# Semi-leptonic B-meson Decays

A thesis submitted towards partial fulfilment of
BS-MS Dual Degree Programme

by

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under the guidance of

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# Certificate

This is to certify that this thesis entitled "SEMI-LEPTONIC B-MESON DECAYS" submitted towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research Pune represents original research carried out by " Siddhartha Das" at "The Institute of Mathematical Sciences", under the supervision of "Dr. Nita Sinha" during the academic year 2013-2014.

| | |
|---|---|
| Student | Supervisor |
| SIDDHARTHA DAS | NITA SINHA |

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# Abstract

The aim of the project was to study and derive differential decay rate for $B \to D^* \tau \bar{\nu}$ semileptonic decay. At later stage, the differential decay rate for $B \to D \tau \bar{\nu}$ with an additional operator added to hamiltonian of SM is calculated. For both cases of semileptonic decays of B mesons : $B \to D^* \tau \bar{\nu}$ and $B \to D \tau \bar{\nu}$, mass of $\tau$-lepton is considered but neutrino is taken to be massless because of its negligible mass in comparison to other elementary particles involved in decay process. At some point of time, the study to methods to determine neutrino mass hierarchy was also done.

# Contents

# Chapter 1

# Introduction

The branch of Physics that studies the nature of elemenatry particles (in the Universe) and their interactions is called Particle Physics. As of today, partciles are considered to excitations of quantum fields and interact through their dynamics. Today, the widely accepted model that describes elementary particles and interactions is the Standard Model.

## 1.1 Standard Model

The Standard Model of particle physics is the 'model' that describes the weak, electromagnetic and strong interactions between leptons and quarks, the basic particles of the Standard Model [1] . The 'Standard Model' asserts that the material in the Universe is made up of elementary fermions interacting through fields, of which they are the sources. The particles associated with the interaction of fields are called bosons. The elementary particles (quarks and leptons) interact via four known basic forces - gravitational, electromagnetic, strong, strong and weak - that can be characterized on the baisi of the following four criteria : the types of particles that experience the force, the relative strength of the force, the range over which the force is effective, and the nature of the particles that mediate the force. The leptons do not have any strong interactions, as they lack 'color charge', which allows quarks to bind together either mesonically or baryonically.

| Type | Relative Strength | Field Particle |
|---|---|---|
| Strong | 1 | Gluons |
| Electromagnetic | $10^{-2}$ | Photons |
| Weak | $10^{-6}$ | $W^{\pm}Z^{0}$ |
| Gravitational | $10^{-38}$ | Graviton |

The elemtary particles zoo is depicted in Fig. 1.1.

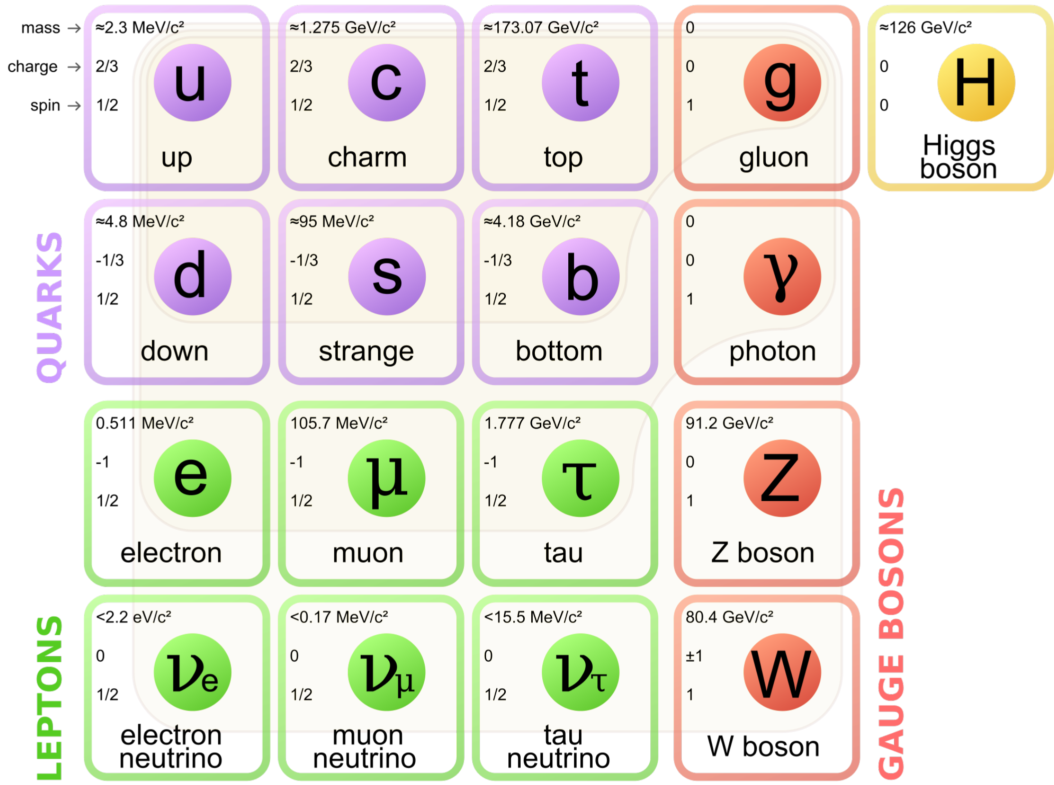


Figure 1.1: **Standard Model of elementary particles**

### CKM Matrix

In extended SM model, in order to explain neutrino osciallations neutrinos are atributed with masses. But since neutrinos are too light in comparison to other leptons and quarks, they are taken to be massless in many cases and hence they can be considered effectively to be degenrated in mass, i.e. the weak eigenstates of neutrinos can be defined to coincide with mass eigenstates of leptons. There exists sufficient freedom in case of the quarks to do the same for three up-type quarks $(u, c, t)$ but then the weak eigenstates $d', s', b'$ are not the same as the quark mass eignestates.

Cabibo-Kobayashi-Maskawa (CKM/KM) matrix is the unitary martix that contains the information on the strength of flavor-changing weak decays. It gives the transformation between quark mass eignestates and weak eigenstates [2].

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1.1}$$

The CKM matrix can also be characterized in terms of 3 real rotation angles, Euler angles, $(\theta_{12}, \theta_{23}, \theta_{13})$ and a real phase $\delta_{13}$,

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{\iota\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\iota\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\iota\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\iota\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\iota\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{1.2}$$

where cosines and sines of angles, $\theta_{ij}$ are denoted as $c_{ij}$ and $s_{ij}$ resp.

The non-zero value of rela phase, $\delta_{13}$ allows for the Standard Model mechanism of CP violation in weak decays. As can be observed, $|V_{ub}| = \sin\theta_{13}$ comes as a multiple with $e^{\iota\delta_{13}}$ in every element of the matrix containg the phase, the requirement for CP-violation to occur implies $|V_{ub}| = \sin\theta_{13}$ to be non-zero.

## 1.2 $B$-Mesons

$B$-mesons are composed of a bottom antiquark and either an up ($B^+$), down ($B^0$) strange (${B_s}^0$) or charm quark (${B_c}^+$). Because of short lifetime of the top quark, the combination of a bottom anti-quark with top qurak is not considered to be possible. Whereas, combination of a bottom anti-quark and a bottom quark is not considered as a B-meson, but rather called *bottomonium*. Each B-meson has an antiparticle that is composed of a bottom quark and an up($B^-$), down ($\bar{B}^0$), strange ($\bar{B_s}^0$) or charm antiquark ($\bar{B_c}^-$) respectively.

The study of semileptonic B-meson decays is interesting because of many reasons. They are some some of the simplest decays for theoretical and experimental studies. Semi-leptonic decay plays an important role in determining the CKM matrix element $|V_{ub}|$ and $|V_{cb}|$[3]. These parameters are coupled with the physics of quark-flavor and quark-mass, and have important inference for the breakdown of CP symmetry. The leptonic tensor of semileptonic B-decays is completely known, while the hadronic tensor is not completely calculable due to non-perturbative QCD effects [[4],[5]].

B physics is seen as an prominent area for investigation of new physics effects at low-energies. Currently, the large discrepancy between a SM prediction and experimental measurements is observed in the branching ratio of charged current mediated $B \to \tau \bar{\nu_\tau}$ decay, where large mass of $\tau$ lifts helicity suppression arising in leptonic B decays [6]. However, the recent reports claim that new results from BaBar measurements on the purely leptonic $B \to \tau \bar{\nu_\tau}$ decay mode show a better consistency with the SM, within the uncertainties [7]. Even small significant systematic deviations have been observed in the semileptonic $B \to D^* \tau \bar{\nu_\tau}$ rates. As the final state is is a vector meson, it has rich spin structure and so $B \to D^* \tau \bar{\nu_\tau}$ decay mode can provide number of tests of such SM deviations (or New Physics).

# Chapter 2

# Semileptonic Decay of B-mesons

In the phenomenonally diverse phenomenology of weak interactions, leptonic and semileptonic decays of hadrons (mesons and baryons) have a special standing. In both type of decay modes, the final state particles include a single charged lepton, which is the clearest experimental signature for a process mediated by weak interaction ($W$-boson). The exclusive semi-leptonic bottom meson decays $B \to D^*\tau\bar{\nu}_\tau$ including non-zero lepton mass effects in the kinematics and dynamics are discussed in 2.1. Here general formalism for the non-zero lepton mass case is developed. In 2.2 we consider effective hamiltonian comprising the SM term and an additional operator for the $b \to cl\bar{\nu}_l$ in $B \to D\tau\bar{\nu}_\tau$ decay mode.

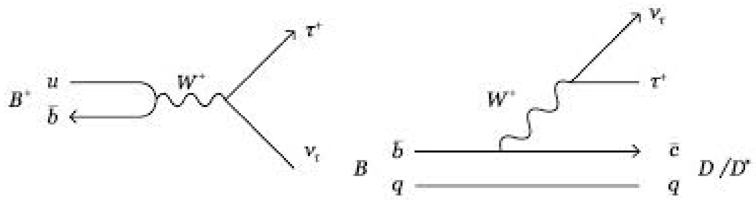


Figure 2.1: **Leptonic and Semileptonic Decays of B meson**

## 2.1 $B \to D^*\tau\bar{\nu}_\tau$

When leptonic mass effects are to be included in an analysis of semi-leptonic (s.l.) decays, we need to consider two different aspects. One aspect is kinematical in that the kinematics of the decay process change. The other aspect is of dynamical nature : When the lepton acquire mass one probes the scalar (or time-component) hadronic current form factor which is not accessible in the zero lepton mass case in addition to the 3-vector (or space-component) current form factors also measurable in the lepton mass zero case [8].

The $b \to cl\bar{\nu}_l$ effective hamiltonian comprising the SM model is given by

$$H^{SM}{}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}[\bar{c}\gamma_\mu(1-\gamma^5)b\bar{\nu}_l\gamma^\mu(1-\gamma^5)l] \tag{2.1}$$

### 2.1.1 Form Factors and Helicity Structures in Semileptonic Decays

The s.l. decay of a meson is a mediated by a virtual $W$, a charged boson. In this process, a quark (in decays of $\bar{B}$ mesons a $b$-qurak) decays to a lighter qurak, e.g. a $c$-quark, and emits a virtual $W$. The virtual $W$ decays to a lepton-neutrino pair in the semileptonic decay. As discussed later, the form of weak current, the coupling of the $W$ to the quarks and the leptons, is well known in the standard model. However, effects of strong interactions complicate the theoretical predictions of the matrix elements in s.l. decays. These interactions are not calculable perturbatively; attempts have been made to calculate the effects of the strong interactions using lattice QCD or QCD sum rules.

The effects of the stron interactions can be paramterized in terms of a set of form factors. We define form factors by expanding the particle-particle current matrix elements along set of covariants. The number of form factors needed to describe a particular decay depends on the spins of the initial and final state mesons. In this decay process - pseudoscalar to vector meson - there are four form factors. However, for negligible lepton masses, $l = (e, \mu)$, only three form factors contribute to the rate.

Let us define the invariant form factors governing the decay $\bar{B}^0 \to D^{*+}l^-\bar{\nu}$. There is no unique way of writing down invariant form factors. The object is to [parametrize the weak-hadronic current for the transistion $\bar{B}^0 \to D^{*+}l^-\bar{\nu}$. Here, a $b$-quark is destroyed and a $c$-quark is created. We can describe this by operator

$$J^\mu = \bar{c}\gamma^\mu(1-\gamma^5)b \tag{2.2}$$

acting to the right on the initial state, the $B$-meson, and to the left of the final state, the $D^{*+}$ meson, giving the weak hadronic current.

Let the momenta of $B$, $D^*$, $l$, $\nu$ with $p_B$, $p_{D^*}$, $k_l$, $k_\nu$ respectively, while $q \equiv p_B - p_{D^*} = k_l + k_\nu$. Let $\theta$ be angle between the $D^*$ and $\tau$ three-momenta in the $\tau - \bar{\nu}_\tau$ rest frame frame, as well as $\chi$, between the pale of the charged lepton and antineutrino momenta, and the decay plane of the $D^*$. Helicity basis vectors of the $D^*$ (vector) meson are denoted as $\varepsilon^\alpha$, while assuming standard lepton interactions, we can use $\hat{\varepsilon}$ for the four basis vectors describing the total helicity of the charged leptopn-neutrino system.

The current is then parametrized by a set of form factors,

$$
\begin{aligned}
H^\mu &= \langle D^{*+}(p_{D^*},\varepsilon)|\bar{c}\gamma^\mu(1-\gamma^5)b|\bar{B}^0(p_B)\rangle \\
&= \frac{2\iota\epsilon^{\mu\nu\alpha\beta}}{m_B+m_{D^*}}\varepsilon_\nu{}^* p_{D^*\alpha}p_{B\beta}V(q^2)-(m_B+m_{D^*})\varepsilon^{*\mu}A_1(q^2)+ \\
&\frac{\varepsilon^*.p_B}{m_B+m_{D^*}}(p_B+p_{D^*})^\mu A_2(q^2)-\frac{\varepsilon^*.p_B}{m_B+m_{D^*}}(p_B-p_{D^*})^\mu A_3(q^2) \quad (2.3)
\end{aligned}
$$

This is the most general form allowed under requirement that weak hadronic current should transform under Lorentz transformations and that it has to be linear in $\varepsilon^*$, the polarization of the $D^{*+}$. The invariant form factors can only be functions of the available Loretnz scalars in the process. Here, there is only one such scalar, which can be chosen to be $q^2$ or $w$, where $w$ is a 4-velocity transfer $w = v_B.v_D$, $v_B = p_B/m_B$ and $v_{D^*} = p_{D^*}/m_{D^*}$.

The hadronic current, $H^\mu$, can be also written in the form,

$$H^\mu = \varepsilon_\nu{}^* T^{\nu\mu} \quad (2.4)$$

where,

$$
\begin{aligned}
T^{\nu\mu} =&\frac{2\iota\epsilon^{\mu\nu\alpha\beta}}{m_B+m_{D^*}}p_{D^*\alpha}p_{B\beta}V(q^2)-(m_B+m_{D^*})g^{\mu\nu}A_1(q^2)+ \\
&\frac{p_B{}^\nu}{M_B+M_{D^*}}(p_B+p_{D^*})^\mu A_2(q^2)-\frac{p_B{}^\nu}{m_B+m_{D^*}}(p_B-p_{D^*})^\mu A_3(q^2)
\end{aligned}
\quad (2.5)
$$

In the rest frame of the $B$ meson with $z$ axis along the trajectory of the $D^*$, a suitable basis for the lepton-neutrino system. In the rest frame of the $B$-meson with $z$-axis along the trajectory of the $D^*$, a suitable basis for the lepton pair helicities is

$$
\begin{aligned}
\hat{\varepsilon}_\mu(\pm) &= \frac{1}{\sqrt{2}}(0,\pm 1,-\iota,0), \\
\hat{\varepsilon}_\mu(0) &= \frac{1}{\sqrt{q^2}}(|\mathbf{p}|,0,0,-q_0), \\
\hat{\varepsilon}_\mu(t) &= \frac{1}{\sqrt{q^2}}(q_0,0,0,-|\mathbf{p}|),
\end{aligned}
\quad (2.6)
$$

where, $q_0 = (m_B{}^2 - m_{D^*}{}^2 + q^2)/2m_B$ and

$$|\mathbf{p}| = \frac{\lambda^{1/2}(m_B{}^2, m_{D^*}{}^2, q^2)}{2m_B} \quad (2.7)$$

with $\lambda(a,b,c) = a^2+b^2+c^2-2(ab+bc+ca)$.

The helicity components of the currents in 2.6 can be referred as to the four helicities of the virtual $W$ ($W_{off-shell}$). Three of these are orthogonal to its momentum, i.e. $q^{\mu}\hat{\varepsilon}_{\mu}(m) = 0$ for $m = \pm, 0$, and thus make up the spin-1 part of the $W_{off-shell}$. The spin-0 (time-) component $m = t$ has the property $\hat{\varepsilon}_{\mu}(t) \propto q_{\mu}$ and clearly does not contribute to the s.l. decays in the lepton mass zero limit. The four helicity components have the orthonormality property

$$\hat{\varepsilon}^{*}_{\mu}(m)\hat{\varepsilon}^{\mu}(m') = g_{mm'}, \ (m, m' = t, \pm, 0), \tag{2.8}$$

and satisfy the completeness relation

$$\sum_{m,m'} \hat{\varepsilon}_{\mu}(m)\hat{\varepsilon}^{*}_{\nu}(m')g_{mm'} = g_{\mu\nu} \tag{2.9}$$

where $g_{mm'}$ =diag$(+, -, -, -)$

Similarly a convenient helicity basis for the $D^*$ is,

$$\varepsilon_{\alpha}(\pm) = \pm\frac{1}{\sqrt{q^2}}(0, 1, \pm\iota, 0),$$

$$\varepsilon_{\alpha}(0) = \frac{1}{m_{D^*}}(|\mathbf{p}|, 0, 0, E_{D^*}), \tag{2.10}$$

where $E_{D^*} = ({m_B}^2 - {m_{D^*}}^2 + q^2)/2m_B$ is the energy of $D^*$ in the $B$ rest frame. These basis vectors satisfy the following normalization

$$\varepsilon^{*}{}_{\alpha}(m)\varepsilon^{\alpha}(m') = -\delta_{mm'} \tag{2.11}$$

and completeness relation

$$\sum_{m,m'} \varepsilon_{\alpha}(m)\varepsilon^{*}{}_{\beta}(m')\delta_{mm'} = -\delta_{\alpha\beta} + \frac{p_{D^*\alpha}p_{D^*\beta}}{{m_{D^*}}^2} \tag{2.12}$$

We can introduce helicity amplitudes, $H_{\pm\pm}$, $H_{00}$ and $H_{0t}$ describing the decay of a pseudo-scalar meson into the three helicity states of a vector meson and four helicity states of the leptonic pair

$$H_{mm}(q^2) = \hat{\varepsilon}(m)^{\mu *}H_{\mu}(m), for \quad m = 0, \pm, \tag{2.13}$$

$$H_{0t}(q^2) = \hat{\varepsilon}(m = t)^{\mu *}H_{\mu}(n = 0). \tag{2.14}$$

Here, $H_{\mu}(m)$ is a corresponding hadronic matrix element, and $m, n$ demote helicity projections of he $D^*$ meson and the leptonic pair in the $B$ rest frame. Only four of the helicity amplitudes, $H_{++}, H_{--}, H_{00}, H_{0t}$, are non-zero.

### 2.1.2 Differential Decay Rates

**Decay Kinematics**

We are dealing with $1 \to 3$ particle decay with momenta (masses) $p_1(M_1) \to p_2(M_2) + l(\mu) + l'(0)$. The decay kinematics of such processes are given by (e.g. [9]).

The differential decay rate is given by

$$\frac{d\Gamma}{dq^2 dE_l} = \frac{{G_F}^2 |V_{bc}|^2}{(2\pi)^3} \frac{1}{8{M_1}^2} L_{\mu\nu} H^{\mu\nu} \tag{2.15}$$

where $L_{\mu\nu}$ is the usual lepton tensor ($\epsilon_{0123} = +1$) and is obtained using Casimir's trick **??** and properties of Gamma matrices.

$$L^{\mu\nu} = {p_l}^{\mu} {p_\nu}^{\nu} + {p_l}^{\nu} {p_\nu}^{\mu} - \frac{q^2 - {m_l}^2}{2} g^{\mu\nu} \pm \iota \epsilon^{\mu\nu\alpha\beta} p_{l,\alpha} p_{\nu,\beta} \tag{2.16}$$

The upper sign refers to the $(l^-, \hat{\nu}_l)$ case, whereas the lower sign refers to the $(l^+, \nu_l)$ case. There are two independent kinematic variabkles. In our discussion we shall use the two complimentary sets $(i) q^2, \cos\theta$ and $(ii) q^2, E_l$, where $q^2$ is the momentum transfer squared, $E_l$ is the lepton's energy in the $B$ rest system and $\theta$ is the polar angle of the lepton in the lepton-neutrino rest system.

The bounds on $q^2$ are given by

$$\mu^2 \leq q^2 \leq (M_1 - M_2)^2 \tag{2.17}$$

where the lower and upper bound corresponds to the $D$ momentum being maximal

$$p_m ax = ({M_1}^4 + {M_2}^4 + {\mu_1}^4 - 2{M_1}^2 {M_1}^2 - 2{M_1}^2 \mu^2 - 2{M_2}^2 \mu^2)^{\frac{1}{2}} / 2M_1 \tag{2.18}$$

and minimal ($p = 0$). The bounds for the lpton energy are given by

$$\mu \leq E_l \leq \frac{{M_1}^2 - {M_2}^2 + {\mu_1}^2}{2M_1} \tag{2.19}$$

where the lower and upper bound correspond to the lepton's momentum being minimal ($p_l = 0$) and maximal ($p_{l\,max}$), respectively. For the cosine of the scattering angle $\theta$ one has

$$\cos\theta = \frac{({M_1}^2 - {M_2}^2 + q^2)(q^2 + \mu^2) - 4q^2 M_1 E_1}{2M_1 |\mathbf{p}| (q^2 - \mu^2)} \tag{2.20}$$

$\cos\theta = \pm 1$ defines the phase space boundaries in the $(q^2, E_l)$-plane.

$$E_l = \frac{1}{2M_1} [q^2 + \mu^2 - \frac{1}{2q^2} ((q^2 - {M_1}^2 + {M_2}^2)(q^2 + \mu^2) \mp 2M_1 |\mathbf{p}| (q^2 - \mu^2))] \tag{2.21}$$

The calculation of the lepton energy spectrum requires the integratic limits ${q_\pm}^2 = {q_\pm}^2(E_l)$, i.e. inverse of above equation. One obtains

$$ {q_\pm}^2 = \frac{1}{a}(b \pm \sqrt{b^2 - ac}) \tag{2.22} $$

where

$$
\begin{aligned}
a &= {M_1}^2 + {\mu_1}^2 - 2M_1E_l & (2.23)\\
b &= M_1E_l({M_1}^2 - {M_2}^2 + \mu^2 - 2M_1E_l) + {M_2}^2 & (2.24)\\
c &= \mu^2[(M_1^2 - {M_2}^2)^2 + \mu^2{M_1}^2 - ({M_1}^2 - {M_2}^2)2M_1E_l] & (2.25)
\end{aligned}
$$

Two-dimensional integration becomes much simpler if one considers two-fold diferential rate with respect to the variables $q^2$ and $\cos\theta$. $E_l$ and $\cos\theta$ are related by

$$ \cos\theta = \frac{2q^2E_l - q_0(q^2 + {m_l}^2)}{|\mathbf{p}|(q^2 - {m_l}^2)} \tag{2.26} $$

Differentiating above equation, we get

$$ \frac{d\cos\theta}{dE_l} = \frac{2q^2}{|\mathbf{p}|(q^2 - {m_l}^2)} \tag{2.27} $$

**Differential decay rate**

Decay distribution differential in the momentum transfer squared $q^2$ and $\cos\theta$,

$$ \frac{d\Gamma}{dq^2\,d\cos\theta} = \frac{{G_F}^2|V_{bc}|^2}{(2\pi)^3}\frac{|\mathbf{p}|}{16{m_B}^2}(1 - \frac{{m_l}^2}{q^2})L_{\mu\nu}H^{\mu\nu} \tag{2.28} $$

where $L_{\mu\nu}, H_{\mu\nu}$ are the leptonic and hadronic current tensors. Using completeness relations 2.9 of the helicity basis vectors we can rewrite $L_{\mu\nu}H^{\mu\nu}$ as

$$
\begin{aligned}
L_{\mu\nu}H^{\mu\nu} &= L_{\mu'\nu'}g^{\mu'\mu}g^{\nu'\nu}H_{\mu\nu}\\
&= \sum_{m,m',n,n'} \left(L_{\mu'\nu'}\hat{\varepsilon}^{\mu'}(m)\hat{\varepsilon}^{\nu'}(n)g_{mm'}g_{nn'}\right)\left(\hat{\varepsilon}^{\mu*}(m')\hat{\varepsilon}^{\nu}(n')H_{\mu\nu}\right) \quad (2.29)
\end{aligned}
$$

The point is that the two Lorentz contractions appearing on the r.h.s of 2.29 can be evaluated in two different Lorentz systems [10]. As shown in [[11],[8]], we can expand the leptonic tensor in terms of a complete set of Wigner's $d^J$ functions, reducing $L_{\mu\nu}H^{\mu\nu}$ to the following compact form

$$
\begin{aligned}
L_{\mu\nu}H^{\mu\nu} = \frac{1}{8} &\sum_{\lambda_l,\lambda_{D^*},\lambda_{l\nu},\lambda'_{l\nu},J,J'} (-1)^{J+J'}|h_{(\lambda_l,\lambda_\nu)}|^2\delta_{\lambda_{D^*}\lambda_{l\nu}}\delta_{\lambda_{D^*}\lambda_{l\nu}}{}'\\
&\times {d^J}_{\lambda_{l\nu},\lambda_l-1/2}(\theta){d^J}_{\lambda'_{l\nu},\lambda_l-1/2}(\theta)H_{\lambda_{D^*}\lambda_{l\nu}}H'_{\lambda_{D^*}\lambda_{l\nu}} \quad (2.30)
\end{aligned}
$$

where $J$ and $J'$ run over 1 and 0. The relevant Wigner's $d^J$-functions for $J = 1/2$ and $J = 1$ are given in A.3. In term, the lepton helicity amplitudes, $h_{(\lambda_l,\lambda_\nu)}$ for a left-handed weak current are given by

$$h_{\lambda_l,\lambda_\nu} = \frac{1}{2}\bar{u}_l(\lambda_l)\gamma^\mu(1-\gamma^5)v_\nu(\lambda_\nu)\hat{\varepsilon}_\mu(\lambda_{l\nu}), \tag{2.31}$$

where for massless right-handed antineutrinos $\lambda_\nu = 1/2$ and $\lambda_{l\nu} = \lambda_l - \lambda\nu$ in the $l - \nu$ center of mass frame by angular momentum conservation. It follows that the two non-vanishing $|h_{(\lambda_l,\lambda_\nu)}|^2$ contributions are

$$\textit{non-flip}(\lambda_W = \mp 1) : |h_{\lambda_l=\mp\frac{1}{2},\lambda_\nu=\pm\frac{1}{2}}|^2 = 8(q^2 - {m_l}^2), \tag{2.32}$$

$$\textit{flip}(\lambda_W = t, 0) : |h_{\lambda_l=\pm\frac{1}{2},\lambda_\nu=\pm\frac{1}{2}}|^2 = 8\frac{{m_l}^2}{2q^2}(q^2 - {m_l}^2), \tag{2.33}$$

Finally using the standard convention for Wigner's d-function, we obtain

$$\frac{d\Gamma}{dq^2 d\cos\theta}(\lambda_l = -1/2) = \frac{{G_F}^2|V_{cb}|^2|\mathbf{p}|q^2}{256\pi^3{m_B}^2}\left(1 - \frac{{m_l}^2}{q^2}\right)^2 \\ [(1-\cos\theta)^2 H^2{}_{++} + (1-\cos\theta)^2 H^2{}_{--} + 2\sin^2\theta H^2{}_{00}] \tag{2.34}$$

$$\frac{d\Gamma}{dq^2 d\cos\theta}(\lambda_l = 1/2) = \frac{{G_F}^2|V_{cb}|^2|\mathbf{p}|q^2}{256\pi^3{m_B}^2}\left(1 - \frac{{m_l}^2}{q^2}\right)^2 \\ [\sin^2\theta(H^2{}_{++} + H^2{}_{--}) + 2(H_{0t} - H_{00}\cos\theta)^2] \tag{2.35}$$

after summation over $\lambda_l$ and/or integration over $\cos\theta$, we get

$$\frac{d\Gamma}{dq^2} = \frac{{G_F}^2|V_{cb}|^2|\mathbf{p}|q^2}{96\pi^3{m_B}^2}\left(1 - \frac{{m_l}^2}{q^2}\right)^2 [(|H_{++}|^2 + |H_{--}|^2 + |H_{00}|^2)\left(1 + \frac{{m_l}^2}{2q^2}\right) \\ + \frac{3{m_l}^2}{2q^2}|H_{0t}|^2] \tag{2.36}$$

where, four non-zero helicity amplitudes are :

$$H_{++} = -(m_B + m_{D^*})A_1(q^2) + 2\frac{p_{D^*}m_B}{m_B + m_{D^*}}V(q^2) \tag{2.37}$$

$$H_{--} = -(m_B + m_{D^*})A_1(q^2) - 2\frac{p_{D^*}m_B}{m_B + m_{D^*}}V(q^2) \tag{2.38}$$

$$H_{00} = -\frac{1}{2m_{D^*}\sqrt{q^2}}\Big(A_1(q^2)(m_B + m_{D^*})({m_B}^2 - {m_{D^*}}^2 - q^2) \\ - 4\frac{{m_B}^2{p_{D^*}}^2}{m_B + m_{D^*}}A_2(q^2)\Big) \tag{2.39}$$

$$H_{0t} = -\frac{m_B p_{D^*}}{m_{D^*}\sqrt{q^2}}\Big(A_1(q^2)(m_B + m_{D^*}) \\ - \frac{1}{2}({m_B}^2 - {m_{D^*}}^2 - q^2)\frac{A_2(q^2)}{m_B + m_{D^*}} + q^2\frac{A_3(q^2)}{m_B + m_{D^*}}\Big) \tag{2.40}$$

## 2.2 $B \to D\tau\bar{\nu}_\tau$

The $b \to cl\bar{\nu}_l$ effective hamiltonian comprising the SM model is given by

$$H^{SM}{}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}[\bar{c}\gamma_\mu(1-\gamma^5)b\bar{\nu}_l\gamma^\mu(1-\gamma^5)l] \tag{2.41}$$

The leptonic sector contribution is same as the previous case. The hadronic natrix elements in $B \to Dl\bar{\nu}_l$ can be parametrized in a standard way [5],

$$\langle D(p')|\bar{c}\gamma_\mu b|B(p)\rangle = F_1(q^2)(p+p')_\mu + \frac{m_B{}^2 - m_D{}^2}{q^2}[F_0(q^2) - F_1(q^2)]q_\mu, \tag{2.42}$$

$$\frac{d\Gamma_l}{dq^2} = \frac{G_F{}^2|V_{cb}|^2\lambda^{1/2}(m_B{}^2, m_D{}^2, q^2)}{192\pi^3 m_B{}^3}\left(1-\frac{m_l{}^2}{q^2}\right)^2 \left(\lambda(m_B{}^2, m_D{}^2, q^2)\left(1+\frac{m_l{}^2}{2q^2}\right)[F_1(q^2)]^2 + m_B{}^4\left(1-\frac{m_D{}^2}{m_B{}^2}\right)^4\frac{3m_l{}^2}{2q^2}[F_0(q^2)]^2\right) \tag{2.43}$$

where, $\lambda(m_B{}^2, m_D{}^2, q^2) = [q^2 - (m_B+m_D)^2]\ [q^2 - (m_B - m_D)^2]$. If

$$|\mathbf{p}| = \frac{\lambda^{1/2}(m_B{}^2, m_D{}^2, q^2)}{2m_B} \tag{2.44}$$

then

$$\frac{d\Gamma_l}{dq^2}|_{SM} = \frac{G_F{}^2|V_{cb}|^2|\mathbf{p}|}{96\pi^3}\left(1-\frac{m_l{}^2}{q^2}\right)^2 \left(4|\mathbf{p}|^2\left(1+\frac{m_l{}^2}{2q^2}\right)[F_1(q^2)]^2 + m_B{}^2\left(1-\frac{m_D{}^2}{m_B{}^2}\right)^4\frac{3m_l{}^2}{2q^2}[F_0(q^2)]^2\right) \tag{2.45}$$

From above equations, we can see that for the massless lepton in the final state the scalar form factor $F_0(q^2)$ does not contribute to the differential branching fraction. There is no interference or mixed terms (containg product of two form factors) as they vanish when integrated over the angle between 3-momenta of $l - \bar{\nu}$ system and $D$-meson.

Now, let us consider the effective hamiltonian comprising the SM term and an additional operator,

$$H_{eff} = H^{SM}{}_{eff} + H^{NP}{}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}[\bar{c}\gamma_\mu(1-\gamma^5)b\bar{\nu}_l\gamma^\mu(1-\gamma^5)l] + g_s[\bar{c}b\bar{\nu}_l(1-\gamma^5)l] \tag{2.46}$$

Hadronic matrix element for NP part, say, is given by :

$$\langle D(p')|\bar{c}b|B(p)\rangle = c' f_0(q^2);\; c' \propto ({m_B}^2 - {m_D}^2) \tag{2.47}$$

Then, the differential decay rate for B to D s.l. decay with ${H^{NP}}_{eff}$ is given by

$$\frac{d\Gamma}{dq^2}|_{NP} = \frac{{g_s}^2|\mathbf{p}|q^2}{96\pi^3 {m_B}^2}(1 - \frac{{m_l}^2}{q^2})^2 c'^2 {f_0}^2(q^2) \tag{2.48}$$

That is,

$$\frac{d\Gamma}{dq^2}|_{SM+NP} = \frac{d\Gamma_l}{dq^2}|_{SM} + \frac{d\Gamma}{dq^2}|_{NP} + \frac{d\Gamma}{dq^2}|_{Int} \tag{2.49}$$

where, $\frac{d\Gamma}{dq^2}|_{Int}$ is contribution because of the interference of SM and additional (NP) operators. In current case, the interference term vanishes. Hence, we have,

$$\begin{aligned} \frac{d\Gamma}{dq^2} &= \frac{d\Gamma}{dq^2}|_{SM} + \frac{d\Gamma}{dq^2}|_{NP} + \frac{d\Gamma}{dq^2}|_{Int} \\ &= \frac{|\mathbf{p}|}{96\pi^3}\big(1 - \frac{{m_l}^2}{q^2}\big)^2 \Big\{ {G_F}^2|V_{cb}|^2\Big(4|\mathbf{p}|^2\big(1 + \frac{{m_l}^2}{2q^2}\big)[F_1(q^2)]^2 \\ &+ {m_B}^2\big(1 - \frac{{m_D}^2}{{m_B}^2}\big)^4 \frac{3{m_l}^2}{2q^2}[F_0(q^2)]^2\Big) + {g_s}^2\frac{q^2}{{m_B}^2}c'^2 {f_0}^2(q^2)\Big\} \end{aligned} \tag{2.50}$$

# Chapter 3

# Determination of Neutrino Mass Hierarchy

Neutrinos experiments have revealed that neutrinos have amss and that, similar to quarks, leptons mix. Neutrino masses were discovered with the observation that neutrinos change favor as prescribed by the mechanism of mass-induced oscillations [12]. But still, the puzzle of neutrino mass-hierarchy, or sign of $\Delta m_{13}{}^2$, is a unsolved problem. There have been recent studies in this direction to help determine the neutrino mass hierarchy. The method discussed in this chapter has been studied in [13]. The idea was to use GLoBES Simulator [14] and data from Low Energy Neutrino Factories (LENF) and check with what precission neutrino mass hierarchy can be determined. *This study forms future work.*

## 3.1 Neutrino Mass Hierarchy

Observations made from atmospheric, solar, reactor and accelerator neutrino experiments show neutrino flavor transitions. Transitions for at least $\mathbf{E}/\mathbf{L}$'s (neutrino energy divided by baseline) are seen. These transitions can not be explained within standard model (SM) as neutrinos are massless according to SM. The extension to SM of article physics is required and the widely accepted extension is to allow the neutrinos to have masses and mixings [12],[13].

Recent experiments, SNO collaboration, KamLAND experiment and other solar neutrino experiments give the range of allowed values for the solar mass squared difference, $\delta m_{21}{}^2$, and the mixing angle, $\theta_{12}$ are

$$+7.3 \times 10^{-5} eV^2 < \delta m_{21}{}^2 < +9.0 \times 10^{-5} eV^2 \quad (3.1)$$

$$0.25 < \sin^2\theta_{12} < 0.37 \quad (3.2)$$

at the 90% confidence level. Maximal mixing, $\sin^2\theta_{12} = 0.5$, has been ruled out greater than $5\sigma$. The solar neutrino data is consistent with with $\nu_e \to \nu_\mu$

and/or $\nu_\tau$. The experiments like Super Kamiokande and K2K long baseline experiments, give range of allowed values for atmospheric mass squared difference, $\delta {m_{32}}^2$, and the mixing angle, $\theta_{23}$, are

$$1.5 \times 10^{-3} eV^2 < |\delta {m_{32}}^2| < 3.4 \times 10^{-3} eV^2 \quad (3.3)$$
$$0.36 < \sin^2\theta_{23} \leq 0.64 \quad (3.4)$$

at the 90% confidence level. The atmospheric data is consistent with $\nu_\mu \to \nu_\tau$ oscillations and sign of $\delta {m_{32}}^2$ is unkown. The constraint on the involvelment of the $\nu_e$ at the atmospheric $\delta m^2$ comes from Chooz reactor experiment, is $0 \leq \sin^2\theta_{13} < 0.04$ at the 90% confidence level at $|\delta m^2_{31}| = 2.5 \times 10^{-3} eV^2$.

The neutrino masses $m_i$, $i = 1, 2, 3$, are defined in the following convenient way : ${m_1}^2 < {m_2}^2$, and $0 < \Delta {m_{12}}^2 \equiv {m_2}^2 - {m_1}^2 < |\Delta {m_{13}}^2|$, where $\Delta {m_{13}}^2 \equiv {m_3}^2 - {m_1}^2$. A positive value of $\Delta {m_{13}}^2$ implies ${m_3}^2 > {m_2}^2$ and so-called normal mass hierarchy, while a negative value of $\Delta {m_{13}}^2$ implies ${m_3}^2 < {m_1}^2$ and so-called inverted mass hierarchy.

One of the goals of the next generation neutrino experiments is to establish the atmospheric mass hierarchy. Data from disapparance experiments of Low energy Neutrinos Factory (LNEF) may prove to be significant in this direction.

Genuine three generation effects make the effective atmospheric neutrino $\delta m^2$ measured by disappearance experiments, in principle, flavor dependent even in vacuum and thus sensitive to the mass hierarchy. This suggests an alternative way to access the mass hierarchy by comparing precisely meausred values for the atmospheric $\delta m^2$ in $\bar{\nu}_e \to \bar{\nu}_e$ (reactor) and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ (accelerator) modes.

## 3.2 Method of Determining Mass Hierarchy

Assuming three active neutrinos only, the survival probability for the $\alpha$-flavor neutrino, in vacuum, is given by

$$\begin{aligned} P(\nu_\alpha \to \nu_\alpha) = P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha) \\ &= 1 - 4|U_{\alpha 3}|^2|U_{\alpha 1}|^2 \sin^2\Delta_{31} - 4|U_{\alpha 3}|^2|U_{\alpha 2}|^2 \sin^2\Delta_{32} \\ &- 4|U_{\alpha 2}|^2|U_{\alpha 1}|^2 \sin^2\Delta_{21} \end{aligned} \quad (3.5)$$

where, $\Delta_{ij} = \delta {m_{ij}}^2 L/4E$, $\delta {m_{ij}}^2 = {m_i}^2 - {m_j}^2$ and $U_{\alpha i}$ are elements of the MNS mixing matrix [?]. The three $\Delta_{ij}$ are not independent since the $\delta {m_{ij}}^2$'s satisfy the constraint, $\delta {m_{31}}^2 = \delta {m_{32}}^2 + \delta {m_{21}}^2$

Let us define an *effective atmospheric mass squarred difference*, $\delta {m_\eta}^2$, which depends linearly on the parameter $\eta$, as follows

$$\delta {m_\eta}^2 = \delta {m_{31}}^2 - \eta \delta {m_{21}}^2 = \delta {m_{32}}^2 + (1 - \eta) \delta {m_{21}}^2$$

so that

$$\Delta_\eta = \Delta_{31} - \eta\Delta_{21} = \Delta_{32} + (1-\eta)\Delta_{21} = \frac{\delta m_\eta{}^2 L}{4E} \tag{3.6}$$

then using the two independent variables, $\Delta_\eta$ and $\Delta_{21}$, 3.5 can be re-written as

$$\begin{aligned} 1 - p(\nu_\alpha \to \nu_\alpha) = 4|U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2)\{\sin^2\Delta_\eta + [r_1 \sin^2(\eta\Delta_{21}) + \\ r_2 \sin^2((1-\eta)\Delta_{21})]\cos 2\Delta_\eta + \frac{1}{2}[r_1\sin(2\eta\Delta_{21}) \\ - r_2 \sin(2(1-\eta)\Delta_{21})]\sin 2\Delta_\eta\} + 4|U_{\alpha 2}|^2|U_{\alpha 1}|^2\sin^2\Delta_{21} \end{aligned} \tag{3.7}$$

where

$$\begin{aligned} r_1 &= \frac{|U_{\alpha 1}|^2}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2} \\ r_2 &= \frac{|U_{\alpha 2}|^2}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2} = 1 - r_1 \end{aligned} \tag{3.8}$$

It can be noticed that the coefficient in front of $\sin 2\Delta_\eta$ is the derivative of the coefficient in front of $\cos 2\Delta_\eta$, with respect to $\eta\Delta_{21}$, upto a constant factor. By choosing $\eta$ so as to set the coefficient in front of $\sin 2\Delta_\eta$ to zero one also minimizes the coefficient in front of $\cos 2\Delta_\eta$. That is, if $\eta$ satisfies

$$\eta = \frac{1}{2\Delta_{21}} \arctan\left(\frac{r_2 \sin 2\Delta_{21}}{r_1 + r_2\cos 2\Delta_{21}}\right) \approx r_2 \tag{3.9}$$

one minimizes the effects of both $\sin 2\Delta_\eta$ and $\cos 2\Delta_\eta$ terms and this $\delta m_\eta{}^2$ with $\eta \approx r_2$ is the effective atmospheric $\delta m^2, \delta m_{eff}{}^2|_\alpha$, measured in $\nu_\alpha$ disappearance experiments. The approximation is $\eta = r_2$ is an excellent provided that $\Delta \ll 1$. Using this approximate solution for $\eta$, the effective atmospheric $\delta m^2$ for the flavor the $\alpha$-flavor is

$$\begin{aligned} \delta m_{eff}{}^2|_\alpha &\equiv \frac{|U_{\alpha 1}|^2 \delta m_{31}{}^2 + |U_{\alpha 2}|^2 \delta m_{32}{}^2}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2} \\ &= r_1 \delta m_{31}{}^2 + r_2 \delta m_{32}{}^2 \end{aligned} \tag{3.10}$$

then the full neutrino survival probabiliy in vacuum, 3.5, can also be written as

$$\begin{aligned} 1 - P(\nu_\alpha \to \nu_\alpha) = 4|U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2)\Big(\sin^2\Delta_{eff} + \{r_1\sin^2(r_2\Delta_{21}) \\ + r_2 \sin^2(r_1\Delta_{21})\}\cos 2\Delta_{eff} - \frac{1}{2}\{r_2\sin^2(2r_1\Delta_{21}) \\ + r_1\sin^2(2r_2\Delta_{21})\}\sin 2\Delta_{eff}\Big) + 4|U_{\alpha 2}|^2|U_{\alpha 1}|^2\sin^2\Delta_{21} \end{aligned} \tag{3.11}$$

where $\Delta_{eff} = \delta m^2_{eff}|_\alpha L/4E$. If the coefficients in front of the $\cos 2\Delta_{eff}$ and $\sin 2\Delta_{eff}$ terms are expanded in powers of $\Delta_{21}$,

$$\{r_1 \sin^2(r_2\Delta_{21}) + r_2 \sin^2(r_1\Delta_{21})\} = r_1 r_2 {\Delta_{21}}^2 + O({\Delta_{21}}^4)\frac{1}{2}$$
$$\{r_2 \sin(2r_1\Delta_{21}) - r_1 \sin(2r_2\Delta_{21})\} = \frac{2}{3} r_1 r_2 (r_2 - r_1){\Delta_{21}}^3 + O({\Delta_{21}}^5) \quad (3.12)$$

and all terms linear in $\Delta_2 1$ have been absorbed into the $\Delta_{eff}$ terms. This shows that ${\delta m_{eff}}^2$, 3.10, is the effective atmospheric $\delta m^2$ to first nontrivial order in ${\delta m_{21}}^2$. It can be noticed that the first term odd in $\Delta_{eff}$ occurs with a coefficient proportional to ${\Delta_{21}}^3$ which, at the first extremum, is a supression factor of order $10^{-4}$. To understand the physical meaning of the effective atmospheric $\delta m^2$ it is useful to write it as follows

$$\begin{aligned} {\delta m_{eff}}^2|_\alpha &= {m_3}^2 - \langle {m_\alpha}^2 \rangle_{12}, \\ \langle {m_\alpha}^2 \rangle_{12} &\equiv \frac{|U_{\alpha 2}|^2 {m_2}^2 + |U_{\alpha 1}|^2 {m_1}^2}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2} \end{aligned} \quad (3.13)$$

$\langle {m_\alpha}^2 \rangle_{12}$ can ne inetrpreted as the $\alpha$-flavor weighted average mass of neutrino states 1 and 2. Thus the effective atmospheric $\delta m^2$ is the difference in the mass squared of the state 3 and this flavor average mass square of states 1 and 2 and is clearly flavor dependent.

The three flavor average mass squares are

$$\begin{aligned} \langle {m_e}^2 \rangle_{12} &= \frac{1}{2}[{m_2}^2 + {m_1}^2 - \cos 2\theta_{12} {\delta m_{21}}^2] \\ \langle {m_\mu}^2 \rangle_{12} &= \frac{1}{2}[{m_2}^2 + {m_1}^2 + (\cos 2\theta_{12} - 2\cos\delta \sin\theta_{13} \sin 2\theta_{12} \tan\theta_{23}){\delta m_{21}}^2] \\ \langle {m_\tau}^2 \rangle_{12} &= \frac{1}{2}[{m_2}^2 + {m_1}^2 + (\cos 2\theta_{12} - 2\cos\delta \sin\theta_{13} \sin 2\theta_{12} \cot\theta_{23}){\delta m_{21}}^2], \end{aligned} \quad (3.14)$$

where the $\tau$-flavor average is given for completeness only. It shows that $\nu_e$ and $\nu_\mu$ disappearance experiments measure *different* ${\delta m_{eff}}^2$'s. In fact the three disappearance ${\delta m_{eff}}^2$ are

$${\delta m_{eff}}^2|_e = \cos^2\theta_{12} {\delta m_{31}}^2 + \sin^2\theta_{12} {\delta m_{32}}^2 \quad (3.15)$$

$${\delta m_{eff}}^2|_\mu = \sin^2\theta_{12} {\delta m_{31}}^2 + \cos^2\theta_{12} {\delta m_{32}}^2 + \cos\delta \sin\theta_{13} \sin 2\theta_{12} \tan\theta_{23} {\delta m_{21}}^2 \quad (3.16)$$

$${\delta m_{eff}}^2|_\tau = \sin^2\theta_{12} {\delta m_{31}}^2 + \cos^2\theta_{12} {\delta m_{32}}^2 - \cos\delta \sin\theta_{13} \sin 2\theta_{12} \cot\theta_{23} {\delta m_{21}}^2 \quad (3.17)$$

If $\sin^2\theta_{12} \to 0$ then ${\delta m_{eff}}^2|_e \to {\delta m_{31}}^2$ and ${\delta m_{eff}}^2|_{\mu,\tau} \to {\delta m_{32}}^2$ as it must, as the mass eigenstate $\nu_1$ is nearly 100% $\nu_e$ in this limit.

$\delta m_{eff}{}^2|_e$, 3.15, is the atmospheric $\delta m^2$ measured by $\nu_e$ d disappearance experiments and $\delta m_{eff}{}^2|_\mu$, 3.16, is the atmospheric $\delta m^2$ measured by $\nu_\mu$ disappearance experiments upto corrections of $O(\delta m_{21}{}^2/\delta m_{32}{}^2)^2$.

The difference in the absolute value of the $e$-flavor and $\mu$-flavor $\delta m_{eff}{}^2$'s is given by

$$|\delta m_{eff}{}^2|_e - |\delta m_{eff}{}^2|_{mu} = \pm \delta m_{21}{}^2 (\cos 2\theta_{12} - \cos\delta \sin\theta_{13} \sin 2\theta_{12} \tan\theta_{23}) \tag{3.18}$$

where + sign (− sign) is for the normal (inverted) hierarchy. Thus by precision measurements of both of these $\delta m_{eff}{}^2$ one can determine the hierarchy and possibly even $\cos\delta$ at very high precision.

# Chapter 4

# Discussions on Semileptonic $B$ meson Decay

We have studied and derived differential decay rate for exclusive semileptonic decays of $B$ mesons including lepton mass effects Chapter-2. $B$ mesons with $\tau$ leptons, whose mass effects can't be ignored, in the final state offer possibilities to explore significant NP contributions that are not possible in processes with light leptons. The large mass-effect of $\tau$ can uplift the helecity suppression of certain leptonic or semileptonic decay amplitudes which are not observable with light leptons in the final state. The $B \to D^*\tau\bar{\nu_\tau}$ decay mode have two detectible particles of non-zero spin in the final state $(D^*, \tau)$ and so offers the possibility of an even more complete investigation of the structure of possible New Physics (NP) contributions to $b \to c\tau\bar{\nu_\tau}$ transitions [15].

Recent measurements of $B$ meson decay modes involving $\tau$-leptons have shown hints of deviations from the SM. Particularly for $b \to c\tau\bar{\nu_\tau}$ modes [16], BaBar has reported recently that

$$\begin{aligned} R(D) &= \frac{\mathcal{B}(\bar{B} \to D\tau^-\bar{\nu_\tau})}{\mathcal{B}(\bar{B} \to Dl^-\bar{\nu_l})} = 0.440 \pm 0.072 \\ R(D^*) &= \frac{\mathcal{B}(\bar{B} \to D^*\tau^-\bar{\nu_\tau})}{\mathcal{B}(\bar{B} \to D^*l^-\bar{\nu_l})} = 0.332 \pm 0.030 \end{aligned} \tag{4.1}$$

The corresponding numbers from Belle are

$$\begin{aligned} R(D) &= \frac{\mathcal{B}(\bar{B} \to D\tau^-\bar{\nu_\tau})}{\mathcal{B}(\bar{B} \to Dl^-\bar{\nu_l})} = 0.35 \pm 0.11 \\ R(D^*) &= \frac{\mathcal{B}(\bar{B} \to D^*\tau^-\bar{\nu_\tau})}{\mathcal{B}(\bar{B} \to D^*l^-\bar{\nu_l})} = 0.43 \pm 0.08 \end{aligned} \tag{4.2}$$

Both the BaBar and Belle results are a bit high compared to SM expectation,

$$
\begin{aligned}
R(D) &= 0.297 \pm 0.017 \\
R(D^*) &= 0.252 \pm 0.003
\end{aligned}
\tag{4.3}
$$

Whereas, the discrepancy between experimental observation and SM prediction for the leptonic decay rate of $B$ meson, $B \to \tau\bar{\nu}$ appears to be within the uncertainties [7] and so SM prediction for this decay mode seems to show better consistency with experimental results.

We also derived differential decay rate for $B \to D\tau\bar{\nu_\tau}$ with effective Hamiltonian containing additional operator along with SM operator. The numerical calculations for derived results are to be done.

# Appendix A

## A.1 Particle Decays

The differential decay rate of a particle of mass M into n bodies in its rest frame is given in terms of the Lorentz-invariant matrix element $\mathcal{M}$ by

$$d\Gamma = \frac{(2\pi)^4}{2M}|\mathcal{M}|^2 d\Phi_n(P; p_1, ....., p_n), \tag{A.1}$$

where $d\Phi_n$ is an element of n-body (Lorentz invariant) phase space is given by

$$d\Phi_n(P; p_1, ........, p_n) = \delta^4\big(P - \sum_{i=1}^{n} p_i\big) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i} \tag{A.2}$$

Given phase space can also be generated recursively,

$$\begin{aligned} d\Phi_n(P; p_1, ........, p_n) &= d\Phi_j(q; p_1, ........, p_j) \\ &\times d\Phi_{n-j+1}(P; q, p_{j+1}, ........, p_n)(2\pi)^3 dq^2 \end{aligned} \tag{A.3}$$

where, $q^2 = (\sum_{i=1}^{j} E_i)^2 - |\sum_{i=1}^{j} \mathbf{p}_i|^2$. The detailed discussion of decay kinematics and phase space for $1 \to 3$ decay are given in [9].

The amplitude of s.l. B decays, $B \to X + l + \bar{\nu}_l$, according to SM is given as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{fb} [\bar{u}(p_l)\gamma^\mu(1-\gamma^5)v(p_{\nu_l})] \langle X(p_X)|f\gamma^\mu(1-\gamma^5)b|B(p_B)\rangle, \tag{A.4}$$

where $f$ denotes the flavor of the quark in the final state, which can be u or c.

## A.2 Casimir's Trick

The notations are followed from [17].
$\langle|\mathcal{M}|^2\rangle \equiv$ average over initial spins, sum over final spins, of $|\mathcal{M}(i \to f)|^2$.
Let,

$$G \equiv [\bar{u}(a)\Gamma_1 u(b)][\bar{u}(a)\Gamma_2 u(b)]^* \tag{A.5}$$

where, $a$, $b$ stands for appropriate spins and momenta.

$$[\bar{u}(a)\Gamma_1 u(b)]^* = [u^\dagger(a)\gamma^0\Gamma_2 u(b)]^\dagger = [u^\dagger(b){\Gamma_2}^\dagger\gamma^\dagger u(a)] \tag{A.6}$$

Then,

$$\begin{aligned}[\bar{u}(a)\Gamma_2 u(b)]^* = \bar{u}(a)\bar{\Gamma_2}u(a); \bar{\Gamma_j} \equiv \gamma^0{\Gamma_2}^\dagger\gamma^0 \\ G = [\bar{u}(a)\Gamma_1 u(b)][\bar{u}(b)\bar{\Gamma_2}u(a)]\end{aligned} \tag{A.7}$$

$$\begin{aligned}\sum_{b\,spins} G &= \bar{u}(a)\Gamma_1\{\sum_{s_b=1,2} u^{(s_b)}(p_b)\bar{u}^{(s_b)}(p_b)\}\Gamma_2 u(a) \\ &= \bar{u}(a)\Gamma_1(\not{p} + m_b c)\bar{\Gamma_2}u(a) \\ &= \bar{u}(a)Qu(a);\ \ Q = \Gamma_1(\not{p} + m_b c)\bar{\Gamma_2}\end{aligned} \tag{A.8}$$

Now,

$$\begin{aligned}\sum_{a\,spins}\sum_{b\,spins} G &= \sum_{s_a=1,2} \bar{u}^{(s_a)}(p_a)Qu^{(s_a)}(p_a) \\ &= \sum_{s_a=1,2} \bar{u}^{(s_a)}(p_a)_i Q_{ij} u^{(s_a)}(p_a)_j \\ &= Q_{ij}\{\sum_{s_a} u^{(s_a)}(p_a)\bar{u}^{s_a}(p_a)\}_{ji} \\ &= Q_{ij}(\not{p}_a + mc)_{ji} \\ &= Tr[Q(\not{p}_a + mc)]\end{aligned} \tag{A.9}$$

$$\sum_{all\ \ spins} [\bar{u}(a)\Gamma_1 u(b)][\bar{u}(a)\Gamma_2 u(b)]^* = Tr[\Gamma_1(\not{p}_b + mc)\bar{\Gamma_2}(\not{p}_a + mc)] \tag{A.10}$$

## A.3 Wigner's $d^J$-functions for J=1/2 and J=1

For $J = \frac{1}{2}$ :

$$d^{1/2}{}_{mm'}(\theta) = \begin{pmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{pmatrix} \tag{A.11}$$

and for $J = 1$ :

$$d^{1}{}_{mm'}(\theta) = \begin{pmatrix} \frac{1}{2}(1+\cos\theta) & -\frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1-\cos\theta) \\ \frac{1}{\sqrt{2}}\sin\theta & \cos\theta & -\frac{1}{\sqrt{2}}\sin\theta \\ \frac{1}{2}(1-\cos\theta) & \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1+\cos\theta) \end{pmatrix} \tag{A.12}$$

The rows and columns are labeled in the order $(1/2, -1/2)$ and $(1, 0, -1)$, respectively.

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