# Topology and Cascade in Power Transmission Network of India

**A Thesis**

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Indian Institute of Science Education and Research Pune
in partial fulfillment of the requirements for the
BS-MS Dual Degree Programme

by

Dinesh Choudhary
Reg. no. 20121003

Indian Institute of Science Education and Research Pune
Dr. Homi Bhabha Road,
Pashan, Pune 411008, INDIA.

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Supervisor: Professor G. Ambika, IISER Pune

# CERTIFICATE

This is to certify that this dissertation entitled ***Topology and Cascade in Power Transmission Network of India*** submitted towards the partial fulfillment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by **Dinesh Choudhary** at Indian Institute of Science Education and Research under the supervision of **Prof. G. Ambika**, Professor Department of Physics during the academic year 2016-2017.

29/3/17

Prof. G. Ambika

IISER Pune

**Committee:**

Prof G. Ambika(IISER Pune)

Prof S. Banerjee(IISER Kolkata)

This thesis is dedicated to my family and friends.

# DECLARATION

I hereby declare that the matter embodied in the report entitled ***Topology and Cascade in Power Transmission Network of India*** are the results of the work carried out by me at the Department of Physics, Indian Institute of Science Education and Research, Pune, under the supervision of **Prof. G. Ambika** and the same has not been submitted elsewhere for any other degree.

Dinesh

Date: 29th March 2017

Dinesh Choudhary

20121003

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# Abstract

The size and complexity of power transmission networks make them good candidates for study using the framework of the complex networks. The topological properties of any network influence the dynamics and stability of the system and vice-versa. The heterogeneity of power transmission network makes it vulnerable to intentional attacks. In this thesis, we use the complex network approach to analyse the topological characteristics of the Indian power transmission network using data generated from power map of India. We compare the topology of Indian power transmission network with that of random graphs of similar sizes.

We address the possible consequences of node removal due to random failure or targetted attacks and study the vulnerability and extent of connectivity loss that can happen in both cases. Then we study the process of cascade due to these attacks in the network. Adopting the local preferential redistribution rule of an overloaded node, we investigate the effect of the failure of nodes with high as well as low loads. We find nodes with low loads are more threatening to the network for strength parameter $> 0.5$. The results of our study are highly relevant to address the strategies required to prevent attacks or stop the spread of cascades in the power transmission network of India.

# Contents

# Chapter 1

# Introduction

Our present society can function pleasantly and with comfort only if its critical infrastructure such as electricity, transportation system, the internet, etc are working efficiently. Among the infrastructures listed above, electricity is the most important one in everyday life. Electricity is the most demanding and powerful form of energy around the world and the network of power transmission is a complex and large-scale real world nonlinear dynamical system. Due to its vast and complex structure, stability and vulnerability of power transmission network become challenging. Even a small perturbation in its network can cause large-scale blackouts [1]. The very high population density of India makes its power transmission network even more complex. In the recent past, India has experienced large-scale blackouts. In January 2001, North India experienced a blackout which affected over 230 million people. Also just four years back, two major blackouts on 30 and 31 July 2012 left most part of India without electricity. The regions affected by 2012 blackouts are shown in figure 1.1. This event affected over 620 million people. If one just googles "The major blackout", Wikipedia shows a very long list of blackouts in the history from the year 1960 to 2017. This means that blackouts are inherent to power transmission networks and can happen unintentionally due to self-generated or external perturbations. These blackouts have drawn the attention of the scientific community and in the last two decades, efforts have been made to study the reasons for such large-scale cascades and to see how to make power transmission network more robust and efficient [2, 3, 4, 5, 6, 7]. Apart from the problem of the blackout, power transmission network systems involve scientific knowledge analysis of complex networks. Therefore it is important to understand the fundamental concepts of the complex network.


Figure 1.1: States in red color are regions in India affected by 2012 blackout.

## 1.1 Complex networks

When we try to study any physical phenomena involving a large number of interacting entities, we need the knowledge of coupled system. The framework of the complex network is perfectly suited to study and understand such phenomena. Leonhard Euler solved the problem of Koenigsberg bridges in 1736 and hence founded the concepts of graph theory [8]. Since then, the theory of graphs or networks has been used in diverse fields like economy [9], sociology [10], internet [11, 12] biology [13], chemistry [14], spread of epidemics [15] and theoretical physics [8].

In graph theory [16] a network or a graph G = $(V, E)$ is defined as an ordered pair of disjoint sets $(V, E)$, where V is a set of vertices with $|V|$ = n is the total number of nodes and E is a set of edges with $|E|$ = e is the total number of edges. The networks or graphs are in general classified into the following types:

- **Undirected graph**:
  A graph in which relation between any pair of two nodes is symmetric. The edge (x,y) is identical to the edge (y,x).

- **Directed graph**:
  A graph in which edges have orientation. The relation between any pair of nodes is ordered. The edges (x,y) and (y,x) can be different. The edge (x,y) is called an arrow directed from node x to node y.

- **Weighted graph**:
  A graph in which every edge (x,y) is assigned a number or weight which represents the cost, traffic, power or capacity depending on the system.

- **Regular graph**:
  A graph where each node has the same number of nearest neighbours. A graph with each node having k number of neighbouring nodes is called k-regular graph.

- **Complete graph**:
  A graph where every node is connected to every other node. The total number of edges in a complete graph is equal to n(n-1)/2.

- **Connected graph**:
  A graph or network is called connected if every unordered pair of nodes has at least one path connecting them.

The adjacency matrix **A** uniquely represents the network configuration with matrix element $a_{ij}$ representing the nature of connection between node i and node j.

$$a_{ij} = \begin{cases} 1 & \text{if node i and node j are connected} \\ 0 & \text{if node i and node j are not connected} \end{cases} \tag{1.1}$$

A graph can be characterised by different topological properties, such as degree distribution, clustering coefficient and characteristic path length.

### 1.1.1 Degree distribution

In graph theory, the degree of a node or a vertex is the number of edges incident to the node. In other words, it gives the information about direct connection of a node to its nearest neighbours. Degree distribution of a network is a measure of the number of nodes having a certain degree k[8]. The degree of $i^{th}$ node is

$$k_i = \sum_{\forall j} a_{i,j} \tag{1.2}$$

The degree distribution represents the global connectivity of a network. A network is homogeneous if nodes have similar degree like in the case of a regular or a random graph, and it is heterogeneous if there are nodes with degree much larger than the average degree of the graph. The networks whose degree distribution follows power law are called scale-free networks.

### 1.1.2 Clustering coefficient

The clustering coefficient of a network is the measure of the extent to which nodes in the network tend to cluster together. Clustering coefficient matrix $C_N$ is a 1-d vector which provides information about the looping in the network, and the matrix element $c_i$ is a ratio of the number of closed triplets to the number of connected triplets of any node.

$$c_i = 2\frac{\sum_{j,k}^{n} a_{jk}}{k_i(k_i - 1)} \tag{1.3}$$

The average clustering coefficient $C_{avg}$ of the network is

$$C_{avg} = \frac{1}{N}\sum_{i=1}^{N} c_i \tag{1.4}$$

$C_{avg}$ varies between 0 to 1. A well-clustered network provides more than one path between any two nodes.

### 1.1.3 Characteristic path length

Duncan Watts and Steven Strogatz [17] in 1998 noted that a graph can be characterised by two independent properties, namely the clustering coefficient and average shortest path. We define a distance matrix $D_{n \times n}$ and its matrix elements $d_{(i,j)}$ as a minimum number of edges required to go from node i to node j. The characteristic path length L of a network is the average of all such shortest paths.

$$L = \frac{1}{n \times (n-1)} \sum_{\forall i,j i \neq j} d_{(i,j)} \tag{1.5}$$

### 1.1.4 Global efficiency

The efficiency of a network is measured by its ease to transfer information between vertices. So a network with shorter characteristic path length will have larger efficiency. Global efficiency $\eta$ is the average of inverse of characteristic path length of every pair of connected nodes.

$$\eta = \frac{1}{n \times (n-1)} \sum_{\forall i,j i \neq j} \frac{1}{d_{ij}} \tag{1.6}$$

## 1.2 Random, scale-free and small-world networks

The way in which nodes or vertices are connected affects their cumulative behaviour because structure always affects the function. Erdos P. and Renyi A. in 1959 [18] introduced the concept of random network, known by their name ER graph. The ER graph starts with N number of isolated nodes and pair of nodes are selected at random with uniform probability to make connections between them. The ER graphs are characterized by average degree, meaning nodes have relatively the same degree. Figure 1.2 shows the degree distribution of a random graph with 2470 nodes and average degree 3.0477.

Most of the real-world networks are scale free. The scale-free network is a network whose degree distribution follows a power law as in equation 1.7. In such network there will be a

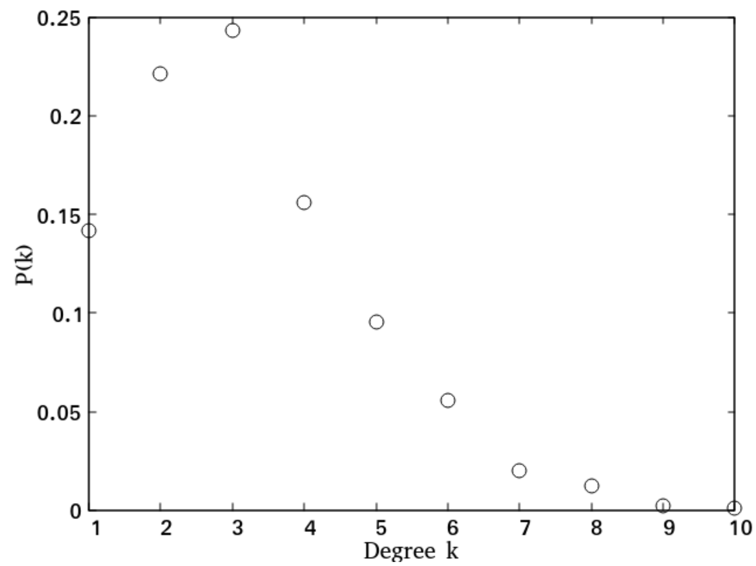


Figure 1.2: Degree distribution of a random network with 2470 nodes and average degree 3.0477

few number of high degree nodes called hubs.

$$P(k) = Ck^{-\lambda} \tag{1.7}$$

Regular graph and random graph are two extreme examples of graphs but real world networks fall somewhere between the two extremes [17]. Another type of network is the small-world network. A small world network is neither random nor regular. It has the clustering coefficient of a regular graph and average shortest path between two nodes of a random graph. In the small world network average path length l varies with number of nodes in the network as: l = $O(log(n))$ for large n.

The world of networks spans from the regular network of a lattice, ring of nodes and fully connected graph where nature of the connection is identical for every unit to random graphs where the nature of connection is completely random. But most of the real world networks have randomness with added patterns on it.

## 1.3 Complex network of power grids

The severe incidents cited in section 1 have motivated the scientific community to extensively study power transmission networks. The complex network approach has been used to study the power transmission networks where substations are considered as nodes and transmission lines connecting different stations are considered as edges. There are a few studies reported in this direction recently. M. Rosas-Casals and B. Corominas-Murtra studied the robustness and reliability of European power transmission network by studying the $\lambda$ parameter of its degree distribution [19]. Paolo Crucitti et al. analyzed efficiency and cascade of Italian electric power transmission network [3] by relating final efficiency to the tolerance parameter and removing nodes randomly and on the basis of the load. Reka Albert et al. investigated the structural vulnerability of the North American power transmission network by removing transmission stations and studying its effect on the network [2]. Different power flow models have been addressed to study the dynamics of power transmission networks [20]. Multilayer property of power transmission networks have been studied by Giuliano Andrea Pagani considering different kV lines as different layers [21].

## 1.4 Indian power transmission network

Compared to the studies on power networks globally, reported studies on the Indian power transmission are very few. After the blackout of 2012, Guidong Zhang et al. with the help of Indian officials used the complex network approach to investigate the power failure in Indian power transmission network [7]. They used a new power flow model including active and reactive power flow to study the cascade. Their investigation revealed the origin and spread of power failure. Himansu Das and group studied the topological properties of Odisha power transmission network and concluded that Odisha power transmission network degree distribution follows exponential function, unlike scale-free networks where degree distribution follows power law [22]. The topology of West Bengal power transmission network has been studied by Himansu Das, Gouri Sankar Panda, Bhagaban Muduli and Pradeep Kumar Rath [23].

Apart from work done by Guidong Zhang et al., the other two investigations of the Indian power transmission network using the complex network approach have considered only regional parts of the Indian power transmission network system. The reason for lacuna of a detailed research on complete Indian power transmission network is the unavailability of data in the useful format. So in the first part of this work, we concentrate on generating this data in the form of adjacency matrix using the power map available as the source. As the first step, the data generation is required even though it is time-consuming. Then we can further study the topological properties, robustness, stability and vulnerability of Indian power transmission network.

In the next chapter, we present the process of data generation and its important characteristics. In the subsequent chapter, we present the results of our study on the topological characteristics. We end by reporting the study on possible cascade processes on this network due to the failure of one or more power stations.

The summary of our work, future scope and references are given at the end.

# Chapter 2

# Data generation

This chapter provides the details about the process of data generation. As discussed in section 1.4, studies on the Indian power network so far have been localized due to lack of data in the useful form. Therefore in this chapter, we present our method of generating data from the power map available on the official web page of Load Despatch Center(https://posoco.in/).

## 2.1 Power map of India

For the work in this thesis, the data representing the electrical power transmission network of India is generated from power map available at an official web page(http://www.srldc.in/var/mandatory_/Compiled%20power%20maps%20of%20all%20Regions-2014.pdf) of Power System Operation Corporation Limited(POSOCO). The POSOCO is responsible for the secure, efficient and reliable operation of the Indian power transmission network. The power map has regional and state wise information of power stations and the transmission lines of different kV wires(66kV, 100kV, 110kV, 132kV, 220kV, 400kV, 765kV and HVDC(high voltage direct current)) joining the different stations. Images of all the five zones are given in figure 2.1, 2.4, 2.3, 2.5 and 2.2.

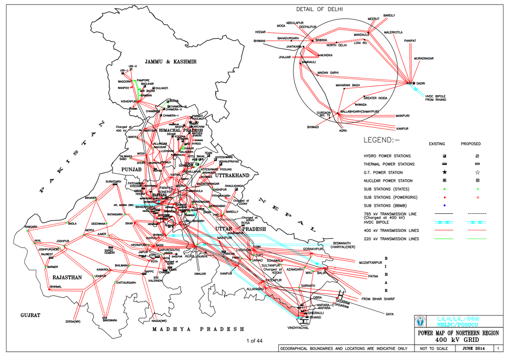


Figure 2.1: Power map of North India

The North India power map in figure 2.1 shows only 400kV and 765kV lines. Therefore we take the data for North India from state maps which include 220kV lines also.

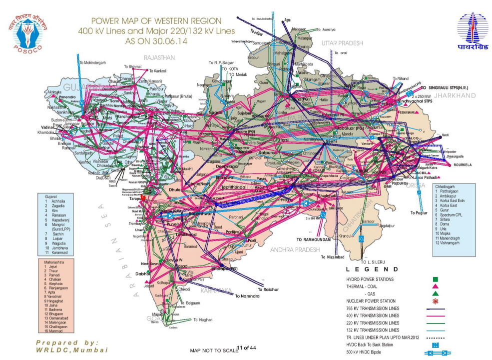


Figure 2.2: Power map of western zone with numbers assigned to stations

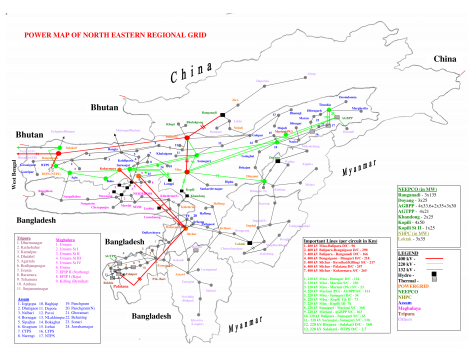


Figure 2.3: Power map of North East India

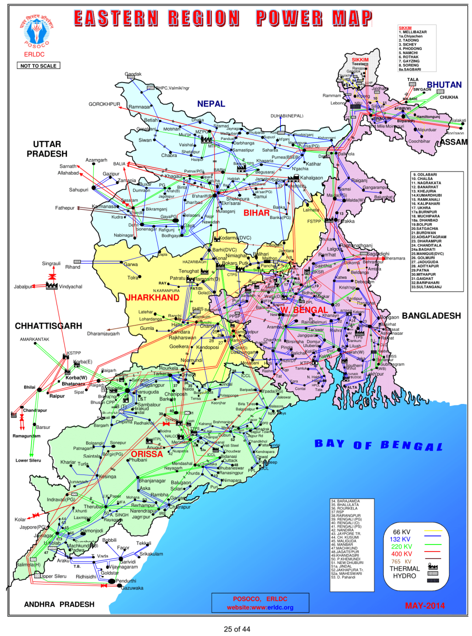


Figure 2.4: Power map of East India

In the map, we assign each power station a unique number starting from 0. We get a total of 2497 stations in 'India after omitting a few due to lack of clarity.

From the map we generate two data sheets, one of which contains the information about the number of transmission lines connecting the station to other stations and nature of the stations, whether a station is a generating station or a distribution station. If it is a generator then its type(thermal, windmill, hydroelectric or nuclear power plant) is also noted. Our second data sheet contains the information of connectivity, like which two stations are connected and through what kV lines. So if we consider the network weighted with different kV lines then we find six layers of network with six different kV lines. Then we get one unweighted network considering all the kV lines as unweighted. Since the total number of stations in lower kV lines like 66kV, 100kV, 110kV and 132kv, and in 765kV are relatively much smaller compared to other kV lines, we consider only 220kV, 400kV and whole network for our further studies.

To find the connectivity of Indian power transmission network system, we use Breadth-First Search (BFS) algorithm [24] with small changes. In particular, we keep count of nodes reached from search key node. This gives us the total number of nodes connected together.

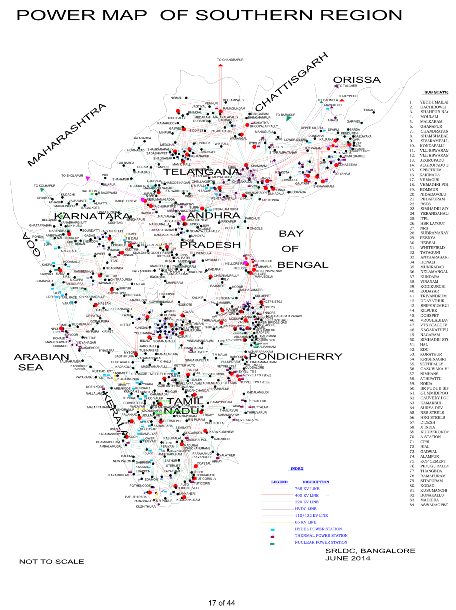


Figure 2.5: Power map of South India

To our surprise, we found many small clusters of stations isolated from other clusters. It is expected to find some isolated clusters in one particular kV line network because these isolated clusters may be connected through any other kV lines. But even on the complete unweighted network, we find a total of 10 isolated colonies. So we consider the single cluster with the highest number of stations as the primary cluster and the others with smaller number of stations as secondary clusters. The frequency and size of secondary clusters are shown in figure 2.6. All further calculations and studies are done considering the primary cluster only.

The reason for getting isolated clusters in the unweighted network is the presence of small independent power sources such as solar, windmills, hydro or nuclear power plant which provides energy for only a small number of stations. Some of such isolated stations in Kerala are shown in figure 2.7 with red circles around the stations which are not connected to the main network. Another reason can be the lack of information about the lower kV lines.

As presented in this chapter, we generated the data on the power transmission network

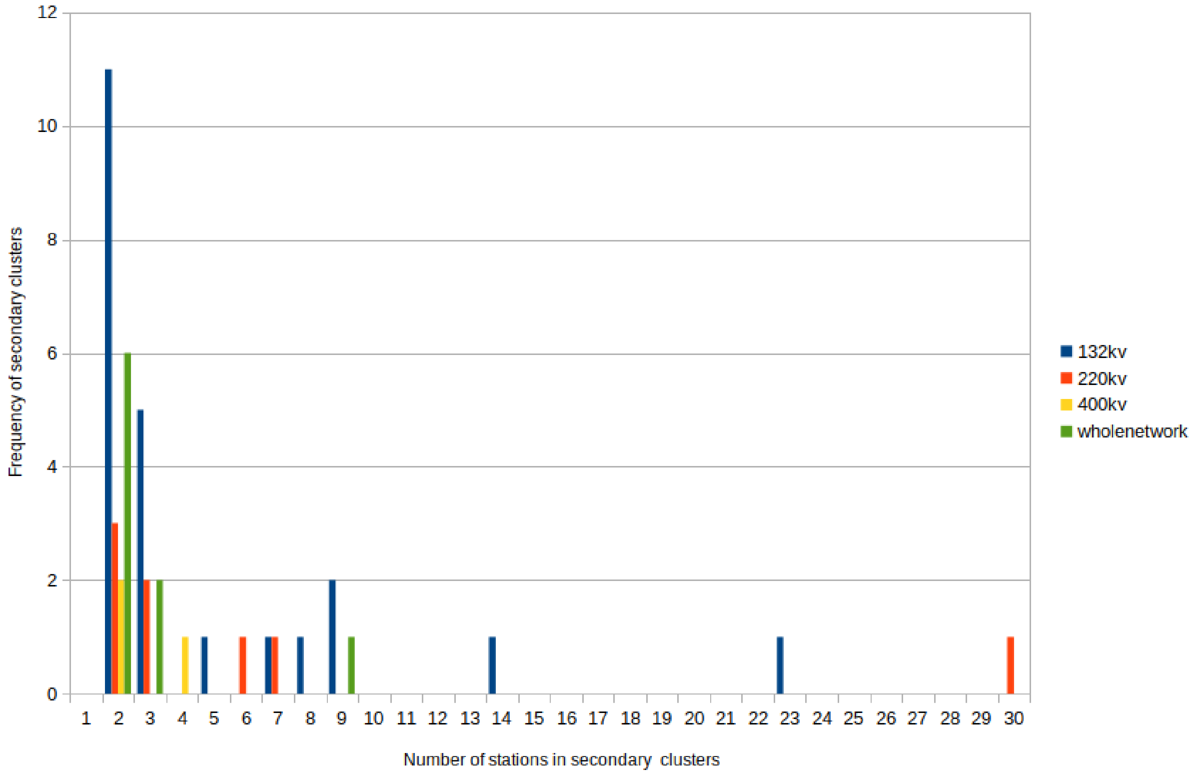


Figure 2.6: Secondary clusters of all kV lines with its size and frequency

of India. The details of the network data generated is summarised in table 2.1.

| Table2.1-Details of all stations and clusters | | | | |
|---|---|---|---|---|
| Networks | Total stations | Number of isolated clusters | Nodes in the largest cluster | Edges in the largest cluster |
| 132kv | 1043 | 29 | 340 | 400 |
| 220kv | 1404 | 10 | 1341 | 1855 |
| 400kv | 413 | 4 | 405 | 613 |
| Whole | 2497 | 10 | 2470 | 3764 |

In the next chapter, we present the results of our study on the topological characteristics of the generated network.

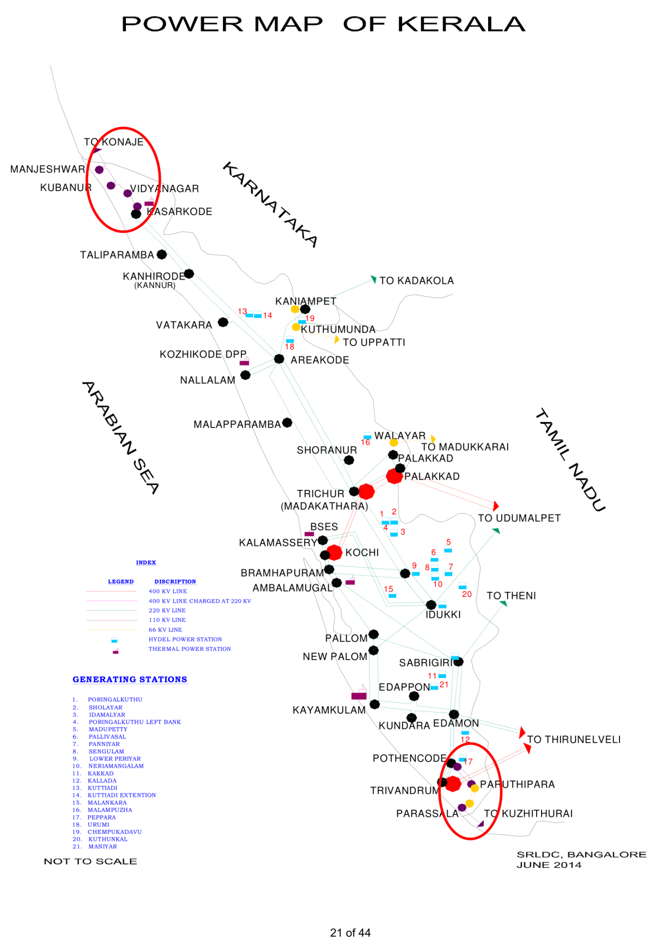


Figure 2.7: Power map of Kerala. The clusters of isolated stations are marked with red circle.

# Chapter 3

# Topological characteristics of Indian power network

This chapter reports the analysis of topological characteristics of the Indian power transmission network. Considering the power transmission network as a complex network, we study its emergent properties like degree distribution, clustering coefficient, characteristic path length and global efficiency.

## 3.1 Modelling power transmission network as a complex network

To represent Indian power transmission network as a complex network, we assume all types of stations(generator,transmission stations and distribution station) to be identical nodes. We consider three networks of the Indian power transmission network, first with 220kv line connection, second with 400kV line connection and third by including all the stations which are connected by any type of transmission lines. All three networks are considered as simple and unweighted networks so that the adjacency matrix A is a symmetric matrix with only zeroes as the diagonal elements.We considered the largest connected component in each network for all the characteristic measurements.

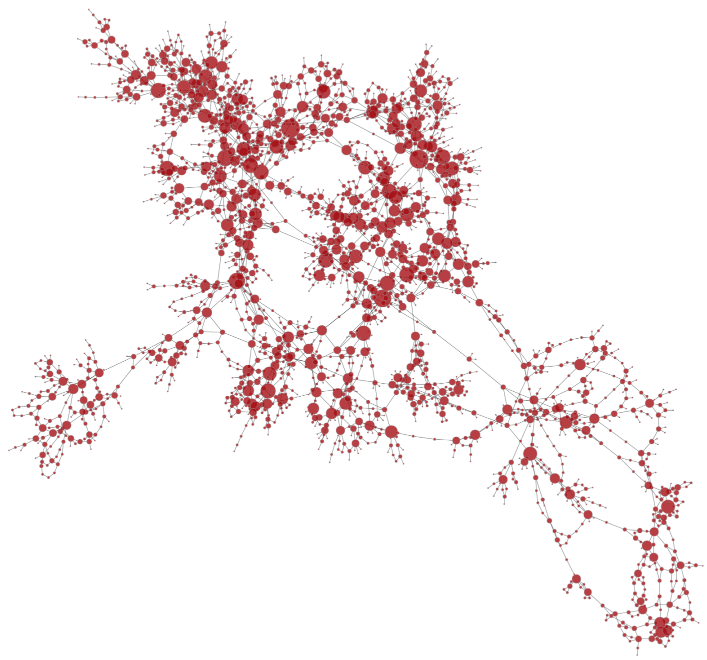

Figure 3.1: Visualization of the whole power transmission network of India made using Graph-tool. The size of each node is indicative of its degree

We generate the visualization of the network from the adjacency matrix using Graph-tool. Figure 3.1 shows the Indian power transmission network with 2470 nodes and 3764 edges. The representations of 220kv and 400kv networks are shown in figures 3.3 and 3.2.

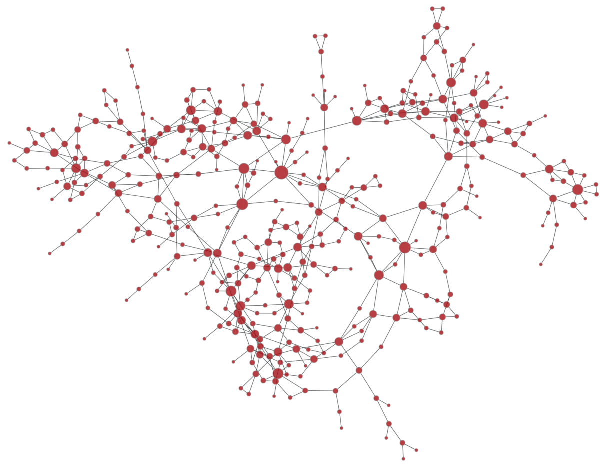

Figure 3.2: Visualization of the 400kv power transmission network of India made using Graph-tool. The size of each node is indicative of its degree

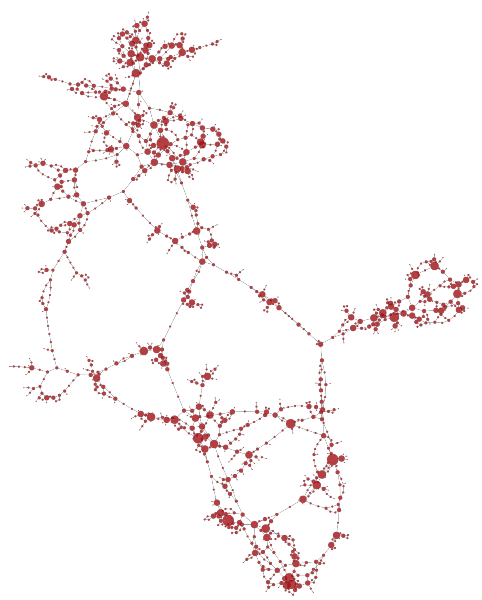

Figure 3.3: Visualization of the 220kv power transmission network of India made using Graph-tool. The size of each node is indicative of its degree

## 3.2 Degree distribution

We calculate the degree distribution of Indian power transmission network and we find that its degree distribution follows an exponential curve. We use Gnuplot to fit the data with an exponential function as

$$P(k) = C \exp^{-\frac{k}{\lambda}} \tag{3.1}$$

where C is normalizing constant and $\lambda$ is the topological parameter. The $\lambda$ value for all three networks(220kv,400kv and whole network) falls between 1.5 and 1.9. The degree distributions for the 220kv, 400kv and whole network are shown in figures 3.4, 3.5 and 3.6 respectively.

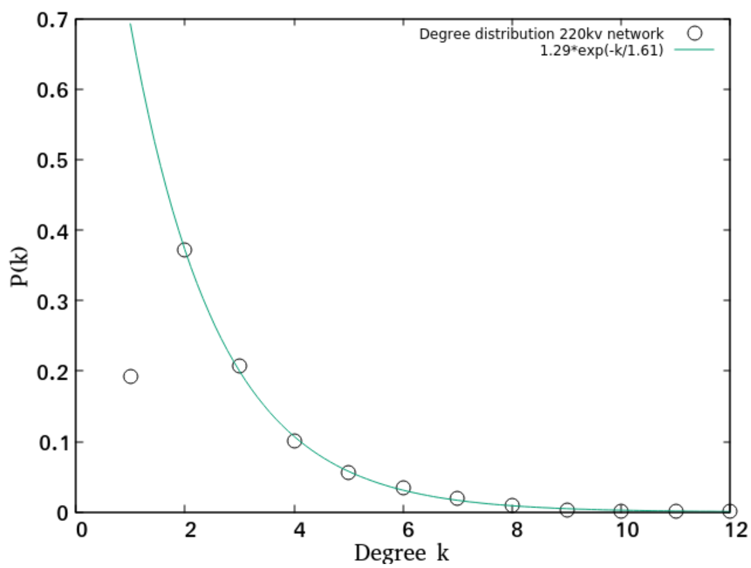


Figure 3.4: Degree distribution of 220kv line network. The x-axis shows degree of nodes and the y-axis is the fraction of node having particular degree. The dots are data from the network and the solid line represents the exponential fit with $\lambda = 1.61$

We note from the reported studies that most of the real world networks have a power law distribution [25, 26]. However, power transmission networks have exponential degree distribution [3, 2, 27, 19]. The table 3.1 shows the comparison of $\lambda$ values of the Indian power transmission network with that of the Iranian, North American and Italian power network.

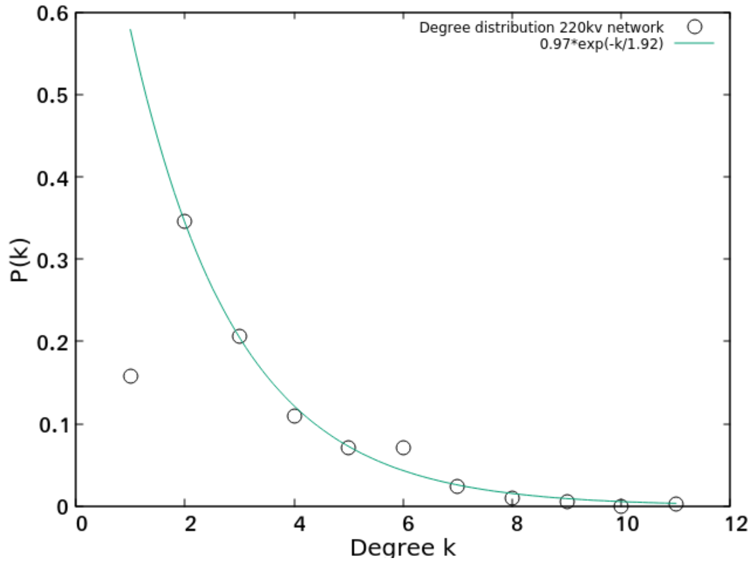


Figure 3.5: Degree distribution of 400kv line network. The x-axis shows the degree of nodes and the y-axis is the fraction of node having a particular degree. The dots are data from the network and the solid line represents the exponential fit with $\lambda = 1.92$

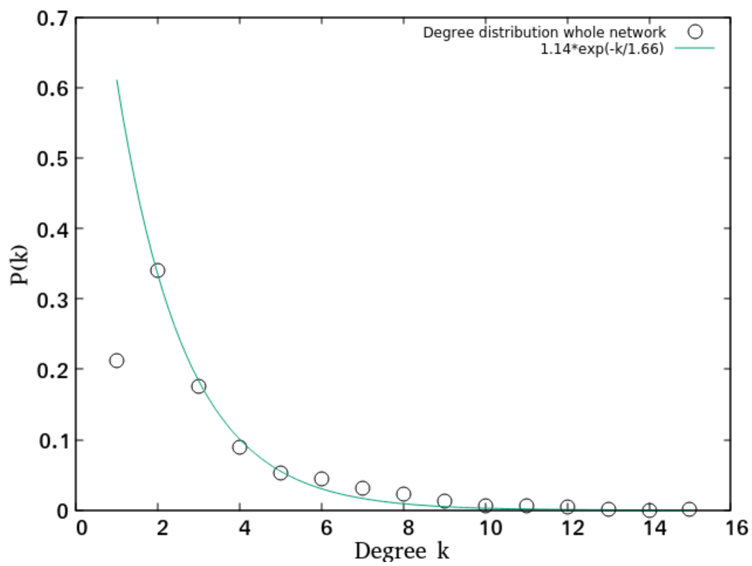


Figure 3.6: Degree distribution of whole Indian power transmission network considering all kv lines. The x-axis shows degree of nodes and the y-axis is the fraction of node having particular degree. The dots are data from the network and the solid line represents the exponential fit with $\lambda = 1.66$

| Networks | Nodes | Edges | $\lambda$ |
|---|---|---|---|
| 220kv | 1341 | 1855 | 1.613 |
| 400kv | 405 | 613 | 1.924 |
| Whole network | 2470 | 3764 | 1.665 |
| Iran [27] | 105 | 142 | 1.587 |
| North America [2] | 14099 | 19657 | 2.0 |
| Italy [3] | 341 | 517 | 1.818 |

Table 3.1: The $\lambda$ values of Indian, Iranian, North American and Italian power network.

## 3.3 Clustering coefficient

The average clustering coefficient of Indian power grid is calculated using equation 1.3 and equation 1.4. The values lie between 0.15 to 0.18. We want to see the extent to which the nodes of the same degree are clustered together, therefore we define clustering index as the measure of clustering among nodes having same degree.

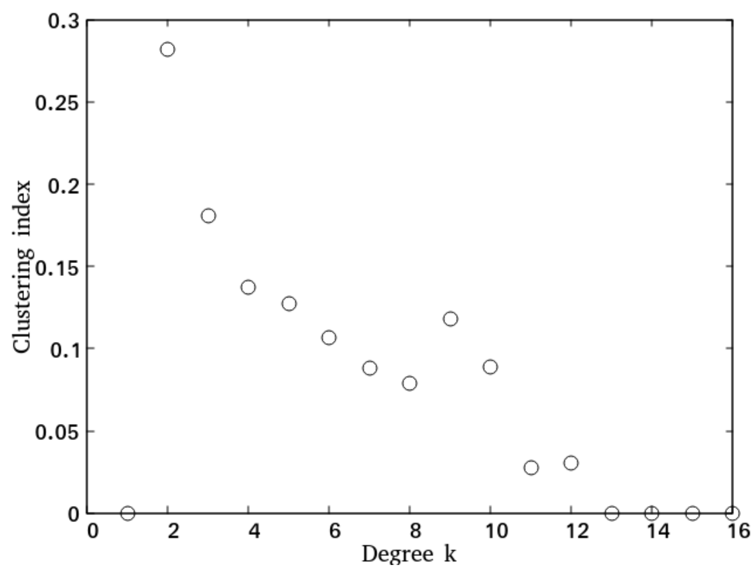


Figure 3.7: Clustering coefficient index of 220kv with clustering index on y-axis and degree in x-axis

From the figures 3.7,3.8 and 3.9 we can conclude that nodes with degree 2 are more probable to connect to nodes of same degree than nodes having different degrees. With the increase in degree, the clustering index of nodes decreases suggesting nodes with higher

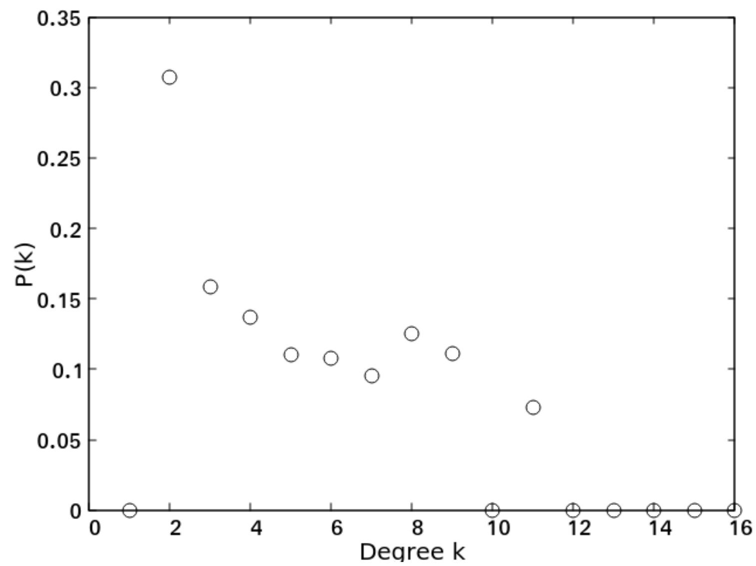


Figure 3.8: Clustering coefficient index of 400kv with clustering index on y-axis and degree in x-axis

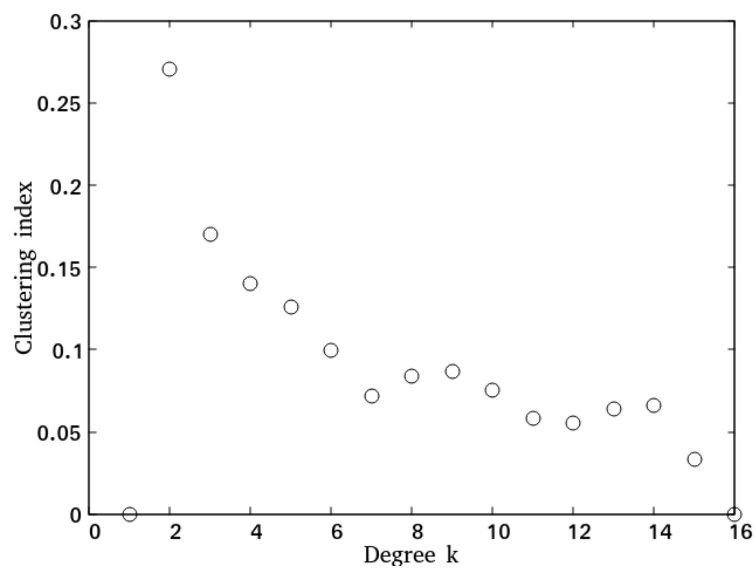


Figure 3.9: Clustering coefficient index of whole Indian network with clustering index on y-axis and degree in x-axis

degree are less probable to cluster together. But we see an increase in clustering index for nodes having degree 8 and 9, different from the expected trend.

## 3.4 Characteristic path length

In a power transmission network, a transmission or distribution station is connected to a generating station through more than one path and power can be transmitted from one station to another through any possible path. The power loss in transmission from one station to another station depends directly on the length of transmission line[28]. Hence we assume that power will flow with the maximum probability between any two stations only through the shortest possible path.

We do not take into account the actual length of connecting line between any two stations. But we consider the edge as a unit length as done in the framework of the complex network. $D_{n\times n}$ is a symmetric matrix with zeroes as diagonal elements for the simple network. The characteristic path length L of a network is the average of all shortest paths between any pair of nodes. We directly used Floyd-Warshall algorithm [29] for calculating the shortest path between every pair of nodes. The characteristic path length of the 220kv, 400kv and whole network is 25, 9 and 13 respectively.

## 3.5 Centrality

The concept of centrality has always been important in the analysis of networks [30, 31]. In the past, researchers have introduced a large number of centrality indices like degree centrality, closeness centrality [32] and betweenness centrality [33, 34]. All the different definitions of centrality are attempts to measure the importance of a node in the network.

### 3.5.1 Degree centrality

The simplest of the centrality indices is degree centrality $C_D$ which counts the number of nodes connected to any nodes. This means larger the degree of a node, more vital that node is to the network. If the load of a node depends on its degree, then the node with highest degree centrality is vital to the network. The degree centrality of a node v is

$$C_D(i) = \sum_{\forall j} a_{i,j} \tag{3.2}$$

### 3.5.2 Closeness centrality

The closeness centrality $C_C$ measures how close a node is to rest of the nodes in the network. The closeness centrality for a node v is inverse of the sum of all the shortest paths from node v to all the other nodes in the network. If the shortest path length between any pair of nodes is infinity(not connected), then this distance is not considered in the calculation of closeness centrality.

$$C_C(v) = \frac{1}{\sum_{t \in \mathbf{V}} d_{(v,t)}} \tag{3.3}$$

where $d_{(v,t)}$ is the shortest path between node v and node t. If one has to study the ability of a node to spread any information to the whole network, then closeness centrality is a better indicator.

### 3.5.3 Betweenness centrality

The betweenness $C_B(v)$ of a node v is defined as the fraction of shortest paths between all the pairs of nodes in a network that goes through the node v. The betweenness centrality is useful to calculate if any type of load flow between nodes travels through the shortest path. Hence betweenness centrality assumes the flow between nodes only through shortest paths.

$$C_B(v) = \sum_{s \neq v \neq t \in \mathbf{V}} \frac{\sigma_{st}(v)}{\sigma_{st}} \tag{3.4}$$

where $\sigma_{st}(v)$ is the number of the shortest paths between node s and node t that goes through node v.

In the case of a power transmission network, it is difficult to formulate actual power flow between stations, so we have assumed that power flows from a generator to load through the shortest path. This assumption allows us to calculate the betweenness centrality for the Indian power network. To calculate the betweenness centrality we used Brandes [34] algorithm.

In the next chapter, we show how the centrality measures are useful to study the vulnerability of Indian power transmission network against node removal.

# Chapter 4

# Vulnerability and cascade in the Indian power transmission network

In this chapter, we present the results of our study on the vulnerability of Indian power transmission network due to the removal of nodes. Since all the stations and substations are connected to each other, failure of any one node can affect the performance of other stations too. Therefore it is important to study the emergent behaviours that can take place in such complex network.

Specificaly we continue the study in two parts. First connectivity loss, where the random failure or targeted attack of nodes and the consiquence changes in the topology are studied. Then the possibility of cascades leading to blackouts due to overloading of the nodes is studied.

## 4.1 Connectivity loss

In this study, we remove one station and try to see how it affects the functioning of the rest of the network. The removal of one station need not have in general much impact on the connectivity of the network. We want to examine if the network disintegrates into small clusters by continuous removal of nodes. In the stable state, there are 2470 stations connected together in the unweighted network. We remove one node and calculate the fraction of the original network that is still connected. We calculate connectivity loss $C_L$ to quantify the decrease in the number of stations connected together, as

$$C_L = 1 - \frac{N_c}{N_o} \tag{4.1}$$

where $N_c$ is the number of stations connected after removing any fraction of nodes and $N_o$ is the total number of stations at the unperturbed state. After removing a certain fraction of nodes there may be a situation where the network will disintegrate into many isolated clusters, then we consider the remaining cluster with the largest number of connected stations as $N_c$. We apply this method to the Indian power transmission network first by removing random nodes corresponding to accidental failure, and then we remove nodes in their decreasing order of degree and betweenness centrality to take care of targeted attacks. We compare all three cases together to see what is the critical fraction of nodes after which the network will disintegrate. We perform this test on all three networks. The results of the study are given in figures 4.1,4.2 and 4.3.

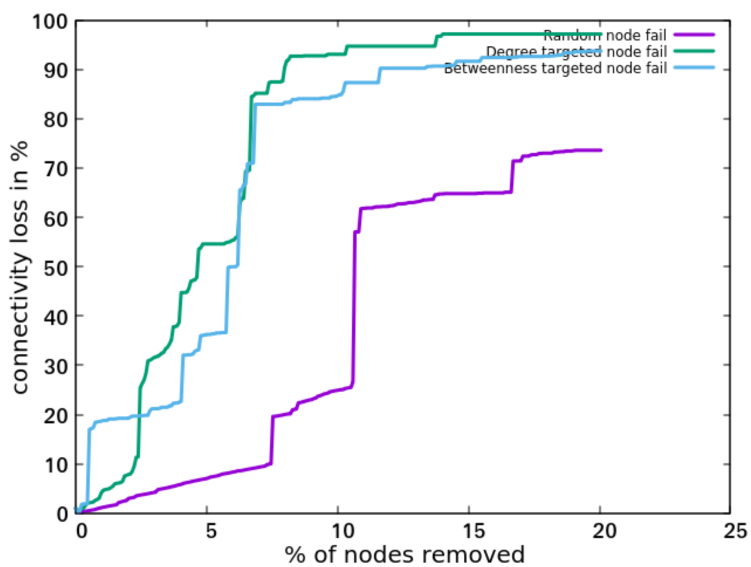


Figure 4.1: Effect of node removal on 220kv network. Connectivity loss in y-axis and % of nodes removed on x-axis.

The curve for random removal of nodes follows close to linear function initially, showing that Indian power transmission network is resilient to the random attack on small number of nodes. But targeted attacks on nodes with high degree or high betweenness have a severe effect on the network. In all three cases for targetted attacks, removal of even 2-3 % of nodes

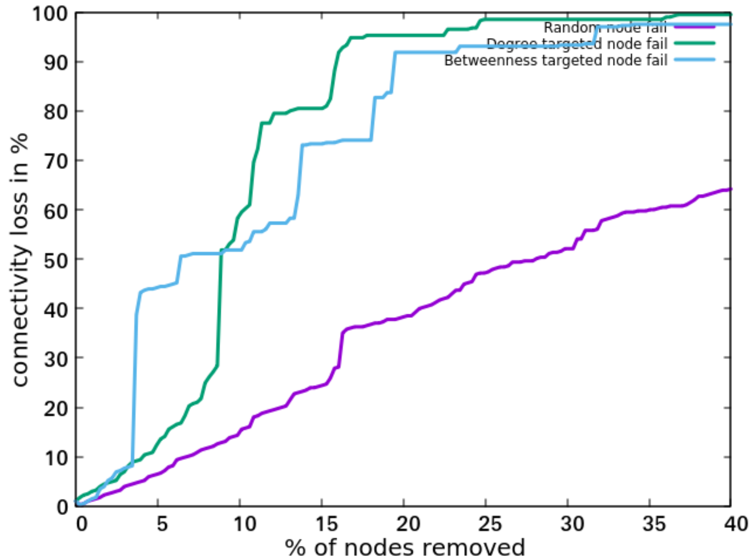


Figure 4.2: Effect of node removal on the 400kv network.Connectivity loss in y-axis and % of nodes removed on x-axis.

results in 30-40% loss in connectivity.

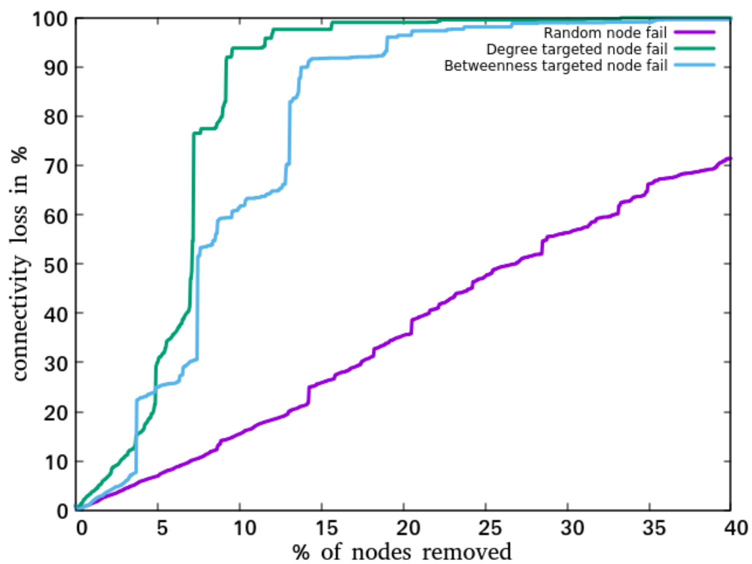


Figure 4.3: Effect of node removal on the whole network.Connectivity loss in y-axis and % of nodes removed on x-axis.

## 4.2 Cascade

In the study of complex networks, the load of a node is generally calculated by degree or centrality of the node. In the case of a power transmission network, we can assume that if any station $n_l$ is connected to a larger number of stations than some other station $n_s$ , then comparatively a larger amount of power will flow through $n_l$ than $n_s$. But sometimes it might be the case that all the stations that are connected to $n_l$ have degree 1, and $n_s$ is connected to the smaller number of stations but which are further connected to other stations, then $n_s$ will have larger load than $n_l$. To study the model of load distribution and cascade failure we use the model presented by Jian-Wei Wang and Li-Li Rong [35].

Jian-Wei Wang and Li-Li Rong consider the degree of neighbouring nodes also in addition to the degree to calculate the load of any node. So the load $L_j$ and capacity $C_j$ of node j depends on its degree $k_j$ and degree of its neighbours $\Gamma_j$. The capacity of any station is the maximum amount of power or load that it can withstand. The capacity of a node is assumed to be proportional to its load.

$$L_j = [k_j(\sum_{m \in \Gamma_j} k_m)]^{\alpha} \tag{4.2}$$

$$C_j = tL_j \tag{4.3}$$

In the equation 4.2, $\alpha$ is a parameter which controls the dependency of load on degrees. The capacity $C_j$ of a node j in equation 4.3 is t times its load $L_j$, where t is tolerance parameter that determines what multiple of load a node can withstand. Now if any node i fails by any means, then its load is redistributed to its set of neighbouring nodes $\gamma_i$, proportional to their loads. Equation 4.4 explains the load redistribution mechanism.

$$\Delta L_{ji} = L_i \frac{L_j}{\sum_{n \in \Gamma_i L_n}} \tag{4.4}$$

We want to see the effect of load redistribution by attacking nodes with both lower load and higher load. So we choose 1 node with high load and examine the cascade for different parameter values and each simulation result is averaged over ten realisations of randomly selected nodes. We also repeat the above procedure for nodes with lower load value to compare the effect of the node failure on both types of nodes. To calculate the effect of the avalanche of size $C_F$, we average the number of nodes failed $N_F$for every parameter and then divide it by the initial number of nodes N. For every parameter value of $\alpha$, we will get a critical tolerance $t_c$ value below which the network will suffer a large blackout.

$$C_F = \langle \frac{N_F}{N} \rangle \tag{4.5}$$

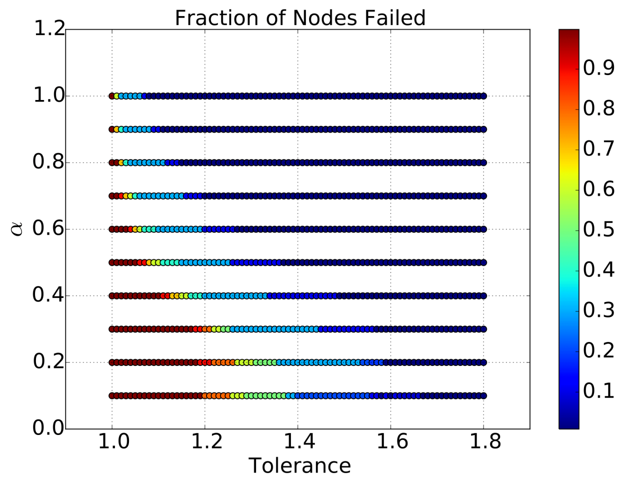


Figure 4.4: Cascade in network by removing node with low load.

Figure 4.4 and 4.5 show the effect of node failure by varying the tolerance parameter t and strength parameter $\alpha$. The colour code indicates the fraction of nodes failed. We see a uniform shift in colour with the change in $\alpha$ and $t_c$. We calculate the critical tolerance value $t_c$ below which a crucial fraction(10%) of the network will be damaged for removal of one node. The dependence of tolerance value $t_c$ on strength parameter $\alpha$ for an attack on both type of nodes, the lower load and higher load is shown in figure 4.6. From figure 4.6, we see an increase in thr critical tolerance value $t_c$ with increasing strength parameter $\alpha$ for the attack on nodes with higher load. But for nodes with lower load, the critical tolerance value $t_c$ decreases with increase in $\alpha$. We can conclude from the figure 4.6 that for $\alpha \leq 0.5$, nodes with higher load are critical to the network and for $\alpha > 0.5$, nodes with lower load are critical for triggering cascade on node removal.

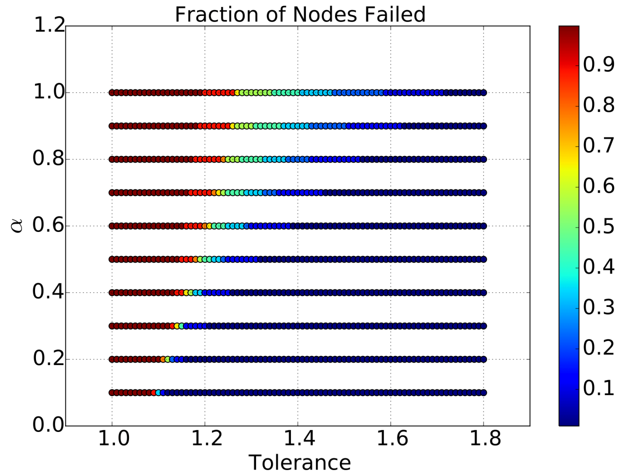


Figure 4.5: Cascade in network by removing node with high load.

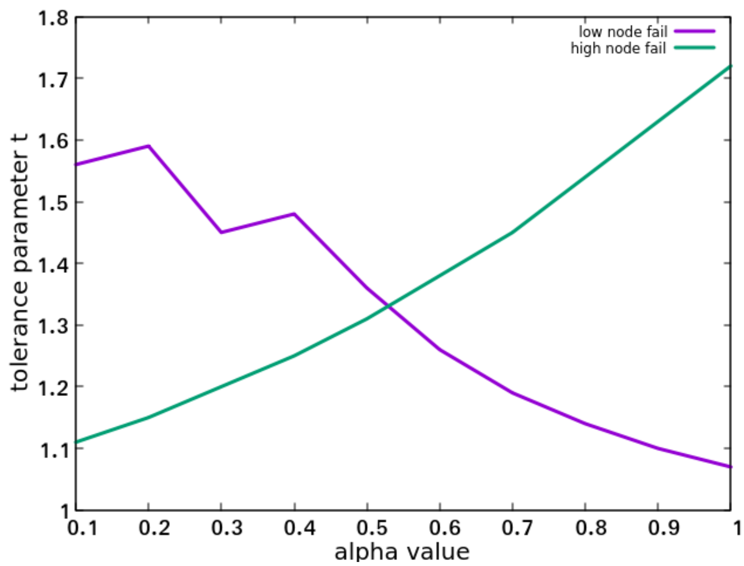


Figure 4.6: Relation between critical tolerance $t_c$ and $\alpha$ value for attacks on low load nodes(purple) and high load nodes(green).

In this chapter, we present the results of the two tests for networks vulnerability against random and targeted attacks on stations. The node removal model tests the effect of node removal on the connectivity of the network and the cascade model studies the effect of an overloaded power station on the network. In the next chapter, we conclude all the results that we obtained on the study of the Indian power transmission network.

# Chapter 5

# Conclusion

Major blackouts in the past and lack of detailed work on Indian power transmission network motivated us to study the Indian power transmission network using complex network analysis. We used the power map of India to generate the complete Indian power network data in the required format, which took around three months.

After generating the data, we first analyse the topological characteristics of Indian power network. The result from section3.2 shows that Indian power transmission network has an exponential degree distribution, with nodes having degree no more than 16. And the clustering index of nodes decreases with increase in their degree, suggesting nodes with lower degree tends to cluster together. We also notice a deviation from this pattern for node with degree 8 and 9. In general, the characteristic path length of Indian power transmission network has value i the range 9-25.

In the table below, we present the main results of the study on the topological characteristics, for the three networks studied. The computed characteristics are compared with that for a random network of same size and edges.

From the table of comparison, it is clear that Indian power transmission network differs substantially from the respective random graphs. This would mean that the growth of Indian power transmission network over time is not purely random. The stations are clustered together with clustering coefficient around 0.15, which is much more than the clustering coefficient of random graphs. The characteristic path length of Indian power transmission

| Table:Comparision of topology of Indian power transmission network with random networks | | | | | | |
|---|---|---|---|---|---|---|
| Networks | Topological characteristics | | | | | |
| - | Node | Edge | Average degree | Clustering coeff. | Characteristic path length | Global efficiency |
| 220kv | 1341 | 1855 | 2.767 | 0.173 | 25.47 | $6.01\times10^{-2}$ |
| Random1 | 1341 | 1836 | 2.767 | $3.91 \times 10^{-4}$ | 4.26 | $1.02 \times 10^{-3}$ |
| 400kv | 405 | 613 | 3.027 | 0.177 | 9.43 | 0.138 |
| Random2 | 405 | 599 | 3.027 | $1.68 \times 10^{-3}$ | 3.68 | $3.66 \times 10^{-3}$ |
| Whole network | 2470 | 3764 | 3.048 | 0.153 | 13.36 | $9.43 \times 10-2$ |
| Random3 | 2470 | 3720 | 3.048 | $9.44 \times 10^{-5}$ | 6.23 | $6.10 \times 10^{-4}$ |

network is $\approx 13$, which is more than that of a random network. The Indian power networks in general have more efficiency than the random graphs.

If we compare the 220kv, 400kv and whole network topology, there are subtle differences between them. The 220kv network is less clustered compare to other two. The characteristic path length of the 400kv network is lower than other two and it is most efficient also. The average degree of 400kv and the whole network are almost similar but slightly higher than that of the 220kv network.

The node removal model shows that Indian power transmission network is resilient to random attacks. But if targeted, only 5-10% of node removal is enough to disintegrate the network completely.

The cascade model based on degree-dependent load redistribution gives some counter-intuitive results for high load and low load nodes. For the strength parameter $\alpha > 0.5$, attacking nodes with low degree(that is low load) will have more cascading effects on the network.

## 5.1 Future work

There is a lot of scope for further study on Indian power transmission network. The complex network is a good tool to simplify the complexity of power flow and study its properties emerging from the topology of the network. Some of the future directions of study are:

1. Considering a dynamical system modellying a power flow on the nodes, we can study the emergence of synchronisation and how node/link removal causes desynchronisation

etc.

2. Considering the whole network as a multilayer network each layer being that of one type of kV line, we can study the network in much more detailed manner.

3. Determining the stability of a dynamical state against local or non-local perturbation by considering the state's basin volume as a measure of its stability is also relevant for future work.

The present study along with the future trends mentioned above can lead to very comprehensive and technically relevant results on the power transmission network of India. This will be useful from an engineering point of view when strategies to prevent attacks or stop the spread of cascade are to be considered in detail.

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