the indices $\pm$ are suppressed). Then we can solve the inhomogeneous equation with non-zero ν :

$$\zeta^{1,2}_{\pm\text{gen}} = \Lambda^{-1}(\zeta^{1,2}_{0\,\pm} + \zeta^{1,2}_\pm(x)) \tag{3.4.6}$$

By explicitly plugging in the above (3.4.4) we get:

$$\mathrm{d}\zeta^{1,2}_\pm(x) = \mp\frac{\mathrm{i}}{2}\,\phi\mathrm{d}\varphi\,(\zeta^{1,2}_0 + \zeta^{1,2}_\pm(x)) \tag{3.4.7}$$

where $\nu = -i\frac{\phi}{2}$ which immediately solves to:

$$\begin{aligned} \partial_r\zeta^{1,2}_\pm &= \partial_u\zeta^{1,2}_\pm = 0 \\ \zeta^{1,2}_\pm(\varphi) &= c\,e^{\mp\mathrm{i}\frac{1}{2}\phi\varphi} - \zeta^{1,2}_{0\,\pm} \end{aligned} \tag{3.4.8}$$

Thus the final solution for the global Killing spinors takes the form :

$$\zeta^{1,2}_{\pm\text{gen}} = \Lambda^{-1}\,c^{1,2}\,e^{\mp\mathrm{i}\frac{1}{2}\phi\varphi} \tag{3.4.9}$$

with $c^{1,2}$ being constant spinor. Like the asymptotic case, the Killing spinors are globally well-defined when $\mathcal{M} = -n^2$, with n being positive integer. More detailed discussions can be found in [14, 23].

3.5 Super BMS$_3$ as a flat Limit of asymptotically super-AdS$_3$ Supergravity

It has been already noticed in the literature that one can obtain the flat asymptotic algebra by an appropriate contraction of two copies of the asymptotic AdS algebras. In [16], we used this method to include all possible supersymmetric extensions of the BMS$_3$ algebras by considering the limit of the two possible combinations for the $R-$ charge generators. One was excluded because the R-generators did not act on the supercharges as they should, so that left us with one well-defined combination of the R-symmetry generators of the two super-Virasoro sectors, which led to a proper $N = 4$ super-BMS$_3$ algebra.Here we have re-derived this algebra as given in 3.3.22 by a direct asymptotic symmetry analysis. This result is in complete agreement with the results of [16] after considering the suitable Sugawara shifts in two generators, as shown in the last section.

3.5.1 Asymptotic symmetry algebra for (2,0) and (0,2) AdS supergravity

There are two inequivalent minimal locally supersymmetric extensions of gravity with negative cosmological constant in three dimensions, which are known as the (2,0) and (0,2) theories. The symmetry algebra for both the theories is shown in in appendix A.5. Here, we shall formulate them as a Chern-Simons theory with appropriate gauge group. The action can be written as a functional of two independent connections $\mathcal{A}_+$ and $\mathcal{A}_-$:

$$I = I[\mathcal{A}_+] + I[\mathcal{A}_-], \tag{3.5.1}$$

where, $I[\mathcal{A}]$ is defined earlier in 3.1.1. Here $x^{\pm} = \frac{u}{l} \pm \varphi$, where, l is the identical AdS radius in both sectors. Thus the (2,0) sector asymptotically only depends on x^+ and the (0,2) sector depends on x^-. We take athe asymptotic gauge fields as:

$$\begin{aligned}
\mathcal{A}_+ &= \Big(L_1 + \frac{r}{l}\,L_0 + \frac{r^2}{4\,l^2}\,L_{-1} - \tfrac{1}{2}\,\mathfrak{L}_+\,L_{-1} - \tfrac{1}{2}\,\psi_+\,\mathcal{Q}^+_- + \tfrac{1}{2}\,\psi_-\,\mathcal{Q}^-_- - i\phi^A_R\,R\Big)\mathrm{d}x^+ + \frac{\mathrm{d}r}{2\,l}L_{-1} \\
\bar{\mathcal{A}}_- &= \Big(\bar{L}_{-1} - \frac{r}{l}\,\bar{L}_0 + \frac{r^2}{4\,l^2}\,\bar{L}_1 - \tfrac{1}{2}\,\mathfrak{L}_-\,\bar{L}_1 - \tfrac{1}{2}\,\bar{\psi}_+\,\bar{\mathcal{Q}}^+_+ + \tfrac{1}{2}\,\bar{\psi}_-\,\bar{\mathcal{Q}}^-_+ - i\bar{\phi}^A_R\,\bar{R}\Big)\mathrm{d}x^- + \frac{\mathrm{d}r}{2\,l}\bar{L}_1.
\end{aligned} \tag{3.5.2}$$

As in the flat case, from the EOM we get the trivial constraints:

$$\partial_- \mathfrak{L}_+ = \partial_- \psi_\pm = \partial_- \phi^A_R = 0, \quad \partial_+ \mathfrak{L}_- = \partial_+ \bar{\psi}_\pm = \partial_+ \bar{\phi}^A_R = 0. \tag{3.5.3}$$

The asymptotic symmetries for these systems are generated by the gauge transformations $\mathcal{A}_\pm = \mathrm{d}\Lambda_\pm + [\mathcal{A}_\pm, \Lambda_\pm]$ for both gauge fields, with the transformation parameters:

$$\begin{aligned}
\Lambda_+ &= \chi^n\, L_n + \epsilon^\alpha_+\, \mathcal{Q}^+_\alpha + \epsilon^\alpha_-\, \mathcal{Q}^-_\alpha + {}^A_R\, R \\
\Lambda_- &= \bar{\chi}^n\, L_n + \bar{\epsilon}^\alpha_+\, \bar{\mathcal{Q}}^+_\alpha + \bar{\epsilon}^\alpha_-\, \bar{\mathcal{Q}}^-_\alpha + \bar{{}^A_R}\, \bar{R}
\end{aligned} \tag{3.5.4}$$

The variation at infinity constraints dependent fields in terms of the independent ones, as well as the variation of various fields appearing in the asymptotic gauge fields. Below, we first present the results in the (2,0) section, where the fields and parameters are only function of x_+:

$$\begin{aligned}
\chi^0 &= -\mathcal{Y}' + \frac{r}{l}\,\mathcal{Y} \\
\chi^- &= \tfrac{1}{2}\,\mathcal{Y}'' - \frac{r}{2\,l}\,\mathcal{Y}' + \big(\frac{r^2}{4\,l^2} - \tfrac{1}{2}\,\mathfrak{L}_+\big)\mathcal{Y} - \tfrac{1}{4}\,(\psi_+\,\epsilon_- - \psi_-\,\epsilon_+) \\
\epsilon^-_+ &= -\epsilon'_+ + \frac{r}{2\,l}\,\epsilon_+ - \tfrac{1}{2}\,\psi_+\,\mathcal{Y} + \tfrac{1}{2}\mathrm{i}\,\phi^A_R\,\epsilon_+ \\
\epsilon^-_- &= -\epsilon'_- + \frac{r}{2\,l}\,\epsilon_- + \tfrac{1}{2}\,\psi_-\,\mathcal{Y} - \tfrac{1}{2}\mathrm{i}\,\phi^A_R\,\epsilon_-
\end{aligned}$$

where we have called $\chi^+ = \mathcal{Y}$, $\epsilon^+_+ = \epsilon_+$ and $\epsilon^+_- = \epsilon_-$. The variations are:

$$\begin{aligned}
\delta\mathfrak{L}_+ &= -\mathcal{Y}''' + 2\,\mathfrak{L}_+\,\mathcal{Y}' + \mathfrak{L}'_+\,\mathcal{Y} + \tfrac{1}{2}\Big(\psi'_+\,\epsilon_- + 3\,\psi_+\,\epsilon'_-\Big) - \tfrac{1}{2}\Big(\psi'_-\,\epsilon_+ + 3\,\psi_-\,\epsilon'_+\Big) \\
&\quad + \tfrac{1}{2}\mathrm{i}\Big(\psi_+\,\epsilon_-\,\phi^A_R + \psi_-\,\epsilon_+\,\phi^A_R\Big) \\
\delta\psi_+ &= 2\,\epsilon''_+ + \psi'_+\,\mathcal{Y} + \tfrac{3}{2}\,\psi_+\,\mathcal{Y}' - \mathrm{i}\Big(\phi^{A\prime}_R\,\epsilon_+ + 2\,\phi^A_R\,\epsilon'_+\Big) - \mathfrak{L}_+\,\epsilon_+ - \tfrac{1}{2}\mathrm{i}\,\psi_+\,\phi^A_R\,\mathcal{Y} - \tfrac{1}{2}\,\lambda^A_R\,\psi_+ \\
&\quad - \tfrac{1}{2}\,\phi^A_R\,\phi^A_R\,\epsilon_+ \\
\delta\psi_- &= -2\,\epsilon''_- + \psi'_-\,\mathcal{Y} + \tfrac{3}{2}\,\psi_-\,\mathcal{Y}' - \mathrm{i}\Big(\phi^{A\prime}_R\,\epsilon_- + 2\,\phi^A_R\,\epsilon'_-\Big) + \mathfrak{L}_+\,\epsilon_- + \tfrac{1}{2}\mathrm{i}\,\psi_-\,\phi_R\,\mathcal{Y} + \tfrac{1}{2}\,\lambda^A_R\,\psi_- \\
&\quad + \tfrac{1}{2}\,\phi^A_R\,\phi^A_R\,\epsilon_- \\
\delta\phi^A_R &= \mathrm{i}\,\lambda^{A\prime}_R - \tfrac{1}{2}\,\mathrm{i}\psi_+\,\epsilon_- - \tfrac{1}{2}\,\mathrm{i}\psi_-\,\epsilon_+
\end{aligned}$$

Now we follow the same procedure as before, The non-zero suoertrace elements are:

$$< L_n, L_m >= \frac{1}{2}\gamma_{nm}, \qquad < \mathcal{Q}^{\pm}_{\alpha}, \mathcal{Q}^{\mp}_{\beta} >= C_{\alpha\beta}, \qquad < R, R >= -\frac{1}{2} \tag{3.5.5}$$

With this we can find out the generic charge and then find out the algebra as before. The non-zero Poisson brackets are:

$$\begin{aligned}
i\{\mathfrak{L}^+_n, \mathfrak{L}^+_m\}_{PB} &= (n-m)\mathfrak{L}^+_{n+m} + \frac{c}{12}n^3\delta_{n+m,0} \\
i\{R_n, R_m\}_{PB} &= \frac{k_l}{2}n\delta_{n+m,0} = \frac{c}{12}n\delta_{n+m,0} \\
i\{\mathfrak{L}^+_n, \psi^{\pm}_{\alpha}\}_{PB} &= \left(\frac{n}{2}-\alpha\right)\psi^{\pm}_{\alpha+n} \mp \frac{1}{2}[\psi^{\pm}R]_{n+\alpha} \\
i\{R_n, \psi^{\pm}_{\alpha}\}_{PB} &= \pm\frac{1}{2}\psi^{\pm}_{\alpha+n} \\
\{\psi^+_{\alpha}, \psi^-_{\beta}\}_{PB} &= \mathfrak{L}^+_{\alpha+\beta} + (\alpha-\beta)R_{\alpha+\beta} + \frac{1}{2}[RR]_{\alpha+\beta} + \frac{c}{6}\alpha^2\delta_{\alpha+\beta,0}
\end{aligned} \tag{3.5.6}$$

where the modes are defined as follows.:

$$\begin{aligned}
\mathfrak{L}_n &= \frac{k_l}{4\pi}\int d\varphi e^{in\varphi}\mathfrak{L}_+, \quad \mathcal{R}_n = \frac{k_l}{4\pi}\int d\varphi e^{in\varphi}\phi^A_R, \\
\psi^{\pm}_{\alpha} &= \frac{k_l}{4\pi}\int d\varphi \psi^{\pm} e^{i\alpha\varphi}, \quad [\psi^{\pm}\mathcal{R}]_{\alpha} = \frac{k_l}{4\pi}\int d\varphi e^{i\alpha\varphi}\psi^{\pm}\phi^A_R, \\
[\mathcal{R}\mathcal{R}]_{\alpha} &= \frac{k_l}{4\pi}\int d\varphi e^{i\alpha\varphi}\phi^A_R\phi^A_R
\end{aligned} \tag{3.5.7}$$

Thus here we also see the same problems as in the flat case: some of the Poisson brackets are pathological. Hence we now need to perform a Sugawara shift:

$$\mathfrak{L}_n \to \hat{\mathfrak{L}}_n = \mathfrak{L}_n + \frac{1}{2}(RR)_n \tag{3.5.8}$$

This gives the right algebra, which, after quantization, looks as:

$$\begin{aligned}
[\hat{\mathfrak{L}}_n, \hat{\mathfrak{L}}_m] &= \frac{c}{12}n^3\delta_{n+m,0} + (n-m)\hat{\mathfrak{L}}_{n+m}, \quad [\mathcal{R}_n, \Psi^{\pm}_{\alpha}] = \pm\frac{1}{2}\Psi^{\pm}_{n+\alpha} \\
\{\Psi^+_{\alpha}, \Psi^-_{\beta}\} &= \hat{\mathfrak{L}}_{\alpha+\beta} + (\alpha-\beta)\mathcal{R}_{\alpha+\beta} + \frac{c}{6}\alpha^2\delta_{\alpha+\beta,0} \\
[\hat{\mathfrak{L}}_n, \mathcal{R}_m] &= -m\mathcal{R}_{n+m}, \quad [\hat{\mathfrak{L}}_n, \Psi^{\pm}_{\alpha}] = \left(\frac{n}{2}-\alpha\right)\Psi^{\pm}_{r+\alpha}, \quad [\mathcal{R}_n, \mathcal{R}_m] = \frac{c}{12}n\delta_{n+m,0}
\end{aligned} \tag{3.5.9}$$

As in the previous case, here the Sugawara shift not only gives the correct algebra for the primary fields with the angular momentum generator, but also absorbs the non-linear term in the fermion anti-commutator.

Similar computation for the $(0,2)$ sector give the results:

$$
\begin{aligned}
\bar{\chi}^0 &= \bar{\mathfrak{Y}}' - \frac{r}{l}\,\mathcal{Y} \\
\bar{\chi}^+ &= \tfrac{1}{2}\,\bar{\mathfrak{Y}}'' - \frac{r}{2\,l}\,\bar{\mathfrak{Y}}' + \big(\frac{r^2}{4\,l^2} - \tfrac{1}{2}\,\mathfrak{L}_-\big)\bar{\mathfrak{Y}} + \tfrac{1}{4}\,(\bar{\psi}_+\,\bar{\epsilon}_- - \bar{\psi}_-\,\bar{\epsilon}_+) \\
\bar{\epsilon}^+_+ &= \bar{\epsilon}'_+ - \frac{r}{2\,l}\,\bar{\epsilon}_+ - \tfrac{1}{2}\,\bar{\psi}_+\,\bar{\mathfrak{Y}} - \tfrac{1}{2}\,\mathrm{i}\bar{\phi}^A_R\,\bar{\epsilon}_+ \\
\bar{\epsilon}^+_- &= \bar{\epsilon}'_- - \frac{r}{2\,l}\,\bar{\epsilon}_- + \tfrac{1}{2}\,\bar{\psi}_-\,\bar{\mathfrak{Y}} + \tfrac{1}{2}\mathrm{i}\,\bar{\phi}^A_R\,\bar{\epsilon}_-
\end{aligned}
$$

where we called $\bar{\chi}^- = \mathcal{Y}$, $\bar{\epsilon}_i^- = \bar{\epsilon}_i$. The variations read:

$$
\begin{aligned}
\delta\mathfrak{L}_- &= -\bar{\mathfrak{Y}}''' + 2\,\mathfrak{L}_-\,\bar{\mathfrak{Y}}' + \mathfrak{L}'_-\,\bar{\mathfrak{Y}} - \tfrac{1}{2}\Big(\bar{\psi}'_+\,\bar{\epsilon}_- + 3\,\bar{\psi}_+\,\bar{\epsilon}'_-\Big) + \tfrac{1}{2}\Big(\bar{\psi}'_-\,\bar{\epsilon}_+ + 3\,\bar{\psi}_-\,\bar{\epsilon}'_+\Big) \\
&\qquad - \tfrac{1}{2}\,\mathrm{i}\Big(\bar{\psi}_+\,\bar{\epsilon}_-\,\bar{\phi}^A_R + \bar{\psi}_-\,\bar{\epsilon}_+\,\bar{\phi}^A_R\Big) \\
\delta\bar{\psi}_+ &= -2\,\bar{\epsilon}''_+ + \bar{\psi}'_+\,\bar{\mathfrak{Y}} + \tfrac{3}{2}\,\bar{\psi}_+\,\bar{\mathfrak{Y}}' + \mathrm{i}\Big(\bar{\phi}^{A\prime}_R\,\bar{\epsilon}_+ + 2\,\bar{\phi}^A_R\,\bar{\epsilon}'_+\Big) + \mathfrak{L}_-\,\bar{\epsilon}_+ - \tfrac{1}{2}\,\mathrm{i}\bar{\psi}_+\,\bar{\phi}^A_R\,\bar{\mathfrak{Y}} - \tfrac{1}{2}\,\bar{\lambda}^A_R\,\bar{\psi}_+ \\
&\qquad + \tfrac{1}{2}\,\bar{\phi}^A_R\,\bar{\phi}^A_R\,\bar{\epsilon}_+ \\
\delta\bar{\psi}_- &= 2\,\bar{\epsilon}''_- + \bar{\psi}'_-\,\bar{\mathfrak{Y}} + \tfrac{3}{2}\,\bar{\psi}_-\,\bar{\mathfrak{Y}}' + \mathrm{i}\Big(\bar{\phi}^{A\prime}_R\,\bar{\epsilon}_- + 2\,\bar{\phi}^A_R\,\bar{\epsilon}'_-\Big) - \mathfrak{L}_-\,\bar{\epsilon}_- + \tfrac{1}{2}\,\mathrm{i}\bar{\psi}_-\,\bar{\phi}^A_R\,\bar{\mathfrak{Y}} + \tfrac{1}{2}\,\bar{\lambda}^A_R\,\bar{\psi}_- \\
&\qquad - \tfrac{1}{2}\,\bar{\phi}^A_R\,\bar{\phi}^A_R\,\bar{\epsilon}_- \\
\delta\bar{\phi}^A_R &= \mathrm{i}\bar{\lambda}^{A\prime}_R + \tfrac{1}{2}\,\mathrm{i}\bar{\psi}_-\bar{\epsilon}_- + \tfrac{1}{2}\,\mathrm{i}\bar{\psi}_-\,\bar{\epsilon}_+
\end{aligned}
$$

The supertrace elements are equal and opposite to the corresponding ones in the unbarred sector. By following a similar procedure, we arrive at the Poisson brackets:

$$
\begin{aligned}
i\{\mathfrak{L}^-_n, \mathfrak{L}^-_m\}_{PB} &= (n-m)\mathfrak{L}^-_{n+m} + \frac{\bar{c}}{12}n^3\delta_{n+m,0} \\
i\{\bar{R}_n, \bar{R}_m\}_{PB} &= \frac{k_l}{2}n\delta_{n+m,0} = \frac{\bar{c}}{12}n\delta_{n+m,0} \\
i\{\mathfrak{L}^-_n, \bar{\psi}^{\pm}_{\alpha}\}_{PB} &= \Big(\frac{n}{2} - \alpha\Big)\,\bar{\psi}^{\pm}_{\alpha+n} \mp \frac{1}{2}[\bar{\psi}^{\pm}\bar{R}]_{n+\alpha} \\
i\{\bar{R}_n, \bar{\psi}^{\pm}_{\alpha}\}_{PB} &= \pm\frac{1}{2}\bar{\psi}^{\pm}_{\alpha+n} \\
\{\bar{\psi}^+_{\alpha}, \bar{\psi}^-_{\beta}\}_{PB} &= \bar{\mathfrak{L}}^+_{\alpha+\beta} + (\alpha-\beta)\bar{R}_{\alpha+\beta} + \frac{1}{2}[\bar{R}\bar{R}]_{\alpha+\beta} + \frac{\bar{c}}{6}\alpha^2\delta_{\alpha+\beta,0}
\end{aligned} \tag{3.5.10}
$$

where the modes are defined as follows.:

$$\bar{\mathfrak{L}}_n = \frac{k_l}{4\pi}\int d\varphi e^{-in\varphi}\mathfrak{L}_+, \quad \bar{\mathcal{R}}_n = \frac{k_l}{4\pi}\int d\varphi e^{-in\varphi}\bar{\phi}^A_R,$$
$$\bar{\psi}^{\pm}_{\alpha} = \frac{k_l}{4\pi}\int d\varphi \bar{\psi}^{\pm} e^{-i\alpha\varphi}, \quad [\bar{\psi}^{\pm}\bar{\mathcal{R}}]_{\alpha} = \frac{k_l}{4\pi}\int d\varphi e^{-i\alpha\varphi}\bar{\psi}^{\pm}\bar{\phi}^A_R,$$
$$[\bar{\mathcal{R}}\bar{\mathcal{R}}]_{\alpha} = \frac{k_l}{4\pi}\int d\varphi e^{-i\alpha\varphi}\bar{\phi}^A_R\bar{\phi}^A_R \tag{3.5.11}$$

It is noteworthy that the definition of Fourier Transform in barred and unbarred sectors come with opposite signs. This is due to the fact that the two sectors depend on $x^+ = \frac{u}{l} + \varphi$ and $x^- = \frac{u}{l} - \varphi$ respectively.
Here also, we need to perform a Sugawara shift as $\bar{\mathfrak{L}}_n \to \hat{\bar{\mathfrak{L}}}_n = \bar{\mathfrak{L}}_n + \frac{1}{2}(\bar{\mathcal{R}}\bar{\mathcal{R}})_n$ to get rid of similar problems as before.

Finally, the asymptotic symmetry algebras for the generators of the barred sector, i.e. of $(0, 2)$ three dimensional AdS theory takes an identical form as the one for $(2, 0)$ three dimensional AdS theories presented in 3.5.9 .

3.5.2 $\mathcal{N} = 4$ super-BMS$_3$ from $\mathcal{N} = (2, 2)$ super-AdS$_3$

Now we shall explicitly show how the correct contraction of the two copies of the AdS gauge field, parameter and algebra reproduce the corresponding quantities of BMS. Before that, let us recall that a In$\ddot{o}n\ddot{u}$-Wigner contraction of two copies of Super-conformal algebra gives us the Super-Poincare algebra. The contraction is defined in the large AdS radius limit $l \to \infty$. The level of the corresponding Chern-Simons actions are related as $k_l = kl$. The generators of the flat algebra are obtained from the AdS ones as,

$$\mathcal{L}_n = L_n - \bar{L}_{-n}, \quad M_n = \frac{L_n + \bar{L}_{-n}}{l}, \quad \mathcal{R} = R - \bar{R}, \quad \mathcal{S} = \frac{R + \bar{R}}{l}$$
$$q^{1\,\pm}_{\alpha} = \sqrt{\frac{2}{l}}\mathcal{Q}^{\pm}_{\alpha}\,, \qquad q^{2\,\pm}_{\alpha} = \sqrt{\frac{2}{l}}\bar{\mathcal{Q}}^{\pm}_{-\alpha}$$

It is easy to check that the asymptotic gauge field and the gauge transformation parameter of the flat theory is obtained from the AdS ones in $l \to \infty$,

$$\mathcal{A} = \mathcal{A}_+ + \mathcal{A}_-, \quad \Lambda = \Lambda_+ + \Lambda_-. \tag{3.5.12}$$

upon the following maps of various fields as,

$$\begin{aligned}\mathcal{M} = \mathfrak{L}_+ + \mathfrak{L}_- \ , \qquad \mathcal{N} = l\,(\mathfrak{L}_+ - \mathfrak{L}_-), \quad \phi = l(\phi^A_R - \bar{\phi}^A_R),\\ \rho = \phi^A_R + \bar{\phi}^A_R, \quad \psi^{A\alpha}_{\pm} = \frac{1}{\sqrt{2l}}\Psi^{1\,\alpha}_{\pm} \ , \qquad \bar{\psi}^{A-\alpha}_{\pm} = \frac{1}{\sqrt{2l}}\Psi^{2\,\alpha}_{\pm}.\end{aligned} \tag{3.5.13}$$

and for the parameters as,

$$\begin{aligned}\epsilon^{\alpha}_{\pm} = \sqrt{\frac{2}{l}}\,\zeta^{1\,\alpha}_{\pm} \ , \qquad \bar{\epsilon}^{-\alpha}_{\pm} = \sqrt{\frac{2}{l}}\,\zeta^{2\,\alpha}_{\pm} \ , \qquad \Upsilon^n = \frac{\chi^n - \bar{\chi}^{-n}}{2},\\ \xi^n = l\frac{\chi^n + \bar{\chi}^{-n}}{2}, \quad \lambda_{\mathcal{R}} = \frac{\lambda^A_R - \bar{\lambda}^A_R}{2} \ , \qquad \lambda_{\mathcal{S}} = l\frac{\lambda^A_R + \bar{\lambda}^A_R}{2}\end{aligned}$$

The constraint relations and the variation of various fields in both theories also follow directly by noticing that,

$$\begin{aligned}\delta\mathcal{A}_{\varphi} = \delta(\mathcal{A}_+ - \mathcal{A}_-)\\ \partial_{\varphi}\Lambda + [\mathcal{A}_{\varphi},\Lambda] = \partial_+\Lambda_+ + [A_+,\Lambda_+] - \big(\partial_-\Lambda_- + [A_-,\Lambda_-]\big)\end{aligned} \tag{3.5.14}$$

where we have used the change of variables as

$$\partial_{\varphi} = \partial_+ - \partial_- \ , \qquad \partial_u = \frac{\partial_+ + \partial_-}{l}. \tag{3.5.15}$$

Therefore we see that given the knowledge of the asymptotic gauge fields and gauge transformation parameters of the two sectors of the AdS theory, one can construct the corresponding ones for the asymptotic flat theory. In a related work [**?**], we have used this relation to find the $N = 8$ Super BMS$_3$ algebra.

Finally, as proposed in [16], the three dimensional $\mathcal{N} = 4$ BMS algebra 3.3.22 can be obtained by two identical copies of asymptotic $(2, 0)$ and $(0, 2)$ AdS$_3$ algebras 3.5.9

with following identification of modes:

$$\begin{aligned}
J_m &= \lim_{\epsilon\to 0}(L_m - \bar{L}_{-m}) \ , & P_m &= \lim_{\epsilon\to 0}\epsilon(L_m + \bar{L}_{-m}) \ , \\
\mathcal{Q}^{1,\pm}_r &= \lim_{\epsilon\to 0}\sqrt{\epsilon}\, Q^{\pm}_r \ , & \mathcal{Q}^{2,\pm}_r &= \lim_{\epsilon\to 0}\sqrt{\epsilon}\, \bar{Q}^{\pm}_{-r} \ , \\
c_1 &= \lim_{\epsilon\to 0}(c - \bar{c}) \ , & c_2 &= \lim_{\epsilon\to 0}\epsilon\,(c + \bar{c}) \ , \\
\mathcal{R}_m &= \lim_{\epsilon\to 0}(R_m - \bar{R}_{-m}) \ , & \mathcal{S}_m &= \lim_{\epsilon\to 0}\epsilon\,(R_m + \bar{R}_{-m}) \ ,
\end{aligned} \tag{3.5.16}$$

where, $\epsilon = \frac{1}{l}$. The above identification follows directly from relation 3.5.13 and definitions of various modes as given in 3.3.18, 3.5.7 and 3.5.11.

We would like to end this section with a comment on the shifts, that we had to perform on the two flat algebra generators $\mathfrak{J}$ and $\mathcal{M}$ to obtain the algebra 3.3.22. While the first one is motivated by the Sugawara shift of the Stress-energy tensor, in presence of a $R-$ symmetry current, the later was proposed to get proper energy bound. But, if we think of the BMS$_3$ algebra as a limit of two copies of AdS$_3$ algebras, then it is obvious. Recall that both sector of AdS$_3$ requires Sugawara shifts and writing these shifts in terms of fields, we have,

$$\hat{\mathfrak{L}}_+ = \mathfrak{L}_+ + \tfrac{1}{2}\phi_A^2 \ , \quad \hat{\mathfrak{L}}_- = \mathfrak{L}_- + \tfrac{1}{2}\bar{\phi}_A^2 \tag{3.5.17}$$

It is easy to check that the BMS$_3$ fields $\mathfrak{J}$ and $\mathcal{M}$, which are the combinations of the $\mathfrak{L}_\pm$ fields, will also take up certain shifts.

$$\mathcal{M} = (\mathfrak{L}_+ + \mathfrak{L}_-) = (\hat{\mathfrak{L}}_+ + \hat{\mathfrak{L}}_-) - \tfrac{1}{2}(\phi_A^2 + \bar{\phi}_A^2) = \hat{\mathcal{M}} - \tfrac{1}{4}(\rho^2 + (\frac{\phi}{l})^2)$$

where we used the definitions of the R- and S-symmetry gauge fields. Taking the limit $l \to \infty$, we finally get the shifts (in terms of the modes) as :

$$\hat{\mathfrak{J}}_n = \mathfrak{J}_n + \tfrac{1}{2}[\mathcal{R}\mathcal{S}]_n \ , \qquad \hat{\mathcal{M}}_n = \mathcal{M}_n + \tfrac{1}{4}[\mathcal{S}\mathcal{S}]_n \tag{3.5.18}$$

This are indeed the correct Sugawara shift for the BMS$_3$ generators that simplify the algebra notably. The most important simplification happens at the level of the anticommutator of the supercharges, as the non-linear term $[\mathcal{S}\mathcal{S}]$ gets immediately absorbed inside $\mathcal{M}$.

CHAPTER 4

Maximally $\mathcal{N}$-extended super-BMS$_3$ algebras and Generalized 3D Gravity Solutions

4.1 Introduction

In the last chapter, we had derived the $\mathcal{N} = 4$ super-BMS$_3$ algebra by asymptotic symmetry analysis. One subtlety of the derivation was the appearance of non-linear terms at intermediate steps, which finally vanished after the necessary Sugawara shifts, thus finally producing a nice linear algebra. However, one important characteristic of the global algebra that we started with was the property that the fermions transformed as $U(1)$ fields under the R-symmetry. In other words, under the R-symmetry, there was no mixing among the different fermions; rather, each transformed into itself, albeit with some scaling factor. In this chapter, we are going to relax this criterion to analyse the more general case, where there can be non-trivial mixing among the different fermions under R-symmetry. In fact, to consider the most general possibility, we will take the fermions to transform under an arbitrary representation, without specifying the structure constants of the algebra. In fact, such a generalization was considered in case of AdS$_3$ by [32] and it had led to a generic superconformal 'algebra' containing non-linear terms. Here we are going to follow the same method for the flat case.

As we will see, this produces the result that the non-linear terms do not generically vanish (except for some very specific case), thus giving rise to an 'algebra' containing non-linear terms in the fermion anti-commutators. This will then obviously affect the different physical results that are derived from the final asymptotic algebra, for example the energy bound. We shall now show these results in details in the present chapter.

4.2 Maximal $\mathcal{N}-$Extended Super-BMS$_3$ algebra with nonlinear extension

Here we present the maximal $\mathcal{N}-$ extended super-BMS$_3$ algebra. The maximally supersymmetric gravity theory that we are considering contains one graviton ${e_\mu}^a$, eight (independent) gravitinos among $\psi^{1,2}_\alpha$ (see below for the range of α), a set of R-symmetry gauge fields ρ^a and a set of internal gauge field $\tilde{\phi}^a$. The theory is invariant under the super-Poincaré algebra:

$$
\begin{aligned}
[J_n, J_m] =&(n-m)J_{n+m}\ , \qquad & [J_n, M_m] &= (n-m)M_{n+m}\ ,\\
[\mathcal{R}^a, \mathcal{R}^b] =&\mathrm{i}\, f^{abc}\mathcal{R}^c\ , & [\mathcal{R}^a, \mathcal{S}^b] &= \mathrm{i}\, f^{abc}\mathcal{S}^c\\
[J_n, r_p^{(1,2),\alpha}] =& \left(\frac{n}{2}-p\right) r_{n+p}^{(1,2),\alpha}, & [\mathcal{S}^a, \mathcal{S}^b] &= [\mathcal{S}^a, r_p^{(1,2),\alpha}] = 0\\
\{r_p^{1,\alpha}, r_q^{1,\beta}\} =&M_{p+q}\eta^{\alpha\beta} - \frac{i}{6\hat{\alpha}}(p-q)(\lambda^a)^{\alpha\beta}S^a_{p+q}, & [\mathcal{R}^a, r_p^{1,\alpha}] &= i(\lambda^a)^\alpha_\beta r_p^{1,\beta},\\
\{r_p^{2,\alpha}, r_p^{2,\beta}\} =&M_{p+q}\eta^{\alpha\beta} + \frac{i}{6\hat{\alpha}}(p-q)(\lambda^a)^{\alpha\beta}S^a_{p+q}, & [\mathcal{R}^a, r_p^{2,\alpha}] &= -i(\lambda^a)^\alpha_\beta r_p^{2,\beta},
\end{aligned}
\tag{4.2.1}
$$

In these equations, J_n, M_n denote the Poincaré generators, m, n run over $(0, 1, -1)$. The fermionic generators $r_p^{1,\alpha}, r_p^{2,\alpha}, \quad p, q = \pm\frac{1}{2}$ transform under a spinor representation R of the internal algebra G, generated by $\mathcal{R}^a$ (which are also R-symmetry generators) and $\mathcal{S}^a$. Generically, we can write the former generators in a representation R as $(\lambda^a)^{\alpha\beta}$, satisfying the same commutation rules, i.e. $[\lambda^a, \lambda^b] = f^{abc}\lambda^c$, where $(\lambda^a)^{\alpha\beta} = -(\lambda^a)^{\beta\alpha}$, f^{abc} are the fully antisymmetric structure constants of the G and the indices $a, b, .. = 1, \dots, D$ while $\alpha, \beta, .. = 1, \dots, d$ with $D = \dim(G)$ and $d = \dim(R_G)$.
The metric $\eta^{\alpha\beta}$ of R can be used to raise and lower spinor indices while the trace of the basis elements can be expressed in terms of the eigenvalue of the second Casimir C_ρ in the representation R. Here $\hat{\alpha} = \frac{C_\rho}{3(d-1)}$ is a constant. This is the maximal $\mathcal{N}-$extended super-Poincaré algebra in 3 dimensions.
In the next section we start from a generic asymptotic gauge field and find the fall-off conditions which are consistent with the maximal $\mathcal{N}-$extended asymptotic symmetry algebra. The required non-zero supertrace elements will have the following form ,

$$
\langle J_m, M_n\rangle = \gamma_{mn}\ , \qquad \langle r_-^\alpha, r_+^\beta\rangle = -\langle r_+^\alpha, r_-^\beta\rangle = 2\eta^{\alpha\beta}\ , \qquad \langle \mathcal{R}^a, \mathcal{S}^b\rangle = \frac{4C_\rho}{d-1}\delta^{ab}.
\tag{4.2.2}
$$

4.2.1 Super-BMS Algebra:

We shall work in the BMS gauge using Eddington-Finkelstein coordinates (u, r, φ). Then the Chern-Simons gauge field can be written in the basis of the global algebra generators as follows:

$$
\begin{aligned}
&\mathcal{A} = \left(M_1 - \frac{1}{4}\mathcal{M}M_{-1} + \frac{1}{24\hat{\alpha}}\rho^a S^a \right) du + \frac{dr}{2}M_{-1} \\
&+ \left(J_1 + rM_0 - \frac{1}{4}\mathcal{M}J_{-1} - \frac{1}{4}\mathcal{N}M_{-1} + \mathfrak{A}\psi^1_\alpha r_1^{-,\alpha} - \bar{\mathfrak{A}}\psi^2_\alpha r^{2,-}_\alpha + \frac{1}{24\hat{\alpha}}\rho^a\mathcal{R}^a + \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\mathcal{S}^a \right) d\varphi \; .
\end{aligned}
\tag{4.2.3}
$$

The various fields $\mathcal{M}, \mathcal{N}, \rho^a, \psi^1_\alpha, \psi^2_\alpha, \tilde{\phi}^a$ depend only on u and φ at null infinity and:

$$
\hat{\alpha} = \frac{C_\rho}{3(d-1)} \; , \qquad \bar{\mathfrak{A}}^2 = \mathfrak{A}^2 = -1/4 \; .
$$

It can be shown easily that the above gauge field encodes the asymptotic flat metric :

$$
\mathrm{d}s^2 = \gamma_{nm}e^n e^m = \mathcal{M}\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + \mathcal{N}\mathrm{d}u\mathrm{d}\varphi + r^2\mathrm{d}\varphi^2, \tag{4.2.4}
$$

where γ_{nm} is the induced metric [1] on this space : $\gamma_{00} = 1, \;\; \gamma_{1,-1} = -2$. It is obvious that the above solution is globally different from Minkowski solution [2].

Finally choosing the gauge: $\mathcal{A} = b^{-1}(a + \mathrm{d})b$ where $b = e^{\frac{r}{2}M_{-1}}$, the components of the gauge field a read:

$$
\begin{aligned}
&a_u = M_1 - \frac{1}{4}\mathcal{M}M_{-1} + \frac{1}{24\hat{\alpha}}\rho^a S^a \; , \\
&a_\varphi = J_1 - \frac{1}{4}\mathcal{M}J_{-1} - \frac{1}{4}\mathcal{N}M_{-1} + \mathfrak{A}\psi^1_\alpha r_1^{-,\alpha} - \bar{\mathfrak{A}}\psi^2_\alpha r_2^{-,\alpha} + \frac{1}{24\hat{\alpha}}\rho^a\mathcal{R}^a + \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\mathcal{S}^a \; .
\end{aligned}
\tag{4.2.5}
$$

Next, we need to compute the gauge variation of this asymptotic field, generated by the

[1] We can calculate the vierbeins as the coefficients of the translation generators :

$$
e^{-1} = -\frac{1}{4}\mathcal{M}\mathrm{d}u - \frac{1}{4}\mathcal{N}\mathrm{d}\varphi + \frac{1}{2}\mathrm{d}r \; , \quad e^0 = r\mathrm{d}\varphi \; , \quad e^1 = \mathrm{d}u \; .
$$

[2] The Minkowski metric in null coordinates is: $\quad ds^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + r^2\mathrm{d}\varphi^2$.

most generic gauge parameter:

$$\Lambda = \zeta^n M_n + \Upsilon^n J_n + \tilde{\lambda}^a_S \mathcal{S}^a + \tilde{\lambda}^a_R \mathcal{R}^a + \zeta^{1,\alpha}_{\pm} r^{1,\alpha}_{\pm} + \zeta^{2,\alpha}_{\pm} r^{2,\alpha}_{\pm} \ , \tag{4.2.6}$$

where $\zeta^n, \Upsilon^n, \tilde{\lambda}^a_S, \tilde{\lambda}^a_R$ are scalar fixed functions of (u, φ) at null infinity [3]

Now to find the algebra, we first need to compute the conserved charges defined in (3.1.12). These can be obtained from the gauge variation equation:

$$\delta a_\varphi = d_\varphi \Lambda + [a_\varphi, \Lambda] \ . \tag{4.2.7}$$

Using the supertraces (4.2.2), we can compute the asymptotic charges $\mathcal{Q}(\lambda)$ of a 3D maximally supersymmetric asymptotically flat solution as,

$$\mathcal{Q}(\lambda) = -\frac{k}{4\pi}\int [\zeta^1 \mathcal{M} + \Upsilon^1 \mathcal{N} + 2\mathfrak{A}\eta^{\alpha\beta}\zeta^1_{+,\alpha}\psi^1_\beta + 2\bar{\mathfrak{A}}\eta^{\alpha\beta}\zeta^2_{+,\alpha}\psi^2_\beta + \tilde{\lambda}^a_R \rho_a + \tilde{\lambda}^a_S \tilde{\phi}_a] \ .$$

Finally we derive the asymptotic algebra by using the relation

$$\{\mathcal{Q}[\lambda_1], \mathcal{Q}[\lambda_2]\}_{PB} = \delta_{\lambda_1} \mathcal{Q}[\lambda_2] \ ,$$

where the variation of the charge follows from (3.1.10). The non-zero Poisson Brackets between the Fourier modes of the charges are:

$$\begin{aligned}
\{\mathfrak{J}_n, \mathfrak{J}_m\} &= \mathrm{i}(n-m)\mathfrak{J}_{n+m}, \qquad \{\mathfrak{J}_n, \mathfrak{M}_m\} = \mathrm{i}(n-m)\mathfrak{M}_{n+m} + \mathrm{i}\frac{c_M}{12}n^3\delta_{n+m,0} \\
\{R^a_n, R^b_m\} &= -f^{abc}R^c_{n+m}, \qquad \{R^a_n, S^b_m\} = \mathrm{i}\, n\hat{\alpha}c_M\delta^{ab}\delta_{n+m,0} - f^{abc}S^c_{n+m} \\
\{\mathfrak{J}_n, \psi^{(1,2),\alpha}_m\} &= \mathrm{i}\left(\frac{n}{2}-m\right)\psi^{(1,2),\alpha}_{n+m} + \frac{k_l}{2k_B}(\lambda^a)^{\beta\alpha}(\psi^{(1,2),\beta}S^a)_{n+m} \\
\{R^a_n, r^{1,\alpha}_p\} &= -(\lambda^a)^\alpha_\beta r^{1,\beta}_{n+p} \ , \qquad \{R^a_n, r^{2,\alpha}_p\} = (\lambda^a)^\alpha_\beta r^{2,\beta}_{n+p} \\
\{\psi^{1,\alpha}_n, \psi^{1,\beta}_m\} &= \frac{c_M}{6}n^2\delta_{n+m,0}\eta^{\alpha\beta} + \mathfrak{M}_{n+m}\eta^{\alpha\beta} - \frac{\mathrm{i}}{6\hat{\alpha}}(n-m)(\lambda^a)^{\alpha\beta}S^a_{n+m} \\
&\quad - \frac{1}{144\hat{\alpha}^2}\{\lambda^a, \lambda^b\}^{\alpha\beta}\frac{1}{4}(S^aS^b)_{n+m} \\
\{\psi^{2,\alpha}_n, \psi^{2,\beta}_m\} &= \frac{c_M}{6}n^2\delta_{n+m,0}\eta^{\alpha\beta} + \mathfrak{M}_{n+m}\eta^{\alpha\beta} + \frac{\mathrm{i}}{6\hat{\alpha}}(n-m)(\lambda^a)^{\alpha\beta}S^a_{n+m} \\
&\quad - \frac{1}{144\hat{\alpha}^2}\{\lambda^a, \lambda^b\}^{\alpha\beta}\frac{1}{4}(S^aS^b)_{n+m} \ .
\end{aligned} \tag{4.2.8}$$

[3]Alternatively, one could derive the correct fall-off conditions for the gauge field and transformation parameter by combining the computation on the two chiral copies of AdS_3 , similarly to what was done in [33]. See appendix A.8.

Here $c_M = 12k = 4/G_{\mathrm{N}}$, $k_l = k\,l$ where l is the AdS radius that needs to be sent to infinity $l \to \infty$. Finally $k_B = \frac{2k_l C_\rho}{d-1}$ and the modes are given by:

$$\begin{aligned}
\mathfrak{J}_n &= \frac{k}{4\pi}\int \mathrm{d}\varphi e^{in\varphi}\mathcal{J}\ , \quad \mathfrak{M}_n = \frac{k}{4\pi}\int \mathrm{d}\varphi e^{in\varphi}\mathcal{M}\ , \quad \psi^{1,\alpha}_n = \frac{k}{4\pi}\int \mathrm{d}\varphi e^{in\varphi}\psi^{1,\alpha}\ , \\
\psi^{2,\alpha}_n &= \frac{k}{4\pi}\int \mathrm{d}\varphi e^{in\varphi}\psi^{2,\alpha}\ , \quad S^a_n = \frac{k}{4\pi}\int \mathrm{d}\varphi e^{in\varphi}\rho^a\ , \qquad R^a_n = \frac{k}{4\pi}\int \mathrm{d}\varphi e^{in\varphi}\tilde{\phi}^a\ .
\end{aligned}$$

Here $\mathfrak{J}_n$ are the modes of the boundary stress tensor and act as spin two generators. Therefore the modes of all the fields should transform with proper weight under $\mathfrak{J}_n$. However, we see the $\{\mathfrak{J}_n, \psi^a_m\}$ Poisson bracket contains an extra non-linear term while the $\{\mathfrak{J}_n, S^a_m\}, \{\mathfrak{J}_n, R^a_m\}$ Poisson brackets are zero. Thus we infer that $\mathfrak{J}_n$ is not the proper mode of the boundary stress tensor. The resolution to this issue is well known. The proper stress tensor modes are obtained by adding quadratic Sugawara-like terms to the modes $\mathfrak{J}_n$. Accordingly, the modes $\mathfrak{M}_n$ also need to be shifted (see [34]). The Sugawara-like shifts read:

$$\mathfrak{J}_n \to \hat{\mathfrak{J}}_n = \mathfrak{J}_n + \frac{1}{24\hat{\alpha}}(R^a S^a)_n\ , \qquad \mathfrak{M}_n \to \hat{\mathfrak{M}}_n = \mathfrak{M}_n + \frac{1}{48\hat{\alpha}}(S^a S^a)_n\ . \quad (4.2.9)$$

The new modes satisfy the following algebra [4],

$$\begin{aligned}
[\hat{\mathfrak{J}}_n, \hat{\mathfrak{J}}_m] &= (n-m)\hat{\mathfrak{J}}_{n+m} + \frac{c_J}{12}n^3\delta_{n+m,0}\ , \quad [\hat{\mathfrak{J}}_n, \hat{\mathfrak{M}}_m] = (n-m)\hat{\mathfrak{M}}_{n+m} + \frac{c_M}{12}n^3\delta_{n+m,0} \\
[\hat{\mathfrak{J}}_n, \psi^{(1,2),\alpha}_m] &= \left(\frac{n}{2} - m\right)\psi^{(1,2),\alpha}_{n+m}\ , \qquad [\hat{\mathfrak{J}}_n, R^a_m] = -mR^a_{n+m}\ , \qquad [\hat{\mathfrak{J}}_n, S^a_m] = -mS^a_{n+m} \\
[R^a_n, R^b_m] &= n\,\hat{\alpha}\,c_R\delta^{ab}\delta_{n+m,0} + if^{abc}R^c_{n+m}\ , \quad [R^a_n, S^b_m] = n\,\hat{\alpha}\,c_M\delta^{ab}\delta_{n+m,0} + if^{abc}S^c_{n+m} \\
[R^a_n, r^{1,\alpha}_p] &= i(\lambda^a)^\alpha_\beta r^{1,\beta}_{n+p}\ , \qquad\qquad [R^a_n, r^{2,\alpha}_p] = -i(\lambda^a)^\alpha_\beta r^{2,\beta}_{n+p} \\
\{\psi^{1,\alpha}_n, \psi^{1,\beta}_m\} &= \frac{c_M}{6}n^2\delta_{n+m,0}\eta^{\alpha\beta} + \hat{\mathfrak{M}}_{n+m}\eta^{\alpha\beta} - \frac{i}{6\hat{\alpha}}(n-m)(\lambda^a)^{\alpha\beta}S^a_{n+m} \\
&\quad - \frac{1}{48\hat{\alpha}}(S^aS^a)_{n+m}\eta^{\alpha\beta} - \frac{1}{144\hat{\alpha}^2}\{\lambda^a, \lambda^b\}^{\alpha\beta}\frac{1}{4}(S^aS^b)_{n+m} \\
\{\psi^{2,\alpha}_n, \psi^{2,\beta}_m\} &= \frac{c_M}{6}n^2\delta_{n+m,0}\eta^{\alpha\beta} + \hat{\mathfrak{M}}_{n+m}\eta^{\alpha\beta} + \frac{i}{6\hat{\alpha}}(n-m)(\lambda^a)^{\alpha\beta}S^a_{n+m} \\
&\quad - \frac{1}{48\hat{\alpha}}(S^aS^a)_{n+m}\eta^{\alpha\beta} - \frac{1}{144\hat{\alpha}^2}\{\lambda^a, \lambda^b\}^{\alpha\beta}\frac{1}{4}(S^aS^b)_{n+m} \qquad (4.2.10)
\end{aligned}$$

with other commutators being zero. This is the most generic quantum maximal $\mathcal{N}$-

[4] We obtain the quantum algebra from the classic Poisson Brackets by using the standard conventions:

$$\{A_n, B_m\}_{PB} = i[A_n, B_m]\ , \quad \{A_n, B_m\}_{PB} = \{A_n, B_m\}\ .$$

extended BMS_3. Here we have introduced two new central terms c_J, c_R in the algebra, that are allowed by Jacobi identity [35]. We also notice that with respect to the modified $\hat{\mathfrak{J}}_n$, all the generators transform appropriately, and the spurious non-linear term in the $[\mathfrak{J}_n, \psi^a_m]$ commutator also vanishes. However extra non-linear terms quadratic in the S^a generators still remain in the anti-commutators (see [32] for the corresponding superconformal algebras). Note that the non-linear terms are a manifestation of the generic choice of representation for the internal symmetries. This is in contrast to the particular case explained below, where for a specific choice of the internal gauge algebra, all the non-linear terms vanish after the Sugawara shifts, and the final asymptotic algebra is linear.

Earlier non-linear extension of the BMS_3 algebra were observed in [36], but in that case they originated by allowing fluctuation in the conformal factor of the boundary metric. In our construction, the boundary metric is always fixed to Minkowski.

Let us end this section with a comment on a special case of $\mathcal{N} = 8$ super-BMS_3 algebra that was studied in [35]. In this case the internal gauge algebra was taken to be $G = SU(2)$ and we chose the fundamental representation F_G, then $(\lambda^a) \sim \sigma^a$ with σ^a Pauli matrices satisfying $\{\sigma^a, \sigma^b\} = 2i\delta^{ab} I$ [5]. It can be seen that for this case, the non-linear terms in the anticommutators cancel (see A.7). This result is consistent with the corresponding superconformal algebra [18], that closes without any non-linear corrections.

4.2.2 BMS Energy Bound

It is well-know that supersymmetry imposes constraints on the energy of supersymmetric states. The bounds are directly obtained from the super algebra. Let us focus only on the NS sector of anti-periodic boundary conditions for the fermions. The global part of the algebra consists of the following generators :

$$(\hat{\mathfrak{J}}_m, \hat{\mathfrak{M}}_m, \psi^{1,\alpha}_{\pm\frac{1}{2}} \psi^{2,\beta}_{\pm\frac{1}{2}}, R^a, S^a), \quad m = 1, 0, 1 \ , \quad \alpha, \beta = 1, \ldots d \ , \quad a = 1, \ldots D \ . \tag{4.2.11}$$

Following [**?**], we consider all possible positive-definite combinations of the super-

[5] σ's are different from λ's, as they are not antisymmetric.

charges

$$\{\psi^{1,\alpha}_{\frac{1}{2}}, \psi^{1,\beta}_{-\frac{1}{2}}\} + \{\psi^{1,\alpha}_{-\frac{1}{2}}, \psi^{1,\beta}_{\frac{1}{2}}\} + \{\psi^{2,\alpha}_{\frac{1}{2}}, \psi^{2,\beta}_{-\frac{1}{2}}\} + \{\psi^{2,\alpha}_{-\frac{1}{2}}, \psi^{2,\beta}_{\frac{1}{2}}\} \geq 0 \,,$$

which explicitly gives:

$$\hat{\mathfrak{M}}_0 \geq -\frac{c_M}{6} + \frac{1}{48\hat{\alpha}}(S^a S^b)_0 \delta_{ab} + \frac{1}{156\alpha^2}\{\lambda^a, \lambda^b\}^{\alpha\beta}\eta_{\alpha\beta}(S^a S^b)_0 \geq -\frac{1}{8G} \,. \tag{4.2.12}$$

As explained in [33], the correct bound is obtained by considering the Sugawara-shifted generators. Note that, due to the non-linear quadratic corrections the energy bound is raised, hence supersymmetric ground states must have a higher energy. The Minkowski vacuum $\hat{\mathfrak{M}}_0 = \mathfrak{M}_0 = -\frac{1}{8G}$, with all other fields vanishing, still saturates the bound. We will use the above bound to constrain the general solutions of 3D supergravity.

4.2.3 Asymptotic Killing Spinors

In order to find fully supersymmetric backgrounds we impose the vanishing of all the fermions and their supersymmetry variations. Among those, the first constraint simply sets the variations of all bosonic fields to zero at null infinity whereas the vanishing of the gravitino variation constitutes the Killing spinor equations, the solutions of which parametrize the fermionic isometries of the background. Since we are interested in (4.2.10) symmetry at null infinity, the only point to appreciate is that, as we have seen in the previous section, we need to perform Sugawara shifts to certain generators to get the correct algebra. With this in hindsight, we begin with a modified gauge field component a_φ, incorporating the Sugawara shifts in the gauge field itself, such that it produce the correct BMS_3 algebra (4.2.10). It takes the following form,

$$\begin{aligned} a_\varphi = J_1 &- \frac{1}{4}\left(\mathcal{M} - \frac{1}{48\alpha}\rho^a\rho^a\right) J_{-1} - \frac{1}{4}\left(\mathcal{N} - \frac{1}{24\alpha}\tilde{\phi}^a\rho^a\right) M_{-1} \\ &+ \mathfrak{A}\psi^1_\alpha r_1^{-,\alpha} - \bar{\mathfrak{A}}\psi^2_\alpha r_2^{-,\alpha} + \frac{1}{24\hat{\alpha}}\rho^a\mathcal{R}^a + \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\mathcal{S}^a \,. \end{aligned} \tag{4.2.13}$$

Thus we now analyze the fermion variations to calculate the asymptotic Killing spinors. For ψ^1_α the variation takes the form:

$$\begin{aligned}
\mathfrak{A}\delta\psi^1_\alpha =& -(\zeta^1_{+,\alpha})'' + \mathfrak{A}\Upsilon^+(\psi_\alpha)' + \frac{3}{2}\mathfrak{A}(\Upsilon^+)'\psi_\alpha + \frac{1}{12\hat{\alpha}}(\lambda^a)^\beta_\alpha\rho^a(\zeta^{+,\beta}_1)' \\
&+ \frac{1}{24\hat{\alpha}}(\lambda^a)^\beta_\alpha(\rho^a)'\zeta^{+,\beta}_1 + \frac{1}{4}\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)\zeta^\alpha_{1,+} - \frac{\mathfrak{A}}{24\alpha}(\lambda^a)^\beta_\alpha\rho^a\Upsilon^+\psi_\beta \\
&+ \mathfrak{A}\psi_\beta(\lambda^a)^\beta_\alpha\lambda^a_R - \frac{1}{8}\frac{1}{144\hat{\alpha}^2}\rho^a\rho^b\{\lambda^a,\lambda^b\}^\delta_\alpha\zeta^1_{+,\delta}\ . \qquad (4.2.14)
\end{aligned}$$

Similar expression holds for $\delta\psi^2_\alpha$. Setting all fermions to zero, the final variation equations for both gravitinos read:

$$\begin{aligned}
\mathfrak{A}\delta\psi^1_\alpha =& (\zeta^1_{+,\alpha})'' - \frac{1}{12\hat{\alpha}}(\lambda^a)^\beta_\alpha\rho^a(\zeta^1_{+,\beta})' - \frac{1}{4}\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)\zeta^1_{+,\alpha} \\
&+ \frac{1}{8}\frac{1}{144\hat{\alpha}^2}\rho^a\rho^b\{\lambda^a,\lambda^b\}^\delta_\alpha\zeta^1_{+,\delta} = 0\ , \\
\bar{\mathfrak{A}}\delta\psi^2_\alpha =& (\zeta^2_{+,\alpha})'' - \frac{1}{12\hat{\alpha}}(\lambda^a)^\beta_\alpha\rho^a(\zeta^2_{+,\beta})' - \frac{1}{4}\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)\zeta^2_{+,\alpha} \qquad (4.2.15) \\
&+ \frac{1}{8}\frac{1}{144\hat{\alpha}^2}\rho^a\rho^b\{\lambda^a,\lambda^b\}^\delta_\alpha\zeta^2_{+,\delta} = 0
\end{aligned}$$

The solutions of the above differential equations are :

$$\begin{aligned}
\zeta^1_{+,\alpha} =& \left(e^{\frac{1}{24\hat{\alpha}}\lambda_a\rho_a\phi}\right)^\beta_\alpha\left[c_{1\beta}\,e^{\frac{\sqrt{\left(\mathcal{M}-\frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)}}{2}\phi} + c_{2\beta}\,e^{-\frac{\sqrt{\left(\mathcal{M}-\frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)}}{2}\phi}\right]\ , \\
\zeta^2_{+,\alpha} =& \left(e^{\frac{1}{24\hat{\alpha}}\lambda_a\rho_a\phi}\right)^\beta_\alpha\left[\tilde{c}_{1\beta}\,e^{\frac{\sqrt{\left(\mathcal{M}-\frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)}}{2}\phi} + \tilde{c}_{2\beta}\,e^{-\frac{\sqrt{\left(\mathcal{M}-\frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)}}{2}\phi}\right]. \qquad (4.2.16)
\end{aligned}$$

Here $c_{i\beta}, \tilde{c}_{i\beta}, (i = 1, 2)$ are constant spinors. The solutions are consistent with the periodicity of ϕ only when $\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a = -n^2$ and n a strictly positive integer and $\lambda_a\rho_a$ is imaginary or zero. These conditions are satisfied for the Minkowski vacuum ($\rho^a = 0, \mathcal{M} = -1$) which is a fully supersymmetric solution. For $n = 0$, the solutions become degenerate and only half the supersymmetries are allowed.

4.3 Generic Bosonic Solutions

Now we shall explore a class of purely bosonic topological 3D gravity solutions, with non-trivial holonomy [31]. As we shall see, these solutions will be cosmological in nature [37, 38]. We shall look for the corresponding bosonic solutions in this theory endowed with maximal $\mathcal{N}$-extended supersymmetry at the null infinity . We shall henceforth restrict our analysis to zero mode solutions, for which all dynamical fields are constants.

Since the asymptotic symmetries are governed by a_φ (4.2.13), we do not modify this field. Also, as we are looking for a pure bosonic solution, we set all fermionic components of the gauge field (4.2.13) to zero. Thus:

$$a_\varphi = J_1 - \frac{1}{4}\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right) J_{-1} - \frac{1}{4}\left(\mathcal{N} - \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\rho^a\right) M_{-1} + \frac{1}{24\hat{\alpha}}\rho^a\mathcal{R}^a + \frac{1}{24\alpha}\tilde{\phi}^a\mathcal{S}^a \ . \tag{4.3.1}$$

We also need to find the gauge transformation parameter Λ that reproduces the right conserved charge corresponding to (4.2.10) via the gauge variation equation (4.2.7). Starting with the most generic gauge parameter (4.2.6) and with a bit of algebra (see appendix A.9 for algebraic details), it can be shown that the required gauge parameter has the following form

$$\begin{aligned}\Lambda =& \xi^1 M_1 + \Upsilon^1 J_1 + \left(\lambda^a_S + \frac{1}{24\hat{\alpha}}\Upsilon^1 R^a\right)\phi^a + \left(\lambda^a_R + \frac{1}{24\hat{\alpha}}\Upsilon^1 R^a - \frac{1}{24\hat{\alpha}}\xi^1 S^a\right)\tau^a \\ & - \frac{1}{4}\Upsilon^1\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}S^aS^a\right)J_{-1} - \frac{1}{4}\left[\Upsilon^1\left(\mathcal{N} - \frac{1}{24\hat{\alpha}}R^aS^a\right) + \xi^1\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}S^aS^a\right)\right]M_{-1}\end{aligned} \tag{4.3.2}$$

while, as stated below (3.1.10), boundary (with respect to the coordinate φ) variations of the various fields in the parameter have already been set to zero.

Now to present a complete stationary circular symmetric bosonic solution of this system endowed with a maximal $\mathcal{N}$-extended asymptotic supersymmetry, we look at the time component a_u of the CS gauge field. First we recall a few relevant points:

- to obtain the generic solution compatible with the asymptotic symmetry, we need to incorporate the chemical potentials into the system [39, 40], which give vacuum expectation value to the time component of the gauge field a_u. These potentials can also be thought of as Lagrange multipliers associated to the dynamical fields of the system defined as the coefficients of the lowest weight components of the symmetry algebra.

- as we have shown in section 3.1.1, the diffeomorphism transformation of gravity is equivalent to the gauge transformation of the CS gauge theory. Thus, the time evolution of the various dynamical components of a_φ is generated by a gauge transformation whose components are now given by the chemical potentials (or Lagrange multipliers). This readily implies [6] that the a_u will have a similar form as (4.3.2),

$$\begin{aligned} a_u =& \mu_M M_1 + \mu_J J_1 + \left(\mu_S^a + \frac{1}{24\hat{\alpha}}\mu_J \rho^a\right)\mathcal{S}^a + \left(\mu_R^a + \frac{1}{24\hat{\alpha}}\mu_J \tilde{\phi}^a + \frac{1}{24\hat{\alpha}}\mu_M \rho^a\right)\mathcal{R}^a \\ -&\frac{1}{4}\mu_J\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)J_{-1} - \frac{1}{4}\left[\mu_J\left(\mathcal{N} - \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\rho^a\right) + \mu_M\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)\right]M_{-1} \ , \end{aligned} \tag{4.3.3}$$

where $\mu_J, \mu_M, \mu_S^a, \mu_R^a$ are the chemical potentials and their boundary variations are taken to zero. We have only turned on the chemical potentials corresponding to bosonic lowest weight generators as we are interested in a pure bosonic solution. This can certainly be generalised to a more generic scenario.

- finally the above solutions have to satisfy appropriate regularity constraints related to the holonomy. In particular, the regularity of the solution requires trivial holomony in presence of a contractible cycle $\mathcal{C}$, i.e.

$$H_{\mathcal{C}} = Pe^{\int_{\mathcal{C}} a_\mu dx^\mu} = \pm I \ . \tag{4.3.4}$$

  For the theory under consideration defined on a 3D manifold $\Sigma \times \mathbb{R}$ we only require the holonomy along time direction to be trivial, i.e. the above condition (4.3.4) must be satisfied for the time component of the gauge field a_u.

Once the holonomy condition (4.3.4) and the energy bound as given in section 4.2.2 are respected, we get a regular solution with required asymptotic falloff properties for

[6] The gauge transformation of a_ϕ by gauge parameter $\Lambda(\mu)$ is :

$$\delta_\mu a_\varphi = d_\varphi \Lambda(\mu) + [a_\phi, \Lambda(\mu)],$$

whereas its time evolution from the equation of motion takes the form:

$$\mathrm{d}_u a_\phi = \mathrm{d}_\phi a_u + [a_\varphi, a_u].$$

These two are identical if $a_u \sim \Lambda(\mu)$.

our system. One last important caveat to notice is that to solve the above holonomy constraint one needs an explicit matrix representation of the symmetry generators, which in general is not known. However, as pointed out in [40, 41], one can exploit the pure-gauge (topological) nature of the solutions to gauge away the components proportional to the supertranslation generators M and internal generators $\mathcal{R}^a$ and $\mathcal{S}^a$, which do not have an explicit matrix representation. [7]. The new component of the gauge field will now depend only on the superrotation generators J (see appendix for their explicit matrix representation) and can be used to impose explicitly the above holonomy condition. To do so, we choose the general gauge group element $g = e^{\lambda_0 M_0}$, which transforms the gauge field component as:

$$\begin{aligned} a_u^g &= g^{-1} a_u g = e^{-\lambda_0 M_0} a_u e^{\lambda_0 M_0} \\ &= a_u + \lambda_0 \mu_J M_1 + \frac{1}{4}\lambda_0 \left[\mu_J \left(\mathcal{M} - \frac{1}{48\hat{\alpha}} \rho^a \rho^a \right) \right] M_{-1} \, , \end{aligned} \tag{4.3.5}$$

where a_u is given as in (4.3.3). Fixing λ_0 and the chemical potential to the values:

$$\lambda_0 = -\frac{\mu_M}{\mu_J} \, , \quad \mu_M = -\frac{\mu_J}{2} \frac{\left(\mathcal{N} - \frac{1}{24\hat{\alpha}} \tilde{\phi}^a \rho^a \right)}{\left(\mathcal{M} - \frac{1}{48\hat{\alpha}} \rho^a \rho^a \right)} \, , \tag{4.3.6}$$

$$\mu_R^a = -\frac{1}{24\hat{\alpha}} \mu_J \tilde{\phi}^a - \frac{1}{24\hat{\alpha}} \mu_M \rho^a \, , \quad \mu_S^a = -\frac{1}{24\hat{\alpha}} \mu_J \rho^a \, , \tag{4.3.7}$$

the time component of the gauge field, now depends only on superrotation generators and hence is representable as a matrix: [8]

$$a_u^g = \mu_J J_1 - \frac{1}{4} \mu_J \left(\mathcal{M} - \frac{1}{48\hat{\alpha}} \rho^a \rho^a . \right) J_{-1} \, , \tag{4.3.8}$$

Finally we can impose the regularity of the solution. Specifically, the gauge field $a_\tau = i a_u^g$ can be diagonalised with eigenvalues

$$\omega = \pm \, i \mu_J \sqrt{\frac{1}{4} \left(\mathcal{M} - \frac{1}{48\hat{\alpha}} \rho^a \rho^a \right)} \, . \tag{4.3.9}$$

Now, in order for this to have a trivial holonomy $\omega = \pm i\pi m$ where $m \in \mathbb{Z}$.

[7] Note that here we are gauging away the generators from the u-component of the gauge field only. The ϕ-cycle is contractible, hence the holonomy condition is trivially satisfied in that direction. Hence the metric will have information about the supertranslations from the a_φ component

[8] Since our initial BMS solution of (4.2.3) does not contain J generators, the holonomy condition is trivially satisfied after above gauge fixing.

This condition fixes the chemical potential μ_J in terms of the fields and an arbitrary integer m to be:

$$|\mu_J| = \frac{2\pi m}{(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a)^{\frac{1}{2}}} \ , \tag{4.3.10}$$

and by the above set of relations (4.3.6) and (4.3.10), all chemical potentials are now fixed in terms of the zero modes of the fields. Thus we obtain the generic 3D bosonic zero mode solution given by (4.3.1) and (4.3.3) in a gravity theory with maximal bulk supersymmetry (4.2.1) and maximal $\mathcal{N}$-extended non-linear asymptotic supersymmetry (4.2.10). Since in our construction we have implicitly assumed $(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a) > 0$, the solution satisfies the energy bound (4.2.12) but there exist no well defined asymptotic Killing spinors (4.2.16). Hence this class of partially gauge fixed solutions are non-supersymmetric and nontrivial only at the boundary. The space time geometry in Bondi coordinates reads:

$$\mathrm{d}s^2 = (M + r^2\mu_J^2)\mathrm{d}u^2 - 2\mu_M \mathrm{d}u\mathrm{d}r + (J + 2r^2\mu_J)\mathrm{d}u\mathrm{d}\varphi + r^2\mathrm{d}\varphi^2 \ , \tag{4.3.11}$$

where,

$$M = \mu_M\left[\mu_J\left(\mathcal{N} - \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\rho^a\right) + \mu_M\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)\right] \ , \quad J = \mu_M\left(\mathcal{N} - \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\rho^a\right) \ . \tag{4.3.12}$$

The chemical potentials appearing in (4.3.11) are fixed as in (4.3.6) and (4.3.10) with $m = 1$ to avoid singularities in space-time. In particular, for $m = 1$, $-\mu_M$ is the inverse Hawking temperature of the space time and μ_J is related to the chemical potential of the angular momentum J of the system. As is clear from (4.3.12), for static configurations with $\mathcal{N} = 0$, the system can have non-zero angular moment due to the presence of the internal gauge fields, a feature that was also observed in [42].

4.3.1 Thermodynamics of the Solution

So far we have presented the space time metric (4.3.11) in the usual Bondi coordinates. In these coordinates, the space time does not have any singularity. To understand the geometry better, following [43], let us rewrite the metric in Schwarzchild-like (ADM) coordinates as,

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \mu_M^2 N^{-2}\mathrm{d}r^2 + r^2(\mathrm{d}\vartheta + N^\vartheta dt)^2 \tag{4.3.13}$$

where we define new coordinates as $t = u - f(r)$ and $\vartheta = \phi - g(r)$ and

$$N^2 = \frac{\tilde{A}^2}{4r^2} - \tilde{B}\ , \qquad N^\vartheta = \frac{\tilde{A}}{2r^2}\ .$$

Here, with (4.3.12) we use compact notations $\tilde{A}$ as the coefficient of $\mathrm{d}u\mathrm{d}\varphi$ and $\tilde{B}$ as the coefficient of $\mathrm{d}u^2$ in the above metric (4.3.11):

$$\tilde{A} = J + 2r^2\mu_J\ , \qquad \tilde{B} = M + r^2\mu_J^2\ . \tag{4.3.14}$$

Let us consider $\big(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\big) \geq 0$, hence a solution satisfying the energy bound (4.2.12). Under this condition (4.3.13) represents a cosmological spacetime . In (t, r, ϑ) coordinate, the function N^2 vanishes at the hypersurface $r = r_c$, $(N^2)_{r=r_c} = 0$. This hypersurface is in fact a cosmological horizon and requiring $r_c > 0$ gives:

$$r_c = \frac{1}{2}\frac{|\mathcal{N} - \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\rho^a|}{\big(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\big)^{\frac{1}{2}}}\ . \tag{4.3.15}$$

To understand the nature of the horizon r_c, we write the above metric in a different coordinate system. For the region of the space time where $r > r_c$, let us define new coordinates (T, X, ϑ) as,

$$T^2 = \frac{r^2 - r_c^2}{\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a}\ , \qquad X = \vartheta + \mu_J t\ . \tag{4.3.16}$$

Similarly for the other region $r < r_c$, we define $(\hat{T}, X, \vartheta)$:

$$\hat{T}^2 = \frac{r_c^2 - r^2}{\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a}\ , \qquad X = \vartheta + \mu_J t\ . \tag{4.3.17}$$

In these coordinates, the space time metric is given by:

$$\begin{aligned}
\mathrm{d}s^2 &= -\mathrm{d}T^2 + \big(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\big)T^2\mathrm{d}X^2 + r_c^2\mathrm{d}\vartheta^2\ , \quad r > r_c \\
&= \mathrm{d}\hat{T}^2 - \big(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\big)\hat{T}^2\mathrm{d}X^2 + r_c^2\mathrm{d}\vartheta^2\ , \quad r < r_c.
\end{aligned} \tag{4.3.18}$$

Thus in the outer region $r > r_c$, we have a cosmological space time of topology $R \times S^1 \times S^1$, a solid torus. Both S^1 factors have periodicity 2π, the radius of the ϑ circle is fixed to r_c, while the radius of the X circle is T dependent. It is also clear that, in the outer region we have closed space-like geodesics whereas in the inner region we can have closed time-like geodesics, as X is a time-like coordinate in the interior. Thus, we readily conclude that $r = r_c$ is a Cauchy horizon [31]. To avoid closed time-like curves, we cut the space-time at $r = r_c$. It can also be checked that $r = r_c$ is also a Killing horizon. Finally, we can compute the Bekenstein-Hawking entropy associated with this class of Cauchy horizons:

$$S = \frac{2\pi r_c}{4G} = \frac{2\pi}{4G}\frac{1}{2}\frac{|\mathcal{N} - \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\rho^a|}{\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)^{\frac{1}{2}}} = \frac{\pi}{4G}\frac{|\mathcal{N} - \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\rho^a|}{\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)^{\frac{1}{2}}} \,. \tag{4.3.19}$$

As expected, the entropy of the system is completely determined by the zero mode solution. Alternatively, the entropy can be found using the Chern-Simons gauge field:

$$\begin{aligned} S &= \frac{k}{2\pi}\int d\varphi \langle a_u, a_\varphi \rangle \\ &= k\left[\mu_J\mathcal{N} + \mu_M\mathcal{M} + \frac{1}{2}\tilde{\phi}^a\mu_S^a + \frac{1}{2}\rho^a\mu_R^a\right] \end{aligned} \tag{4.3.20}$$

and plugging in the expressions (4.3.6), (4.3.10) for the chemical potentials, the entropy reduces to:

$$S = k\pi m\frac{|\mathcal{N} - \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\rho_a|}{\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)^{\frac{1}{2}}} \,, \tag{4.3.21}$$

which matches with (4.3.19) for $m = 1$. The choice of $m = 1$ sector is obvious, as only this sector is connected to the standard cosmological space time (4.3.11).

4.4 Discussion and Outlook

With this chapter we completed the detailed analysis of fall-off conditions necessary to obtain *all* the supersymmetric extensions of the BMS algebras, presented in [35]. In the maximal $\mathcal{N}$-extended super-BMS$_3$ case analyzed here we find non-linearity in the asymptotic algebra and modifications to the energy bounds for asymptotic states. Unlike $\mathcal{N} = 4, 8$ super-BMS$_3$ studied respectively in [33] and appendix A.7 of this paper, the non-linearity does not disappear after Sugawara-shifting the energy-momentum generators[9] . Furthermore, we have shown that circular symmetric solutions that are flat cosmologies, satisfying $\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right) > 0$, are not supersymmetric. Similar results hold for abelian R-symmetry algebra as discussed in [42]. There are three other distinct kinds of solutions [31] that would appear for different conditions on the fields as presented below :

$a)$ $\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right) = 0$: this class corresponds to *null orbifold* solutions [44]. Here the asymptotic Killing spinors (4.2.16) are degenerate and only half of them are independent. Hence this class of solution is only asymptotically half supersymmetric.

$b) -\frac{1}{8G} < \left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right) < 0$: *conical defect* solutions [30, 45], satisfying the energy bound and asymptotically full supersymmetric.

$c)$ $\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right) < -\frac{1}{8G}$: *conical surplus* solutions that do not satisfy the energy bound.

These solutions may not be interesting from a cosmology perspective but are nevertheless non-trivial configurations of 3D gravity. Detailed discussions on the thermodynamics of their R-symmetry-abelian counterparts can be found in [42] and references therein. For the non-abelian R-symmetry cases studied in this chapter, most of the physics will be similar and hence we do not present the details here.

Let us end this chapter with some comments on the outlook. It is known that 3D gravity solutions with non-trivial topology correspond to the stress-energy tensors of a two dimensional theory. It comes from the relation between a Chern-Simons theory with a boundary and an associated chiral Wess-Zumino-Witten model [46–48]. As we have already seen, the non-trivial boundary for the Chern-Simons theory (in our case the torus) comes from generic asymptotic fall off conditions on the gauge fields. It has been shown in [49] for ordinary BMS_3 and in [15] for $\mathcal{N} = 1$ super-BMS_3 that one needs to add a suitable boundary term to the action for the variation principle to go through. The fall off conditions also provide extra constraints to the Wess-Zumino-Witten model. Finding a similar two dimensional description for the $\mathcal{N}$-extended super-BMS_3 obtained in this chapter would provide a complete set of such $2-$dimensional theories that will be dual to 3D asymptotically flat supergravity theories.

The second point is more generic and is related to the issue of understanding the

[9]For anti-periodic boundary conditions of fermionic generators, the non-linearity in energy bound as reported in [42] also disappears after proper modification of generators, as shown in [33].

implications of these infinite dimensional 3-dimensional asymptotic symmetries on the dynamics of the corresponding two dimensional theory. As in 4-dimensional gravity, we know [50–52] that the Ward identities of BMS_4 symmetries are related to bulk gravitational soft theorems. Interestingly, it has been very recently noticed by Barnich [53] that in 4-dimensions there are also boundary degrees of freedom and they are highly constrained by BMS_4. In fact it has been proposed that the classical contribution to the Bekenstein-Hawking entropy comes from these degrees of freedom. In the 3-dimensional case, there is no bulk graviton and hence we do not have a notion of soft theorem but the boundary theory and boundary degrees of freedom do exist. It would be interesting to study the of BMS_3 symmetry on their counting. Although the above issue is not directly related to study of this chapter, but having (maximal)supersymmetry in the theory is technically helpful in counting the corresponding degrees of freedom.

CHAPTER 5

Free-field Realizations of the BMS_3 Algebras and its Extensions:

5.1 Introduction:

Gauge-gravity duality or holography is a well-known concept in physics. Its first concrete realisation is the AdS-CFT correspondence, which taught us to build a dictionary relating a theory of gravity with its dual non-gravitational theory. In particular, it was realised that the asymptotic symmetry algebra in a gravity theory on a manifold with boundary gives the symmetry of the dual non-gravitational theory on that boundary. It is then natural to ask if this correspondence also holds beyond the AdS case, that is, for the dS or flat case.

It has been known in the literature that the asymptotic symmetry algebra on $\mathcal{I}^+$ for gravity theories with flat spacetime asymptotics is given by the Bondi-van der Burg-Metzner Sachs (BMS) algebra. It seems sensible to ask if this algebra encodes information about bulk gravity with flat space asymptotics. There are many interesting works on this [4, 7–9, 11, 12, 54]. One interesting outcome of this in $2+1$-dimensions is that the relevant algebra, known as BMS_3 algebra, is known to be an extension of a single Virasoro algebra by an additional chiral spin-2 generator, along with two independent central charges.

Another related question is whether there are concrete realisations of the BMS algebra in terms of quantum fields. Besides the motivation of flat holography, this could also be interesting from the point of view of conformal field theory. This is particularly exciting in the case of $2+1$-dimensions, since the BMS_3 algebra has a Virasoro algebra as its subalgebra, thus one may expect to utilise the language of CFT to find classes of realisations. Moreover, three dimensional gravity is relatively simple in the bulk and two dimensional field theories (conformal or otherwise) have been studied extensively. So it seems a nice starting point to try to understand flat holography.

In this chapter, we give an explicit realization of the BMS_3 algebra with non vanishing central charges using holomorphic free fields. By adding chiral matter, we can extend this to a realisation having arbitrary values for the two independent central charges. By introducing additional free fields, we then extend our construction to the supersymmetric BMS_3 algebras as well as the non-linear higher-spin BMS_3-W_3 algebra. We also describe an extended system that realises both the $\mathrm{SU}(2)$ current algebra and the BMS_3 via the Wakimoto representation. However, in this case, in order to introduce a central extension, new non-central operators get introduced.

5.2 Free-Field Realization of Pure BMS_3:

Let us start with the pure BMS_3 algebra. We want to find an infinite dimensional Fock space representation of the aboce algebra. We introduce a holomorphic coordinate z (not to be literally taken as the coordinate of an underlying space or spacetime). Let us construct the canonical fields:

$$\begin{aligned} T(z) &= \sum_{n\in\mathbb{Z}} \mathfrak{J}_n z^{-n-2} \\ \mathcal{M}(z) &= \sum_{n\in\mathbb{Z}} \mathcal{M}_n z^{-n-2} \end{aligned} \tag{5.2.1}$$

We are now going to use holomorphic free fields to construct the two fields $T(z), \mathcal{M}(z)$. This is only a techinical device. In principle, we could keep working with the modes $\mathfrak{J}_n, \mathcal{M}_n$ and construct them in terms of the infinitely many pairs of modes satisfying the canonical commutation relations. However, it is easier to use the holomorphic approach and find the realisation in terms of the fields constructed out of these infinite set of modes [55]. The algebra in terms of modes can be written as an operator product expansion in terms of the fields:

$$\begin{aligned} T(z)T(w) &\sim \frac{1}{2}\frac{c_1}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} \\ T(z)\mathcal{M}(w) &\sim \frac{1}{2}\frac{c_2}{(z-w)^4} + \frac{2\mathcal{M}(w)}{(z-w)^2} + \frac{\partial \mathcal{M}(w)}{z-w} \\ \mathcal{M}(z)\mathcal{M}(w) &\sim 0 \end{aligned} \tag{5.2.2}$$

where, as per convention, we write only the singular terms on the right hand side. The field T gives rise to a Virasoro sub-algebra. The OPE of $\mathcal{M}$ with T tells us that $\mathcal{M}$ is a field of dimension 2 under the Virasoro algebra. However, it is not a primary field as the OPE contains a central term.

In order to give a free field representation of this algebra, we start with a bosonic $\beta - \gamma$ system satisfying the operator product expansion:

$$\gamma(z)\beta(w) \sim \frac{1}{z-w} \tag{5.2.3}$$

Such a system has played important roles in many areas of string theory and conformal field theory [56]. The conformal dimensions of β, γ are taken as $(p, 1-p)$ for any integer p. For this, one starts with a basic pair of dimensions $(1, 0)$ and then twists the energy-momentum tensor suitably by adding derivatives of the ghost-number current $:\beta\gamma:$. For our purposes, we will work with (β, γ) having dimensions $(2, -1)$. The energy-momentum tensor twisted to achieve this turns out to be

$$T_{\beta,\gamma} = -2 : \beta\partial\gamma : - : \gamma\partial\beta : \tag{5.2.4}$$

As expected, the OPE of this twisted energy-momentum tensor with the β and γ fields are given by:

$$\begin{aligned} T_{\beta,\gamma}(z)\beta(w) &\sim \frac{2\beta(w)}{(z-w)^2} + \frac{\partial\beta(w)}{z-w} \\ T_{\beta,\gamma}(z)\gamma(w) &\sim \frac{-\gamma(w)}{(z-w)^2} + \frac{\partial\gamma(w)}{z-w} \end{aligned} \tag{5.2.5}$$

From the OPE of $T_{\beta,\gamma}$ with itself, its central charge is calculated to be 26.

Now we note that the pair of spin-2 fields $(T_{\beta,\gamma}(z), \beta(z))$ generate an algebra close to the BMS$_3$ algebra if we identify these with the BMS$_3$ generators $(T(z), \mathcal{M}(z))$. This is because the OPE of $T_{\beta,\gamma}$ with itself and with β forms a BMS$_3$ algebra with $c_1 = 26, c_2 = 0$. However a generic BMS$_3$ requires non-vanishing c_2. This does not arise here because β is primary with respect to our chosen $T_{\beta,\gamma}$.

To get rid of this problem and introduce a non-vanishing central charge c_2, we now twist the energy-momentum tensor:

$$T(z) = T_{\beta,\gamma} - a\partial^3\gamma \tag{5.2.6}$$

where a is an arbitrary constant. As we discussed in the introduction, this twist is not of the form $T(z) \to T(z) + \frac{1}{2}\partial J(z)$ for a primary current $J(z)$. In this case, $J(z)$ would be proportional to $\partial^2\gamma$, which, being the descendant of a primary field γ, is definitely not primary. As a consequence, it is apriori not evident that the above twist preserves the Virasoro algebra. What is obvious, though, is that it will induce a fourth-order pole, which is the central term, in the $T(z)\beta(w)$ OPE.

It is important to note here that the OPE of $T(z)$ with itself gives rise to poles upto the fifth order at intermediate stages via the cross-terms between $T_{\beta,\gamma}$ and $\partial^3\gamma$.However, if the twisted energy-momentum tensor $T(z)$ still has to satisfy the Virasoro algebra, the third and fifth order poles must vanish completely, while the second and first order poles should depend only on $T(z)$ as a whole, and not separately on $\partial^3\gamma$. However, none of these requirements can be manipulated by a choice of the coefficient a, as all the cross-terms are proportional to a. All these imply that the system is quite overdetermined. Surprisingly, when one performs the actual calculation, it is found that all the unwanted terms indeed cancel and the structure of the OPE is preserved!

$$T(z)T(w) \sim \frac{1}{2}\frac{26}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} \tag{5.2.7}$$

The twist $\partial^3\gamma$ that we have added has changed neither the OPE structure, nor the value of the central charge! It has just modified the right hand side so that the final expression is now expressed in terms of T.

Now choosing $\mathcal{M}(z) = \beta(z)$, we find that the OPE of β with T is modified due to the introduction of the twist, giving the result:

$$T(z)\mathcal{M}(w) \sim \frac{1}{2}\frac{12a}{(z-w)^4} + \frac{2\mathcal{M}(w)}{(z-w)^2} + \frac{\partial\mathcal{M}(w)}{z-w} \tag{5.2.8}$$

Also, due to the first-order nature of the ghost system, we have:

$$\mathcal{M}(z)\mathcal{M}(w) \sim 0 \tag{5.2.9}$$

Combining the above results, one can realise that together $T(z)$ and $\mathcal{M}(z)$ define a BMS_3 algebra with central charges $c_1 = 26$ and $c_2 = 12a$. Note the freedom in the choice of c_2 to any non-zero value by tuning the value of the coefficient a. This freedom is a manifestation of the fact that within the BMS_3 algebra, the central charge c_2 can be changed by scaling $\mathcal{M}$.

This construction gives us a fixed value of the central charge c_1 namely 26. However, we may couple to this sysytem any chiral conformal field theory, having energy-momentum tensor T_{matter} with central charge c_0. Then the total energy-momentum tensor will be

$$T(z) = T_{\text{mattter}} + T_{\beta,\gamma} - a\partial^3\gamma \tag{5.2.10}$$

which will result in the total central charge $c_1 = c_0 + 26$ and no change in the structure of any OPE. So to summarise, starting with a (β, γ) system of spin $(2, 1)$, and adding necessary twists, we have constructed an explicit realisation of the BMS_3 algebra having completely arbitrary central charges.

5.3 Free-Field Realization of super- BMS_3:

Here we consider the minimal supersymmetric generalisation of the BMS_3 algebra. We have given the algebra before. In addition to the $T(z)$ and $\mathcal{M}(z)$ fields, this now contains the additional chiral field $\Psi(z) = \sum_r \Psi_r z^{-r-\frac{3}{2}}$. The algebra can be written in terms of operator product expansion as:

$$
\begin{aligned}
T(z)T(w) &\sim \frac{1}{2}\frac{c_1}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} \\
T(z)\mathcal{M}(w) &\sim \frac{1}{2}\frac{c_2}{(z-w)^4} + \frac{2\mathcal{M}(w)}{(z-w)^2} + \frac{\partial \mathcal{M}(w)}{z-w} \\
T(z)\Psi(w) &\sim \frac{\frac{3}{2}\Psi(w)}{(z-w)^2} + \frac{\partial\Psi(w)}{z-w} \\
\Psi(z)\Psi(w) &\sim \frac{1}{3}\frac{c_2}{(z-w)^3} + \frac{\mathcal{M}(w)}{z-w}
\end{aligned}
\tag{5.3.1}
$$

where the remaining OPE s are non-singular.

We now want to write a free-field realization of this algebra. To do this, we supplement the β-γ system of the previous section by a Grassmann-odd $\mathfrak{b}-\mathfrak{c}$ ghost system of spins $\left(\frac{3}{2}, -\frac{1}{2}\right)$. These have the OPEs:

$$
\mathfrak{b}(z)\mathfrak{c}(w) \sim \frac{1}{z-w}, \qquad \mathfrak{b}(z)\mathfrak{b}(w) \sim 0, \qquad \mathfrak{c}(z)\mathfrak{c}(w) \sim 0
$$

This system can be shown to have central charge -15. Now we choose $T(z)$ to be the canonical energy-momentum tensor for the fields (β, γ) and $(\mathfrak{b}, \mathfrak{c})$ of dimensions $(2, -1)$ and $\left(\frac{3}{2}, -\frac{1}{2}\right)$ respectively. As before, we twist the energy-momentum tensor by $-a\partial^3\gamma$ and choose $\mathcal{M}(z) = \beta(z)$. We thus obtain the bosonic part of the super-BMS$_3$ algebra with the central charges $c_1 = 26 - 15 = 11$ and $c_2 = 12a$.

Now our aim is to represent the supersymmetry generator $\Psi(z)$. As this has dimension $\frac{3}{2}$, a natural guess is to realise it as $\mathfrak{b}$. However, it would then produce the OPE $\Psi(z)\Psi(w) \sim 0$, which is not what we want here. So we try to add terms of the same dimension namely $\frac{3}{2}$ to $\mathfrak{b}$ so as to produce both $\mathcal{M}(z)$ and a central term on the RHS of $\Psi(z)\Psi(w)$. Now since we have chosen $\mathcal{M}(z) = \beta(z)$, the first requirement is achieved by adding a term of the form $\beta\mathfrak{c}$ in Ψ. For the central term, we have to add a term proportional to $\partial^2\mathfrak{c}$. These give rise to the required terms in the OPE, however, we again have the problem of getting many additional terms from the square of the individual terms in Ψ as well as from the cross terms. What is non-trivial here is that again all these extra terms vanish! There is no contribution to the OPE from the square of the individual terms, as each term contains only one of a pair of canonically conjugate variables, hence OPE of each such term with itself is zero. Similarly the additional cross term vanishes as well. Now we can easily adjust the coefficients of the terms in Ψ to give the correct $\Psi(z)\Psi(w)$ OPE:

$$\Psi(z) = \mathfrak{b}(z) + \frac{1}{2} : \beta\mathfrak{c} : (z) + a\partial^2\mathfrak{c}(z) \tag{5.3.2}$$

This, along with the generators $T(z)$ and $\mathcal{M}(z)$ of the three-dimensional BMS algebra constitute the generators of the super-BMS$_3$ algebra:

$$\begin{aligned} T(z) &= -\frac{3}{2} : \mathfrak{b}\partial\mathfrak{c} : (z) + \frac{1}{2} : \mathfrak{c}\partial\mathfrak{b} : (z) - 2 : \beta\partial\gamma : (z) - : \gamma\partial\beta : (z) - a\partial^3\gamma(z) \\ \mathcal{M}(z) &= \beta(z) \\ \Psi(z) &= \mathfrak{b}(z) + \frac{1}{2} : \beta\mathfrak{c} : (z) + a\partial^2\mathfrak{c}(z) \end{aligned} \tag{5.3.3}$$

However, now we have a potential problem: the OPE's of $\Psi(z)$ with $T(z)$ and $\mathcal{M}(z)$ have not yet been checked, and there is no longer any freedom to adjust any of the generators.

The OPE of $T(z)$ with $\Psi(w)$ must produce the correct poles so that Ψ is a primary field. The new terms added can potentially disrupt the structure of the OPE, however, it so happens that the non-primary contributions exactly cancel, resulting in the desired OPEs:

$$\begin{aligned} T(z)T(w) &\sim \frac{\frac{15}{2}}{(z-w)^4} + \frac{2T(z)}{(z-w)^2} + \frac{\partial T(w)}{z-w} \\ T(z)\mathcal{M}(w) &\sim \frac{6a}{(z-w)^4} + \frac{2\mathcal{M}(z)}{(z-w)^2} + \frac{\partial \mathcal{M}(w)}{z-w} \\ T(z)\Psi(w) &\sim \frac{\frac{3}{2}\Psi(w)}{(z-w)^2} + \frac{\partial\Psi(w)}{z-w} \\ \Psi(z)\Psi(w) &\sim \frac{4a}{(z-w)^3} + \frac{\mathcal{M}(w)}{z-w} \end{aligned} \tag{5.3.4}$$

Finally the OPE of $\mathcal{M}(z)$ with $\Psi(w)$ is zero as Ψ is independent of γ. Thus we have given a free-field realisation of the super-BMS$_3$ algebra with the central charges $c_1 = 15$ and $c_2 = 12a$.

We can couple this system to a bosonic CFT of chiral matter with central charge c_0 to change $T(z)$, while $\mathcal{M}(z)$ and $\Psi(z)$ remain fixed. Then the first central charge of the super-BMS$_3$ algebra has an arbitrary value $c_1 = c_0 + 15$, while the second one c_2 is proportional to a free parameter a and is therefore arbitrary.

5.4 Free-Field Realization of $\mathcal{N}=2$ super- BMS$_3$:

Here again we start with a bosonic $\beta-\gamma$ system of dimensions $(2,-1)$ and introduce two independent pairs of Grassmann-odd $\mathfrak{b}-\mathfrak{c}$ ghost systems of dimensions $\left(\frac{3}{2},-\frac{1}{2}\right)$. The fermionic ghost fields satisfy the OPE:

$$\mathfrak{b}^a(z)\mathfrak{c}^b(w) \sim \frac{\delta^{ab}}{z-w} \tag{5.4.1}$$

The fields $T(z)$ and $\mathcal{M}(z)$ are defined as before. The fermion fields are:

$$\Psi^a(z) = \sum_r \Psi^a_r z^{-r-\frac{3}{2}} \tag{5.4.2}$$

Ψ satisfies the OPEs:

$$\begin{aligned} T(z)\Psi^a(w) &\sim \frac{\frac{3}{2}\Psi^a(w)}{(z-w)^2} + \frac{\partial\Psi^a(w)}{z-w} \\ \Psi^a(z)\Psi^b(w) &\sim \frac{1}{3}\frac{c_2}{(z-w)^3}\delta^{ab} + \frac{\mathcal{M}(w)}{z-w}\delta^{ab} \end{aligned} \tag{5.4.3}$$

Now we choose the $\mathcal{N}=2$ super-BMS$_3$ generators in terms of the ghost fields as:

$$\begin{aligned} T(z) =& -\frac{3}{2} : \mathfrak{b}^1\partial\mathfrak{c}^1 : (z) + \frac{1}{2} : \mathfrak{c}^1\partial\mathfrak{b}^1 : (z) - \frac{3}{2} : \mathfrak{b}^2\partial\mathfrak{c}^2 : (z) + \frac{1}{2} : \mathfrak{c}^2\partial\mathfrak{b}^2 : (z) \\ & - 2 : \beta\partial\gamma : (z) - : \gamma\partial\beta : (z) - \lambda\partial^3\gamma(z) \\ \mathcal{M}(z) =& \beta(z) \\ \Psi^1(z) =& \mathfrak{b}^1(z) + \frac{1}{2} : \beta\mathfrak{c}^1 : (z) + a\partial^2\mathfrak{c}^1(z) \\ \Psi^2(z) =& \mathfrak{b}^2(z) + \frac{1}{2} : \beta\mathfrak{c}^2 : (z) + a\partial^2\mathfrak{c}^2(z) \end{aligned} \tag{5.4.4}$$

This choice gives us the desired OPEs with the central charges $c_1 = -4$ and $c_2 = 12\lambda$. As before, c_1 can be made arbitrary by adding independent canonical free fields to the system.

5.5 Free-Field Realization of $\mathcal{N}=4$ super- BMS$_3$:

Now we present the free-field realisation of the most generic centrally-extended version of the $\mathcal{N}=4$ super-BMS$_3$ algebra that we have presented earlier. Let us first express the algebra in terms of the OPEs of the various fields:

$$\begin{aligned}
T(z)T(w) &\sim \frac{\frac{c_1}{2}}{(z-w)^4}+\frac{2T(w)}{(z-w)^2}+\frac{\partial T(w)}{z-w}, \qquad \mathcal{R}(z)\mathcal{R}(w)\sim \frac{c_3}{3}\frac{1}{(z-w)^2}\\
T(z)\mathcal{M}(w) &\sim \frac{\frac{c_2}{2}}{(z-w)^4}+\frac{2\mathcal{M}(w)}{(z-w)^2}+\frac{\partial \mathcal{M}(w)}{z-w}, \qquad \mathcal{S}(z)\mathcal{R}(w)\sim \frac{c_2}{3}\frac{1}{(z-w)^2}\\
T(z)\mathcal{S}(w) &\sim \frac{\mathcal{S}(w)}{(z-w)^2}+\frac{\partial\mathcal{S}(w)}{z-w}, \qquad T(z)\mathcal{R}(w)\sim \frac{\mathcal{R}(w)}{(z-w)^2}+\frac{\partial\mathcal{R}(w)}{z-w}\\
\mathcal{M}(z)\mathcal{R}(w) &\sim \frac{\mathcal{S}(w)}{(z-w)^2}+\frac{\partial\mathcal{S}(w)}{z-w}, \qquad \mathcal{R}(z)\Psi^{1,\pm}(w)\sim \pm\frac{\Psi^{1,\pm}}{z-w}\\
T(z)\Psi^{a\pm}(w) &\sim \frac{3}{2}\frac{\Psi^{a\pm}(w)}{(z-w)^2}+\frac{\partial\Psi^{a\pm}(w)}{z-w}, \qquad \mathcal{R}(z)\Psi^{2,\pm}(w)\sim \mp\frac{\Psi^{2,\pm}}{z-w}\\
\Psi^{1,\pm}(z)\Psi^{1,\mp}(w) &\sim \frac{1}{2}\left[\frac{\mathcal{M}(z)}{z-w}+\frac{1}{2}\left\{\frac{2\mathcal{S}(w)}{(z-w)^2}+\frac{\partial\mathcal{S}}{z-w}\right\}+\frac{c_2}{3}\frac{1}{(z-w)^3}\right]\\
\Psi^{2,\pm}(z)\Psi^{2,\mp}(w) &\sim \frac{1}{2}\left[\frac{\mathcal{M}(z)}{z-w}-\frac{1}{2}\left\{\frac{2\mathcal{S}(w)}{(z-w)^2}+\frac{\partial\mathcal{S}}{z-w}\right\}+\frac{c_2}{3}\frac{1}{(z-w)^3}\right]
\end{aligned} \tag{5.5.1}$$

We now want to give a free-field representation of this algebra. For this we use the fields (β_2,γ_{-1}), (β_1,γ_0) and four pairs of fermionic fields $(\mathfrak{b}^{a,\alpha},\mathfrak{c}^{a,\alpha})$ where $a=1,2$ and $\alpha=\pm$. With these fields, we define

$$\begin{aligned}
T_{(2,-1)} &= -2\beta_2\partial\gamma_{-1}-\gamma_{-1}\partial\beta_2, \qquad T_{(1,0)}=-\beta_1\partial\gamma_0\\
T^{a,\alpha}_{\left(\frac{3}{2},-\frac{1}{2}\right)} &= -\frac{3}{2}\mathfrak{b}^{a,\alpha}\partial\mathfrak{c}^{a,\alpha}+\frac{1}{2}\mathfrak{c}^{a,\alpha}\partial\mathfrak{b}^{a,\alpha}
\end{aligned} \tag{5.5.2}$$

Then we construct the various $\mathcal{N}=4$ fields as follows:

$$\begin{aligned}
T &= T_{(2,-1)}+T_{(1,0)}+\sum_{i=1}^{2}\sum_{\alpha=1}^{2}T^{a,\alpha}_{\left(\frac{3}{2},-\frac{1}{2}\right)}-\lambda\partial^3\gamma^1_{-1}, \qquad \mathcal{M}=\beta_2\\
\mathcal{R} &= \partial\gamma_0+\kappa\partial\beta_1\gamma_{-1}+\kappa\beta_1\partial\gamma_{-1}+\sum_{a=1}^{2}\mathfrak{b}^{a,\alpha}(\sigma_3)_{\alpha\beta}\mathfrak{c}^{\alpha\beta}, \qquad \mathcal{S}=-\kappa\beta_1\\
\Psi^{a,\alpha} &= \frac{1}{2}\left[\mathfrak{b}^{a,\alpha}+\beta_2(\sigma_1)^{\alpha}_{\beta}\mathfrak{c}^{a,\beta}+\rho\partial\beta_1(i\sigma_2)^{\alpha}_{\beta}\mathfrak{c}^{a,\beta}+2\rho\beta_1(i\sigma_2)^{\alpha}_{\beta}\partial\mathfrak{c}^{a,\beta}+\eta(\sigma_1)^{\alpha}_{\beta}\partial^2\mathfrak{c}^{a,\beta}\right]
\end{aligned} \tag{5.5.3}$$

Now with the identifications $\lambda=\frac{c_2}{12},\kappa=\frac{c_2}{3},\rho=\frac{c_2}{6},\eta=\frac{c_2}{6}$, one can show that the above fields correctly reproduce the $\mathcal{N}=4$ super-BMS$_3$ algebra. The central charges

c_1 and c_3 get fixed to the values -32 and 12 respectively. However, we can make these two arbitrary by adding independent free fields to our system.

5.6 Free-Field Realization of $\mathcal{N}=8$ super- BMS$_3$:

Let us now start with the $\mathcal{N}=8$ algebra and express it in terms of OPEs.

$$\begin{aligned}
T(z)T(w) &\sim \frac{\frac{c_1}{2}}{(z-w)^4}+\frac{2T(w)}{(z-w)^2}+\frac{\partial T(w)}{z-w}, \qquad \mathcal{R}^i(z)\mathcal{R}^j(w)\sim\frac{i\epsilon^{ijk}\mathcal{R}^k}{z-w}+\frac{c_3}{3}\frac{\delta^{ij}}{(z-w)^2}\\
T(z)\mathcal{M}(w) &\sim \frac{\frac{c_2}{2}}{(z-w)^4}+\frac{2\mathcal{M}(w)}{(z-w)^2}+\frac{\partial \mathcal{M}(w)}{z-w}, \qquad \mathcal{R}^i(z)\mathcal{S}^j(w)\sim\frac{i\epsilon^{ijk}\mathcal{S}^k}{z-w}+\frac{c_2}{3}\frac{\delta^{ij}}{(z-w)^2}\\
T(z)\mathcal{S}^i(w) &\sim \frac{\mathcal{S}^i(w)}{(z-w)^2}+\frac{\partial\mathcal{S}^i(w)}{z-w}, \qquad T(z)\mathcal{R}^i(w)\sim\frac{\mathcal{R}^i(w)}{(z-w)^2}+\frac{\partial\mathcal{R}^i(w)}{z-w}\\
\mathcal{M}(z)\mathcal{R}^i(w) &\sim \frac{\mathcal{S}^i(w)}{(z-w)^2}+\frac{\partial\mathcal{S}^i(w)}{z-w}, \qquad \mathcal{R}^i(z)\Psi^{A,a,+}(w)\sim-\frac{1}{2}\frac{(\sigma^i)^a_b\Psi^{A,b,+}}{z-w}\\
T(z)\Psi^{A,a,\pm}(w) &\sim \frac{3}{2}\frac{\Psi^{A,a\pm}(w)}{(z-w)^2}+\frac{\partial\Psi^{A,a,\pm}(w)}{z-w}, \qquad \mathcal{R}^i(z)\Psi^{A,a,-}(w)\sim\frac{1}{2}\frac{(\bar\sigma^i)^a_b\Psi^{A,b,-}}{z-w}\\
\Psi^{A,a,\pm}(z)\Psi^{B,b,\mp}(w) &\sim \frac{1}{2}\delta^{AB}\left[\frac{\delta^{ab}\mathcal{M}(z)}{z-w}-(\sigma^i)_{ab}\left\{\frac{2\mathcal{S}^i(w)}{(z-w)^2}+\frac{\partial\mathcal{S}^i}{z-w}\right\}+\frac{c_2}{3}\frac{\delta_{ab}}{(z-w)^3}\right]
\end{aligned} \tag{5.6.1}$$

Now for a free-field realizaion of this, we introduce one pair of conjugate bosonic ghost-fields (β_2,γ_{-1}), three pairs of comjugate bosonic ghost fields (β_1^i,γ_0^i) for $i=1,2,3$ and eight pairs of fermionic ghost fields $\left(\mathfrak{b}^{A,a,\alpha},\mathfrak{c}^{A,a,\alpha}\right)$ where both A and a run over 1 and 2 while $\alpha=\pm$. Then the $\mathcal{N}=8$ fields can be constructed as follows:

$$\begin{aligned}
T &= T_{(2,-1)}+\sum_{i=1}^{3}T^i_{(1,0)}+\sum_{A=1}^{2}\sum_{a=1}^{2}\sum_{\alpha=\pm}T^{A,a,\alpha}_{\left(\frac{3}{2},-\frac{1}{2}\right)}-\lambda\partial^3\gamma_{-1}, \qquad \mathcal{M}=\beta_2\\
\mathcal{R}^i &= \partial\gamma_0^i+\kappa\left(\beta_1^i\partial\gamma_{-1}+\partial\beta_1^i\gamma_{-1}\right)+i\epsilon^{ijk}\gamma_0^j\beta_1^k\\
&\quad+\frac{1}{2}\mathfrak{c}^{A,a,\alpha}(\sigma^i)_{ab}(\sigma_3)_{\alpha\beta}\mathfrak{b}^{A,b,\beta}, \qquad \mathcal{S}^i=-\kappa\beta_1^i\\
\Psi^{A,a,+} &= \frac{1}{2}\left[\mathfrak{b}^{A,a,+}+\beta_2(\sigma_1)^+_-\mathfrak{c}^{A,a,-}+\rho(\sigma^j)_{ae}\left\{\partial\beta_1^i(i\sigma_2)^+_-\mathfrak{c}^{A,e,-}\right.\right.\\
&\quad\left.\left.+2\beta_1^i(i\sigma_2)^+_-\partial\mathfrak{c}^{A,e,-}\right\}+\eta(\sigma_1)^+_-\partial^2\mathfrak{c}^{A,a,-}\right]\\
\Psi^{A,a,-} &= \frac{1}{2}\left[\mathfrak{b}^{A,a,-}+\beta_2(\sigma_1)^-_+\mathfrak{c}^{A,a,+}+\rho(\bar\sigma^j)_{ae}\left\{\partial\beta_1^i(i\sigma_2)^-_+\mathfrak{c}^{A,e,+}\right.\right.\\
&\quad\left.\left.+2\beta_1^i(i\sigma_2)^-_+\partial\mathfrak{c}^{A,e,+}\right\}+\eta(\sigma_1)^-_+\partial^2\mathfrak{c}^{A,a,+}\right]
\end{aligned} \tag{5.6.2}$$

where the parameters are fixed in terms of c_2 as $\lambda = \frac{c_2}{12}, \eta = \frac{c_2}{6}, \rho = -\frac{c_2}{12}, \kappa = \frac{c_2}{12}$. This gives $c_1 = -88$ and $c_3 = 24$.

5.7 Free-Field Realization of Spin 3 BMS$_3$ Algebra:

Let us now find a free-field realisation of the W_3 BMS$_3$ algebra. This is the ordinary BMS$_3$ algebra, supplemented by W_n and V_n. The algebra is [57, 58]:

$$
\begin{aligned}
[\mathfrak{J}_n, \mathfrak{J}_m] &= (n-m)\mathfrak{J}_{n+m} + \frac{c_1}{12}(n^3-n)\delta_{n+m,0} \\
[\mathfrak{J}_n, \mathcal{M}_m] &= (n-m)\mathcal{M}_{n+m} + \frac{c_2}{12}(n^3-n)\delta_{n+m,0} \\
[\mathfrak{J}_n, W_m] &= (2n-m)W_{n+m}, \qquad [\mathfrak{J}_n, V_m] = (2n-m)V_{n+m} \\
[\mathcal{M}_n, W_n] &= (2n-m)V_{n+m} \\
[W_n, W_m] &= \frac{1}{30}\Big[(n-m)(2n^2+2m^2-nm-8)\mathfrak{J}_{n+m} \\
&\quad + \frac{192}{c_2}(n-m)\Lambda_{n+m} - \frac{96\left(c_1+\frac{44}{5}\right)}{c_2^2}(n-m)\Theta_{n+m} \\
&\quad + \frac{c_1}{12}n(n^2-1)(n^2-4)\delta_{n+m,0}\Big] \\
[W_n, V_m] &= \frac{1}{30}\Big[(n-m)(2n^2+2m^2-nm-8)\mathcal{M}_{n+m} \\
&\quad + \frac{96}{c_2}(n-m)\Theta_{n+m} + \frac{c_2}{12}n(n^2-1)(n^2-4)\delta_{n+m,0}\Big]
\end{aligned}
\tag{5.7.1}
$$

where $\Theta_n = \sum_m \mathcal{M}_m\mathcal{M}_{n-m}$ and $\Lambda_n = \sum_m : \mathfrak{J}_m\mathcal{M}_{n-m} : -\frac{3}{10}(n+2)(n+3)\mathcal{M}_n$. As Λ contains a bilinear of two non-commuting operators L and $\mathcal{M}$, it is necessary to specify normal ordering in the definition. It is also important to note that Λ contains a term linear in $\mathcal{M}$.

Converting the modes into fields, the generators of this algebra are our familiar spin-2 fields $T(z)$ and $\mathcal{M}(z)$, complemented by a pair of spin-3 fields $W(z)$ and $V(z)$.

The free-field realisation involves the pairs of conjugate bosonic fields: (β_2, γ_{-1}) of dimensions $(2,-1)$ and (β_3, γ_{-2}) of dimensions $(3,-2)$. Taking into account the various pole structures, we can show that the following representation of the operators reproduces the correct algebra:

$$\begin{aligned}
T(z) &= -2 : \beta_2 \partial \gamma_{-1} : - : \partial \beta_2 \gamma_{-1} : -3 : \beta_3 \partial \gamma_{-2} : -2 : \partial \beta_3 \gamma_{-2} : -a \partial^3 \gamma_{-1} \\
W(z) &= \frac{1}{\sqrt{15}} \Big[3 : \beta_3 \partial \gamma_{-1} : + : \partial \beta_3 \gamma_{-1} : +5 : \beta_2 \partial^3 \gamma_{-2} : + : \partial^3 \beta_2 \gamma_{-2} : + \frac{9}{2} : \partial^2 \beta_2 \partial \gamma_{-2} : \\
&\quad \frac{15}{2} : \partial \beta_2 \partial^2 \gamma_{-2} : + \frac{8}{a} \left(: \beta_2 \left(: \beta_2 \partial \gamma_{-2} : \right) : + : \beta_2 \left(: \partial \beta_2 \gamma_{-2} : \right) : \right) \\
&\quad + \frac{a}{2} \partial^5 \gamma_{-2} + \frac{68}{15a} \beta_3 \Big] \\
\mathcal{M}(z) &= \beta_2, \qquad V(z) = -\frac{1}{\sqrt{15}} \beta_3
\end{aligned} \tag{5.7.2}$$

It is noteworthy that there are nested normal-ordered products in $W(z)$ which will generate non-trivial contributions to the linear terms in $\mathcal{M}$ that are crucial to obtain the correct algebra. Also the composite fields are defined as:

$$\begin{aligned}
\Lambda(w) &=: T\mathcal{M} : (w) - \frac{3}{10} \partial^2 \mathcal{M}(w) \\
\Theta(w) &=: \mathcal{M}\mathcal{M} : (w)
\end{aligned}$$

These fields give rise to the OPEs:

$$
\begin{aligned}
T(z)T(w) &\sim \frac{50}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{(z-w)} \\
T(z)\mathcal{M}(w) &\sim \frac{6a}{(z-w)^4} + \frac{2\mathcal{M}(w)}{(z-w)^2} + \frac{\partial \mathcal{M}(w)}{(z-w)} \\
T(z)W(w) &\sim \frac{3W(w)}{(z-w)^2} + \frac{\partial W(w)}{(z-w)} \\
T(z)V(w) &\sim \frac{3V(w)}{(z-w)^2} + \frac{\partial V(w)}{(z-w)} \\
\mathcal{M}(z)W(w) &\sim \frac{3V(w)}{(z-w)^2} + \frac{\partial V(w)}{(z-w)} \\
W(z)W(w) &\sim \frac{100}{3(z-w)^6} + \frac{2T(w)}{(z-w)^4} + \frac{\partial T(w)}{(z-w)^3} \\
&\quad + \frac{1}{(z-w)^2}\left[\frac{2}{60}\left(\frac{16}{a}\Lambda - \frac{1088}{15a^2}\Theta\right) + \frac{3}{10}\partial^2 T\right](w) \\
&\quad + \frac{1}{(z-w)}\left[\frac{1}{60}\left(\frac{16}{a}\partial\Lambda - \frac{1088}{15a^2}\partial\Theta\right) + \frac{1}{15}\partial^3 T\right](w) \\
W(z)V(w) &\sim \frac{4a}{(z-w)^6} + \frac{2\mathcal{M}(w)}{(z-w)^4} + \frac{\partial \mathcal{M}(w)}{(z-w)^3} \\
&\quad + \frac{1}{(z-w)^2}\left[\frac{2}{60}\left(\frac{16}{a}\Theta\right) + \frac{3}{10}\partial^2 T\right](w) \\
&\quad + \frac{1}{(z-w)}\left[\frac{16}{60a}\partial\Theta + \frac{1}{15}\partial^3 T\right](w)
\end{aligned}
\tag{5.7.3}
$$

When converted to modes, this reproduces the W_3-BMS$_3$ algebra that we have written earlier.

We have obtained the central charges $c_1 = 100$ and $c_2 = 12a$. Unlike the previous cases, it would be quite non-trivial to extend the above construction to allow for an arbitrary value of c_1.

5.8 An SU(2) Generalisation of BMS$_3$ and its Wakimoto Representation:

Here we shall use the affine current algebra symmetry to obtain a generalization of the BMS$_3$ algebra, which can be called "SU(2)-BMS$_3$" algebra. It contains as subalgebras both the BMS$_3$ algebra and the SU(2) affine Lie algebra. Then we will use the Wakimoto free-field representation [59] as well as our previous method to give a free-field representation of the full SU(2)-BMS$_3$ algebra.

In a CFT affine symmetry arises from the mode expansion of conserved currents.

$$J^a(z) = \sum_{n=-\infty}^{\infty} J^a_n z^{-n-1}, \qquad \bar{J}^a(\bar{z}) = \sum_{n=-\infty}^{\infty} \bar{J}^a_n \bar{z}^{-n-1}$$

where the index $a = 1, ..., D$, and D is the dimension of the Lie algebra. The energy-momentum tensor is constructed out of these currents using the Sugawara method [60]:

$$T_J(z) = \frac{1}{2(k+g)} : J^a(z) J^a(z) : \tag{5.8.1}$$

Here k is the level of the affine algebra and g is the dual Coxeter number. The currents transform as dimension 1 primary fields under the Virasoro symmetry. Hence the operator product expansions are:

$$\begin{aligned} T_J(z)T_J(w) &\sim \frac{\frac{c}{2}}{(z-w)^4} + \frac{2T_J(w)}{(z-w)^2} + \frac{\partial T_J(w)}{z-w} \\ J^a(z)J^b(w) &\sim \frac{k\delta^{ab}}{(z-w)^2} + \frac{if^{abc}J^c(w)}{z-w} \\ T_J(z)J^a(w) &\sim \frac{J^a(w)}{(z-w)^2} + \frac{\partial J^a(w)}{z-w} \end{aligned} \tag{5.8.2}$$

where $c = \frac{kD}{(k+g)}$ is the Virasoro central charge and f^{abc} are the structure constants of the Lie algebra.

Let us consider the case of SU(2). This algebra has structure constants $f^{abc} = \sqrt{2}\epsilon^{abc}$. In this basis $J^{\pm} = \frac{1}{\sqrt{2}}(J_1 \pm iJ^2)$, $J^0 = \sqrt{2}J^3$. The current algebra is:

$$\begin{aligned} J^+(z)J^-(w) &\sim \frac{k}{(z-w)^2} + \frac{J^0(w)}{z-w} \\ J^0(z)J^{\pm}(w) &\sim \frac{\pm 2J^{\pm}}{z-w} \\ J^0(z)J^0(w) &\sim \frac{2k}{(z-w)^2} \end{aligned} \tag{5.8.3}$$

All other OPEs are regular, particularly $J^+(z)J^+(w)$ has no singularity. This fact will be useful later on.

The Sugawara construction in this basis is:

$$T_J(z) = \frac{1}{2(k+2)} \left[\frac{1}{2} : J^0 J^0 : (z) + : J^+ J^- : (z) + : J^- J^+ : (z) \right] \tag{5.8.4}$$

Now to combine this with BMS, we need a slight variation of this construction. It is known that there exists a twisted version of the SU(2) algebra [61] in two-dimensional gravity, in which (J^+, J^0, J^-) have conformal dimensions $(2, 1, 0)$ respectively. This is achieved by modifying the above Sugawara energy-momentum tensor as:

$$T(z) = T_J(z) - \frac{1}{2}\partial J^0(z) \tag{5.8.5}$$

It can be shown easily that after this twist, the conformal dimensions of the currents are modified as above. So now we have a potential method of defining a combined SU(2)-BMS$_3$ algebra. This is because $T(z)$ and $J^+(z)$ together form a pair of spin-2 holomorphic fields of which $T(z)$ satisfies a Virasoro algebra, $J^+(z)$ has a non-singular operator product expansion with itself, and also J^+ is a spin-2 primary under T. Thus we have all the ingredients to define a BMS$_3$ algebra with $c_2 = 0$. This makes it possible to define the SU(2)-BMS$_3$ algebra with non-zero central extensions by introducing a c_2 term in the $T - J^+$ OPE:

$$\begin{aligned}
T(z)T(w) &\sim \frac{\frac{c_1}{2}}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{(z-w)} \\
T(z)\Psi^+(w) &\sim \frac{\frac{c_2}{2}}{(z-w)^4} + \frac{2J^+(w)}{(z-w)^2} + \frac{\partial J^+(w)}{(z-w)} \\
J^0(z)J^\pm(w) &\sim \frac{\pm 2J^\pm}{z-w}, \qquad J^+(z)J^+(w) \sim 0 \\
J^+(z)J^-(w) &\sim \frac{k}{(z-w)^2} + \frac{J^0(w)}{z-w} \\
J^0(z)J^0(w) &\sim \frac{2k}{(z-w)^2}, \qquad T(z)J^-(w) \sim \frac{\partial J^-(w)}{z-w} \\
T(z)J^0(w) &\sim \frac{2k}{(z-w)^3} + \frac{J^0(w)}{(z-w)^2} + \frac{\partial J^0(w)}{z-w}
\end{aligned} \tag{5.8.6}$$

Let us now introduce the Wakimoto representation of the SU(2) affine Lie algebra. [59], [60] This allows us to construct the affine SU(2) Lie algebra at arbitrary level k. Here the three holomorphic SU(2) currents $J^\pm(z), J^0(z)$ are constructed using (β, γ) fields of spins $(1, 0)$ and a free scalar field φ with a background charge depending on the real number k:

$$
\begin{aligned}
J^{+}(z) &= \beta(z) \\
J^{0}(z) &= \frac{i\sqrt{2}}{\alpha_{+}}\partial\varphi(z) + 2 : \gamma\beta : (z) \\
J^{-}(z) &= \frac{-i\sqrt{2}}{\alpha_{+}} : \partial\varphi\gamma : (z) - k\partial\gamma(z) - : \beta\gamma\gamma : (z)
\end{aligned}
\tag{5.8.7}
$$

As we mentioned before, k is the level of the affine algebra and $\alpha_{+} = \frac{1}{\sqrt{k+2}}$ is proportional to the background charge of the scalar field φ. Using the canonical OPEs, it is easy to show that the above spin-1 currents satisfy the affine SU(2) Lie algebra at arbitrary level k.

The energy-momentum tensor is given by:

$$
\begin{aligned}
T_{J}(z) &= \frac{1}{2(k+2)}\left[\frac{1}{2} : J^{0}J^{0} : (z) + : J^{+}J^{-} : (z) + : J^{-}J^{+} : (z)\right] \\
&= - : \beta\partial\gamma : (z) - \frac{1}{2} : \partial\varphi\partial\varphi : (z) - \frac{i\alpha_{+}}{\sqrt{2}}\partial^{2}\varphi(z)
\end{aligned}
\tag{5.8.8}
$$

Thus in the Wakimoto form, the energy-momentum tensor splits into two pieces, corresponding to that of a β-γ system and of a scalar field φ with a background charge. Comparison with the canonical form of the energy-momentum tensor for φ shows that the background charge is $\frac{-\alpha_{+}}{2}$. As β is one of the currents in the Wakimoto representation, its conformal dimension is 1. Hence the conformal dimension of its conjugate field γ is zero.

Now in the Wakimoto representation, we will perform a twist to change the spins of (J^{+}, J^{0}, J^{-}) to $(2, 1, 0)$. This can be implemented by changing the spins of (β, γ) to $(2, -1)$ [62]. So we must implement this twist in the Wakimoto representation, as well as the $\partial^{3}\gamma$ twist as in the previous section.

Performing both the twists, the final energy-momentum tensor takes the form:

$$
T = -2 : \beta\partial\gamma : - : \gamma\partial\beta : -a\partial^{3}\gamma - \frac{1}{2} : \partial\varphi\partial\varphi : -\frac{i}{\sqrt{2}}\left(\alpha_{+} + \frac{1}{\alpha_{+}}\right)\partial^{2}\varphi \tag{5.8.9}
$$

This gives the required OPEs:

$$\begin{aligned}
T(z)T(w) &\sim \frac{1}{2}\frac{c_1}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} \\
T(z)J^+(w) &\sim \frac{1}{2}\frac{12a}{(z-w)^4} + \frac{2J^+(w)}{(z-w)^2} + \frac{\partial J^+(w)}{z-w} \\
J^+(z)J^+(w) &\sim 0 \\
T(z)J^0(w) &\sim \frac{12a\gamma(w)}{(z-w)^4} + \frac{2k}{(z-w)^3} + \frac{J^0(w)}{(z-w)^2} + \frac{\partial J^0(w)}{z-w} \\
T(z)J^-(w) &\sim \frac{-6a : \gamma\gamma : (w)}{(z-w)^4} + \frac{\partial J^-(w)}{z-w}
\end{aligned} \tag{5.8.10}$$

Thus in this chapter we have given the free-field representations of BMS_3, super-BMS_3 and certain related algebras explicitly.

CHAPTER 6

Conclusions:

- We have deduced the correct scalings of the superconformal generators to reproduce the super-BMS_3 algebras by Innonu-Wigner contraction.
- We started with a set of global generators and used the Chern-Simons formulation to perform the asymptotic symmetry analysis (assuming the transformation of fermions as $\mathrm{U}(1)$ under R-symmetry) and derived the $\mathcal{N} = 4$ super-BMS_3 algebra. We have shown that after necessary modifications, the final algebra exactly matches with the one derived by contraction.
- We then generalised the transformation of the fermions under R-symmetry from $\mathrm{U}(1)$ to any generic representation, and show that this results in significant change in the final asymptotic algebra, in particular, the presence of non-linear terms therein.
- We also derived various interesting physical results such as energy bounds, Killing spinors, gravity solutions etc for the above cases.
- Finally, we presented the free field realisations of the (super) BMS_3 algebras and a few other related systems.

APPENDIX A

A.1 $N = 2$ GCA$_2$ and BMS$_3$ algebras

The GCA$_2$ algebra has been studied in detail in the literature, see for instance [63]. We will briefly review the relevant results and comment on the isomorphism with the BMS$_3$ algebra, which at the supersymmetric level gets lifted. One starts from linear combinations of holomorphic/anti-holomorphic Virasoro generators which maintain the mode number[8], and scales asymmetrically as follows:

$$\mathcal{M}_m = \lim_{\epsilon\to 0} \epsilon\,(L_m \mp \bar{L}_m)\ , \qquad \mathfrak{J}_m = \lim_{\epsilon\to 0}(L_m \pm \bar{L}_m)\,. \tag{A.1.1}$$

The commutators turn out to be:

$$\begin{aligned}
[\mathcal{M}_m, \mathcal{M}_n] &= 0 \\
[\mathcal{M}_m, \mathfrak{J}_n] &= (m-n) \lim_{\epsilon\to 0} \epsilon\,(L_{m+n} - \bar{L}_{m+n}) + \ldots \\
[\mathfrak{J}_m, \mathfrak{J}_n] &= (m-n) \lim_{\epsilon\to 0} (L_{m+n} + \bar{L}_{m+n}) + \ldots
\end{aligned} \tag{A.1.2}$$

Thus the algebra closes, and if we fix the signs in (A.1.1) to be minus in the definition of P_m and plus sign in the definition of J_m then we recover the BMS$_3$ algebra.

An N=2 generalisation of the GCA algebra was presented in Ref.[17]. It involves an asymmetric scaling of the form:

$$\Psi^1_r = \lim_{\epsilon\to 0} \epsilon\,(Q_r \mp \bar{Q}_r)\ , \qquad \Psi^2_r = \lim_{\epsilon\to 0}(Q_r \pm \bar{Q}_r) \tag{A.1.3}$$

The choice of upper/lower signs is immaterial as it simply corresponds to a sign change for . The resulting algebra is:

$$
\begin{aligned}
&[\mathcal{M}_m, \Psi^1_r] = 0 \ , && [\mathcal{M}_m, \Psi^2_r] = \Big(\frac{m}{2} - r\Big)\Psi^1_{m+r} m \\
&[\mathfrak{J}_m, \Psi^1_r] = \Big(\frac{m}{2} - r\Big)\Psi^1_{m+r} \ , && [\mathfrak{J}_m, \Psi^2_r] = \Big(\frac{m}{2} - r\Big)\Psi^2_{m+r} \\
&\{\Psi^1_r, \Psi^1_s\} = 0 \ , \qquad \{\Psi^1_r, \Psi^2_s\} = 2\mathcal{M}_{r+s} + \dots \ , && \{\Psi^2_r, \Psi^2_s\} = 2\mathfrak{J}_{r+s} + \dots
\end{aligned} \tag{A.1.4}
$$

We can now examine whether this algebra is isomorphic to the $\mathcal{N} = 2$ super-BMS algebra in Eq.(**??**). Clearly it is not: the supercharge anti-commutators can be diagonalised to find that one of them has a negative right-hand-side. This shows that the $\mathcal{N} = 2$ super-GCA of Ref.[17] is not equivalent to the $\mathcal{N} = 2$ super-BMS$_3$ algebra. Thus the BMS/GCA correspondence does not hold in the supersymmetric case.
In [23], this asymptotic superalgebra is studied in detail and proven to arise from a 'twisted' novel supersymmetric theory in 3 dimensions.

One may try to scale the super-generators symmetrically:

$$
\Psi^+_r = \lim_{\epsilon\to 0} \sqrt{\epsilon} Q_r, \qquad \Psi^-_r = \lim_{\epsilon\to 0} \sqrt{\epsilon}\bar{Q}_r \tag{A.1.5}
$$

This is similar (except for the fact that mode number is preserved) to the symmetric scaling used in super-BMS, but the bosonic generators are scaled according to GCA and the resulting algebra therefore contains:

$$
\begin{aligned}
&[\mathcal{M}_m, \Psi^\pm_r] = 0 \ , && [\mathfrak{J}_m, \Psi^\pm_r] = \Big(\frac{m}{2} - r\Big)\Psi^\pm_{m+r} \\
&\{\Psi^\pm_r, \Psi^\pm_s\} = \pm(\mathcal{M}_{r+s} + \dots) \ , && \{\Psi^+_r, \Psi^-_s\} = 0
\end{aligned} \tag{A.1.6}
$$

We see that the RHS has a negative sign in front of $\mathcal{M}$ for one of the generators. Therefore this also cannot be identified with the super-BMS$_3$ algebra.

One might be tempted to redress this by inserting a factor of i:

$$
\Psi^+_r = \lim_{\epsilon\to 0} \sqrt{\epsilon}\, Q_r \ , \qquad \Psi^-_r = \lim_{\epsilon\to 0} \mathrm{i}\sqrt{\epsilon}\, \bar{Q}_r \tag{A.1.7}
$$

but unfortunately this implies that the hermiticity condition on Ψ^- is violated. Thus we really get nothing new.

Finally, let us comment on a proposal in Ref.[64]. These authors propose to recover an "inhomogeneous SGCA" by defining:

$$\Psi^1_r = \lim_{\epsilon\to 0} \epsilon\,(Q^+_n \pm \mathrm{i}Q^-_{-n})\ , \qquad \Psi^2_r = \lim_{\epsilon\to 0}(Q^+_n \mp \mathrm{i}Q^-_{-n})\ . \tag{A.1.8}$$

Unfortunately this suffers from an analogous defect to Eq.(A.1.7) above, namely the supercharges do not satisfy ${\Psi^i_r}^\dagger = \Psi^i_{-r}$. Instead one finds that ${\Psi^1_r}^\dagger \sim \frac{1}{\epsilon}\Psi^2_{-r}$ so the hermiticity properties are incompatible with the scaling.

A.2 Inequivalent $\mathcal{N} = 2$ super-BMS$_3$ from (1,1) and (2,0) Virasoro algebras

There is another way of constructing $\mathcal{N} = 2$ super BMS$_3$ algebra than the one presented in the main draft. To obtain this algebra we need to consider only one sector of superconformal algebra, which for definiteness can be taken to be the holomorphic sector, for the supercharges and the R-symmetry generators. The R-generators can be scaled in either way as $\mathcal{R}_m = \lim_{\epsilon\to 0} \epsilon\, R_m$ or $\mathcal{S}_m = R_m$, while the remaining generators are scaled as usual.

Let's consider the first scaling. The commutation relations (2.2.5) will still be valid except that there is no generator corresponding to $\mathcal{S}$, so we find the algebra obtained by setting $\mathcal{S} = 0$ there:

$$\begin{aligned}
&[\mathcal{M}_m, \mathcal{R}_n] = 0\ , \qquad [\mathfrak{J}_m, \mathcal{R}_n] = -n\,\mathcal{R}_{m+n}\ , \quad [\mathcal{R}_m, \mathcal{R}_n] = 0\\
&[\mathcal{M}_m, \Psi^{+,i}_r] = 0\ , \qquad [\mathfrak{J}_m, \Psi^{+,i}_r] = \left(\frac{m}{2} - r\right)\Psi^{+,i}_{m+r}\\
&[\mathcal{R}_m, \Psi^{+,i}_r] = 0\ , \qquad \{\Psi^{+\,1}_r, \Psi^{+,2}_s\} = \tfrac{1}{2}\,\mathcal{M}_{r+s} + \tfrac{1}{4}\,(r-s)\,\mathcal{R}_{r+s} + \frac{c_2}{12}\,r^2\,\delta_{r+s,0}
\end{aligned} \tag{A.2.1}$$

This is a consistent algebra, which differs from the $\mathcal{N} = 2$ BMS$_3$ algebra we found before because of the presence of the $\mathcal{R}$ generator. However this is not an R-symmetry since it does not rotate the supercharges but instead commutes with them.

Next consider the second scaling, i.e. the generator $\mathcal{S}_m = R_m$. The algebra will now look like:

$$
\begin{aligned}
&[\mathcal{M}_m, \mathcal{S}_n] = 0\ , \quad [\mathfrak{J}_m, \mathcal{S}_n] = -n\,\mathcal{S}_{m+n}\ , \qquad [\mathcal{S}_m, \mathcal{S}_n] = \frac{c_1}{3}\, m\, \delta_{m+n,0}\ , \\
&[\mathcal{M}_m, \mathcal{Q}_r^{+,i}] = 0\ , \qquad\qquad [\mathfrak{J}_m, \Psi_r^{+,i}] = \left(\frac{m}{2} - r\right) \Psi^{+,i}_{m+r} \\
&[\mathcal{S}_m, \Psi_r^{+,1}] = \ \Psi^{+\,1}_{m+r}\ , \qquad\qquad [\mathcal{S}_m, \Psi_r^{+,2}] = -\ \Psi^{+\,2}_{m+r} \\
&\{\Psi_r^{+\,1}, \Psi_s^{+,2}\} = \tfrac{1}{2}\mathcal{M}_{r+s} + \frac{c_2}{12}\, r^2\, \delta_{r+s,0}
\end{aligned}
\tag{A.2.2}
$$

This time the generator $\mathcal{S}$ can be considered an R-symmetry generator since it rotates the supercharges, but it does not appear on the RHS of the anticommutator of two $\mathcal{Q}$'s. Hence, although this seems to be a valid alternate super BMS$_3$ algebra, it is not as rich as the one presented in the main draft. Similar behavior will hold for higher extended algebras.

A.3 Conventions

Here we list all the necessary notations. The antisymmetric Levi-Civita symbol has component $\epsilon_{012} = +1$ and the tangent space metric is the 3D Minkowski metric

$$
\eta_{ab} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\tag{A.3.1}
$$

The Γ-matrices satisfying the three dimensional Clifford algebra $\{\Gamma_a, \Gamma_b\} = 2\eta_{ab}$ are:

$$
\Gamma_0 = i\sigma_2\,, \qquad \Gamma_1 = \sigma_1\,, \qquad \Gamma_2 = \sigma_3\,,
\tag{A.3.2}
$$

with σ_i the Pauli matrices:

$$
\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\ , \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\ , \qquad \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\ .
\tag{A.3.3}
$$

Finally, the charge conjugation matrix $C = i\sigma_2$, or explicitly

$$C_{\alpha\beta} = \varepsilon_{\alpha\beta} = C^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} . \tag{A.3.4}$$

The fermion indices α, β run over $-, +$ (contrarily to [23] where they run over $+, -$). The supercharges are also taken to be Grassmann quantities, as the fermion parameters and the gravitini. All spinors taken here are Majorana and the Majorana conjugate of a spinor ψ^α is $\bar{\psi}_\alpha = C_{\alpha\beta}\psi^\beta$. Our conventions imply that we can use the identities

$$\Gamma_a\Gamma_b = \epsilon_{abc}\Gamma^c + \eta_{ab}\not{\mathbb{K}} , \qquad \Gamma^{a\alpha}{}_\beta \Gamma_a{}^\gamma{}_\delta = 2\delta^\alpha_\delta\delta^\gamma_\beta - \delta^\alpha_\beta\delta^\gamma_\delta , \tag{A.3.5}$$

$$C^T = -C , \qquad C\Gamma_a = -(\Gamma_a)^T C \tag{A.3.6}$$

In verifying the closure of the supersymmetry algebra on the fields and the off-shell invariance of the action, the three dimensional Fierz relation is useful.

$$\zeta\bar{\eta} = -\frac{1}{2}\bar{\eta}\,\zeta\not{\mathbb{K}} - \frac{1}{2}(\bar{\eta}\Gamma^a\zeta)\Gamma_a , \tag{A.3.7}$$

Other useful identities are:

$$\bar{\psi}\Gamma_a\,\eta = \bar{\eta}\,\Gamma_a\,\psi$$
$$\bar{\psi}\Gamma_a\,\epsilon = -\bar{\epsilon}\,\Gamma_a\,\psi$$

where ψ, η are Grassmannian one-forms, while ϵ is a Grassmann paramter. It is sometimes convenient to change basis of the tangent space to one more suited for the $isl(2)$ algebra in the bosonic sector of flat space supergravity. We do this by choosing a map to bring the generators of $SO(2,1)$ ($[J_a, J_b] = \epsilon_{abc}J^c$) to those of $SL(2,\mathbf{R})$ satisfying $[L_n, L_m] = (n-m)L_{n+m}$. This defines a matrix $U^a{}_n$ as a map from the tangent space metric η_{ab} with $a, b = \{0,1,2\}$ to the metric γ_{nm} defined in (A.3.10) with $n, m = \{-1, 0, +1\}$, satisfying

$$L_n = J_a\,U^a{}_n . \tag{A.3.8}$$

An explicit representation of $U^a{}_n$ that does the job is for instance

$$U^a{}_n = \begin{pmatrix} -1 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} . \tag{A.3.9}$$

In this basis the gamma matrices satisfy a Clifford algebra with

$$\{\tilde{\Gamma}_m, \tilde{\Gamma}_n\} = 2\gamma_{nm} \equiv 2 \begin{pmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 0 \end{pmatrix} \qquad \text{with: } n, m = -1, 0, +1 . \tag{A.3.10}$$

A real representation for the gamma matrices with n, m indices can be obtained by taking $\tilde{\Gamma}_n = U^a{}_n \Gamma_a$, or explicitly:

$$\tilde{\Gamma}_{-1} = -(\sigma_1 + i\sigma_2) = \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix} , \tag{A.3.11}$$

$$\tilde{\Gamma}_0 = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \tag{A.3.12}$$

$$\tilde{\Gamma}_{+1} = \sigma_1 - i\sigma_2 = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} . \tag{A.3.13}$$

In addition to the Clifford algebra, the gamma matrices now satisfy the commutation relations

$$[\tilde{\Gamma}_n, \tilde{\Gamma}_m] = 2(n - m)\tilde{\Gamma}_{n+m} , \tag{A.3.14}$$

which is just the $sl(2, \mathbf{R})$ algebra.

A.4 Calculation of Poisson Brackets for BMS$_3$ Algebra:

Here let us show the detailed calculation of one of the Poisson brackets, namely the $\{\mathfrak{J}_n, \mathcal{M}_m\}$. The rest will follow similarly. Using the convention of the Fourier transform between the modes and the fields as given earlier, the expression for the charge

$$\mathcal{C} = -\frac{k}{4\pi}\int \big[\Upsilon^{+}\mathfrak{J}+T\mathcal{M}+(\Psi^{1}_{+}\zeta^{1+}_{-}-\Psi^{1}_{-}\zeta^{1+}_{+})-(\Psi^{2}_{+}\zeta^{2+}_{-}-\Psi^{2}_{-}\zeta^{2+}_{+})+i\lambda_{\mathcal{R}}\phi+i\lambda_{\mathcal{S}}\rho\big]d\varphi. \tag{A.4.1}$$

can also be expressed in terms of modes as

$$\mathcal{C} = -\frac{2}{k}\sum_{n}\big[\Upsilon^{+}_{-n}\mathfrak{J}_{n}+T_{-n}\mathcal{M}_{n}+(\Psi^{1}_{+n}\zeta^{1+}_{-n}-\Psi^{1}_{-n}\zeta^{1+}_{+n})-(\Psi^{2}_{+n}\zeta^{2+}_{-n}-\Psi^{2}_{-n}\zeta^{2+}_{+n})+i\lambda^{\mathcal{R}}_{-n}\phi_{n}+i\lambda^{\mathcal{S}}_{-n}\rho_{-n}\big]. \tag{A.4.2}$$

We use the above two equivalent definitions of charge on the RHS and LHS of the following equation respectively:

$$\{\mathcal{C}[_1],\mathcal{C}[_2]\}_{PB} = \delta_{_1}\mathcal{C}[_2], \tag{A.4.3}$$

$$\text{LHS} = \frac{4}{k^2}\sum_{n}\sum_{m}\Upsilon^{+}_{-n}T_{-m}\{\mathfrak{J}_{n},\mathcal{M}_{m}\}$$

$$\text{RHS} = -\frac{k}{4\pi}\int d\varphi T\delta_{\lambda}\mathcal{M}_{|\Upsilon^{+}}$$

Now using the variation of $\mathcal{M}$ that we found earlier, the RHS equals

$$-\frac{k}{4\pi}\int d\varphi T[-2(\Upsilon^{+})''' + 2\mathcal{M}(\Upsilon^{+})' + \mathcal{M}'\Upsilon^{+}]$$

Now we convert the generators as well as the generic parameters from fields to modes by Fourier transformations, and use integration by parts. This gives the RHS as

$$-\frac{4}{k^2}in^3k\delta_{n+m,0} - \frac{4}{k^2}i(n-m)\mathcal{M}_{n+m}$$

Equating this to the LHS gives the required result.

A.5 The $(0,2)$ and $(2,0)$ AdS sectors

We present the $N=(2,0)$ and $(0,2)$ superconformal algebras. These are the global bulk algebras for $N=(2,0)$ and $(0,2)$ asymptotically AdS super gravity theories.

$$[J_a, J_b] = \epsilon_{abc} J^c, \qquad [J_a, Q^{\pm}_{\alpha}] = \frac{1}{2}(\Gamma_a)^{\beta}_{\alpha} Q^{\pm}_{\beta}, \qquad [R, Q^{\pm}_{\alpha}] = \pm\frac{1}{2} Q^{\pm}_{\alpha} \tag{A.5.1}$$

$$\{Q^{+}_{\alpha}, Q^{-}_{\beta}\} = -\frac{1}{2}(\Gamma^a)_{\alpha\beta} J_a - \frac{1}{2} C_{\alpha\beta} R, \qquad \{Q^{\pm}_{\alpha}, Q^{\pm}_{\beta}\} = 0 \tag{A.5.2}$$

The algebra in the other sector is exactly the same, albeit with barred gnerators.

Here, $a, b = 0, 1, 2$ and $\alpha, \beta = \pm\frac{1}{2}$.

The action is invariant off-shell under the supersymmetry transformation laws $\delta A = \mathrm{d}\lambda + [A, \lambda]$ with $\lambda = \varepsilon^{\alpha}_{\pm} \mathcal{Q}^{1,\pm}_{\alpha} + \vartheta^{\alpha}_{\pm} \mathcal{Q}^{2\pm}_{\alpha}$. In terms of the fields these transformations read:

$$\delta e_{\mu}{}^{a} = -\frac{1}{2} \sum_{\beta=\pm} \left(\bar{\varepsilon_{\beta}} \Gamma^a \psi_{\mu\,-\beta} + \bar{\vartheta_{\beta}} \Gamma^a \eta_{\mu\,-\beta} \right) \tag{A.5.3}$$

$$\delta\psi^{1\alpha}_{\pm\mu} = D_{\mu} \varepsilon^{\alpha}_{\pm} = \mathrm{d}\varepsilon^{\alpha}_{\pm} + \tfrac{1}{2}\, \omega^a\, (\Gamma_a)^{\alpha}{}_{\gamma}\, \varepsilon^{\gamma}_{\pm} \pm \tilde{\phi}\, \varepsilon^{\alpha}_{\pm}\,, \tag{A.5.4}$$

$$\delta\psi^{2\alpha}_{\pm\mu} = D_{\mu} \vartheta^{\alpha}_{\pm} = \mathrm{d}\vartheta^{\alpha}_{\pm} + \tfrac{1}{2}\, \omega^a\, (\Gamma_a)^{\alpha}{}_{\gamma}\, \vartheta^{\gamma}_{\pm} \pm \tilde{\phi}\, \vartheta^{\alpha}_{\pm} \tag{A.5.5}$$

$$\delta\tilde{\rho} = -\tfrac{1}{4}(\bar{\psi}^1_{+}\, \varepsilon_{-} - \bar{\psi}^1_{-}\, \varepsilon_{+} + \bar{\psi}^2_{+}\, \vartheta_{-} - \bar{\psi}^2_{-}\, \vartheta_{+}) \tag{A.5.6}$$

A.6 Construction of the supertrace elements

In this appendix, we shall describe the construction of the supertrace element for a given algebra. Below, we present the computation for (2,0) AdS algebra, that is presented in the last appendix. Super trace element is computed from non-degenerate bilinear form of a given algebra. For this, we construct a quadratic scalar combination of all the generators and impose that it commutes with all the generators, so that it is a Casimir operator. The construction of this quadratic scalar invariant is quite easy. Let us focus on the (2,0) algebra first, and find its non-zero supertrace elements. Now let us start with the most generic possible bilinear form W :

$$W = a\eta^{ab}J_aJ_b + bC^{\alpha\beta}Q^+_\alpha Q^-_\beta + \bar{b}C^{\alpha\beta}Q^-_\alpha Q^+_\beta + cC^{\alpha\beta}Q^+_\alpha Q^+_\beta + \bar{c}C^{\alpha\beta}Q^-_\alpha Q^-_\beta + dRR \tag{A.6.1}$$

By demanding that W commutes with all the generators of the (2,0) super algebra, we can fix the factors $(a, b, \bar{b}, c, \bar{c}, d)$. In this process, we need to make sure that the final Casimir is non-degenerate. Using various relations among the commutators/anti-commutators, one can show that the parameters get fixed as

$$a = b = \bar{b} = -d, \qquad c = \bar{c} = 0 \tag{A.6.2}$$

So overall, the invariant becomes:

$$W = a(\eta^{ab}J_aJ_b + C^{\alpha\beta}Q^+_\alpha Q^-_\beta + C^{\alpha\beta}Q^-_\alpha Q^+_\beta - RR) \tag{A.6.3}$$

From W, we extract all the supertrace elements by taking the inverse of the matrices $\eta^{\alpha\beta}$, $C^{\alpha\beta}$ and I:

$$< J_a, J_b >= \frac{1}{a}\eta_{ab}, \qquad < Q^+_\alpha, Q^-_\beta >=< Q^-_\alpha, Q^+_\beta >= \frac{1}{a}C_{\alpha\beta}, \qquad < R, R >= -\frac{1}{a} \tag{A.6.4}$$

Similarly for the $(0, 2)$ sector, the supertrace elements are given as:

$$< \bar{J}_a, \bar{J}_b >= \frac{1}{\bar{a}}\eta_{ab}, \qquad < \bar{Q}^+_\alpha, \bar{Q}^-_\beta >=< \bar{Q}^-_\alpha, \bar{Q}^+_\beta >= \frac{1}{\bar{a}}C_{\alpha\beta}, \qquad < \bar{R}, \bar{R} >= -\frac{1}{\bar{a}} \tag{A.6.5}$$

The overall factors a, $\bar{a}$ correspondond to the overall normalization in the action. For the bosonic action to contain the Einstein-Hilbert term, these factors get fixed as $a = -\bar{a} = -2$.

Note that as the super-Poincare generators can be expressed as linear combinations of these superconformal generators, the supertrace of the super-Poincare generators also get fixed by this analysis.

A.7 $\mathcal{N}=8$ Super-BMS$_3$

In this appendix, we demonstrate how the $N=8$ super-BMS$_3$ algebra does not get non-linear extension in the supercharges anticommutators. To do so, we prove this is the case for the asymptotic symmetry algebra for 3D AdS gravity with $N=(4,4)$ supersymmetry. The gravitinos transform under the defining representation of the SU(2) R-symmetry. The global super conformal algebra reads:

$$
\begin{aligned}
&[L_n, L_m] = (n-m)L_{n+m}\ , && [R^i, R^j] = \mathrm{i}\epsilon^{ijk}\, R^k\ ,\\
&[L_n, Q^{a\pm}_\alpha] = \left(\frac{n}{2}-\alpha\right) Q^{a,\pm}_{n+\alpha}\ , && [L_n, R^i] = 0\ ,\\
&[R^i, Q^{a\,+}_\alpha] = -\tfrac{1}{2}\,(\sigma^i)^a{}_b Q^{b+}_\alpha\ , && [R^i, Q^{a\,-}_\alpha] = +\tfrac{1}{2}\,(\bar\sigma^i)^a{}_b Q^{b-}_\alpha\ ,\\
&\{Q^{a,+}_\alpha, Q^{b,-}_{\underline{\}}}} = \delta^{ab} L_{\alpha+\beta} - (\alpha-\beta)\left(\sigma^i\right)^{ab} R^i\ , && \{Q^{a,\pm}_\alpha, Q^{b,\pm}_{\underline{\}}}} = 0\ .
\end{aligned}
$$

The asymptotic gauge field we start from has the form:

$$
A = \left(L_1 + \frac{r}{l}L_0 + \frac{r^2}{4l^2}L_{-1} - \frac{1}{2}\mathfrak{L}_+ L_{-1} - \frac{1}{2}\psi_{a,+}Q^{a,+}_- + \frac{1}{2}\psi_{a,-}Q^{a,-}_- + i\phi^i R^i\right)\mathrm{d}x^+
$$

Let us take the supertrace elements as

$$
\langle L_n, L_m\rangle = \gamma_{nm}\ , \quad \langle Q^{a,+}_\alpha, Q^{a,-}_\beta\rangle = \langle Q^{a,-}_\alpha, Q^{a,+}_\beta\rangle = C_{\alpha\beta}\ , \quad \langle R_i, R_j\rangle = -\delta_{ij}\ .
$$

and the generic gauge parameter

$$
\lambda = \chi^n L_n + \epsilon^\alpha_{a,+} Q^{a,+}_\alpha + \epsilon^\alpha_{a,-} Q^{a,-}_\alpha + \lambda^i R^i\ .
$$

From the gauge variations, we first compute the constraint equations:

$$
\begin{aligned}
\chi^0 &= -Y' + \frac{r}{l}Y\ ,\\
\chi^- &= \frac{1}{2}Y'' - \frac{r}{2l}Y' + \left(\frac{r^2}{4l^2} - \frac{1}{2}\mathfrak{L}_+\right)Y - \frac{1}{4}\sum_{a=1,2}\left(\psi_{a,+}\epsilon_{a,-} - \psi_{a,-}\epsilon_{a,+}\right)\ ,\\
\epsilon^-_{a,+} &= -\epsilon'_{a,+} + \frac{r}{2l}\epsilon_{a,+} - \frac{1}{2}\psi_{a,+}Y + \frac{i}{2}\phi^i_R\epsilon_{b,+}\left(\sigma^i\right)^b_a\ ,\\
\epsilon^-_{a,-} &= -\epsilon'_{a,-} + \frac{r}{2l}\epsilon_{a,-} + \frac{1}{2}\psi_{a,-}Y - \frac{i}{2}\phi^i_R\epsilon_{b,-}\left(\bar\sigma^i\right)^b_a\ .
\end{aligned}
$$

where $\epsilon^{+}_{a,\pm}=\epsilon_{a,\pm}$ and $\chi^{+}=Y$.

The other variation equations read:

$$
\begin{aligned}
\delta\mathcal{L}_{+}=&-Y'''+2\mathcal{L}_{+}Y'+\mathcal{L}'_{+}Y+\frac{1}{2}\left(\psi'_{a,+}\epsilon_{a,-}+3\psi_{a,+}\epsilon'_{a,-}\right)-\frac{1}{2}\left(\psi'_{a,-}\epsilon_{a,+}+3\psi_{a,-}\epsilon'_{a,+}\right)\\
&+\frac{i}{2}\left[\psi_{a,+}\epsilon_{b,-}\phi^{i}\left(\bar{\sigma}^{i}\right)^{b}_{a}+\psi_{a,-}\epsilon_{b,+}\phi^{i}\left(\sigma^{i}\right)^{b}_{a}\right]\ ,\\
\delta\psi_{a,+}=&2\epsilon''_{a}+\left(\psi'_{a,+}Y+\frac{3}{2}\psi_{a,+}Y'\right)-i\left[\phi^{i\prime}\epsilon_{b,+}\left(\sigma^{i}\right)^{b}_{a}+2\phi^{i}\epsilon'_{b,+}\left(\sigma^{i}\right)^{b}_{a}\right]-\mathfrak{L}_{+}\epsilon_{a,+}\\
&-\frac{i}{2}\psi_{b,+}\phi^{i}Y\left(\sigma^{i}\right)^{b}_{a}+\frac{1}{2}\lambda^{i}\psi_{b,+}\left(\sigma^{i}\right)^{b}_{a}-\frac{1}{2}\phi^{i}\phi^{j}\epsilon_{c,+}\left(\sigma^{j}\right)^{c}_{b}\left(\sigma^{i}\right)^{b}_{a}\ ,\\
\delta\psi_{a,-}=&-2\epsilon''_{a,-}+\left(\psi'_{a,-}Y+\frac{3}{2}\psi_{a,-}Y'\right)-i\left[\phi^{i\prime}\epsilon_{b,-}\left(\bar{\sigma}^{i}\right)^{b}_{a}+2\phi^{i}\epsilon'_{b,-}\left(\bar{\sigma}^{i}\right)^{b}_{a}\right]+\mathfrak{L}_{+}\epsilon_{a,-}\\
&+\frac{i}{2}\phi^{i}\left(\bar{\sigma}^{i}\right)^{b}_{a}\psi_{b,-}Y+\frac{1}{2}\phi^{i}\phi^{j}\epsilon_{c,-}\left(\bar{\sigma}^{i}\right)^{b}_{a}\left(\bar{\sigma}^{j}\right)^{c}_{b}-\frac{1}{2}\lambda^{i}\psi_{b,-}\left(\bar{\sigma}^{i}\right)^{b}_{a}\ ,\\
i\delta\phi^{i}=&\lambda^{i\prime}-\epsilon_{ijk}\phi^{j}\lambda^{k}+\frac{1}{2}\psi_{a,+}\epsilon_{b,-}\left(\sigma^{i}\right)^{ab}R_{i}+\frac{1}{2}\psi_{a,-}\epsilon_{b,+}\left(\sigma^{i}\right)^{ba}R_{i}\ .
\end{aligned}
$$

The charges are obtained from :

$$\delta\mathcal{C}=-\frac{k}{4\pi}\int\mathrm{d}\phi\langle\lambda,\delta A_{\phi}\rangle\ .$$

Hence we get

$$
\begin{aligned}
\mathcal{C}&=-\frac{k}{4\pi}\int\mathrm{d}\phi\left[\mathcal{L}_{+}Y+\frac{1}{2}\epsilon_{a,+}\psi_{a,-}-\frac{1}{2}\epsilon_{a,-}\psi_{a,+}-i\lambda_{i}\phi_{i}\right]\\
&=-\frac{2}{k}\left[\sum_{n}L_{n}Y_{-n}+\sum_{\alpha}\frac{1}{2}\epsilon^{-\alpha}_{a,+}\hat{\psi}^{\alpha}_{a,+}-\sum_{\alpha}\frac{1}{2}\epsilon^{-\alpha}_{a,-}\hat{\psi}^{\alpha}_{a,-}-i\sum_{n}\lambda^{-n}_{i}R^{n}_{i}\right]
\end{aligned}
$$

We then derive the asymptotic algebra by using the relation

$$\{\mathcal{C}[\lambda_{1}],\mathcal{C}[\lambda_{2}]\}_{PB}=\delta_{\lambda_{1}}\mathcal{C}[\lambda_{2}]$$

The Poisson brackets are

$$
\begin{aligned}
i\{L_n, L_m\} &= \frac{n^3 k}{2}\delta_{n+m,0} + (n-m)L_{n+m,0} \\
i\{L_n, \hat{\psi}^{a,+}_{\alpha}\} &= \left(\frac{n}{2} - \alpha\right)\hat{\psi}^{a,+}_{n+\alpha} - \frac{1}{2}\left(\hat{\psi}^{b,+}\phi^i\right)_{n+\alpha}(\sigma^i)^b_a \\
i\{R^i_n, \hat{\psi}^{\alpha}_{a,-}\} &= \frac{1}{2}\hat{\psi}^{n+\alpha}_{b,-}(\sigma^i)^b_a \qquad i\{R^i_n, \hat{\psi}^{\alpha}_{a,+}\} = -\frac{1}{2}\hat{\psi}^{n+\alpha}_{b,+}(\sigma^i)^b_a \\
i\{R^i_n, R^j_m\} &= \frac{nk}{2}\delta_{n+m,0} + i\epsilon_{ijk}R^k_{n+m} \\
i\{L_n, R^j_m\} &= 0 \\
\{\hat{\psi}^{\alpha}_{a,+}, \hat{\psi}^{\beta}_{b,-}\} &= \alpha^2 k\delta_{\alpha+\beta}\delta_{ab} + L_{\alpha+\beta}\delta_{ab} + \frac{1}{2}(R^iR^i)_{\alpha+\beta}\delta_{ab} - (\alpha-\beta)R^i_{\alpha+\beta}(\bar{\sigma}^a_b)\ ,
\end{aligned}
$$

Where the modes are defined as follows:

$$
\begin{aligned}
L_n &= \int \mathrm{d}\theta e^{-in\theta}\mathcal{L}_+\ , \qquad & R^i_n &= \int \mathrm{d}\theta e^{-in\theta}\phi^i\ , \\
\hat{\psi}^{a,+}_{\alpha} &= \int \mathrm{d}\theta e^{-i\alpha\theta}\psi^{a,-}\ , \qquad & \hat{\psi}^{a,-}_{\alpha} &= \int \mathrm{d}\theta e^{-i\alpha\theta}\psi^{a,+}\ .
\end{aligned}
$$

By adding the Sugawara term

$$
L_n \to L'_n = L_n + \frac{1}{2}(R^iR^i)_n
$$

the $i\{L_n, R^j_m\}$ gets modified as

$$
i\{\hat{L}_n, R^j_m\} = -mR^j_{n+m}\ ,
$$

and the supercharge anti-commutator takes the form:

$$
\{\hat{\psi}^{\alpha}_{a,+}, \hat{\psi}^{\beta}_{b,-}\} = \alpha^2 k\delta_{\alpha+\beta}\delta_{ab} + L'_{\alpha+\beta}\delta_{ab} - (\alpha-\beta)R^i_{\alpha+\beta}(\bar{\sigma}^a_b)\ .
$$

Note that the second and third term in the previous anti-commutator combined give the modified Sugawara generator $L'_{\alpha+\beta}$ so that the non-linear terms are absent in the final Poisson bracket. Thus, we see that the asymptotic AdS algebra will not have any non-linearity in the R-symmetry charges. As a consequence, the corresponding asymptotic flat $\mathcal{N}=8$ Super-BMS$_3$ algebra will also present no non-linearity.

A.8 AdS analysis and flat-space identifications

$$
\begin{aligned}
[L_n, L_m] =&(n-m)L_{n+m} \ , & [L_n, R^\alpha_p] &= \left(\frac{n}{2}-p\right) R^\alpha_{n+p} \\
[L_n, T^a] =&0 \ , & \{R^\alpha_p, R^\beta_q\} &= L_{p+q}\eta^{\alpha\beta} - \frac{\mathrm{i}}{6\hat{\alpha}}(p-q)(\lambda^a)^{\alpha\beta}T^a\delta_{p+q,0} \ , \\
[T^a, R^\alpha_p] =&\mathrm{i}(\lambda^a)^\alpha_\beta R^\beta_p \ , & [T^a, T^b] &= \mathrm{i}f^{abc}T^c \ ,
\end{aligned}
$$

and similarly for the anti-chiral sector. The structure constants of the above algebra are the same as defined in section 4.2. We begin with two such identical copies of Superconformal algebras. To get the asymptotic quantum algebra, let us begin with the gauge fields and generic variation parameters for the two copies of AdS:

$$
\begin{aligned}
A &= \left[L_1 + \frac{r}{l}L_0 + \left(\frac{r^2}{4l^2} - \frac{1}{2}\mathfrak{L}_+\right)L_{-1} + \mathfrak{A}Q_\alpha R^{-\alpha} + \frac{1}{2}\frac{k_l}{k_B}\phi^a T^a\right]\mathrm{d}x^+ + \frac{\mathrm{d}r}{2l}L_{-1} \ , \\
\bar{A} &= \left[\bar{L}_{-1} - \frac{r}{l}\bar{L}_0 + \left(\frac{r^2}{4l^2} - \frac{1}{2}\bar{\mathfrak{L}}_-\right)\bar{L}_1 + \bar{\mathfrak{A}}\bar{Q}_\alpha \bar{R}^{+\alpha} + \frac{1}{2}\frac{k_l}{k_B}\bar{\phi}^a \bar{T}^a\right]\mathrm{d}x^- + \frac{\mathrm{d}r}{2l}\bar{L}_1 \ ,
\end{aligned}
$$

where $k_l = \frac{c}{6}$, where c is the central charge of the quantum superconformal algebra. Asymptotic gauge transformations $\delta A = \delta\lambda + [A, \lambda]$ generate the asymptotic symmetries of the theory. The generic variation parameters are:

$$
\begin{aligned}
\lambda &= \chi^n L_n + \epsilon_{+,\alpha}R^{+,\alpha} + \epsilon_{-,\alpha}R^{-,\alpha} + \omega^a T^a \ , \\
\bar{\lambda} &= \bar{\chi}^n \bar{L}_n + \bar{\epsilon}_{+,\alpha}\bar{R}^{+,\alpha} + \bar{\epsilon}_{-,\alpha}\bar{R}^{-,\alpha} + \bar{\omega}^a \bar{T}^a \ .
\end{aligned}
$$

AdS unbarred Sector Variation:

Here we present the constraints on the parameters and the variations of the independent fields:

$$
\begin{aligned}
\chi^0 =& \frac{r}{l}\chi_1 - \chi_1' \ , \\
\chi_{-1} =& -\frac{r}{2l}\chi_1' + \frac{1}{2}\chi_1'' + \left(\frac{r^2}{4l^2} - \frac{1}{2}\mathfrak{L}_+\right)\chi_1 + \frac{\mathfrak{A}}{2}Q_\alpha\epsilon_+^\alpha \ , \\
\epsilon_{-,\alpha} =& -\epsilon_{+,\alpha}' + \mathfrak{A}Q_\alpha\chi_1 + \frac{k_l}{2k_B}\phi^a\epsilon_{+,\beta}(\lambda^a)_\alpha^\beta + \frac{r}{2l}\epsilon_{+,\alpha} \ , \\
\delta\mathfrak{L}_+ =& -\chi_1''' + \mathfrak{L}_+'\chi_1 + 2\mathfrak{L}_+\chi_1' - 3\mathfrak{A}Q_\alpha\epsilon_{+,\alpha}' - \mathfrak{A}Q_\alpha'\epsilon_{+,\alpha} + \mathfrak{A}\frac{k_l}{k_B}Q_\alpha\phi^a\epsilon_{+,\beta}(\lambda^a)_\alpha^\beta \ , \\
\mathfrak{A}\delta Q_\alpha =& -\epsilon_{+,\alpha}'' + \mathfrak{A}Q_\alpha'\chi_1 + \frac{3}{2}\mathfrak{A}Q_\alpha\chi_1' + \frac{k_l}{2k_B}(\lambda^a)_\alpha^\beta\left[2\phi^a\epsilon_{+,\beta}' + (\phi^a)'\epsilon_{+,\beta}+\right] + \frac{1}{2}\mathfrak{L}_+\epsilon_{+,\alpha} \\
& - \mathfrak{A}\frac{k_l}{2k_B}(\lambda_a)_\alpha^\beta\phi^a Q_\beta\chi_1 - \frac{k_l^2}{4k_B^2}\phi^a\phi^b(\lambda^a)_\beta^\gamma(\lambda^b)_\alpha^\beta\epsilon_{+,\gamma} + \mathfrak{A}\omega^a Q_\beta(\lambda^a)_\alpha^\beta \ , \\
\delta\phi^a =& 2\frac{k_B}{k_l}(\omega^a)' + \phi^b\omega^c f^{abc} + 2\mathfrak{A}Q_\alpha\epsilon_{+,\beta}(\lambda^a)^{\alpha\beta} \ .
\end{aligned}
$$

AdS Barred Sector Variation:

Similar computations for the barred sector will give:

$$
\begin{aligned}
\bar{\chi}_0 =& -\frac{r}{l}\chi_{-1} + \bar{\chi}_{-1}' \ , \\
\bar{\chi}_1 =& -\frac{r}{2l}\bar{\chi}_{-1}' + \frac{1}{2}\bar{\chi}_{-1}'' + \left(\frac{r^2}{4l^2} - \frac{1}{2}\bar{\mathfrak{L}}_{-1}\right)\bar{\chi}_{-1} - \frac{\bar{\mathfrak{A}}}{2}\bar{Q}_\alpha\bar{\epsilon}_-^\alpha \ , \\
\bar{\epsilon}_{+,\alpha} =& \bar{\epsilon}_{-\alpha}' + \bar{\mathfrak{A}}\bar{\chi}_{-1}\bar{Q}_\alpha - \frac{r}{2l}\bar{\epsilon}_{-,\alpha} + \frac{1}{12\alpha}(\lambda^a)_\alpha^\beta\bar{\phi}^a\bar{\epsilon}_{-,\beta} \ , \\
\delta\bar{\mathcal{L}}_- =& -\bar{\chi}_{-1}''' + \bar{\mathcal{L}}_-'\bar{\chi}_{-1} + 2\bar{\mathfrak{L}}_{-1}\bar{\chi}_{-1}' + 3\bar{\mathfrak{A}}\bar{Q}_\alpha(\bar{\epsilon}^{-,\alpha})' + \bar{\mathfrak{A}}\bar{Q}_\alpha'\bar{\epsilon}^{-,\alpha} + \bar{\mathfrak{A}}\frac{k_l}{k_B}(\lambda^a)^{\beta\alpha}\bar{Q}_\alpha\bar{\phi}^a\bar{\epsilon}_{-,\beta} \ , \\
\bar{\mathfrak{A}}\delta\bar{Q}_\alpha =& \bar{\epsilon}_{-,\alpha}'' + \bar{\mathfrak{A}}\bar{\chi}_{-1}\bar{Q}_\alpha' + \frac{3\bar{\mathfrak{A}}}{2}(\bar{\chi}_{-1})'\bar{Q}_\alpha + \frac{k_l}{2k_B}(\lambda^a)_\alpha^\beta[2\bar{\phi}^a\bar{\epsilon}_{-,\beta}' + (\bar{\phi}^a)'\bar{\epsilon}_{-,\beta}] - \frac{1}{2}\bar{\mathfrak{L}}_-\bar{\epsilon}_{-,\alpha} \\
& + \bar{\mathfrak{A}}\frac{k_l}{2k_B}\bar{\phi}^a(\lambda^a)_\alpha^\beta\bar{\chi}_{-1}\bar{Q}_\beta + \frac{k_l^2}{4k_B^2}\bar{\phi}^a\bar{\phi}^b(\lambda^a)_\alpha^\beta(\lambda^b)_\beta^\gamma\epsilon_{-,\gamma} + \bar{\mathfrak{A}}\bar{\omega}^a(\lambda^a)_\alpha^\beta\bar{Q}_\beta \ , \\
\delta\bar{\phi}^a =& 2\frac{k_B}{k_l}(\bar{\omega}^a)' + \bar{\phi}^b\bar{\omega}^c f^{abc} - 2\bar{\mathfrak{A}}\bar{Q}_\alpha\bar{\epsilon}_{-,\beta}(\lambda^a)^{\alpha\beta} \ .
\end{aligned}
$$

Identification with flat fields and generators:

Using these above relations, one can find the corresponding constraints and variations for the gauge field $\mathcal{A}$ (4.2.3) and gauge transformation parameter Λ (4.3.2) that gives the asymptotic symmetry for the 3D flat space time. Specifically:

$$\mathcal{A} = A + \bar{A} \,, \qquad \Lambda = \lambda + \bar{\lambda} \,,$$

$$\begin{aligned} J_n &= L_n - \bar{L}_{-n} \,, \qquad & M_n &= \frac{L_n + \bar{L}_{-n}}{l} \,, \qquad & r^1_{\pm,\alpha} &= \sqrt{\frac{2}{l}} R_{\pm,\alpha} \,, \\ r^2_{\pm,\alpha} &= \sqrt{\frac{2}{l}} \bar{R}_{\pm,-\alpha} \,, \qquad & \mathcal{R}^a &= T^a - \bar{T}^a \,, \qquad & \mathcal{S}^a &= \frac{T^a + \bar{T}^a}{l} \,. \end{aligned}$$

Using this identification the map for the charges is the following:

$$\begin{aligned} \mathcal{M} &= \mathcal{L}_+ + \bar{\mathcal{L}}_- \,, \qquad & \mathcal{N} &= l(\mathcal{L}_+ - \bar{\mathcal{L}}_-) \,, \qquad & \psi^1_{\pm\alpha} &= \sqrt{\frac{l}{2}} Q_{\pm\alpha} \,, \\ \psi^2_{\pm\alpha} &= \sqrt{\frac{l}{2}} \bar{Q}_{\mp\alpha} \,, \qquad & \rho^a &= \phi^a + \bar{\phi}^a \,, \qquad & \tilde{\phi}^a &= l(\phi^a - \bar{\phi}^a) \,, \end{aligned}$$

and the parameters are scaled as:

$$\begin{aligned} \xi^n &= \frac{l}{2}(\chi^n + \bar{\chi}^{-n}) \,, \qquad & \Upsilon^n &= \frac{1}{2}(\chi^n - \bar{\chi}^{-n}) \,, \qquad & \lambda^a_S &= \frac{l}{2}(\omega^a + \bar{\omega}^a) \,, \\ \lambda^a_R &= \frac{1}{2}(\omega^a - \bar{\omega}^a) \,, \qquad & \zeta^1_{\pm,\alpha} &= \sqrt{\frac{l}{2}} \epsilon_{\pm,\alpha} \qquad & \zeta^2_{\pm,\alpha} &= \sqrt{\frac{l}{2}} \bar{\epsilon}_{\pm,-\alpha} \,. \end{aligned}$$

The modes of the charges are defined as follows:

$$\begin{aligned} \mathfrak{J}_m &= \lim_{l\to\infty} (\mathcal{L}^+_m - \bar{\mathcal{L}}^-_m) \,, \qquad & \mathfrak{M}_n &= \lim_{l\to\infty} \frac{1}{l}(\mathcal{L}^+_n + \bar{\mathcal{L}}^-_{-n}) \\ S^a_n &= \lim_{l\to\infty} \frac{1}{l}(\phi^a_n + \bar{\phi}^a_{-n}) \,, \qquad & R^a_n &= \lim_{l\to\infty} (\phi^a_n - \bar{\phi}^a_{-n}) \,, \\ \psi^{1,\alpha}_{\pm} &= \lim_{l\to\infty} \sqrt{\frac{2}{l}} Q^\alpha_\pm \,, \qquad & \psi^{2,\alpha}_{\pm} &= \lim_{l\to\infty} \sqrt{\frac{2}{l}} \bar{Q}^\alpha_\mp \,, \\ c_J &= \lim_{l\to\infty} (c - \bar{c}) \,, \qquad & c_M &= \lim_{l\to\infty} \frac{1}{l}(c + \bar{c}) \,. \end{aligned}$$

Using these identifications, the final Asymptotic symmetry algebra for flat 3D space time has been obtained in (4.2.8).

A.9 Asymptotic gauge Field and gauge parameter for maximal extended super-BMS$_3$

The above form of asymptotic gauge fields needs to be modified for the right asymptotic algebra (4.2.10), as mentioned in section (4.4). The modified most generic gauge field was introduced in (4.3.1). Here, we find the right gauge transformation parameter that finally gives us (4.2.10). The most generic transformation parameter has the form ,

$$\Lambda = \zeta^n M_n + \Upsilon^n J_n + \tilde{\lambda}^a_S \mathcal{S}^a + \tilde{\lambda}^a_R \mathcal{R}^a + \zeta^{1,\alpha}_\pm r^{1,\alpha}_\pm + \zeta^{2,\alpha}_\pm r^{2,\alpha}_\pm \ , \tag{A.9.1}$$

The constraints and variations are given by:

$$\begin{aligned}
\Upsilon^0 &= -(\Upsilon^1)' \ , \qquad\qquad \xi^0 = -(\xi^1)' \ , \\
\Upsilon^{-1} &= \frac{1}{2}\left[(\Upsilon^1)'' - \frac{1}{2}\Upsilon^1\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)\right] \ , \\
\delta\mathcal{M} &= \left(\frac{1}{24\alpha}\rho^a\delta\rho^a\right) - 4(\Upsilon^{-1})' - \left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)\Upsilon^0 \ , \\
\xi^{-1} &= -\frac{1}{2}\left[-(\xi^1)'' + \frac{1}{2}\Upsilon^1\left(\mathcal{N} - \frac{1}{24\hat{\alpha}}\rho^a\tilde{\phi}^a\right) + \frac{1}{2}\xi^1\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)\right] \ , \\
\delta\mathcal{N} &= \frac{1}{24\hat{\alpha}}\delta(\tilde{\phi}^a\rho^a) - 4(\xi^{-1})' - \left(\mathcal{N} - \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\rho^a\right)\Upsilon^0 - \left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)\xi^0 \ , \\
\delta\tilde{\phi}^a &= 24\hat{\alpha}(\tilde{\lambda}^a_R)' + i\tilde{\phi}^b\tilde{\lambda}^c_R f^{abc} \ , \\
\delta\rho^a &= 24\hat{\alpha}(\tilde{\lambda}^a_S)' + i\left(\tilde{\phi}^b\tilde{\lambda}^c_S f^{abc} - \tilde{\lambda}^b_R\rho^c f^{abc}\right) \ ,
\end{aligned}$$

Let us choose (A, B, C, D) generic constants :

$$\begin{aligned}
\tilde{\lambda}^a_R &= \lambda^a_R + A\tilde{\phi}^a + B\rho^a \\
\tilde{\lambda}^a_S &= \lambda^a_R + C\tilde{\phi}^a + D\rho^a
\end{aligned}$$

The variation of the charge reads

$$\begin{aligned}
\delta C =& -\frac{k}{4\pi}\int d\phi \langle \Lambda, \delta a_\phi\rangle \\
=& -\frac{k}{4\pi}\int d\phi \left[\frac{1}{2}\xi^1\delta\mathcal{M} + \frac{1}{2}\Upsilon^1\delta N + \frac{1}{2}(\delta\tilde{\phi}^a\lambda_S^a + \delta\rho^a\lambda_R^a)\right] \\
&+ \frac{k}{4\pi}\int d\phi \frac{1}{48\alpha}\left[\rho^a\delta\rho^a\xi^1 + \tilde{\phi}^a\delta\rho^a\Upsilon^1 + \rho^a\delta\tilde{\phi}^a\Upsilon^1\right] \\
&- \frac{k}{4\pi}\int d\phi \frac{1}{2}\left[C\tilde{\phi}^a\delta\tilde{\phi}^a + D\rho^a\delta\tilde{\phi}^a + A\tilde{\phi}^a\delta\rho^a + B\rho^a\delta\rho^a\right]
\end{aligned}$$

The above variation simplifies to our required form

$$\delta C = -\frac{k}{4\pi}\int d\phi \left[\frac{1}{2}\xi^1\delta\mathcal{M} + \frac{1}{2}\Upsilon^1\delta N + \frac{1}{2}(\delta\tilde{\phi}^a\lambda_S^a + \delta\rho^a\lambda_R^a),\right]$$

for the following choice

$$A = D = \frac{1}{24\hat{\alpha}}\Upsilon^1, \qquad B = \frac{1}{24\hat{\alpha}}\xi^1, \qquad C = 0$$

It can be checked that the above charge rightly reproduces the algebra (4.2.10). Finally, inserting back the constraints, we get the expression for the transformation parameter:

$$\begin{aligned}
\Lambda =& \xi^1 M_1 + \Upsilon^1 J_1 + \left(\lambda_S^a + \frac{1}{24\hat{\alpha}}\Upsilon^1\tilde{\phi}^a\right)\mathcal{S}^a + \left(\lambda_R^a + \frac{1}{24\hat{\alpha}}\Upsilon^1\tilde{\phi}^a + \frac{1}{24\hat{\alpha}}\xi^1\rho^a\right)\mathcal{R}^a \\
&- (\xi^1)' M_0 - (\Upsilon^1)' J_0 + \frac{1}{4}\left[2(\Upsilon^1)'' - \Upsilon^1\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)\right] J_{-1} \\
&- \frac{1}{4}\left[-2(\xi^1)'' + \Upsilon^1\left(\mathcal{N} - \frac{1}{24\hat{\alpha}}\tilde{\phi}^a\rho^a\right) + \xi^1\left(\mathcal{M} - \frac{1}{48\hat{\alpha}}\rho^a\rho^a\right)\right] M_{-1}\ .
\end{aligned}$$

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LIST OF PAPERS ON WHICH THE THESIS IS BASED:

1. Free-field realisations of the BMS_3 algebra and its extensions
   Nabamita Banerjee, Dileep Jatkar, Sunil Mukhi, **Turmoli Neogi**
   JHEP 1606 (2016) 024

2. Extended supersymmetric BMS_3 algebras and their free field realisations
   Nabamita Banerjee, Dileep Jatkar, Ivano Lodato, Sunil Mukhi, **Turmoli Neogi**
   JHEP 1611 (2016) 059

3. N=4 Supersymmetric BMS_3 algebras from asymptotic symmetry analysis
   Nabamita Banerjee, Ivano Lodato, **Turmoli Neogi**
   Phys.Rev. D96 (2017) no.6, 066029

4. Maximally N-extended super-BMS_3 algebras and Generalized 3D Gravity Solutions
   Nabamita Banerjee, Arindam Bhattacharjee, Ivano Lodato, **Turmoli Neogi**
   arXiv:1807.06768