# Bootstrapping Cosmological Correlators

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by

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# Certificate

This is to certify that this dissertation entitled Bootstrapping Cosmological Correlatorstowards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by Kushan Panchal at Indian Institute of Science Education and Research under the supervision of Diptimoy Ghosh, Assistant Professor, Department of Physics , during the academic year 2018-2023.

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This thesis is dedicated to my parents and my Nana-Nani

# Declaration

I hereby declare that the matter embodied in the report entitled Bootstrapping Cosmological Correlators are the results of the work carried out by me at the Department of Physics, Indian Institute of Science Education and Research, Pune, under the supervision of Diptimoy Ghosh and the same has not been submitted elsewhere for any other degree.

Kushan

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# Abstract

Both Conformal and Boostless Bootstrap techniques have been applied by many in the literature to compute pure scalar and graviton inflationary correlators. In this thesis, our focus will primarily be on mixed graviton and scalar correlators. We start by reviewing single-field inflation and then move ahead to developing an EFT of Inflation (EFToI) with some general assumptions, clarifying various subtleties related to power counting. We verify explicitly the soft limits for mixed correlators, showing how they are satisfied for higher derivative operators beyond the Maldacena action. We clarify some confusion in the literature related to the soft limits for operators that modify the power spectra of gravitons or scalars. We then proceed to apply the boostless bootstrap rules to operators that do not modify the power spectra. Towards the end, we give a prescription that gives correlators for $\alpha$ vacua directly once we have the correlator for the Bunch-Davies vacuum. This enables us to bypass complicated in-in calculations for $\alpha$ vacua

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# Chapter 1

# Introduction

It is believed that the structure in our universe was seeded by quantum mechanical fluctuations generated during an epoch of exponential expansion known as inflation [1, 2, 3]. During this period, these quantum fluctuations were stretched to super-horizon distances with amplitudes freezing post horizon exit [4, 5]. Inflation generates correlations between these fluctuations, which seed the late-time cosmological observables such as temperature correlations on the CMB. Although current observational reach is limited to deducing the scalar power spectrum and its tilt [6, 7, 8], one can expect that in the future, higher point scalar correlators, as well as correlators involving spinning fields such as the graviton, will also be measured.

There are a wide variety of models for Inflation [9, 10] and one can compute observables for each of them. However, one can adopt a more general (model-independent) approach by constructing an Effective Field Theory of Inflation (EFToI) [11, 12, 13] consistent with symmetries. This allows us to go beyond the minimally coupled canonical single field slow-roll inflation [14]. In this thesis, we consider higher derivative operators which contribute to the mixed three-point correlation functions (i.e. $\langle\zeta\zeta\gamma\rangle$, $\langle\gamma\gamma\zeta\rangle$, which we compute using the in-in formalism). It is important that the EFT respects soft limits [14, 15, 16] provided that we do not violate any of the assumptions implicit in their derivations. We clearly state under what assumptions these theorems are expected to hold and explicitly check (for mixed correlators) that they are satisfied for operators that modify both scalar as well as tensor power spectra. This provides important consistency checks for our calculations.

It is also interesting to bootstrap these correlators from a pure boundary perspective without referring to the bulk evolution. This approach has a lot of advantages, for instance, it was shown in [17, 18] that using certain field redefinitions (that vanish at the boundary) some operators can be removed from the EFT without affecting the late-time correlators. This redundancy, by construction, is not present once we have a purely boundary perspective and therefore can potentially lead to a lot of simplifications. There is a large amount of literature on both the conformal/pure de-sitter bootstrap [19, 20, 21, 22] as well as the "boostless bootstrap" approach, the latter being more recent [23, 24, 25]. The boostless bootstrap program focuses on properties such as the analytical structure of the correlators, soft limits etc. The analytical structure of correlators on its own gives a lot of information such as the initial state [26, 27, 28] and the flat space amplitude [16, 21] corresponding to the interaction. This technique has had considerable success for pure graviton correlators (sometimes in conjunction with tools like spinor helicity formalism and results related to parity and the cosmological optical theorem [29, 30, 24]). In this thesis, we follow the rules entailed in [23] to bootstrap three-point mixed correlators arising from operators in our EFToI that do not modify power spectra, and check to what extent the Boostless Bootstrap works. In doing so, we separately consider local and non-local interactions.

Finally, we give a prescription for obtaining the correlators for $\alpha$- vacua (which is the most general family of vacuum states consistent with de Sitter isometries [31]) once we have the answers for the Bunch Davies (BD) vacuum. This offers significant computational benefit since one does not have to repeat the cumbersome in-in calculation for $\alpha$- vacua.

**Imporant Note:** The Introduction and Conclusion, as well as the results in Sections 5.2, and 6.4 to **??**, are mostly taken from my pre-print [32] written in collaboration with Prof. Diptimoy Ghosh and Farman Ullah.

# Chapter 2

# de-Sitter and Conformal Bootstrap

The de-Sitter metric is the metric given by:

$$-dt^2 + e^{2Ht}(dx^2 + dy^2 + dz^2) \tag{2.1}$$

where we denote the scale factor by $a(t) = e^{Ht}$, $H$ being Hubble's constant . One can also replace $t$ by $\eta$, the conformal time coordinate defined by $d\eta = dt/a(t)$. A scalar field of mass $m$ in dS would have the quadratic lagrangian:

$$S_2 = \int d^4x\, a^4 \;\; \frac{1}{2}\left(\frac{(\partial_\mu \phi)^2}{a^2} - m^2\phi^2\right) \tag{2.2}$$

with the mode functions given by [33]:

$$u_k(\eta) = \frac{H}{2} e^{\nu + \frac{1}{2}\pi} - \eta^{3/2} \mathscr{H}_\nu^{(2)}(-k\eta) \qquad \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \tag{2.3}$$

$\mathscr{H}_\nu^{(2)}$ being the Hankel function of the 2nd kind. For each field, there's generally a "scaling dimension" associated to it in the literature denoted by $\Delta = \frac{3}{2} + \nu$. So, we have $\Delta = 3$ for massless fields for whom,

$$u_k(\eta) = \frac{H}{\sqrt{2k^3}}(1 - ik\eta)e^{ik\eta} \tag{2.4}$$

Note that the choice of mode functions is such that they resemble the Minkowski mode functions, $e^{ik\eta}$ in the far past ($\eta \to -\infty$ ). The choice of vacuum corresponding to this choice of mode function is the Bunch Davies (BD) vacuum. However, this is not a compulsion and one can in general have any Bogolyubov transformation of BD as the vacuum. There's a single parameter family of vacua, called $\alpha$ vacua which respects the dS isometries, with BD being the $\alpha = 0$ case.

Their mode functions are defined as:

$$u_k(\alpha,\eta) = \cosh\alpha \; u_{k,BD}(\eta) - i\sinh\alpha \; u_{k,BD}(\eta)^* \tag{2.5}$$

## 2.1 de-Sitter symmetry group

The de-Sitter symmetry group can easily be guessed to have 10 elements: 3 spatial translations, 3 rotations, 1 scaling and 3 Special Conformal Transformations (SCTs, also called boosts colloquially). The generators for these can be found by solving the killing vectors of the metric:

$$ds^2 = -\frac{1}{H^2\eta^2}\left(-d\eta^2 + dx^2 + dy^2 + dz^2\right) \tag{2.6}$$

$$\begin{aligned} T_i = -i\partial_i \quad & \text{3 Translations} \\ J_k = -\frac{i}{2}\varepsilon_{ijk}(x_i\partial_j - x_j\partial_i) \quad & \text{3 Rotations} \\ S = -ix^\mu\partial_\mu \quad & \text{Scaling} \\ U_j = -i(\eta^2 - \vec{x}^2)\partial_j - i(2x^jx^i)\partial_i - i(2\eta x^j)\partial_0 \quad & \text{3 SCT} \end{aligned}$$

We have the following important commutation relations:

$$\begin{aligned} [T_i, S] &= -iT_i \\ [J_i, S] &= -iJ_i \\ [U_i, S] = iU_i \quad [U_i, U_j] &= 0 \\ [T_i, U_j] = 2i\varepsilon_{ijk}J_k - 2\delta_{ij}S \quad [J_i, U_j] &= i\varepsilon_{ijk}U_k \end{aligned}$$

If we take the linear combinations $A_i = \frac{T_i+U_i}{2}$ and $B_i = \frac{T_i-U_i}{2}$, we get from the resulting algebra that the $A_i$ s act like rotations along with $J_i$ s and $B_i$ s and S together are like 4 Lorentz boosts, hence the group is locally isomorphic to SO(1,4). Another important thing to notice is that in the limit $\eta \to 0$, the generators look exactly like the generators of the 3-dimensional Euclidean CFT group. Hence, correlators at the boundary of de-Sitter can be bootstrapped using the machinery of CFT.

## 2.2 The OPE

The operator product expansion gives us asymptotic expressions for the product of 2 or more CFT operators. OPEs are generally algebraically derivable in 2-d CFTs where one of the operators is the energy-momentum tensor. However, more generally,in a CFT we have the state-operator correspondence i.e. every state which is an eigenstate of the dilatation operator with eigenvalue $\Delta$ can be represented by an operator $O_\Delta$ acting on the vacuum $|\Omega\rangle$ (see radial quantization in [34]). For our purposes, we need the following result:

$$\mathscr{O}_1(x)\mathscr{O}_2(0)\,|\Omega\rangle = \sum_k \frac{\tilde{\mathscr{O}}_k(0)}{x^k} \qquad (2.7)$$
$$\text{where we have} \quad k = \Delta_1 + \Delta_2 - \Delta_k$$

## 2.3 Ward identities and the 2-point function

Momentum space scaling and SCT identities for scalars with scaling dimensions $\{\Delta_a\}$ are :( here $\Delta_t = \sum_a \Delta_a$)

$$\left(2d - \Delta_t + \sum_a p_a \frac{\partial}{\partial_a}\right) \langle 0|\,\phi(p_1)\phi(p_2)..\phi(p_a)\,|0\rangle = 0 \qquad (2.8)$$

$$\left(\sum_a 2(\Delta_t - d)\frac{\partial}{\partial p_{a\mu}} - 2p_{a\nu}\frac{\partial}{\partial p_{a\nu}}\frac{\partial}{\partial p_{a\mu}} + p_{a\mu}\frac{\partial}{\partial p_{a\nu}}\frac{\partial}{\partial p_{a\nu}}\right) \langle 0|\,\phi(p_1)\phi(p_2)...\phi(p_a)\,|0\rangle = 0 \qquad (2.9)$$

For the 2-point correlator, the scaling identity by itself is sufficient to fix the correlator:

$$\langle \phi_\Delta(\vec{k}_1)\phi_\Delta(\vec{k}_2)\rangle \sim k_1^{2\Delta-3}\delta^{(3)}(\vec{k}_1 + \vec{k}_2) \qquad (2.10)$$

For the 3-point correlator, we can expand the operator $\mathscr{O}^\mu$ , the differential operator of the SCT idenitity as $\mathscr{O}^\mu = p_1^\mu \mathscr{O}_1 + p_2^\mu \mathscr{O}_2$ and we define operators $K_{ij} = K_i - K_j$ where

$$K_i = \frac{\partial^2}{\partial p_i^2} - \frac{2\Delta_i - d - 1}{p_i}\frac{\partial}{\partial p_i}$$

The triple K integral :

$$F(p_1, p_2, p_3) = \int_0^\infty dx x^\alpha \prod_i p^{\beta_i} K_{\beta_i}(p_i x) \qquad \alpha = \frac{d}{2} - 1 \;\; \beta_i = \Delta_i - \frac{d}{2}$$

satisfies $K_{ij}F = 0$ and hence it satisfies both the SCT and scaling ward identities. This solution to the ward identities is unique after demanding OPE consistency (i.e. only BD vacuum-type poles). The integral converges when $|\alpha| > \sum_i |\beta_i| - 1$. The other cases need some regulation to the solutions, details of which can be found in [35].

## 2.4 Bootstrap of a Mixed 3-pt correlator in dS

This section reviews the results in [19]. We consider an example of bootstrapping $< OOT_{ij} >$ using full de-sitter isometries. $O$ and $T_{ij}$ can be considered as the dual CFT operators for $\phi$ and $\gamma_{ij}$ which have dimensions equal to $\Delta_{\mathscr{O}} = \Delta_{\phi} = 3$ and the same for $T_{ij}$. This notation is borrowed from the Wavefunction Formalism [36]. We can bootstrap without using these operators, but using this formalism makes it easier to use the CFT language. Now the operators transform under an SCT transformation as:

$$
\begin{aligned}
&O(k) \to O(k) + (b\cdot k)\partial_k^2 O(k) - 2K_i b_j \partial_i \partial_j O(k)\\
&T_{ij}(k) \to T_{ij} + (b\cdot k)\partial_k^2 T_{ij}(k) - 2k_i b_j \partial_i \partial_j T_{ij}(k) + 2(b_j\partial_a - b_a\partial_j)T_{ai} + 2(b_i\partial_a - b_a\partial_i)T_{aj}
\end{aligned}
$$

and have the 2-point and 3-point correlations:

$$
\langle \zeta(k_1)\zeta(-k_1)\rangle = \frac{1}{\langle O(k_1)O(-k_1)\rangle} \quad \langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = \frac{O(k_1)O(k_2)O(k_3)}{\langle O(k_1)O(-k_1)\rangle\langle O(k_2)O(-k_2)\rangle\langle O(k_3)O(-k_3)\rangle} \tag{2.11}
$$

We assume a schematic form based on permutation symmetry of $k_1, k_2$:

$$
\begin{aligned}
\langle O(k_1)O(k_2)T_{ij}(k_3)\rangle &= k_{1i}k_{1j}F_A(k_1,k_2,k_3) + k_{2i}k_{2j}F_A(k_2,k_1,k_3) + k_{1i}k_{2j}F_B(k_1,k_2,k_3)\\
&\quad + k_{2i}k_{1j}F_B(k_1,k_2,k_3) + \delta_{ij}F_C(k_1,k_2,k_3)\\
&\implies \langle O(k_1)O(k_2)T_{ij}(k_3)\rangle e_3^{ij} = -2S
\end{aligned}
$$

where $S = \frac{1}{2}(F_A(k_1,k_2,k_3) + F_A(k_2,k_1,k_3) - 2F_B(k_1,k_2,k_3))$.

Using the ward identity of the transformations above, we get the following equations after a lot of simplifications:

$$
\left(\frac{\partial^2}{\partial k_1^2} - \frac{2\partial}{k\partial k_1}\right)S = \left(\frac{\partial^2}{\partial k_2^2} - \frac{2\partial}{k\partial k_2}\right)S = \left(\frac{\partial^2}{\partial k_3^2} - \frac{2\partial}{k\partial k_3}\right)S \tag{2.12}
$$

$$
(\vec{k}_2\cdot\vec{k}_3)k_1\partial_{k_1}S - (\vec{k}_1\cdot\vec{k}_3)k_2\partial_{k_2}S + (k_1^2 - k_2^2)S - \frac{3}{2}(k_1^3 - k_2^3) = 0 \tag{2.13}
$$

We can guess the solutions of these equations to be of the form

$$
S = \int_0^\infty dz \frac{m_{abc}}{z^2}(1 - ak_1\eta)e^{iak_1\eta}(1 - bk_2\eta)e^{ibk_2\eta}(1 - ck_3\eta)e^{ick_3\eta}
$$

where a, b,c can be $\pm 1$. If we fix c=1, Equation 2.13 gives constraints on $m_a bc$ :

$$\sum_{a,b} m_{a,bc} a^3 = \sum_{a,b} m_{a,bc} a^3 = 1 \tag{2.14}$$

The z dependence comes from the scale invariance identity. Hence we have 8 different solutions. We can eliminate 7 of them by considering the OPE limit and Maldacena consistency conditions:

- **Maldacena soft limit**: We can convert the $< OOT_{ij} >$ correlator to the $< \zeta\zeta\gamma_{ij} >$ correlator and then apply the condition (discussed in detail in Section 5.1)

  $$\lim_{k_1 \to 0} \langle \gamma(k_1)\zeta(k_2)\zeta(k_3)\rangle = -k_{2i}k_{2j}\frac{d}{dk_2^2}\langle \zeta(k_2)\zeta(-k_2)\rangle$$

  .

- **OPE**: We consider two limits here. First we consider $k_1 << k_2, k_3$ . Using

  $$O(0)T_{ij}(x) \approx \frac{x_i x_j}{x^5} O(0) + \frac{(x_i\partial_j + x_j\partial_i)O(0)}{x^3} + \frac{\partial_i\partial_j O(0)}{x} + \dots$$

  We get that

  $$\lim_{k_1 \to 0} \langle O(k_1)O(k_2)T_{ij}(k_3)\rangle \approx \frac{k_{1i}k_{1j}k_1^3}{k_3^2}$$

  . Secondly, we consider $k_3 << k_1, k_2$ . Using

  $$O(0)O(x) \approx \frac{x_i x_j}{x^5} T_{ij}(0) + \dots$$

  we get that

  $$\lim_{k_3 \to 0} \langle O(k_1)O(k_2)T_{ij}(k_3)\rangle \approx \frac{k_{3i}k_{3j}k_3^3}{k_2^2}$$

  .

Note that to compare our solutions to these OPE limits, we only consider the terms non-analytic in the large momenta as they will give the dominant terms in the position space. This procedure finally fixes our correlator to give $\langle \gamma\zeta\zeta\rangle$ calculated in Section 3. The calculations can also be done using spinor helicity variables, as shown in [19]

## 2.5 4 point functions in dS

We follow the analysis of [21] in this section. To bootstrap four-point correlators, we again invoke the ward identities given in equation 2.9. Using the notation $C(u,v,s)$ for the 4-point correlator, with $s = \left|\vec{k}_1 + \vec{k}_2\right|$, $u = \frac{s}{k_1+k_2}$ and $v = \frac{s}{k_3+k_4}$, the quantity $G(u,v) = sC(u,v,s)$ would satisfy the following equation:

$$(\Delta_u - \Delta_v)G(u,v) = 0 \qquad \Delta_u = u^2(1-u^2)\partial_u^2 - 2u^3\partial_u \tag{2.15}$$

To get the simplest possible contact diagram, i.e with the vertex having no derivatives, we take the solution with the simplest possible total energy pole structure (i.e. the $u+v$ pole structure). Then we have

$$G_0(u,v) = \frac{uv}{u+v}$$

Higher derivative answers can be obtained from this answer by the ansatz:

$$G^{HOD}(u,v) = \sum_n c_n(\Delta_u)^n G_0(u,v) \tag{2.16}$$

which for example is $c_0 = 1, c_1 = 1, c_2 = 1/4$ and rest all 0 , for $(\nabla\phi)^4$. Now for exchange diagrams, we start again with the simplest possible vertex, $\phi^3$. Denote the correlator by E(u,v) and $H(u,v) = sE(u,v)$. Then, for a massive particle of mass $M$ being exchanged, we have:

$$\left(\Delta_u + \frac{M^2}{H^2} - 2\right)H(u,v) = G_0(u,v) \tag{2.17}$$

This relation is derived basically from the structure of the in-in integral which is given by:

$$E(u,v) \sim \mathrm{Re}\left(\iint d\eta d\eta' e^{i(k_1+k_2)\eta} e^{i(k_3+k_4)\eta'} G_F(s,\eta,\eta')\right) \tag{2.18}$$

where $G_F(s,\eta,\eta')$ is the propagator for the exchanged particle. Noting that the propagator satisfies the EOM we're led to the relation:

$$‘\left(s^2\partial_s^2 + s\partial_s - s^2\partial_{k_1+k_2}^2 + \frac{M^2}{H^2} - \frac{9}{4}\right)E(u,v) = 0 \tag{2.19}$$

which leads us to Equation 2.17. Hence, we note that this relation is not derived from a complete CFT/ boundary perspective. To solve the equation, we keep in mind the following:

- we expect the amplitude to be the residue of the total energy pole/discontinuity. One can already see this by taking the limit before solving the equation which gives us:

$$\lim_{u+v\to 0}\frac{v^2}{1-v^2}\frac{\partial^2 H(u,v)}{\partial u^2}=\frac{uv}{u+v} \tag{2.20}$$

$$\implies E(u,v)\to\frac{-k_T\log k_T}{s_{flat}} \tag{2.21}$$

hence giving $1/s_{flat}$ i.e. the flat space amplitude in the high energy limit.

- The answer should be symmetric in u,v and should match at u=v.
- The answer should not contain any poles in the limit $u,v=1$ as these pertain to excited states [26, 28] and we are considering BD vacuum.

The solution to equation 2.17 respecting the conditions above, can be written using the power series method as:

$$H(u,v)=\begin{cases}\sum_{n,m}c_{mn}u^{2m+1}(\frac{u}{v})^n+g(u,v) & \text{if u} \leq \text{v}\\ \sum_{n,m}c_{mn}v^{2m+1}(\frac{v}{u})^n+g(v,u) & \text{if v} \leq \text{u}\end{cases} \tag{2.22}$$

$$c_{mn}=\frac{(n+1)(n+2)...(n+2m)}{((n+\frac{1}{2})^2+\mu^2)((n+\frac{3}{2})^2+\mu^2)....((n+2m+\frac{1}{2})^2+\mu^2)}$$

where $\mu^2=\frac{M^2}{H^2}-\frac{1}{4}$. The form of g(u,v) is quite complicated and can be found in [21]. Similarly, one can have spinning particles being exchanged for which we can find the correlator by applying projection tensors and raising lowering operators, details of which can be found in [31]. One must now convert the $\Delta=2$ answers to $\Delta=3$ ones. For this, we have weight-shifting operators, which depend on the spin of the particles being exchanged.

Denoting the massless external field by $\phi_0$, For scalar exchange of a massive field, say $X$, the interaction $X\phi^2$ can be mapped to $X(\nabla\phi_0)^2$ i.e to an interaction with 2 more derivatives through:

$$(\nabla_\mu\phi_0(k_1)\nabla_\nu\phi_0(k_2))=s^2U_{12}\phi(k_1)\phi(k_2) \tag{2.23}$$

$$U_{12}(y)=\frac{1}{2}\left(1-\frac{k_1k_2}{k_1+k_2}\partial_{k_1+k_2}\right)\left(\frac{1-u^2}{u^2}\partial_u(uy)\right) \tag{2.24}$$

which, after some tedious calculations, gives us a relation between the correlators:

$$E(u,v,\phi_0)=s^3U_{12}U_{34}E(u,v,\phi) \tag{2.25}$$

For spinning exchange fields, we have to have separate weight-shifting operators for each helicity component, shown in [21] in great detail.

The approach is generalizable to interactions with an arbitrary number of derivatives which a big advantage. The main limitation of this analysis is that it is not a pure boundary calculation as it has multiple references to in-in formalism and Bunch-Davies initial conditions (while fixing the pole structures). Hence, it is difficult to solve equations like Equation 2.17 for $\alpha$ vacua since the ansatz would depend on additional variables like $k_1 - k_2$ and $k_3 - k_4$. However, as we'll see in later sections, a direct bootstrap for $\alpha$ vacua is not necessary.

# Chapter 3

# Single Field Inflation

## 3.1 Slow roll Inflation

In single-field inflation, we have a single scalar field sourcing the metric fluctuations about a pure de-sitter background. The most famous single-field inflation is slow roll inflation. The inflaton field, denoted by $\phi(t)$ "rolls" on a potential $V(\phi)$ in a pure de-sitter background. The EOM for the field is given by:

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + \frac{\partial V}{\partial \phi} = 0 \tag{3.1}$$

For slow roll inflation, we require that the kinetic energy term of the inflaton is very small compared to the "Hubble friction" generated by the second term in the equation above. We have the following two small parameters characterizing slow roll inflation(all defined in $M_{pl} = 1$ units) [14]:

$$\varepsilon = \frac{-\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \sim \left(\frac{V'}{V}\right)^2 \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} + \varepsilon = \frac{V''}{V} \tag{3.2}$$

We now consider perturbations of the field about the solution to Equation 3.1. Note that we chose that to be our zeroth order solution since the background metric is isotropic and homogeneous, which motivates the need to take a solution which depends only on $t$. Before doing any calculations, we need to fix a gauge. In unitary gauge, which is a gauge where $\delta\phi = 0$, we can take a general metric of the form:

$$\begin{aligned} ds^2 &= -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \\ h_{ij} &= a^2 e^{2\zeta} e^{\gamma_{ij}} \quad \partial_i \gamma_{ij} = \gamma_{ii} = 0 \end{aligned} \tag{3.3}$$

Note that the variables $\zeta$ and $\gamma_{ij}$ are gauge invariant fields (see Appendix A.1). We also have the flat gauge, which is defined by $\delta\phi \neq 0$ and

$$
\begin{aligned}
ds^2 =& -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \\
h_{ij} =& a^2 e^{\gamma_{ij}} \quad \partial_i \gamma_{ij} = \gamma_{ii} = 0
\end{aligned}
\tag{3.4}
$$

so that one degree of freedom of the scalar part of the metric is replaced by the scalar field now. The exact relation between these two scalar degrees of freedom is given by:

$$
\zeta = -\psi + H\frac{\delta\phi}{\dot{\phi}} \tag{3.5}
$$

## 3.2 The ADM formulation

The ADM formalism aims to develop a Hamiltonian or initial value formulation of general relativity. For that, we imagine the 3+1 dimensional spacetime being foliated by spatial 3-dimensional surfaces with the spatial metric $g_{ij}$. Taking $n_\mu$ to be the normal vector, i.e. the vector perpendicular to the hypersurfaces which are foliated the spacetime, we can define an induced metric on the hypersurface through:

$$
g^{\mu\nu} = -n^\mu n^\nu + h^{\mu\nu} \tag{3.6}
$$

which is basically like a completeness relation. We define a time vector $t^\mu = (1,0,0,0)$ and this enables us to define the shift and lapse functions:

$$
\begin{aligned}
\alpha = -g_{\mu\nu} t^\mu n^\mu \qquad \beta^\mu = h^{\mu\nu} t_\nu \\
\implies h_{\mu\nu} = g_{\mu\nu} + \frac{1}{\alpha^2}\left(t_\nu - \beta_\mu\right)\left(t_\mu - \beta_\nu\right)
\end{aligned}
\tag{3.7}
$$

Now, for our case, we have the hypersurfaces as the 3d spatial slices, so $n^\mu = t^\mu/\sqrt{-g_{00}}$. This gives us $h_{ij} = g_{ij}$. We can decompose some of the operators living in 3+1 dimensional spacetime in terms of purely spatial tensors. We define ${}^{(3)}R_{ijkl}$ as the Riemann tensor coming only from the spatial part of the metric. We also define extrinsic curvature tensor as :

$$
K_{ij} = \nabla_i n_j = \frac{1}{2}\left(\dot{h_{ij}} - \nabla_i N_j - \nabla_j N_i\right) \tag{3.8}
$$

This enables us to write

$$R = {}^{(3)}R + \frac{1}{2}K_{ij}K^{ij} - \frac{1}{2}K^2 + \text{Boundary term} \tag{3.9}$$

The action for a canonical single field model is then written as:

$$S = \int d^4x \left( {}^{(3)}R + \frac{1}{2}K_{ij}K^{ij} - \frac{1}{2}K^2 + \frac{1}{N}(\dot{\phi} - N_i\partial^i\phi)^2 - \frac{1}{N}(\partial_i\phi)^2 \right) \tag{3.10}$$

Solving the constraint equations i.e. EOM for $N$ and $N_i$ to first order in $\zeta, \gamma_{ij}$ gives us:

$$\begin{aligned} N &= 1 + \frac{\dot{\zeta}}{H} \\ N^i &= \partial_i(-a^{-2}\frac{\zeta}{H} + \varepsilon\partial^{-2}\dot{\zeta}) \end{aligned} \tag{3.11}$$

Putting it back into the action gives us the actions for $\gamma, \zeta$ till cubic order in these fields. We do not need to solve for $N, N_i$ beyond the first order in $\zeta, \gamma$. This is because, for example, for the 3rd order action, the 2nd order expressions of $\zeta, \gamma$ would multiply the first order equation of motion for $N, N_i$ which vanishes. Similarly, the 3rd order expressions would multiply the zeroth order EOM which vanishes. Hence the first order solutions to $N, N_i$ are enough. This is unsurprisingly, not the case for the fourth-order action and beyond. We get the quadratic actions:

$$S_{\zeta\zeta} = \frac{1}{2}\int d^4x\, \varepsilon \left( a^3\dot{\zeta}^2 - a(\partial\zeta)^2 \right) \tag{3.12}$$

$$S_{\gamma\gamma} = \frac{1}{8}\int d^4x\, \left( a^3\dot{\gamma_{ij}}^2 - a(\partial\gamma_{ij})^2 \right) \tag{3.13}$$

The action for $\zeta$ is like that of a massless field in de-Sitter, while the graviton action is almost the same as the flat space one. However, there are small spectral tilts for the power spectra of these fields due to the time dependence of the mode functions. Since all quantities are evaluated at the time of horizon exit $t_0$ defined by $k \sim 1/a(t_0)H(t_0)$ we have (after restoring the $M_{pl}$ factors):

$$\langle\zeta(\boldsymbol{k_1})\zeta(\boldsymbol{k_2})\rangle = (2\pi)^3\delta^3(\boldsymbol{k_1}+\boldsymbol{k_2})\frac{H^2}{4\varepsilon k^3}\left(\frac{k}{aH}\right)^{n_s-1} \quad \langle\gamma^{h_1}(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\rangle = (2\pi)^3\delta^3(\boldsymbol{k_1}+\boldsymbol{k_2})\frac{H^2}{M_{pl}^2k^3}\left(\frac{k}{aH}\right)^{n_t} \tag{3.14}$$

$$n_s - 1 = \frac{d\ln\left(k^3\langle\zeta_{-\vec{k}}\zeta_{\vec{k}}\rangle\right)}{d\ln k} = \frac{1}{H}\frac{d\log\langle\zeta\zeta\rangle}{dt_0} = 2\eta - 6\varepsilon \qquad n_t = \frac{d\ln\left(k^3\langle\gamma_{-\vec{k}}\gamma_{\vec{k}}\rangle\right)}{d\ln k} = \frac{1}{H}\frac{d\log\langle\gamma\gamma\rangle}{dt_0} = -2\varepsilon \tag{3.15}$$

## 3.3 Cubic Maldacena action

Similar to the quadratic action calculation above, one can calculate the cubic actions for calculating 3-point functions for $\gamma, \zeta$. Note that since $R$ is a gauge invariant quantity, we can calculate the action in the flat gauge in terms of $\delta\phi$ and then convert the answer to one involving $\zeta$. This has the benefit of giving the lagrangian ordered in powers of $\varepsilon$[14]. For example, calculating the action in unitary gauge directly gives:

$$S_{\gamma\zeta\zeta} = \int d^4x\, a\varepsilon\gamma_{ij}\partial_i\zeta\partial_j\zeta + \frac{1}{2}a^3\varepsilon^2\partial^2\gamma_{ij}\partial_i\partial^{-2}\dot{\zeta}\partial_j\partial^{-2}\dot{\zeta} + \frac{1}{2}a^3\varepsilon^2\dot{\gamma}_{ij}\partial_i\partial^{-2}\dot{\zeta}\partial_j\zeta + \text{EOM terms} \tag{3.16}$$

where the EOM terms are the terms arising from the field redefinition, i.e. the conversion between $\phi$ and $\zeta$. These terms can always be pushed to a higher order in perturbations according to the field redefinition theorem. The in-in calculation for the 3-pt correlator for the vertex which is leading order in $\varepsilon$ above yields:

$$\langle\gamma^h(\boldsymbol{k_1})\zeta(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle = \frac{H^4}{4M_{pl}^4\varepsilon}\frac{1}{(\prod_{a=1}^3 k_a^3)}e^h_{ij}(k_1)k_{2i}k_{3j}\left[-k_T + \frac{\sum k_ik_j}{k_T} + \frac{k_1k_2k_3}{k_T^2}\right] \tag{3.17}$$

**A comment on Non-Local terms:** One should note that the source of these non-localities is rooted in the fact that not all metric components are dynamic variables. Since, we have constraint equations for these non-dynamical variables, one plugs in their formal solution in the action which can potentially involve inverse differential operators since the constraint equations are differential equations. Though it may appear that the non-locality is always $\varepsilon$ suppressed, this is not true. That would mean that without a source scalar field, we have no non-localities. However, even with only the graviton present, we have non-localities for example, in the fourth order action coming from the constraint solutions:

$$N = ... + \frac{1}{2H}\partial_j\partial^{-2}\left(\partial_j\gamma_{ik}\dot{\gamma}_{ik}\right) + .... \tag{3.18}$$

which is not $\varepsilon$ suppressed.

The cubic action for various combinations of the fields, for the sake of completeness. at leading order in $\varepsilon$, are as follows:

$$S_{\zeta\zeta\zeta} = \int \varepsilon^2 \left( a^3 \zeta \dot{\zeta}^2 + a\zeta (\partial \zeta)^2 - 2a^3 \dot{\zeta} \partial_i \partial^{-2} \zeta \partial_i \zeta \right) \tag{3.19}$$

$$S_{\zeta\zeta\gamma} = \int \varepsilon \, a \gamma_{ij} \partial_i \zeta \partial_j \zeta \tag{3.20}$$

$$S_{\zeta\gamma\gamma} = \int \frac{\varepsilon}{8} \left( a^3 \zeta \dot{\gamma}_{ij} \dot{\gamma}_{ij} + a\zeta (\partial \gamma_{ij})^2 - 2a^3 \partial_i \partial^{-2} \dot{\zeta} \dot{\gamma}_{ab} \partial_i \gamma ab \right) \tag{3.21}$$

# Chapter 4

# Effective Field Theory of Inflation

The Effective field theory of Inflation is an attempt to unify all models of inflation by constructing the most general low energy action consistent with symmetries.

As mentioned above, we are finally interested in a theory of fluctuations about this time-dependent background and therefore time diffeomorphism while still being a symmetry is now non-linearly realised on the fluctuations i.e they are spontaneously broken. One can then choose a gauge to write the EFT in the so-called unitary gauge which has no inflaton fluctuations $\delta\phi(\vec{x},t) = 0$. In this gauge, all fluctuations go into the metric. The transformation rule for $\delta\phi(\vec{x},t)$ under $x^\mu \to x^\mu + \varepsilon^\mu$,

$$\delta\tilde{\phi}(\tilde{x}) = \delta\phi(x) - \varepsilon^0\dot{\bar{\phi}}(t) \tag{4.1}$$

indicates that unitary gauge just fixes the time diffs, and therefore, the EFT will only involve terms which are invariant under spatial diffeomorphisms. We are allowed to write full diff invariant terms, e.g.,${}^{(4)}R^{(4)}_{\mu\nu}R^{\mu\nu}$, terms with free upper time indices, e.g., $\delta g^{00}$ (terms with lower free time indices are not invariant under spatial diffs) and terms describing the slicing. Hence we have:

$$S_{EFT} = \int d^4x\sqrt{-g}\mathscr{L}({}^{(4)}R_{\mu\nu\sigma\rho}, {}^{(3)}R_{ijkl}, \nabla_\mu, \delta K_{ij}, g^{00}, \partial_t, ...) \tag{4.2}$$

So far, we have identified the correct degrees of freedom for the action. Now, like for any sensible EFT, one must identify the correct expansion parameter(s) to organise the terms. The EFToI is an expansion in the number of metric perturbations and the number of derivatives on the metric perturbations which (before any canonical normalization) have dimension 0. We start by splitting the action into three parts,

$$S_{EFT} = S_0 + S_2 + S_{\geq 3} \tag{4.3}$$

where,

$$S_0 = \int d^4x\, \sqrt{-g} M_{pl}^2 \left( \frac{{}^{(4)}R}{2} - M_{pl}^2 \lambda(t) - c(t)\delta g^{00} + D\delta K \right)$$

$$S_2 = \int d^4x\, \sqrt{-g} M_{pl}^2 \left( m_1 \left( \delta K^i_j \delta K^j_i \right) + m_2 (\delta K)^2 + m_3 \left( {}^{(3)}R \delta g^{00} \right) + M_2^2 (\delta g^{00})^2 + M_3 \left( \delta g^{00} \right) \delta K \right.$$
$$\left. + \frac{1}{M_4} {}^{(3)}R_{ij} \delta K^{ij} + \frac{1}{M_5} {}^{(3)}R \delta K + \frac{1}{M_6^2} {}^{(3)}R^2 + \frac{1}{M_7^2} {}^{(3)}R_{ij}^2 + \frac{1}{M_8^2} {}^{(3)}R_{ijkl}^2 \right) + .... \tag{4.4}$$

where, $S_n$ has terms which start from $n^{th}$ order in perturbations. Here, $S_0$ is Einstein-Hilbert action coupled with a scalar field with $\lambda(t) = -3H^2 + \dot{H}$ and $c(t) = -\dot{H}$ so that the background/zeroth-order equations of motion are satisfied. The dots in the above equations indicate the same terms but with more derivatives. $\{m_i\}$ have mass dimension zero whereas $\{M_i\}$ have mass dimension one. We have not made any assumption about how the EFT operators are generated from the UV, therefore, every operator comes with a different Mass scale, $M_i$. It is important to keep in mind that finally, we want a theory in terms of the scalar fluctuations $\zeta$ and the tensor fluctuation $\gamma_{ij}$. The EFT derivative power counting is clear only in terms of these variables and to illustrate this consider the operator $\partial_i \delta g^{00} \partial^i \delta g^{00}$. This naively looks like a leading two-derivative term and therefore, is not suppressed or enhanced by any mass scale. However, in terms of $\zeta$, depending on the constraint solution, it can actually generate higher derivative terms. Therefore, some terms in the relevant part of Lagrangian might actually turn out to be irrelevant. Since we are ignorant about the short distance physics i.e. $k \gg H$ and are interested in the modes which have exited the horizon and have left an imprint on the CMB, the typical length/energy scale is $H$. Hence an expansion in $H/M$ is obtained for our EFToI. Assuming for concreteness that all dimensional coefficients are order unity and $H \ll M_{4,5,6,7}$, one can easily arrange the action as a perturbative series. For instance, the quadratic action, $S_2$, is

$$S_2 \sim \left( m_1 + m_2 + m_3 + \frac{M_2^2}{H^2} + \frac{M_3}{H} + \frac{H}{M_4} + \frac{H^2}{M_5^2} + \frac{H^2}{M_6^2} + \frac{H^2}{M_7^2} \right) + .... \tag{4.5}$$

For some explicit calculations, we take the action $S_0 + S_2$ and write only the operators which have at most 2 derivatives on the metric :

$$S_0+S_2 = \int d^4x\,\sqrt{-g}M_{pl}^2\left(\frac{^{(4)}R}{2}+m_1\delta K^i_j\delta K^j_i+m_2(\delta K)^2+m_3^{(3)}R\delta g^{00}+D\delta K-M_{pl}^2\lambda(t)-c(t)g^{00}\right.$$
$$\left.+M_1g^{ij}\partial_i\delta g^{00}\partial_j\delta g^{00}+M_2^2(\delta g^{00})^2+M_3\delta g^{00}\delta K\right) \tag{4.6}$$

The leading (two derivatives) quadratic (scalar and tensor) actions are given by [14] :

$$S_{\zeta\zeta} =\frac{1}{2}\int d^4x\,\varepsilon\left(a^3\dot{\zeta}^2-a(\partial\zeta)^2\right) \tag{4.7}$$
$$S_{\gamma\gamma} =\frac{1}{8}\int d^4x\,\left(a^3\dot{\gamma_{ij}}^2-a(\partial\gamma_{ij})^2\right) \tag{4.8}$$

i.e the action derived for a canonical scalar field minimally coupled to gravity. This is an element of the class of actions defined by equation (4.6) where the action only contains the first three terms of $S_0$. In passing we point out that from (4.7) and (4.8) one can easily see that the existence of $\zeta$ is tied to the fact that inflation is quasi de Sitter ($\varepsilon \neq 0$), and such a variable does not exist in pure de Sitter ($\varepsilon = 0$). On the other hand, the tensor perturbation $\gamma_{ij}$ is also well defined in de Sitter. Coming back to (4.6), there is a large number of unknown parameters but as usual, the EFT parameters have to be determined/constrained experimentally. Unfortunately in cosmology, the number of observables is very small. This is due to the fact that unlike in flat space EFTs, we do not have experimental control. Therefore, as we will show below, one can constrain or fix only a handful of parameters in the EFToI:

- If a mass term is generated for the scalar perturbations in the EFT ($c_1\dot{\zeta}^2-c_2(\partial\zeta)^2-c_3\zeta^2$, where $m^2=c_3/c_1$ ) then from the observed tilt of the scalar spectrum one can show that $\frac{m}{H}\ll 1$, where $m$ is the mass of the fluctuations. This can be easily inferred from the expression for power spectrum for a massive scalar in de-Sitter.

$$P_\zeta(k)=\frac{H^2}{2k^3}\left(\frac{k}{aH}\right)^{3-2\nu} \tag{4.9}$$

  where, $\nu=\sqrt{\frac{9}{4}-\frac{m^2}{H^2}}$. Now, since we know (through observations) that the spectral tilt is very close to unity, therefore $\nu\sim\frac{3}{2}\implies\frac{m^2}{H^2}\ll 1$. This allows us to strongly constrain the coefficients of operators which generate a mass term for $\zeta$.

- For operators which contribute both to the sound speed and non-gaussianities, we can use experimental bounds to deduce the bounds on $c_s$ or the operator coefficients. Further constraints on $c_s$ or the coefficients can be found by applying the partial wave unitarity bound by going to the flat gauge [11, 37].

To illustrate the points mentioned above, we take two examples of quadratic actions of the type mentioned in Equation (4.6). We first solve for the ADM constraint variables up to 1st order in $\zeta$ (See Appendix A.1). To simplify calculations, we take $m_2 = -m_1$. We then consider the cases:

- $\boldsymbol{M_3 = 0, m_1, m_2 \neq 0}$. The canonical 2 derivative terms in the action are given by:

$$S_0 + S_2 = \int M_{pl}^2 \left( a^3 \left( \varepsilon + 4 \left( \frac{M_2}{H} \right)^2 \right) \dot{\zeta}^2 - a\varepsilon (\partial \zeta)^2 \right) \tag{4.10}$$

  i.e. the usual canonical action with a different sound speed. The partial wave unitarity bound in the flat space limit is satisfied in the limit $c_s \to 1$ [11] , which gives $M_2 \ll \sqrt{\varepsilon} H$. For this theory, which has no mass term, we have the spectral tilt:

$$n_s - 1 = \frac{1}{H} \frac{d}{dt_*} \langle \zeta(\boldsymbol{k}) \zeta(-\boldsymbol{k}) \rangle = \frac{1}{H} \frac{d}{dt_*} \frac{H^2(t_*)}{4\varepsilon(t_*) M_{pl}^2 c_s(t_*) k^3} = 2\eta - 6\varepsilon + 4c_s^2 \left( \frac{M_2}{H} \right)^2 \left( \frac{\eta}{\varepsilon} - 1 \right) \tag{4.11}$$

  where the asterisk denotes the time of horizon crossing. Because of the $c_s$ constraint, the spectral tilt is already small. As was shown in [11], a small speed of sound ($c_s \ll 1$) also implies large interactions (non-gaussianities). These are derivative interactions (e.g., $\dot{\pi}^3$) and they naturally produce equilateral non-gaussianity since, due to derivatives their contribution to the squeezed limit is negligible. There are experimental constraints on $c_s$ from bounds on equilateral non-gaussianity, $f_{NL}^{equil}$,

$$c_s \gg 0.028 \tag{4.12}$$

- $\boldsymbol{M_3 \neq 0, m_1 = m_2 = 0, M_2 = 0}$. Since calculating the action for arbitrarily large values of $M_3$ is difficult, we take the case where $g = M_3/H \ll 1$. For this we get upto 1st order in $\varepsilon$ and $g$,

$$S_0 + S_2 = \int M_{pl}^2 \left( a^3 \varepsilon \dot{\zeta}^2 - a(\varepsilon - 2g)(\partial \zeta)^2 \right) \tag{4.13}$$

  Again, taking into account that mass terms only start appearing at $\mathscr{O}(g^2)$, the spectral tilt is small and we won't explicitly calculate it here. Going to the Stueckelberg gauge [11], we can write the lagrangian in the flat space limit, in terms of the Stueckelberg boson $\pi$ as :

$$\mathscr{L}_\pi = M_{pl}^2 \varepsilon H^2 \left(\dot{\pi}^2 - (\partial\pi)^2 \left(1 - \frac{M_3}{H\varepsilon}\right)\right) - 2\frac{M_3 M_{pl}^2 c_s^3}{\left(\sqrt{M_{pl}^2 \dot{H}}\right)^3} \partial^2\pi(\dot{\pi}^2 + (\partial\pi)^2) + \ (\varepsilon\text{ -suppressed}) \tag{4.14}$$

This lagrangian tells us that we must have $M_3 > 0$ for $c_s < 1$ and the partial wave unitarity bound for the tree level $\pi\pi$ amplitude gives:

$$\frac{M_3^2}{\varepsilon^3 M_{pl}^2 c_s^2}\left(c_s^4 + 4(1 + c_s^2)^2\right) < \pi \tag{4.15}$$

where, in the line above, the $\pi$ is the numerical constant.

While all this is for the scalar action, the graviton case is simpler to deal with since the only two operators contributing to the two derivative quadratic EFT are ${}^{(3)}R$ and $\delta K_{ij}\delta K^{ij}$ which can be removed by suitable field re-definitions of $g_{\mu\nu}$ [17, 15]. We also note that one can write the EFT in some other gauge, for instance, the flat gauge where $\delta\phi \neq 0$ [14]. As discussed before, doing calculations in flat gauge can sometimes lead to simplifications as it can sometimes directly give us an EFT ordered in $\varepsilon$ [14]. However, this simplification is only present for gauge invariant operators (like ${}^{(4)}R$, ${}^{(4)}R_{\mu\nu}{}^{(4)}R_{\mu\nu}$, etc). One has to be careful while doing the calculations in this gauge and converting them to the unitary gauge answers. This is pointed out using an example in Appendix A.5.

# Chapter 5

# Soft Limits

## 5.1 Overview

Soft limits provide us a connection between $n$-point and $(n-1)$-point correlators. Unlike the soft theorems in flat space-time, the soft theorems for inflationary correlators are basically rooted in the evolution of modes once they cross a length(or energy) scale which is intrinsic to the given space-time i.e $H$. Soft theorems rely primarily on three assumptions:

- There being only a single degree of freedom i.e. a single field inflation model.
- The solution to the background equations of motion must be an attractor solution. This is true for slow roll models but can be violated in models such as ultra slow roll inflation.
- Bunch-Davies initial conditions

The argument goes as follows. We take a 3-point correlator for the purposes of this proof. Suppose one of the modes, $k_1$, is soft i.e. $k_1 \ll k_2, k_3$. Then $k_1$ exits the horizon long before the other two modes and acts as a classical background for them. This can be explicilty seen in the spatial part of the metric which is proportional to $e^{2\zeta}$. Hence, the soft mode acts like ( to a leading order in $k_1/k_2$ or $k_1/k_3$ ) a re-scaling of coordinates given by :

$$x^i \to x^i e^{\zeta} \approx x^i(1+\zeta_q) \quad \Longrightarrow \; k^i \to k^i(1-\zeta_q) \tag{5.1}$$

This leads to the infinitesimal change in the correlators:

$$\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = \langle\zeta(k_1)\zeta(-k_1)\rangle\left(3-k_2\frac{\partial}{\partial k_2}-k_3\frac{\partial}{\partial k_3}\right)\langle\zeta(k_2)\zeta(k_3)\rangle \tag{5.2}$$

$$\to -(n_s-1)\langle\zeta(k_1)\zeta(-k_1)\rangle\langle\zeta(k_2)\zeta(-k_2)\rangle \tag{5.3}$$

Similarly, for soft tensor modes, we have

$$x^i \approx x^i + \frac{\gamma_{ij}}{2}x^j \quad \Longrightarrow k^i \to k^i - \frac{\gamma_{ij}}{2}k^j \quad \Longrightarrow k^2 \to k^2 - \gamma_{ij}k^ik^j \tag{5.4}$$

which leads to the change in correlators:

$$\langle\gamma^h(k_1)\zeta(k_2)\zeta(k_3)\rangle \to -e^h_{ij}k_{2i}k_{2j}\langle\gamma^h(k_1)\gamma^h(-k_1)\rangle\frac{\partial}{\partial(k_2^2)}\langle\zeta(k_2)\zeta(-k_2)\rangle \tag{5.5}$$

Analogously we also have :

$$\langle\gamma^{h_1}(k_1)\gamma^{h_2}(k_2)\zeta(k_3)\rangle \to -n_t\langle\gamma^{h_1}(k_1)\gamma^{h_2}(k_2)\rangle\langle\zeta(k_3)\zeta(-k_3)\rangle \tag{5.6}$$

$$\langle\gamma^h(k_1)\gamma^{h_2}(k_2)\gamma^{h_3}(k_3)\rangle \to -e^h_{ij}k_{2i}k_{2j}\langle\gamma^{h_1}(k_1)\gamma^{h_1}(-k_1)\rangle\frac{\partial}{\partial(k_2^2)}\langle\gamma^{h_2}(k_2)\gamma^{h_3}(-k_2)\rangle \tag{5.7}$$

For first order corrections in $q$, where $q$ is the soft mode, consider the quadratic maldacena action calculated above. if we perform a coordinate transformation [17]:

$$x^i \to x^i - b^ix^2 + 2(b\cdot x)x^i + \delta x(t)$$

$$\delta \dot{x}(t) = -\frac{2b^i}{a^2H} + O(\varepsilon)$$

The infinitesimal change in the action, upto first orfer in epsilon, is then given by:

$$\delta S_2 = \int d^4x\left(6a^3\varepsilon(b\cdot x)\dot{\zeta}^2 - 2a\varepsilon(b\cdot x)(\partial\zeta)^2 + 4a\varepsilon\frac{\dot{\zeta}}{H}b - i\partial_i\zeta\right) \tag{5.8}$$

We now take $S_3$ i.e. the cubic Maldacena action and decompose $\zeta = \zeta_L + \zeta_S$ where L,S indicate long and short modes. This is just like writing a fourier transform but here its more rudimentary, we are just separating the short modes from the long modes. Ignoring any time derivatives and double spatial derivatives for $\zeta_L$ we get the action for 1 long and 2 short modes to be:

$$S_3 = \int d^4x\left(3a^3\varepsilon\zeta_L{\dot{\zeta}_S}^2 - a\varepsilon\zeta_L(\partial\zeta_S)^2 + 2a\varepsilon\frac{\dot{\zeta}}{H}\partial_i\zeta_L\partial_i\zeta_S\right) \tag{5.9}$$

We equate the two expressions above and find that a scaling of coordinates and an SCT with $b_i = -\frac{1}{2}\partial_i \zeta_L$ is equivalent to introducing a long mode in the background. We thus have [17]:

$$\lim_{q\to 0}\langle \zeta(q)\zeta(k_1)\zeta(k_2)....\zeta(k_n)\rangle = -P(q)\left(3(n-1)+k_n\frac{\partial}{\partial k_n}+\frac{1}{2}q_i D_i\right)\langle \zeta(k_1)\zeta(k_2)....\zeta(k_n)\rangle$$
$$D_i \cdot q_i = \sum_{a=1}^{n}\left(6\vec{q}\cdot\vec{\partial}_a - \vec{q}\cdot\vec{k}_a\partial_a^2 + 2\vec{k}_a\cdot\partial_a(\vec{q}\cdot\partial_a)\right) \tag{5.10}$$

Note that from these expressions, we find that in order to get a non-trivial soft limit, we must break scale invariance as well as invariance under de-sitter boosts. This is because the right-hand sides of the soft limits are proportional to the ward identities of scaling and de-sitter boosts. The spectral tilts of the power spectra of $\gamma, \zeta$ give us the non-trivial contributions for 3-point functions while for O(q) corrections for 3-point functions, we get 0. We also note that for tensor correlators, these results might not be extendable as the polarization tensors are functions of the momenta which can make things tricky while taking derivatives w.r.t the momenta.

These relations rely on the fact that there is a negligible contribution to the three-point function when all the modes are within the horizon. In fact, these theorems assume that the contribution to the correlators only starts becoming sizable when the long mode has left the horizon and has already frozen acting as a classical background. This is always the case when the initial state is the BD vacuum. The BD three-point correlators only have a total energy pole which comes from integrals of the form

$$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle \sim \int_{-\infty}^{0}\tau^m e^{iK\tau}d\tau \tag{5.11}$$

where, $K = k_1 + k_2 + k_3$. To compute such an integral, a regularisation scheme is required. This integral can be regularized by taking $\tau \to \tau(1-i\varepsilon)$, and therefore, damping the contribution in the far past. It only starts giving appreciable contribution once $K\tau \sim -1$ which in the equilateral case ($k_1 = k_2 = k_3$) corresponds to the epoch of horizon crossing for all the modes and in the squeezed case ($k_1 \ll k_2 \approx k_3$, relevant for soft theorems) corresponds to epoch of horizon crossing for the short modes since $K \sim k_s$. Therefore, in the squeezed limit the in-in contribution remains negligible till the epoch of the horizon crossing for the short modes. By this time the long mode has already frozen and therefore, the soft theorems must hold. There exist cases where the soft theorems are violated even in single field models [38, 39, 40, 28], for instance, if one starts in an excited initial state, for e.g., $\alpha$-vacuum [31] then apart from total energy pole, one also has a $(k_1 + k_2 - k_3)$ kind of pole structure arising from mode-mixing. Therefore the integral contains terms,

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle \sim \int_{-\infty}^{0} \tau^m e^{i(k_1+k_2-k_3)\tau} d\tau \tag{5.12}$$

which, in the squeezed limit ($k_1 \ll k_2 \approx k_3$) starts giving appreciable contribution when $k_1 \tau \sim -1$ i.e when the long mode is near horizon exit. Therefore, the soft theorem proof does not hold in this case.

### 5.1.1 Soft theorem for the Inflaton field?

In the limit $\varepsilon \to 0$, the action in terms of $\delta\phi$ reduces to that of a spectator massless field in dS. Only one term in the maldacena action survives which is given by:

$$\begin{aligned} S_2 =& \frac{1}{2}\left(\dot{\phi}^2 - (\partial\phi)^2 - V''(\phi)\phi^2\right) \\ S_3 =& \frac{1}{3!} V'''(\phi)\phi^3 \end{aligned} \tag{5.13}$$

i.e. the canonical scalar field action. Note that we have the expressions $V''(\phi) = 3\eta H^2$ and $V'''(\phi) = -\frac{3}{2}\frac{H^2\dot{\eta}}{\sqrt{2\varepsilon}M_{pl}}$, and hence this field has a small mass (compared to Hubble). This gives us a small spectral tilt [23, 41, 42]:

$$n_s - 1 \approx 2\left(\eta + \frac{\dot{\eta}}{H}(log2 - 2 + \gamma_E)\right) \tag{5.14}$$

The three-point function of a field with the cubic vertex given above can be calculated to be:

$$\langle \zeta\zeta\zeta \rangle = -\frac{H^3\dot{\eta}}{16M_{pl}^4\varepsilon^2}\frac{1}{\prod_{a=1}^3 k_a^3}\left[\sum k_a^3(\log(k_1+k_2+k_3) - 1 + \gamma_E) + k_1k_2k_3 - \sum_{a\neq b} k_a^2 k_b\right] - \sum_{a<b} 2\eta P(k_a)P(k_b) \tag{5.15}$$

where the last term comes from the relation between $\phi$ and $\zeta$ i.e. a field redefinition term. It is easy to check that the soft limit of 5.15 satisfies the soft limit for three scalar fields with $n_s$ given by 5.14. Note that this analysis only makes sense in the $\varepsilon \to 0$ limit since the cubic vertex is of the same order in slow roll as the leading order cubic vertex for $\zeta$. The soft limit is of course still for $\zeta$ only since it is $\zeta$ that enters the metric and modifies the background spacetime.

We notice that the limit under consideration gives us an interaction that is Lorentz invariant in the flat space limit (i.e. $a \to 1$). Now from that we can guess that the interaction with 1 graviton + 2 scalars would be $\gamma_{ij}\partial_i\phi\partial_i\phi$ which is the leading order cubic term ($\mathscr{O}(\varepsilon)$) in $S_{\gamma\zeta\zeta}$. The soft limit for this correlator is the same as before.

## 5.2 Explicit Checks

In this section, we explicitly calculate and verify the soft limits mentioned above. We start with the scalar three-point function calculated for the action in 4.6 with only $M_2 \neq 0$ as a check for calculations with $c_s \neq 1$ while for the mixed correlators, we take the canonical minimally coupled quadratic action for simplicity of calculation. Although the scalar soft theorems have been explicitly checked [43] the mixed correlator ones have not been explicitly checked for higher derivative EFToI operators and therefore, according to our knowledge, this is the first such an attempt.

**Soft limit for $\langle\zeta\zeta\zeta\rangle$**

The terms which contribute to the soft 3-point correlator $\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle|k_1 \to 0$ are given by:

$$S_0 + S_2 = \int M_{pl}^2 \left[ \left( \varepsilon^2 \zeta_c \dot{\zeta_c}^2 a^3 + \varepsilon^2 \zeta_c (\partial\zeta_c)^2 a \right) + 4\,(3\varepsilon - 2\eta) \left(\frac{M_2}{H}\right)^2 \zeta_c \dot{\zeta_c}^2 a^3 \right] \tag{5.16}$$

where the first two terms are from the Maldacena cubic action. It is important to note that the final Maldacena cubic action is derived after performing a field redefinition [14]

$$\zeta = \zeta_c + \frac{1}{2}(2\varepsilon - \eta)\zeta_c^2 + \frac{\dot{\zeta}_c \zeta_c}{H} + \ldots.$$

which removes terms proportional to the equation of motion. Since $M_2$ also gives quadratic corrections to $\zeta$ action, the above field redefinition generates additional cubic terms. Therefore, the last term in 5.16 is a combination of the cubic part generated by $M_2$ and the terms generated by the redefinition. The soft correlator is given by (where $n_s$ is given by 4.11),

$$\begin{aligned}\langle\zeta(\boldsymbol{k_1})\zeta(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_{\boldsymbol{k_1}\to 0} &= \left(\frac{H^2}{4\varepsilon c_s M_{pl}^2}\right)^2 \left(\frac{11}{2}\varepsilon - 2\eta + \frac{\varepsilon}{2}c_s^2 + 4\frac{M_2^2}{H^2}c_s^2\left(\frac{3}{2} - \frac{\eta}{\varepsilon}\right)\right)\frac{1}{k_1^3}\frac{1}{k_2^3} \\ &= -(n_s - 1)\langle\zeta(\boldsymbol{k_1})\zeta(-\boldsymbol{k_1})\rangle\langle\zeta(\boldsymbol{k_2})\zeta(-\boldsymbol{k_2})\rangle\end{aligned} \tag{5.17}$$

### 5.2.1 Soft limits for mixed correlators

We classify the operators in the EFT into two classes, Cubic and Purely Cubic operators. Cubic operators are those which contribute to $\langle\zeta\zeta\rangle$ or $\langle\gamma\gamma\rangle$, as well as the mixed bispectra, while purely cubic operators contribute only to the latter. We take two cubic operators from 4.4 as examples:

**(1)** $\int d^4x\sqrt{-g}\frac{M_{pl}^2}{M_5}{}^{(3)}R\boldsymbol{\delta K}$ :

Since $\delta K$ contains both $\delta N$ and $N^i$, i.e. the ADM constraint variables, they get modified and are now given by:

$$\delta N = \frac{\dot\zeta}{H}+\frac{2\partial^2\zeta a^{-2}}{M_5 H} \tag{5.18}$$

$$N^i = \partial_i\left(-\frac{\zeta}{H}a^{-2}+\varepsilon\partial^{-2}\dot\zeta+\frac{2\varepsilon\zeta a^{-2}}{M_5}\right) \tag{5.19}$$

Note that the two equations above are valid only when $\frac{H}{M_5}\ll 1$ i.e. they're 1st order expressions in $1/M_5$. Using this, the corrections to the quadratic and cubic actions for $\gamma\gamma\zeta$ are given up to first order in $1/M_5$ by:

$$\mathscr{O}_{\zeta\zeta} = \int 4\frac{M_{pl}^2}{M_5}a\partial^2\zeta\left(\varepsilon\dot\zeta-\frac{\partial^2\zeta}{H}a^{-2}\right) \tag{5.20}$$

$$\mathscr{O}_{\gamma\gamma\zeta} = \int\frac{M_{pl}^2}{4M_5}\left(a\frac{\dot\gamma_{ij}\dot\gamma_{ij}}{H}\partial^2\zeta-2a^{-1}\frac{\partial_l\gamma_{ij}\partial_l\gamma_{ij}\partial^2\zeta}{H}+\varepsilon a\partial_l\gamma_{ij}\partial_l\gamma_{ij}\dot\zeta-2\varepsilon\gamma_{ij}\partial_l\dot\gamma_{ij}\partial_l\zeta\right) \tag{5.21}$$

$$\tag{5.22}$$

The correction to the scalar power spectrum is given by,

$$\delta P_\zeta = -\frac{M_{pl}^2}{M_5}\frac{H^3}{4\varepsilon^2 M_{pl}^4 k^3}(5-3\varepsilon) \tag{5.23}$$

For perturbation theory to work here, the quadratic correction $\delta P_\zeta$ must be small compared to $P_\zeta(k)=\frac{H^2}{4\varepsilon M_{pl}^2 k^3}$. We henceforth assume that $\frac{H}{M_5\varepsilon}\ll 1$ so that the enhancement to the power spectrum remains small. This also ensures that we can expand the action (as well as the ADM constraints) in powers of $M$ and the analysis above holds. In the soft limit, $\mathscr{O}_{\gamma\gamma\zeta_{k\to 0}}=0$ i.e its contribution to the mixed correlator in the soft limit vanishes at leading order. However, the following "exchange diagram" diagram gives a non-zero contribution:

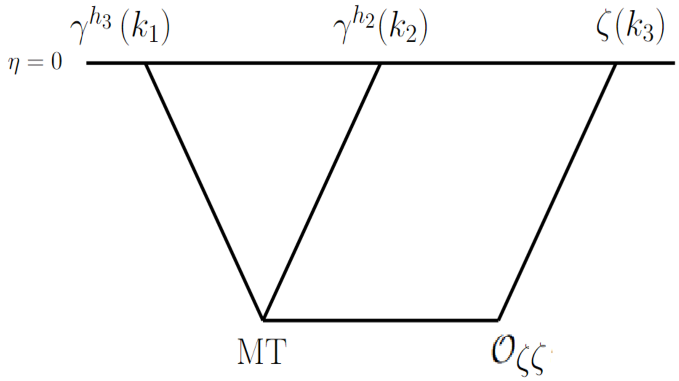


Figure 5.1: The MT (Maldacena term) vertex is the usual cubic $\gamma\gamma\zeta$ vertex of the Maldacena action

where the Maldacena term refers to the mixed cubic vertex ($\zeta\gamma\gamma$) computed in [14]. This 3-point correlator is a sum of two terms,

$$\langle\gamma(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_{\boldsymbol{k_3}\to 0} = 2\left(\langle\gamma(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_{LR,\boldsymbol{k_3}\to 0} - \langle\gamma(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_{RR,\boldsymbol{k_3}\to 0}\right) \quad (5.24)$$

where $R$ and $L$ indicate the time and anti-time orderings of the interaction Hamiltonian respectively. For instance, RR means both vertices are time ordered (See [44] for more details on these notations and conventions). The $RR$ contribution is given by [1]

$$\begin{aligned}\langle\gamma(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_{RR,\boldsymbol{k_3}\to 0} = &\left(\frac{H^6}{M_{pl}^6 k_1^3 k_2^3 k_3^3}\right)\left(\frac{M_{pl}^2\Lambda}{8\varepsilon H M_7^2}\right) e_{ij}^{h_1}(k_1) e_{ij}^{h_2}(k_2)\Bigg[k_1^2 k_2^2\left[\frac{5-3\varepsilon}{4k_T} + \frac{(3-\varepsilon)k_3}{k_T^2} - \right.\\ &\left.\frac{\varepsilon k_3^2}{k_T^3} - \frac{5k_3^3}{k_T^4} - \frac{8k_3^4}{k_T^5}\right] - (\boldsymbol{k_1}\cdot\boldsymbol{k_2})\left[\frac{5-3\varepsilon}{4}k_T + \varepsilon k_3 I - \frac{5-3\varepsilon}{2}k_3 + (3-\varepsilon)\frac{k_3^2}{k_T} - k_1 k_2\left(\frac{5-3\varepsilon}{4k_T} + \frac{5-3\varepsilon}{2k_T^2} + \right.\right.\\ &\left.\frac{k_3^2}{k_T^3} - 2\frac{k_3^3}{k_T^4}\right) + (k_2 k_3 + k_1 k_3)\left(\frac{5-3\varepsilon}{4k_T} + \frac{(1+\varepsilon)k_3}{2k_T^2} + \frac{k_3^2}{k_T^3} + \frac{2k_3^3}{k_T^4}\right) + k_1 k_2 k_3\left(\frac{5-3\varepsilon}{4k_T^2} + \frac{(1+\varepsilon)k_3}{k_T^3} + \frac{3k_3^2}{k_T^4}\right.\\ &\left.\left.+\frac{8k_3^3}{k_T^5}\right)\right] - \left[k_2^2(\boldsymbol{k_1}\cdot\boldsymbol{k_3})\left(\frac{5-3\varepsilon}{4k_T} + \frac{5-3\varepsilon}{4k_T^2}(k_1+2k_3) + \frac{k_1k_2(5-3\varepsilon)+k_3^2}{k_T^3} + \frac{3k_1k_3^2 - 2k_3^3}{k_T^4} - \frac{8k_3^3 k_1}{k_T^5}\right)\right.\\ &\left.-(k_1\leftrightarrow k_2)\right]\Bigg]\end{aligned} \quad (5.25)$$

[1]The results given from here on do not include the divergent terms of the type: $\lim_{\eta\to 0}\frac{cos(k\eta)}{\eta}$. These are ignored while calculating contact diagrams since they contribute only to the imaginary part of the $L$ and $R$ vertices and so, they get cancelled. Here, however, these terms come from both the vertices of the "exchange" diagram and get multiplied, because of which they contribute to the real part. However, one can easily check that the contributions from $RR, RL, LR$ and $LL$ add up to 0.

where $I = \gamma_E + \log(-k_T\eta_0)$ , $\eta_0 \to 0$ and $k_T = \sum_{a=1}^{3} k_a$. It is easy to show that the $LR$ contribution is simply given by,

$$\langle\gamma(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_{LR} = \frac{B_{MT}\,\delta P_\zeta(k_3)}{4P_\zeta(k_3)} \tag{5.26}$$

Therefore, the final 3-point function is given by:

$$\langle\gamma(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_{RR} = \frac{B_{MT}\,\delta P_\zeta(k_3)}{2P_\zeta(k_3)} - 2\langle\gamma(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_{RR} \tag{5.27}$$

where $B_{MT}$ is the mixed Bispectrum of the Maldacena cubic action. One can easily check that:

$$\langle\gamma(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_{\boldsymbol{k_3}\to 0} = -\frac{n_t P_\gamma(k_1) P_\zeta(k_3)}{2P_\zeta(k_3)} - \frac{1}{2} n_t P_\gamma(k_1)\delta P_\zeta(k_3) = -n_t P_\gamma(k_1)\delta P_\zeta(k_3) \tag{5.28}$$

i.e the soft limit 5.6 is satisfied.

**(2)** $\int d^4x\sqrt{-g}\frac{M_{pl}^2}{M_7^2}{}^{(3)}R_{ij}{}^{(3)}R_{ij}$:
This operator gives the corrections:

$$\mathscr{O}_{\gamma\gamma\zeta} = \int \frac{M_{pl}^2}{M_7^2} a^{-1}\left[-\frac{1}{4}\zeta\partial^2\gamma_{ij}\partial^2\gamma_{ij} + \frac{\dot{\zeta}}{4H}\partial^2\gamma_{ij}\partial^2\gamma_{ij}\right] \tag{5.29}$$

$$O_{\zeta\zeta} = \int 6\frac{M_{pl}^2}{M_7^2} a^{-1}(\partial^2\zeta)^2 \quad O_{\gamma\gamma} = \int \frac{M_{pl}^2}{4M_7^2} a^{-1}\partial^2\gamma_{ij}\partial^2\gamma_{ij} \tag{5.30}$$

The corresponding corrections to the power spectra are,

$$\delta P_\zeta = 15\frac{M_{pl}^2}{M_7^2}\frac{H^4}{8M_{pl}^4\varepsilon^2} \quad \delta P_\gamma = 5\frac{M_{pl}^2}{M_7^2}\frac{H^4}{2M_{pl}^4 k^3}\delta_{h_1 h_2} \tag{5.31}$$

Again, as also noted in [24] one should be worried about the $\frac{\Delta\langle\zeta\zeta\rangle}{\langle\zeta\zeta\rangle_0} \sim \frac{M_{pl}^2}{M_7^2} H^2/\varepsilon M_{pl}^2$ enhancement of the power spectrum and thus we can remove the correction by taking an extra $({}^{(3)}R)^2$ term or (for the purposes of this thesis )assume that this ratio is small, which places a lower bound on $M_7$. As pointed out in [24], after the field redefinition $\gamma_{ij} \to \gamma_{ij} - \dot{\gamma}_{ij}\zeta/H + ...$ which removes the terms proportional to the equation of motion in the Maldacena mixed cubic action, and after integration by parts, we get:

$$\mathscr{O} = \int \sqrt{-g}\frac{M_{pl}^2}{M_7^2}{}^{(3)}R_{ij}{}^{(3)}R_{ij} + \frac{1}{2}\frac{M_{pl}^2}{M_7^2}\zeta\partial^2\frac{\dot{\gamma}_{ij}}{H}\partial^2\gamma_{ij}a^{-1} = -\int \frac{1}{4}\varepsilon\zeta\frac{M_{pl}^2}{M_7^2}\partial^2\gamma_{ij}\partial^2\gamma_{ij}a^{-1} + .... \tag{5.32}$$

After taking the soft limit $\zeta_{\boldsymbol{k_3}\to 0}$, the terms represented by dots go to 0 at leading order as mentioned in [24]. The term proportional to $\varepsilon$ however, does contribute and we have:

$$\Delta_A\langle\zeta(\boldsymbol{k_3})\gamma^{h_1}(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\rangle_{\boldsymbol{k_3}\to 0} = -\frac{5}{8}\frac{M_{pl}^2}{M_7^2}\left(\frac{H}{M_{pl}}\right)^6\frac{1}{k_1^3}\frac{1}{k_3^3}\delta_{h_1h_2} \tag{5.33}$$

This term 5.32 is not mentioned in [24] and there, the 3-point function in the soft limit is shown to vanish at $\mathscr{O}((k_1/k_3)^0)$. The difference lies in the order of calculations, i.e. whether you compute and simplify the operator first and then do the in-in computation or just do the in-in computation for all the terms and add the answers as done in [24]. The difference in the answers arises because $H$ is not a constant when we simplify the operators but is taken to be time-independent while doing the in-in integrals. The reason for this is that the in-in integral pick up most of the contribution near horizon crossing and therefore, it is a good approximation to take $H(t) = H_*$ i.e Hubble at horizon crossing.[2] To get the correct soft limit, we should thus always try to eliminate $H$ dependence in the action and get everything ordered in powers of $\varepsilon, \eta$, as in 5.32, as much as possible before calculating the correlator. This prescription is also followed in standard Maldacena action calculations [14]. Proceeding with the calculations, we again have the "exchange diagrams" as before as shown below. The left one gives a contribution:

$$\Delta_B\langle\zeta(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\gamma^{h_3}(\boldsymbol{k_3})\rangle_{\boldsymbol{k_1}\to 0} = 2\left(\frac{B_{MT}|_{k_3\to 0}\delta P_\gamma(k_1)}{2P_\gamma(k_1)}\right) - 2\langle\zeta(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\gamma^{h_3}(\boldsymbol{k_3})\rangle_{RR,\boldsymbol{k_1}\to 0} \tag{5.34}$$

$$= 2\left(\frac{5}{8}\frac{M_{pl}^2}{M_7^2}\frac{H^6}{M_{pl}^6}\frac{1}{k_1^3k_3^3}\delta_{h_1h_2}\right) + \frac{15}{8}\frac{M_{pl}^2}{M_7^2}\frac{H^6}{M_{pl}^6}\frac{1}{k_1^3k_3^3}\delta_{h_1h_2} \tag{5.35}$$

where the first term in the last line just follows from the Maldacena soft limit and the second term is calculated in Appendix A.2. Calculating the modified spectral tilt of $\langle\gamma\gamma\rangle$ yields:

$$\widetilde{n_t} = \frac{1}{H}\partial_t \log\langle\gamma\gamma\rangle = -2\varepsilon - 5\varepsilon\frac{M_{pl}^2}{M_7^2}\frac{H^2}{M_{pl}^2} \tag{5.36}$$

[2] for instance if the operator is $\mathscr{O} = \int a^3 M_{pl}(M_2/H)^2\dot{\zeta}^3$ and we want to compute $\langle\zeta\zeta\zeta\rangle$ we'll compute it as follows

$$\langle\zeta\zeta\zeta\rangle \sim \left(\frac{M_2}{H_*}\right)^2\int_{-\infty}^{0} d\eta\, k_1^2k_2^2k_3^2\eta^4 e^{ik_T\eta}$$

i.e. we keep $H_*$ outside the integral. There are also multiple factors of $H_*$ (and $M_{pl}$), which come from $a = -1/H\eta$ and the mode functions that are also taken outside.

Adding both the contributions $\Delta_A$ and $\Delta_B$ gives:

$$\langle\zeta(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\gamma^{h_3}(\boldsymbol{k_3})\rangle_{A+B,\ \boldsymbol{k_1}\to 0} = -\widetilde{n_t}(P_\gamma(k_1)+\delta P_\gamma(k_1))P_\zeta(k_3) \tag{5.37}$$

The second diagram is just like the one we considered for ${}^{(3)}R\delta K$, and hence adding it to the previous answer gives:

$$\langle\zeta(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\gamma^{h_3}(\boldsymbol{k_3})\rangle_{\boldsymbol{k_1}\to 0} = -\widetilde{n_t}(P_\gamma(k_1)+\delta P_\gamma(k_1))(P_\zeta(k_3)+\delta P_\zeta(k_3)) \tag{5.38}$$

where the equality holds at $\mathscr{O}((H/M_{pl})^6)$. These calculations for the soft limits of $\langle\gamma\gamma\zeta\rangle$ also hold for $\langle\gamma\zeta\zeta\rangle$ and one can verify it for ${}^{(3)}R\delta K$ (see Appendix A.3).

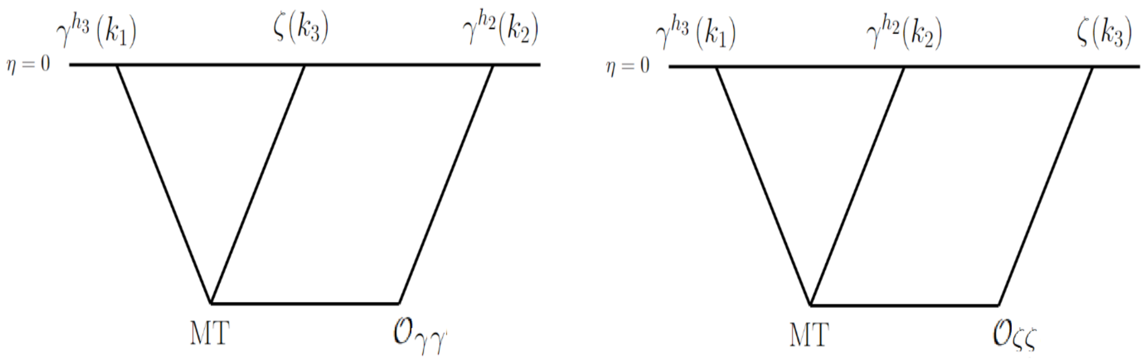


Figure 5.2: The MT (Maldacena term) vertex is the usual cubic vertex of the Maldacena action and $\mathscr{O}_{\gamma\gamma}, \mathscr{O}_{\zeta\zeta}$ the respective quadratic correction operators from ${}^{(3)}R_{ij}^{(3)}R_{ij}$

**Purely Cubic Operators:** We have shown how the soft limits change at the leading order in soft momenta for cubic operators. However, when we take purely cubic operators, we see from the derivative structure of $\delta K, \delta K_{ij}, {}^{(3)}R_{ij}, {}^{(3)}R$ (which are the building blocks for purely cubic operators) that we just have [3]:

$$e_3^3\langle\zeta_q\gamma\gamma\rangle|_{\boldsymbol{q}\to 0} = e_3^3\langle\zeta_q\zeta\gamma\rangle|_{\boldsymbol{q}\to 0} = 0 \tag{5.39}$$

$$\mathscr{B}(\gamma_q\gamma\zeta)|_{\boldsymbol{q}\to 0} = \mathscr{B}(\gamma_q\zeta\zeta)|_{\boldsymbol{q}\to 0} = \widetilde{0} \tag{5.40}$$

where $\widetilde{0}$ represents the fact that the correlator is 0 as a function without taking $\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3}=0$. Hence these operators only give $\mathscr{O}(q^2)$ corrections on the RHS of the soft limits 5.6 and 5.3.

[3]Note that 5.40 doesn't work for operators involving terms with 4 indices like ${}^{(3)}R_{ijkl}$. In that case, the RHS of 5.40 is the same as 5.39. However, in this thesis, we're only dealing with operators constructed from ${}^{(3)}R_{ij}, {}^{(3)}R, \delta K_{ij}, \delta K, \delta g^{00}$ and $\widetilde{N}_i$ for which there's always a $\partial^2$ or $\partial_t$ acting on $\gamma_{ij}$, due to which the given limit holds.

We have thus shown explicitly how the soft limits are obeyed for various higher derivative operators and models (i.e. $c_s = 1$ or otherwise). These provide a consistency check for models beyond the Maldacena action and as we'll see next, these have an important role in the boostless bootstrap of correlators.

# Chapter 6

# Boostless Bootstrap

As mentioned in the previous section, one can have EFTs of Inflation where de-sitter boosts are broken . Even in slow roll inflation, de-sitter boosts are broken where the breaking is proportional to the slow roll parameters. For such theories, we have to move on from the Conformal bootstrap program. Naturally, just like the S-matrix bootstrap doesn't give us the full answer when Lorentz boosts are broken , we don't expect that we would be able to completely booootstrap the correlators using a purely-boundary perspective. As we shall see, we do indeed retain some of the constraints such as soft limits and the BD initial conditions. For this chapter, we mainly follow the analysis presented in [23].

## 6.1 Flat space amplitude limit

Under the assumption that the modes we're working with are de-sitter mode functions, we get the following for any operator [25]:

$$\lim_{k_T \to 0} \langle \zeta_1 \zeta_2 \zeta_3 .. \zeta_n \rangle = \frac{(-1)^n H^{p+n-1} (p-1)!}{2^{n-1}} \frac{\mathrm{Re}(i^{n+p+1} A_n)}{k_T^p \prod_{a=1}^n k_a^2} \tag{6.1}$$

where $k_T = \sum_a k_a$ and $p = \sum_{vertices}(\text{degree} + \text{no. of derivatives}) - 3$. For $n = 3$, i.e. cubic correlators, we have $p$ equal to the total no. of derivatives in the Hamiltonian. A brief proof is given below for contact diagrams with $n$ vertices. The proof can be easily extended to exchange diagrams by viewing them as contact diagrams connected to each other.

**Proof:** Let us consider a generic operator given by

$$\mathscr{O} = \int a^{4-s} \prod_{a=1}^{n} \zeta^{s_a} d^3x\, d\eta = (-H\eta)^{4-s} \prod_{a=1}^{n} \zeta^{s_a} d^3x\, d\eta \tag{6.2}$$

where $s_i$ is the total no. of derivatives on the $i^{th}$ vertex of the operator and s=$\sum_a s_a$. Now, we have the expression for the correlator:

$$\langle \zeta_{k_1} \zeta(k_2)...\zeta(k_n)\rangle = 2\mathrm{Re}\left[ i\int d^3x \int_{-\infty}^{0} \prod_{a=1}^{n} \partial_{s_a} \left( \frac{H^2}{2k_a^3}(1-ik_a\eta)e^{ik_a\eta}e^{i\vec{k_a}\cdot\vec{x}} \right) d\eta \right] \tag{6.3}$$

$$= 2\mathrm{Re}\left[ \frac{A_n}{(\prod_a k_a^2)}(-i)^n H^{2n+s-4} \int_{-\infty}^{0} \eta^{s+n-4} e^{ik_T\eta} d\eta \right] + ...... \tag{6.4}$$

$$=_{k_T\to 0} \frac{(-1)^n H^{p+n-1}(p-1)!}{2^{n-1}} \frac{\mathrm{Re}(i^{n+p+1}A_n)}{k_T^p \prod_{a=1}^{n} k_a^2} \tag{6.5}$$

where $p = n+s-3$, the terms represented by dots in the second line are subleading in powers of $\eta$ and so they'll not give the leading pole in $k_T$. For $n = 3$ we have $p = s$. QED

For $p = 0$ we will have a logarithm instead of a pole. For such an interaction, we will have

$$Disc[\langle \zeta_{k_1} \zeta(k_2)\zeta(k_3)\rangle_{Re\quad axis}] = 2\pi i[\text{Principle Value}] \;\sim\; A_3/e_3^2 \tag{6.6}$$

## 6.2 Pole Structure of Correlators

In this section, we further review the pole structure of correlators coming from different initial states. For BD vacuum, the only possible pole structure for contact diagrams is the total energy pole $k_T$ coming from the integrals of the type $\int e^{ik_T\eta}$. However, as we have shown in previous sections, we are free to choose any initial state. One obvious choice will be $\alpha$ vacua. For the cubic scalar inflationary vertex, we have the following correlator [20]:

$$\langle \zeta(\boldsymbol{k_1})\zeta(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_\alpha = \frac{H^4}{32M_{pl}^4\varepsilon^2}\frac{1}{(\prod_{a=1}^3 k_a^3)}\Bigg[2(\varepsilon-\eta)\sum_a k_a^3 + \varepsilon\left(\sum_a k_a^3 + \sum_{a\neq b} k_a^2 k_b + 8\sum_{a>b}\frac{k_a^2k_b^2}{k_T}\right) +$$
$$\sinh^2 2\alpha\left(2(\varepsilon-\eta)\sum_a k_a^3 + \varepsilon\left(\sum_a k_a^3 + \sum_{a\neq b} k_a^2k_b + \sum_{a>b} 8\frac{k_a^2k_b^2}{k_2+k_3-k_1} + 8\frac{k_a^2k_b^2}{k_3+k_1-k_2} + 8\frac{k_a^2k_b^2}{k_1+k_2-k_3}\right)\right)\Bigg] \tag{6.7}$$

Here, the "physical" poles of the type $k_1+k_2-k_3$ cannot be obviously tied to a physical process like scattering. This is because our momentum-conserving delta function is like for 3 particles all with incoming momenta. Hence, following the same convention, the notion of "energy conservation" here would correspond to the total energy being 0 (with the one energies of the particles actually being negative of the given energies )and not the other 3 combinations. Note that reversing the momentum directions would not change the magnitude, and so, we can't associate a set of momenta $(\vec{k}_1,\vec{k}_2,-\vec{k}_3)$ to the set of "energies" $(-k_1,-k_2,-k_3)$. Moving to 4-point functions, we take the following exchange correlator for the $\dot{\zeta}^3/\eta$ interaction as an example [28]:

$$\begin{aligned}\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\zeta(k_4)\rangle \sim \frac{s}{\prod_{a=1}^4 k_a}\Bigg(&\frac{2}{k_T^3(k_1+k_2+s)^3}+\frac{6}{k_T^4(k_1+k_2+s)^2}+\frac{12}{k_T^5(k_1+k_2+s)}+\\ &\frac{1}{(k_1+k_2+s)^3(k_3+k_4+s)^3}+\text{cyclic}\Bigg)\end{aligned} \tag{6.8}$$

where $s=|\vec{k}_1+\vec{k}_2|$ as defined before. For $\alpha$ vacua, we will again have extra poles like $k_1+k_2-s$. We can also take excited states as initial states. In a way, even $\alpha$ vacua are excited states w.r.t the BD vacuum. However, we can even take states like $a_p^\dagger\,|0\rangle_{BD}$. There might be subtleties involved in projecting the vacuum of the interacting theory onto this state instead of the vacuum of the free theory (i.e. BD vacuum)in the far past. These are discussed in [28]. Ignoring them, calculating the 3-point correlator for $\dot{\zeta}^3/\eta$ for this state gives:

$$\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\zeta(k_4)\rangle \sim \frac{1}{\prod_a k_a}\left(\frac{\delta^3(\vec{k}_1+\vec{p})\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)}{(p_2+p_3-p)^3}+\frac{\delta^3(0)}{k_T^3}+\text{cyclic}+....\right) \tag{6.9}$$

where we have omitted some combinatorial factors for brevity. Hence, we get disconnected structures in this case. Note that we can't single out an initial state uniquely based on the pole structures. A very obvious example of this is the family of $\alpha$ vacua. Another more non-trivial example is taking the Coherent State, defined by [26, 28](with appropriate normalisation):

$$a_{\vec{p}}\,|C\rangle = C(\vec{p})\,|C\rangle\,,\quad C(\vec{p})+C^*(-\vec{p})=0\quad\ \forall\vec{p}$$

as the initial state. We see that :

$$\begin{aligned}\langle C|\,\zeta(k_1)\zeta(k_2)\,|C\rangle &=_{BD}\langle 0|\,\zeta(k_1)\zeta(k_2)\,|0\rangle_{BD}=\frac{H^2}{4\varepsilon k_1^3}\delta^3(\vec{k}_1+\vec{k}_2)\\ \langle C|\,\zeta(k_1)\zeta(k_2)\zeta(k_3)\,|C\rangle &=_{BD}\langle 0|\,\zeta(k_1)\zeta(k_2)\zeta(k_3)\,|0\rangle_{BD}\end{aligned} \tag{6.10}$$

where the second line is valid for all interaction hamiltonians.

## 6.3 Boostless Bootstrap rules

We use the following rules entailed in [23] to boostrap 3-point functions:

- Symmetry between identical bosons. For instance, $\langle \zeta(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\gamma(\boldsymbol{k_3})\rangle$ should be symmetric under $k_2 \leftrightarrow k_3$.

- The amplitude limit mentioned previously.

- Manifest Locality Test(MLT)

$$\left.\frac{\partial \mathscr{B}}{\partial k_1}\right|_{k_1=0} = \left.\frac{\partial \mathscr{B}}{\partial k_2}\right|_{k_2=0} = \left.\frac{\partial \mathscr{B}}{\partial k_3}\right|_{k_3=0} = 0 \tag{6.11}$$

  where $\mathscr{B}(k_1,k_2,k_3) \sim \langle \zeta_1\zeta_2\zeta_3\rangle_{tr} k_1^3 k_2^3 k_3^3$, "tr" signifying that the tensor contractions have been trimmed. This equation is valid only for local operators. This can be seen easily by taking the most general local operator $\mathscr{O} \sim \zeta \cdot \zeta^{(m)} \cdot \zeta^{(n)}$ where $n,m$ denote the no. of derivatives on $\zeta$. Suppose $\boldsymbol{q}$ is the soft mode. Note that if all three $\zeta$'s had derivatives then the MLT would be trivially satisfied due to powers of $q$ coming from the derivatives. Hence the non-trivial contribution comes when we have $\boldsymbol{q}$ associated with the $\zeta$ with no derivatives. In this case, the main term to focus on is:

$$\langle \zeta_{\boldsymbol{q}\to 0}\zeta\zeta\rangle_{tr} \sim \frac{\partial}{\partial q}\int_{-\infty}^{0} d\eta (1-iq\eta)e^{ik_T\eta}|_{\boldsymbol{q}\to 0} + \frac{\partial}{\partial q}\int_{-\infty}^{0} d\eta (1+iq\eta)e^{-ik_T\eta}|_{\boldsymbol{q}\to 0} = 0$$

  where we have omitted the mode functions involving the other 2 momenta. The analysis is easily extended to correlators with $\gamma$.

- We're taking BD vacuum as the initial state. So there are no poles except the $k_T$ and $1/k_a^3$ type poles coming from the mode functions.

- The soft limits discussed in Chapter 5. We are primarily focused on operators present in the Maldacena action and Purely Cubic operators. This is because for cubic operators not present in the Maldacena action that modify the power spectra, we have strange exchange diagrams contributing to the power spectrum and it is more economical to directly do the in-in calculation there.

**Boostraping Maldacena action $\langle \gamma\zeta\zeta\rangle$:** In the $\varepsilon \to 0$ limit, we have the "lorentz invariant" interactions remaining as discussed before. Hence, the amplitude can be directly bootstrapped for this interaction to give :

$$A[1,2,3^h] \sim \frac{[21]^2[31]^2}{[23]^2} \text{ or } \frac{\langle 23\rangle^2}{\langle 31\rangle^2\langle 12\rangle^2} = e^h_{ij}k_{2i}k_{3j} \tag{6.12}$$

where the middle expression is the expression from spinor helicity [45] and $h$ represents the helicity of the graviton. From the amplitude and bose symmetry rule, we can write an ansatz (where $e_1 = k_2 + k_3$, $e_2 = k_2k_3$ and $e_3 = k_1k_2k_3$) :

$$\langle \gamma^h(k_1)\zeta(k_2)\zeta(k_3)\rangle = \frac{A_0 e^h_{ij}k_{2i}k_{3j}}{k_T^2 e_3^3}\left(e_3 + k_T(A_1e_2 + A_2e_1^2) + A_3k_T^2e_1 + A_4k_T^3\right) \tag{6.13}$$

The MLT relations give:

$$\begin{aligned} &\textbf{MLT for } k_1: A_1 = 1 \quad A_2 = A_4 \\ &\textbf{MLT for } k_2: A_3 + A_4 = 0 \quad 1 + A_1 + 2A_2 = 0 \end{aligned} \tag{6.14}$$

while the soft limit for $k_1$ gives:

$$A_0 = \frac{1}{4\varepsilon} \tag{6.15}$$

which fixes our correlator completely to be:

$$\langle \gamma^h(k_1)\zeta(k_2)\zeta(k_3)\rangle = \frac{H^4}{4\varepsilon M_{pl}^4}\frac{e^h_{ij}k_{2i}k_{3j}}{k_T^2 e_3^3}\left(e_3 + k_T(e_2 - e_1^2) + k_T^2e_1 - k_T^3\right) \tag{6.16}$$

$$= \frac{H^4}{4\varepsilon M_{pl}^4}\frac{e^h_{ij}k_{2i}k_{3j}}{k_T^2 e_3^3}\left(e_3 + k_T^2\sum_{a<b}k_ak_b - k_T^3\right) \tag{6.17}$$

This method can similarly be extended to the pure graviton and pure scalar correlators. As explained in [23], we still require the SCT ward identities to completely fix the scalar correlator. For $\langle\gamma\gamma\zeta\rangle$ the interaction is non-local and boost breaking. We shall deal with such interaction next section.

## 6.4 Relevant terms for $\langle\gamma\gamma\zeta\rangle$

| Term | No. of Derivatives | $\mathcal{O}(\varepsilon)$ | $\mathcal{O}(\Lambda)$ or $\mathcal{O}(M_{pl})$ |
|---|---|---|---|
| ${}^{(3)}R$ | 2 | 1 | $M_{pl}^2$ |

| | | | |
|---|---|---|---|
| ${}^{(3)}R\delta g^{00}$ | 2 | 1 | $\Lambda^2$ |
| $\delta K_{ij}\delta K_{ij}$ | 3 | $1,\epsilon$ | $M_{pl}^2$ |
| $\delta K_{ij}\delta K_{ij}\delta g^{00}$ | 3 | 1 | $\Lambda^2$ |
| $\delta K_{ij}\delta K_{jk}\delta K_{ki}$ | 3 | $1,\epsilon$ | $\Lambda$ |
| $\delta K_{ij}\delta K_{ji}\delta K$ | 3 | $1,\epsilon$ | $\Lambda$ |
| ${}^{(3)}R_{ij}\delta K_{ij}$ | 3 | $1,\epsilon$ | $\Lambda$ |
| ${}^{(3)}R\delta K$ | 3 | $1,\epsilon$ | $\Lambda$ |
| ${}^{(3)}R_{ij}\delta K_{ij}\delta g^{00}$ | 3 | 1 | $\Lambda$ |
| ${}^{(3)}R_{ij}^{(3)}R_{ij}$ | 4 | 1 | 1 |
| ${}^{(3)}R_{ij}^{(3)}R_{ij}\delta g^{00}$ | 4 | 1 | 1 |
| ${}^{(3)}R_{ij}\delta K_{jk}\delta K_{ki}$ | 4 | $1,\epsilon$ | 1 |
| ${}^{(3)}R_{ij}^{(3)}R_{jk}\delta K_{ki}$ | 5 | $1,\epsilon$ | $1/\Lambda$ |
| ${}^{(3)}R_{ij}^{(3)}R_{ij}\delta K$ | 5 | $1,\epsilon$ | $1/\Lambda$ |
| ${}^{(3)}R_{ij}^{(3)}R\delta K_{ij}$ | 5 | $1,\epsilon$ | $1/\Lambda$ |
| ${}^{(3)}R_{ij}^{(3)}R_{jk}^{(3)}R_{ki}$ | 6 | 1 | $1/\Lambda^2$ |
| ${}^{(3)}R_{ij}^{(3)}R_{ij}^{(3)}R$ | 6 | 1 | $1/\Lambda^2$ |

Table 6.1: Quadratic and cubic operators and their contributions to $\langle\gamma\gamma\zeta\rangle$ . The 3rd column shows what powers of $\epsilon$ can be present in the terms generated by the operators, while the last column shows the coefficients of the operators in terms of some energy scale $\Lambda$. The term in red is redundant as it can be removed using $\int A(t)^{(3)}R_{ij}K_{ij} = \int A(t)^{(3)}RK + \dot{A}(t)^{(3)}R/2N +$ total derivative, while the term in blue is present in the Maldacena action. Here $\delta g^{00} = g^{00}+1$.

As mentioned before, While writing the operators in Table 6.1, we use covariant objects (i.e. which are covariant, at least w.r.t. the 3d metric). We also take the operators defined in [18] :

$$V = \frac{\dot{\delta N} - N^i \partial_i N}{N} \tag{6.18}$$

$$A_\mu = \frac{h_\mu^\nu \nabla_\nu N}{N} \tag{6.19}$$

where $h_{\mu\nu}$ is the 3d spatial metric. Note that using $A_0$ and $V$ we can obtain $\dot{\delta N}$ and $\widetilde{N^i}\partial_i N$, so we'll use them instead. Also, using $\tilde{N}_i$ and $\partial_i N$ we can obtain $A_i$'s contribution to various operators. and so, we will not use $A_i$ explicitly. For $\langle \gamma\gamma\zeta \rangle$, only $\dot{\delta N}$ is relevant, which is just a higher derivative operator derived from $\delta N$ and hence it has not been included in Table 6.1. All the higher derivative operators will of course be suppressed by an energy or mass scale $\Lambda$. Some of the operators give non-local terms which we shall discuss below.

## 6.5 Purely Cubic Local Terms

We take the purely cubic vertex

$$R_{ij}^{(3)} \delta K_{ij} \delta g^{00} = \int a \frac{\Lambda}{H} \partial^2 \gamma_{ij} \dot{\gamma}_{ij} \dot{\zeta} \tag{6.20}$$

Direct in-in calculation gives:

$$\langle \zeta(\boldsymbol{k_1}) \gamma^{h_2}(\boldsymbol{k_2}) \gamma^{h_3}(\boldsymbol{k_3}) \rangle = \frac{\Lambda}{H} e_{ij}^{h_2}(\boldsymbol{k_2}) e_{ij}^{h_3}(\boldsymbol{k_3}) k_1^2 k_2^2 k_3^2 \frac{H^6}{\varepsilon M_{pl}^6 k_1^3 k_2^3 k_3^3} \left[ \frac{2}{(k_1+k_2+k_3)^3} + 3 \frac{k_2+k_3}{(k_1+k_2+k_3)^4} \right] \tag{6.21}$$

we have $p = 4$ for this vertex and [1]

$$A[1^0, 2^{h_2}, 3^{h_3}] \sim e_{ij}^{h_2}(\boldsymbol{k_2}) e_{ij}^{h_3}(\boldsymbol{k_3}) k_1 k_2 k_3 (k_2 + k_3) \tag{6.22}$$

where we have kept the normalization arbitrary (which contains information about things like $\Lambda$ and $\varepsilon$ dependence). From this we write an ansatz using the first rule(where $k_T = k_1 + k_2 + k_3$ ,

[1] note that there's no way to bootstrap the amplitude since any function of the form $[23]^4 f(k_1, k_2, k_3)$, where f is a degree 4 polynomial, is a valid $p = 4$ amplitude. Here [ ] is the relevant helicity bracket.

$e_1 = k_2 + k_3$, $e_2 = k_2 k_3$ and $e_3 = k_1 k_2 k_3$):

$$\langle\zeta(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\gamma(\boldsymbol{k_3})\rangle \sim \frac{e_{ij}^{h_2}(\boldsymbol{k_2})e_{ij}^{h_3}(\boldsymbol{k_3})}{k_T^4 e_3^3}\Big[e_3 k_1 k_2 k_3(k_2+k_3) + k_T(A_1 e_2^3 + A_2 e_2^2 e_1^2 + A_3 e_2 e_1^4 + A_4 e_1^6) +$$
$$k_T^2(A_5 e_2^2 e_1 + A_6 e_2 e_1^3 + A_7 e_1^5) + k_T^3(A_8 e_2^2 + A_9 e_2 e_1^2 + A_{10} e_1^4) + k_T^4(A_{11} e_2 e_1 + A_{12} e_1^3)$$
$$+k_T^5(A_{13} e_2 + A_{14} e_1^2) + A_{15} k_T^6 e_1 + A_{16} k_T^7\Big] \tag{6.23}$$

We get the following set of equations after applying MLT and soft limits for various momenta:

$$\text{Soft limit for } k_2, k_3\text{:}\quad A_4 = A_7 = A_{10} = A_{12} = A_{14} = A_{15} = A_{16} = 0 \tag{6.24}$$
$$\text{MLT for } k_2, k_3\text{:}\quad A_3 = A_6 = A_9 = A_{11} = A_{13} = 0 \tag{6.25}$$
$$\text{Soft limit for } k_1\text{:}\quad A_2 + A_5 + A_8 = 0 \tag{6.26}$$
$$\text{MLT for } k_1\text{:}\quad A_1 = 0 \quad 3A_2 + 2A_5 + A_8 = 0 \tag{6.27}$$

which fixes our correlator to be:

$$\langle\zeta(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\gamma(\boldsymbol{k_3})\rangle \sim \frac{e_{ij}^{h_2}(\boldsymbol{k_2})e_{ij}^{h_3}(\boldsymbol{k_3})}{k_T^4 e_3^3} e_3^2\left[A_2 k_T + e_1\right] \tag{6.28}$$

which means we're able to fix the bispectra up to an overall factor and another arbitrary constant. This expression agrees with the explicit calculation 6.21 with $A_2 = 2/3$.

## 6.6 Non-local terms

We have the following non-local terms (see Appendix A.4) at various orders in $\varepsilon$ and energy scales:

| Operator | $\mathscr{O}(\varepsilon)$ | $\mathscr{O}(H/M_{pl})$ or $\mathscr{O}(H/\Lambda)$ |
|---|---|---|
| $\delta K_{ij}\delta K_{ij}$ | $\varepsilon$ | $H/M_{pl}$ |
| $\delta K_{ij}\delta K_{jk}\delta K_{ki}$ | $\varepsilon$ | $(H/M_{pl})^2(\Lambda/M_{pl})$ |
| $\delta K_{ij}\delta K_{jk}{}^{(3)}R_{ki}$ | $\varepsilon$ | $(H/M_{pl})^3$ |
| $\delta K_{ij}{}^{(3)}R_{jk}{}^{(3)}R_{ki}$ | $\varepsilon$ | $(H/M_{pl})^3(H/\Lambda)$ |

(For the sake of brevity we have not included the odd parity terms but their contributions are of the same order as the last 3 terms). The second term in the table gives:

$$\int\sqrt{-g}\,\Lambda\delta K_{ij}\delta K_{jk}\delta K_{ki} = \int -\frac{3}{4}a^3\Lambda\varepsilon\dot{\gamma}_{ij}\dot{\gamma}_{jk}\partial_i\partial_k\partial^{-2}\dot{\zeta} \tag{6.29}$$

for which the explicit in-in calculation yields:

$$\langle \zeta(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\gamma^{h_3}(\boldsymbol{k_3})\rangle = -\frac{3}{2}\frac{H^6}{M_{pl}^6}\left(\frac{\Lambda}{H}\right) e_{ij}^{h_2}(\boldsymbol{k_2})e_{jm}^{h_3}(\boldsymbol{k_3})k_{1i}k_{1m}\frac{k_2^2k_3^2}{k_T^3e_3^3} \tag{6.30}$$

The non-local term in 6.29 naively doesn't seem to have a proper flat space counterpart, but as pointed out in [23], it can be considered to come from a toy model:

$$S_{flat} = \int d^4x \;\; \varepsilon(\partial_\mu\zeta)^2 - \frac{1}{2}(\partial_i X)^2 + \frac{M_{pl}^2}{8}(\partial_\mu\gamma_{ij})^2 - \frac{3\Lambda\varepsilon\dot{\zeta}_0}{8}X + \frac{3\Lambda\varepsilon}{8\dot{\zeta}_0}X\dot{\zeta}^2 + \Lambda\dot{\gamma}_{ij}\dot{\gamma}_{jk}\partial_i\partial_k X \tag{6.31}$$

Here $\zeta_0(t) = -\sqrt{2\varepsilon}\bar{\phi}(t)$ is the background value of the scalar field. Integrating out the field $X$ above gives us an EFT with the desired non-local term, which gives an amplitude (with no of derivatives, $p=3$):

$$A[1^0, 2^{h_2}, 3^{h_3}] = e_{ij}^{h_2}(\boldsymbol{k_2})e_{jm}^{h_3}(\boldsymbol{k_3})k_{1i}k_{1m}\frac{k_2k_3}{k_1} \tag{6.32}$$

Taking the ansatz as before(the soft limit for $k_2, k_3$ has already been taken):

$$\langle \zeta(\boldsymbol{k_1})\gamma(\boldsymbol{k_2})\gamma(\boldsymbol{k_3})\rangle \sim \frac{e_{ij}^{h_2}(\boldsymbol{k_2})e_{jm}^{h_3}(\boldsymbol{k_3})k_{1i}k_{1m}}{k_T^3e_3^3}\left[\frac{k_2k_3}{k_1}e_3 + Ak_Te_1e_2 + Bk_T^2e_2\right] \tag{6.33}$$

applying MLT with respect to $k_2$ and $k_3$ fixes $A = B = 0$ and hence, the correlator up to an overall factor, matching with 6.30. Note that as the value of $p$ increases, we'll get more and more unknown parameters. Specifically for the non-local terms, we cannot use the MLT w.r.t the non-local momenta, which removes one condition and increases the no. of arbitrary constants. From the conditions we have used, one can simply find that:

- For local terms and odd $p$ with $p \geq 3$, one has either $(p+1)(p-3)/4+1$ or $(p-3)^2/4$ parameters (which correspond to either the polarization tensors contracted with each other or with external momenta) that can't be fixed (excluding the overall factor). For even $p$, this number is either $(p^2-2p-4)/4$ or $(p-4)(p-2)/4$.
- For non-local terms and odd $p$ with $p \geq 3$, one has either $(p^2-5)/4$ or $(p-3)(p-1)/4$ parameters that can't be fixed (excluding the overall factor). For even $p$, this number is either $(p^2-4)/4$ or $(p-2)^2/4$.

We want to point out that the source of these non-localities is rooted in the fact that not all metric components are dynamical variables. Since, we have constraint equations for these non-dynamical variables, one plugs in their formal solution in the action which can potentially involve inverse differential operators since the constraint equations are differential equations.

## 6.7 Extending results to $\langle\gamma\zeta\zeta\rangle$

All operators in this case have enough derivatives on $\gamma$ and $\zeta$ so that the soft limits 5.39, 5.40 are still valid. A similar bootstrap analysis can be carried out for non-local and local terms separately. One again finds that the non-local terms start appearing at $\mathcal{O}(\varepsilon)$. The operators are summarized in Table 6.2 below

| Term | No. of Derivatives | $\mathcal{O}(\varepsilon)$ | $\mathcal{O}(\Lambda)$ or $\mathcal{O}(M_{pl})$ |
|---|---|---|---|
| $c(t)\delta g^{00}$ | 0 | $\varepsilon$ | $M_{pl}^2$ |
| $\delta K_{ij}\delta K_{ij}$ | 2 | $1,\varepsilon$ | $M_{pl}^2$ |
| $\delta K_{ij}\widetilde{N}_i\partial_j\delta g^{00}$ | 2 | $1,\varepsilon$ | $\Lambda^2$ |
| ${}^{(3)}R_{ij}\delta K_{ij}$ | 3 | $1,\varepsilon$ | $\Lambda$ |
| $\delta K_{ij}\delta K_{jk}\delta K_{ki}$ | 3 | $1,\varepsilon,\varepsilon^2$ | $\Lambda$ |
| $\delta K_{ij}\delta K_{ji}\delta K$ | 3 | $1,\varepsilon,\varepsilon^2$ | $\Lambda$ |
| ${}^{(3)}R_{ij}\delta K_{ij}\delta g^{00}$ | 3 | $1,\varepsilon$ | $\Lambda$ |
| $\delta K_{ij}\partial_i\delta g^{00}\partial_j\delta g^{00}$ | 3 | 1 | $\Lambda$ |
| ${}^{(3)}R_{ij}\widetilde{N}_i\partial_j\delta g^{00}$ | 3 | $1,\varepsilon$ | $\Lambda$ |

| | | | |
|---|---|---|---|
| ${}^{(3)}R_{ij}\delta K_{ij}\delta K$ | 4 | $1,\varepsilon,\varepsilon^2$ | 1 |
| ${}^{(3)}R_{ij}\delta K_{jk}\delta K_{ki}$ | 4 | $1,\varepsilon,\varepsilon^2$ | 1 |
| ${}^{(3)}R_{ij}\partial_i\delta g^{00}\partial_j g^{00}$ | 4 | 1 | 1 |
| ${}^{(3)}R_{ij}^{(3)}R_{ij}\delta g^{00}$ | 4 | 1 | $1/\Lambda$ |
| ${}^{(3)}R_{ij}^{(3)}R_{jk}\delta K_{ki}$ | 5 | $1,\varepsilon$ | $1/\Lambda$ |
| ${}^{(3)}R_{ij}^{(3)}R_{ij}\delta K$ | 5 | $1,\varepsilon$ | $1/\Lambda$ |
| ${}^{(3)}R_{ij}^{(3)}R_{jk}^{(3)}R_{ki}$ | 6 | 1 | $1/\Lambda^2$ |
| ${}^{(3)}R_{ij}^{(3)}R_{ij}^{(3)}R$ | 6 | 1 | $1/\Lambda^2$ |

Table 6.2: Quadratic and cubic operators and their contributions to $\langle\gamma\zeta\zeta\rangle$. Again, the blue terms are Maldacena terms, and the red term is removable by the identity mentioned below Table 6.1.

We take the term (which is the operator $\mathscr{O}_3$ in Appendix A.4) :

$$\mathscr{O} = \delta K_{ij}\partial_i\delta g^{00}\partial_j\delta g^{00} \sim \int a\frac{1}{\Lambda}\dot{\gamma}_{ij}\partial_i\dot{\zeta}\partial_j\dot{\zeta} \tag{6.34}$$

The explicit in-in calculation gives:

$$\langle\gamma^h(\boldsymbol{k_1})\zeta(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle = 6\frac{H^7}{\varepsilon^2 M_{pl}^6\Lambda}\frac{e^h_{ij}(\boldsymbol{k_1})k_{2i}k_{3j}}{e_3 k_T^5} \tag{6.35}$$

We have the corresponding flat space amplitude:

$$A[1^h,2^0,3^0] \sim e_3 e^h_{ij}(\boldsymbol{k_1})k_{2i}k_{3j} \tag{6.36}$$

which gives us the ansatz for the correlator to be:

$$\langle \gamma^h(\boldsymbol{k_1})\zeta(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle \sim \frac{e^h_{ij}(\boldsymbol{k_1})k_{2i}k_{3j}}{k_T^5 e_3^3}\Big[e_3^2 + k_T(A_1 e_2^2 e_1 + A_2 e_2 e_1^3 + A_3 e_1^5) + k_T^2(A_4 e_2^2 + A_5 e_2 e_1^2 + A_6 e_1^4)$$
$$k_T^3(A_7 e_2 e_1 + A_8 e_1^3) + k_T^4(A_9 e_2 + A_{10} e_1^2) + A_{11} k_T^5 e_1 + A_{12} k_T^6\Big] \tag{6.37}$$

Soft limits and MLTs give the following equations:

$$\text{Soft limit } \boldsymbol{k_1} \to 0 \begin{cases} A_1 + A_4 = 0 \\ A_2 + A_5 + A_7 + A_9 = 0 \\ A_3 + A_6 + A_8 + A_{10} + A_{11} + A_{12} = 0 \end{cases} \tag{6.38}$$

$$\text{MLT for } k_1 \begin{cases} 4A_1 + 3A_4 = 0 \\ 4A_2 + 3A_5 + 2A_7 + A_9 = 0 \\ 4A_3 + 3A_6 + 2A_8 + A_{10} - A_{12} = 0 \end{cases} \tag{6.39}$$

$$\text{Soft limit } \boldsymbol{k_2} \to 0 \quad \text{Already satisfied due to the tensor structure} \tag{6.40}$$

$$\text{MLT for } k_2 \begin{cases} 5A_3 + A_2 + 2A_6 = 0 \\ A_3 = 0 \\ A_5 + 4A_6 + 3A_8 = 0 \\ A_7 + 3A_8 + 4A_{10} = 0 \\ A_9 + 2A_{10} + 5A_{11} = 0 \\ A_{11} + 6A_{12} = 0 \end{cases} \tag{6.41}$$

which leads to the following correlator with just one arbitrary parameter:

$$\langle \gamma^h(\boldsymbol{k_1})\zeta(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle \sim \frac{e^h_{ij}(\boldsymbol{k_1})k_{2i}k_{3j}}{k_T^5 e_3^3}\Big[e_3^2 + A_{10}\Big(-2k_T e_2 e_1^3 + k_T^2(2e_2 e_1^2 + e_1^4) + k_T^3(e_2 e_1 - 2e_1^3)$$
$$+ k_T^4(-2e_2 + e_1^2)\Big)\Big] \tag{6.42}$$

which matches the explicit result 6.35 for $A_{10} = 0$.

## 6.8 Going to $\alpha$ vacua

In our calculations, we are not compelled to fix the initial condition (and the subsequent evolution) by taking the Bunch-Davies (BD) vacuum. One can take the well-known family of $\alpha$ vacua [31, 36] as well since they respect the symmetries of the quasi dS background. Bootstrapping $\alpha$-vacua answers directly using the BB is difficult since we don't have the soft limit conditions such as 5.40, because of the pole structures mentioned in Section 6.2. However, once we have bootstrapped $\mathscr{B}(k_1,k_2,k_3)$(as defined above) for BD, we can extend the result to $\alpha$ vacua easily. For a (k-independent) Bogolyubov transformation (BT), just by noting the form of the mode functions, which are given by :

$$\begin{gathered} u_k(\eta) = \alpha(1-ik\eta)e^{ik\eta} + \beta(1+ik\eta)e^{-ik\eta} \\ |\alpha|^2 - |\beta|^2 = 1 \end{gathered} \tag{6.43}$$

we can give an ansatz for the BT bispectra as follows:

$$\begin{aligned} \mathscr{B}_{BT}(k_1,k_2,k_3,\{\boldsymbol{k}\}) = \mathrm{Re}\Bigg[(\alpha+\beta)^3\Bigg(&\psi_3'(k_1,k_2,k_3,\{\boldsymbol{k}\})\alpha^{*3} + \sum_{\text{cyclic}}\psi_3'(-k_1,k_2,k_3,\{\boldsymbol{k}\})\alpha^{*2}\beta^* \\ &+ \sum_{\text{cyclic}}\psi_3'(-k_1,-k_2,k_3,\{\boldsymbol{k}\})\alpha^*\beta^{*2} + \psi_3'(-k_1,-k_2,-k_3,\{\boldsymbol{k}\})\beta^{*3}\Bigg)\Bigg] \end{aligned} \tag{6.44}$$

where $\psi_3'$ is the trimmed cubic wavefunction coefficient in BD vacuum [25, 24]. From the cosmological optical theorem, if we have odd parity interactions i.e. odd number of momenta contracted with the polarization tensors, the correlator for BD is 0 and we need the wavefunction coefficients to get the final answer for BT states. However, for even parity interactions, we have $\mathscr{B}_{BD} = \psi_3'$ and in this case we can get the answers for BT states directly from the BD answers. Putting $\alpha = \cosh\alpha$ and $\beta = i\,\sinh\alpha$ , we get the $\alpha$ vacua result. Using this equation to bootstrap $\langle\gamma\gamma\gamma\rangle$ in $\alpha$ vacua for the Maldacena action, we take the well-known result for BD which was bootstrapped in [23] (also explicitly calculated in [14]), and get :

$$\langle\gamma^{h_1}(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\gamma^{h_3}(\boldsymbol{k_3})\rangle_\alpha = -\frac{2H^4}{M_{pl}^4}\frac{1}{(\prod_{a=1}^3 k_a^3)}e^{h_1}_{ii'}e^{h_2}_{jj'}e^{h_3}_{kk'}t_{ijk}t_{i'j'k'}\Bigg[\left(-k_T+\frac{\sum k_ik_j}{k_T}+\frac{k_1k_2k_3}{k_T^2}\right)+$$
$$\sinh^2 2\alpha\left(-(-k_1+k_2+k_3)+\frac{k_2k_3-k_1k_2-k_1k_3}{(-k_1+k_2+k_3)}-\frac{k_1k_2k_3}{(-k_1+k_2+k_3)^2}\right)+\text{cyclic}\Bigg]$$
$$t_{ijk} = k_{2i}\delta_{jl}+k_{3j}\delta_{li}+k_{1k}\delta_{ij} \tag{6.45}$$

which agrees with the explicit in-in result in [46]. We also get the following expression for the pure scalar correlator:

$$\langle\zeta(\boldsymbol{k_1})\zeta(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_\alpha = \frac{H^4}{32M_{pl}^4\varepsilon^2}\frac{1}{(\prod_{a=1}^3 k_a^3)}\Bigg[2(\varepsilon-\eta)\sum_a k_a^3+\varepsilon\left(\sum_a k_a^3+\sum_{a\neq b}k_a^2k_b+8\sum_{a>b}\frac{k_a^2k_b^2}{k_T}\right)+$$
$$\sinh^2 2\alpha\Bigg(2(\varepsilon-\eta)\sum_a k_a^3+\varepsilon\Bigg(\sum_a k_a^3+\sum_{a\neq b}k_a^2k_b+\sum_{a>b}8\frac{k_a^2k_b^2}{k_2+k_3-k_1}+8\frac{k_a^2k_b^2}{k_3+k_1-k_2}+$$
$$8\frac{k_a^2k_b^2}{k_1+k_2-k_3}\Bigg)\Bigg)\Bigg] \tag{6.46}$$

which agrees with the calculation done in [36]. This demonstrates that the precription given above indeed works.

Similarly, using the BD results, we get the following for mixed correlators for Maldacena action in $\alpha$ vacua:

$$\langle\gamma^{h}(\boldsymbol{k_1})\zeta(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle_\alpha = \frac{H^4}{4M_{pl}^4\varepsilon}\frac{1}{(\prod_{a=1}^3 k_a^3)}e^{h}_{ij}(k_1)k_{2i}k_{3j}\Bigg[\left(-k_T+\frac{\sum k_ik_j}{k_T}+\frac{k_1k_2k_3}{k_T^2}\right)+$$
$$\sinh^2 2\alpha\left(-(-k_1+k_2+k_3)+\frac{k_2k_3-k_1k_2-k_1k_3}{(-k_1+k_2+k_3)}-\frac{k_1k_2k_3}{(-k_1+k_2+k_3)^2}\right)+\text{cyclic}\Bigg] \tag{6.47}$$

$$\langle\zeta(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\gamma^{h_3}(\boldsymbol{k_3})\rangle_\alpha = \frac{H^4}{8M_{pl}^4}\frac{1}{(\prod_{a=1}^3 k_a^3)}e_{ij}^{h_2}(k_2)e_{ij}^{h_3}(k_3)\left[\left(-\frac{1}{4}k_1^3+\frac{1}{2}k_1(k_2^2+k_3^2)+4\frac{k_2^2k_3^2}{k_T}\right)\right.$$
$$\left.+\sinh^2 2\alpha\left(-\frac{1}{4}k_1^3+\frac{1}{2}k_1(k_2^2+k_3^2)+4\frac{k_2^2k_3^2}{(k_2+k_3-k_1)}+4\frac{k_2^2k_3^2}{(k_3+k_1-k_2)}+4\frac{k_2^2k_3^2}{(k_1+k_2-k_3)}\right)\right] \tag{6.48}$$

To take an example of an odd parity interaction we can take the Weyl action and calculate the un-trimmed wavefunction coefficient $\psi_3$ (up to some numerical factors):

$$S_3 = \int d\eta d^3x a^{-5}\left(\partial_\eta\Pi^+_{ij}\partial_\eta\Pi^+_{jk}\partial_\eta\Pi^+_{ki} - \partial_\eta\Pi^-_{ij}\partial_\eta\Pi^-_{jk}\partial_\eta\Pi^-_{ki}\right) \tag{6.49}$$

$$\text{where}\;\; \partial_\eta\Pi^\pm_{ij} = \frac{1}{2}\left(\partial_\eta(a\partial_\eta\gamma_{ij}) \mp i\varepsilon_{jab}\partial_b\partial_\eta\gamma_{ia}\right) \tag{6.50}$$

$$\psi_3 \sim \frac{k_1^2k_2^2k_3^2}{k_T^6}\left(\varepsilon_{iab}\varepsilon_{jcd}\varepsilon_{kfg}e^{h_1}_{jb}e^{h_2}_{kd}e^{h_3}_{ig}k_{1a}k_{2c}k_{3f} - \sum_{cyclic} k_2k_3\varepsilon_{jab}k_{1b}e^{h_1}_{ia}e^{h_2}_{ki}e^{h_3}_{kj}\right) \tag{6.51}$$

We can simplify the last equation using the relation: $\varepsilon_{iab}k_be^h_{ja} = -ikhe_{ij}$. However, we must keep in mind that while using the ansatz 6.44 we have to take the trimmed part as the one before we use the relation above to simplify 6.51, i.e we consider the unsimiplified levi-cevita contractions to be the tensor contractions. Hence we don't flip the signs of $k$'s generated from this contraction while using the ansatz. We also note that 6.51 has multiple tensor contractions and for each contraction, the trimmed part obeys 6.44, so we can just add up the answers. This finally gives the following correlators for arbitrary helicities $h_i = \pm 1$:

$$\langle\gamma^{h_1}(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\gamma^{h_3}(\boldsymbol{k_3})\rangle_\alpha \sim \sinh 4\alpha e^{h_1}_{ij}e^{h_2}_{jk}e^{h_3}_{ki}\left[\frac{3}{k_T^6}(h_1+h_2+h_3+h_1h_2h_3)\right.$$
$$\left. - \sum_{cyclic}\frac{1}{(-k_1+k_2+k_3)^6}(h_1-h_2-h_3+h_1h_2h_3)\right] \tag{6.52}$$

$$\implies \langle\gamma^-(\boldsymbol{k_1})\gamma^+(\boldsymbol{k_2})\gamma^+(\boldsymbol{k_3})\rangle_\alpha \sim \sinh 4\alpha e^{h_1}_{ij}e^{h_2}_{jk}e^{h_3}_{ki}\frac{1}{(k_2+k_3-k_1)^6} = -\langle\gamma^+(\boldsymbol{k_1})\gamma^-(\boldsymbol{k_2})\gamma^-(\boldsymbol{k_3})\rangle_\alpha \tag{6.53}$$

Again, these results match with the explicit calculations done in [47].

We see that the prescription saves us the effort of doing the cumbersome in-in calculation which involves simplifying a lot of integrals.

# Chapter 7

# Conclusion

In this work, we aimed to understand the mixed graviton and scalar bispectra in the EFT of inflation. A summary of the main results of the thesis is as follows:

- Following the methods prescribed in [11], we wrote a general EFToI and attempted to organize terms in the order of the number of derivatives on them w.r.t the metric perturbations. We also clarified the energy scale counting in terms of $H/M_i$ where $M_i$'s are the UV cutoffs/high mass scales appearing in our EFTs.

- We gave some general constraints on the EFT parameters, namely the bounds due to small spectral tilt, unitarity bound in flat space limit and experimental bounds of non-gaussianities [11]. We gave 2 simple examples where these bounds constrained some of the arbitrary EFT coefficients.

- We explicitly checked the soft limits 5.5, 5.6 and 5.3 for EFT operators, which change both the quadratic and cubic action for $\zeta$ or $\gamma$. These limits as we checked, are obeyed for cubic operators at leading order in soft momenta and leading order in the "couplings" (i.e. $\varepsilon, \eta$ etc.) and $H/M_{pl}$. We clarified some confusion in the literature related to what diagrams to take and how to organise terms in order to get the correct soft limits. Hence, as expected from the general derivation of the soft limits [17, 15] (see Section 5.1), they can be extended to models beyond the standard Maldacena action [14].

- We discussed/reviewed some conformal bootstrap methods and we can readily see that the approach is definitely less interaction dependent than the boostless bootstrap. However, the calculations become complicated for arbitrary interactions and hence, these methods might not be very economical.

- We attempted to bootstrap the three-point correlators from purely cubic operators, i.e. operators that do not change the quadratic action, by noticing that they don't contribute to the soft limit at leading order in soft momenta (see Equations 5.39, 5.40). Using the bootstrap prescription in [23], we found (as expected) that the number of undetermined parameters increases with the number of derivatives. Furthermore, this bootstrap method heavily relies on knowing the interaction hamiltonian since we use the amplitude of the interaction to determine the residue of the total energy pole [25]. Hence, the bootstrap method is more of an alternative to doing the in-in integrals than an ideal "boundary perspective" bootstrap.
- Since the background symmetry allows us to take $\alpha$ vacua, we have shown that the results of BD can be readily extended to give the $\alpha$ vacua 3-point correlators if the interaction/action is parity even. This helps us in bypassing a lot of in-in integrals.

It will be interesting to explore mixed quartic operators for and beyond the Maldacena action and check the soft limits for these since some exchange diagrams also come into the picture here as they're of a similar order in "couplings" and $H/M_{pl}$ as contact diagrams. The right-hand side of the soft limits might be tricky to evaluate due to the momentum dependence of the polarization tensors. We would also like to extend our $\alpha$ vacua results to four-point correlators and explore the implications of the new kinds of pole structures we get. The approach of calculating 4-point de-sitter correlators and then taking a leg soft to give 3-point inflationary correlators can also be discussed in the context of $\alpha$ vacua. Furthermore, we refer to non-attractor models and the fact that the consistency relations are violated for such models. While the usual Maldacena soft theorems are indeed violated, for the most common non-attractor models, where there is also a shift symmetry (since the potential is a constant), there are other soft theorems that hold. They were derived in [48] using very general tools (Ward identities and OPEs) and rephrased in the EFT language in [49]. These "shift-symmetric" soft theorem now also involve derivatives of the power spectrum with respect to time (on top of the derivative with respect to k in Maldacena's relation). It can be interesting to explore how these new soft theorems can be used for a "non-attractor bootstrap".

# Appendix A

## A.1 Calculating the action in Unitary Gauge

From the definitions of the ADM metric variables, we have $\widetilde{N^i} = -g_{00}N^i = N^2N^i$. We further write $N = 1 + \delta N$ and then consider the following quadratic action:

$$\mathscr{L}_2 = \sqrt{-g}M_{pl}^2\Bigg(\frac{1}{2}R^{(4)} + m_1\delta K^i_j\delta K^j_i + m_2(\delta K)^2 + D\delta K - M_{pl}^2\lambda(t) - c(t)g^{00}$$
$$+M_1 g^{ij}\partial_i(g^{00}+1)\partial_j(g^{00}+1) + M_2^2(g^{00}+1)^2 + M_3(g^{00}+1)\delta K + m_3^{(3)}R(g^{00}+1)\Bigg) \tag{A.1}$$

where $c(t)$,$\lambda(t)$ are as defined before. Taking $\widetilde{N^i} = \partial_i\chi$, the equations of motion for $N$ and $\widetilde{N^i}$ gives

$$\delta N\left((m_1+3m_2-1)H - M_3\right) + (m_1+m_2)\partial^2\chi = \dot{\zeta}(m_1+3m_2-1) \tag{A.2}$$

$$\partial^2\chi = \frac{-1}{(m_1+3m_2-1)H - M_3}\left(-\partial^2\zeta a^{-2} + c(t)\delta N + 4M_2^2\delta N - 4a^{-2}M_1\partial^2\delta N\right) - 3\dot{\zeta} \tag{A.3}$$

$$+\frac{3\dot{\zeta}H(m_1+3m_2-1)}{(m_1+3m_2-1)H - M_3} - \frac{3\dot{\zeta}H(m_1+3m_2-1)M_3}{((m_1+3m_2-1)H - M_3)^2} \tag{A.4}$$

To solve these equations, one will have to take $m_1 + m_2 = 0$ so that the equations separate.

## A.2 Calculating the RR vertex for ${}^{(3)}R_{ij}^{(3)}R_{ij}$

The expression for the diagram where both the vertices are time ordered, i.e. RR vertices [44] after taking the soft limit is given by:

$$\langle \gamma^{h_1}(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})|_{\boldsymbol{k_3}\to 0}\rangle|_{RR} = \frac{H^6}{M_{pl}^6 k_1^9 4\varepsilon k_3^3}\left(\frac{M_{pl}^2\varepsilon}{M_7^2(4\cdot 8)} 2\cdot 2\cdot 2\cdot 2\, e_{ij}^{h_1}(\boldsymbol{k_1})e_{ij}^{h_2}(\boldsymbol{k_2})\right)$$
$$\left[\int_{-\infty}^{0}\left(k_1^2k_2^2e^{2ik_1\eta}-(k_1\cdot k_2)\frac{(1-ik_1\eta)(1-ik_2\eta)}{\eta^2}e^{2ik_1\eta}\right)\left(\int_{\eta}^{0}(1+ik_2\eta')k_2^4(1-ik_2\eta')\right)d\eta d\eta' +\right.$$
$$\left.\int_{-\infty}^{0}\left(k_1^2k_2^2-(k_1\cdot k_2)\frac{(1-ik_1\eta)(1+ik_2\eta)}{\eta^2}\right)\left(\int_{-\infty}^{\eta}(1-ik_2\eta')^2k_2^4e^{2ik_2\eta}\right)d\eta d\eta'\right] \tag{A.5}$$

where we have taken $\boldsymbol{k_3}\to 0$ but have not yet put $\boldsymbol{k_1}=-\boldsymbol{k_2}$ for clarity. All the combinatorial and numerical/coupling factors are in the bracket in the first line. Note that for the Maldacena vertex, we have not taken the non-local term [14], as that term is 0 (or rather subleading) in the soft limit. After putting $\boldsymbol{k_1}=-\boldsymbol{k_2}$ and simplifying we get:

$$\langle \gamma^{h_1}(\boldsymbol{k_1})\gamma^{h_2}(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})|_{k_3\to 0}\rangle|_{RR} = -\frac{15c_1H^6}{16M_{pl}^6k_1^3k_3^3} \tag{A.6}$$

## A.3 Soft limit of $\langle\gamma\zeta\zeta\rangle$ for ${}^{(3)}R\delta K$

The “exchange diagram” is the same as the right diagram in Figure 2 and we have, to the 1st order in $c_1$:

$$O_{2\zeta} = \int 4a\frac{M_{pl}^2}{M_5}\partial^2\zeta\left(\varepsilon\dot{\zeta}-\frac{\partial^2\zeta}{H}a^{-2}\right) \tag{A.7}$$

$$O_{\gamma\zeta\zeta} = \text{Maldacena terms } +4\frac{M_{pl}^2}{M_5}\left(2\frac{\gamma_{ij}a^{-1}}{H}\partial_i\partial_j\zeta\partial^2\zeta - a\gamma_{ij}\partial_i\partial_j\zeta\dot{\zeta}\right) \tag{A.8}$$

This gives a correction to the 3-point function, in the limit $\boldsymbol{k_1}\to 0$

$$\langle \gamma^h(\boldsymbol{k_1})\zeta(\boldsymbol{k_2})\zeta(\boldsymbol{k_3})\rangle = B_{LL}+B_{RR}-B_{RL}-B_{LR}+B_{\text{contact}} \tag{A.9}$$

where the contact vertex is the one from $\mathscr{O}_{\gamma\zeta\zeta}$. One finds that $B_{RR}+B_{LL}=B_{\text{contact}}$ and we can easily see that

$$B_{LR}(k_1 \to 0) = B_{RL}(k_1 \to 0) = \frac{1}{2}\frac{B_{MT}(k_1 \to 0, k_2 = k_3)\delta P_\zeta(k_3)}{P_\zeta(k_3)} = \frac{3}{4}e^h_{ij}(\boldsymbol{k_1})\frac{1}{k_2^2}k_{2i}k_{2j}P_\gamma(k_1)\delta P_\zeta(k_2) \tag{A.10}$$

and hence the soft limit (Equation 5.3) is satisfied.

## A.4 Purely Cubic Operators

Here, we give explicit expressions for operators that contribute to the mixed correlators. Note that for these calculations, we have taken 4.7, 4.8 as the quadratic actions, i.e. the Maldacena quadratic action. $\Lambda_i$'s are the UV cutoffs for each term while $c_i's$ are dimensionless quantities.

**Purely Cubic Operators for $\langle\gamma\gamma\zeta\rangle$**

$$\mathscr{O}_1 = \int \sqrt{-g}\Lambda_1^2 \delta K_{ij}\delta K_{ij}\delta g^{00} = \int \Lambda_1^2 a^3 \frac{1}{2H}\dot{\gamma}_{ij}\dot{\gamma}_{ij}\dot{\zeta} \tag{A.11}$$

$$\mathscr{O}_2 = \int \sqrt{-g}\Lambda_2 {}^{(3)}R_{ij}\delta K_{ij}\delta g^{00} = \int \Lambda_2 a \frac{1}{2H}\partial^2\gamma_{ij}\dot{\gamma}_{ij}\dot{\zeta} \tag{A.12}$$

$$\mathscr{O}_3 = \int \sqrt{-g} c_3 {}^{(3)}R_{ij} {}^{(3)}R_{ij}\delta g^{00} = \int c_3 a^{-1}\frac{1}{2H}\partial^2\gamma_{ij}\partial^2\gamma_{ij}\dot{\zeta} \tag{A.13}$$

$$\mathscr{O}_4 = \int \sqrt{-g} c_4 {}^{(3)}R_{ij}\delta K_{jk}\delta K_{ki} = \int c_4 a \left(\frac{1}{2}\partial^2\gamma_{ij}\dot{\gamma}_{ij}\dot{\zeta} - \frac{1}{2H}a^{-2}\partial^2\gamma_{ij}\dot{\gamma}_{jk}\partial_i\partial_k\zeta\right. \tag{A.14}$$

$$\left. + \frac{1}{2}\varepsilon\partial^2\gamma_{ij}\dot{\gamma}_{jk}\partial_i\partial_k\partial^{-2}\dot{\zeta} - \frac{1}{4}\dot{\gamma}_{ij}\dot{\gamma}_{jk}\partial_i\partial_k\zeta - \frac{1}{4}\dot{\gamma}_{ij}\dot{\gamma}_{ij}\partial^2\zeta\right) \tag{A.15}$$

$$\mathscr{O}_5 = \int \sqrt{-g}\frac{1}{\Lambda_5} {}^{(3)}R_{ij} {}^{(3)}R_{jk}\delta K_{ki} = \int \frac{1}{\Lambda_5}\left(\frac{1}{4H}a^{-3}\partial^2\gamma_{ij}\partial^2\gamma_{jk}\partial_k\partial_i\zeta\right. \tag{A.16}$$

$$\left. - \frac{1}{4}\varepsilon a^{-1}\partial^2\gamma_{ij}\partial^2\gamma_{jk}\partial_k\partial_i\partial^{-2}\dot{\zeta}\right) + \frac{1}{2}a^{-1}\partial^2\gamma_{ij}\dot{\gamma}_{ik}\partial_j\partial_k\zeta + \frac{1}{2}a^{-1}\partial^2\gamma_{ij}\dot{\gamma}_{ij}\partial^2\zeta \tag{A.17}$$

$$\mathscr{O}_6 = \int \sqrt{-g}\frac{1}{\Lambda_6} {}^{(3)}R_{ij} {}^{(3)}R_{ij}\delta K = \int \frac{1}{\Lambda_6}\left(\frac{1}{4H}a^{-3}\partial^2\gamma_{ij}\partial^2\gamma_{ij}\partial^2\zeta - \frac{1}{4}\varepsilon a^{-1}\partial^2\gamma_{ij}\partial^2\gamma_{ij}\dot{\zeta}\right) \tag{A.18}$$

$$\mathscr{O}_7 = \int \sqrt{-g}\frac{1}{\Lambda_7} {}^{(3)}R_{ij}\delta K_{ij} {}^{(3)}R = \int \frac{1}{\Lambda_7}a^{-1}\partial^2\gamma_{ij}\dot{\gamma}_{ij}\partial^2\zeta \tag{A.19}$$

$$\mathscr{O}_8 = \int \sqrt{-g}\frac{1}{\Lambda_8^2} {}^{(3)}R_{ij} {}^{(3)}R_{jk} {}^{(3)}R_{ki} = \int \frac{1}{\Lambda_8^2}\left(-\frac{3}{4}a^{-3}\partial^2\gamma_{ij}\partial^2\gamma_{jk}\partial_k\partial_i\zeta - \frac{3}{4}a^{-3}\partial^2\gamma_{ij}\partial^2\gamma_{ij}\partial^2\zeta\right) \tag{A.20}$$

$$\mathscr{O}_9 = \int \sqrt{-g}\frac{1}{\Lambda_9^2}{}^{(3)}R_{ij}{}^{(3)}R_{ij}{}^{(3)}R = \int -\frac{1}{\Lambda_9^2}a^{-3}\partial^2\gamma_{ij}\partial^2\gamma_{ij}\partial^2\zeta \tag{A.21}$$

$$\mathscr{O}_{10} = \int \sqrt{-g}\Lambda_{10}\delta K_{ij}\delta K_{jk}\delta K_{ki} = \int \frac{3}{4}a^3\Lambda_{10}\dot{\gamma}_{ij}\gamma_{jk}\left(\partial_k\partial_i\frac{\zeta}{H}a^{-2} - \varepsilon\partial_k\partial_i\partial^{-2}\dot{\zeta}\right) \tag{A.22}$$

$$\mathscr{O}_{11} = \int \sqrt{-g}\Lambda_{11}\delta K_{ij}\delta K_{ji}\delta K = \int \frac{3}{4}a^3\Lambda_{11}\dot{\gamma}_{ij}\dot{\gamma}_{ji}\left(\partial^2\frac{\zeta}{H}a^{-2} - \varepsilon\dot{\zeta}\right) \tag{A.23}$$

**Purely Cubic operators for $\langle\gamma\zeta\zeta\rangle$**

$$\mathscr{O}_1 = \int \sqrt{-g}\Lambda_1^2\delta K_{ij}\widetilde{N}_i\partial_j\delta g^{00} = \int \Lambda_1^2\left(-a\frac{\dot{\gamma}_{ij}}{H^2}\partial_i\zeta\partial_j\dot{\zeta} + a^3\frac{\varepsilon}{H}\dot{\gamma}_{ij}\partial_i\dot{\zeta}\partial_j\partial^{-2}\dot{\zeta}\right) \tag{A.24}$$

$$\mathscr{O}_2 = \int \sqrt{-g}\Lambda_2{}^{(3)}R_{ij}\delta K_{ij}\delta g^{00} = \int \Lambda_2\left(-a^{-1}\frac{\partial^2\gamma_{ij}}{H^2}\partial_i\partial_j\zeta\dot{\zeta} + \varepsilon a\frac{\partial^2\gamma_{ij}}{H^2}\partial_i\partial_j\dot{\zeta}\dot{\zeta}\right) \tag{A.25}$$

$$\mathscr{O}_3 = \int \sqrt{-g}\Lambda_3\delta K_{ij}\partial_i\delta g^{00}\partial_j\delta g^{00} = \int 2\Lambda_3 a\dot{\gamma}_{ij}\partial_i\dot{\zeta}\partial_j\dot{\zeta} \tag{A.26}$$

$$\mathscr{O}_4 = \int \sqrt{-g}\Lambda_4{}^{(3)}R_{ij}\widetilde{N}_i\partial_j\delta g^{00} = \int \Lambda_4\left(a^{-1}\frac{\partial^2\gamma_{ij}}{H^2}\partial_i\zeta\partial_j\dot{\zeta} - \varepsilon a\frac{\partial^2\gamma_{ij}}{H}\partial_i\dot{\zeta}\partial_j\partial^{-2}\dot{\zeta}\right) \tag{A.27}$$

$$\mathscr{O}_5 = \int \sqrt{-g}c_5{}^{(3)}R_{ij}\delta K_{ij}\delta K = \int c_5\left(-a^{-3}\frac{\partial^2\gamma_{ij}}{2H^2}\partial_i\partial_j\zeta\partial^2\zeta + \varepsilon a^{-1}\frac{\partial^2\gamma_{ij}}{2H}\partial^2\zeta\partial_i\partial_j\partial^{-2}\dot{\zeta}\right. \tag{A.28}$$

$$\left. + \varepsilon a^{-1}\frac{\partial^2\gamma_{ij}}{2H}\dot{\zeta}\partial_i\partial_j\zeta - \varepsilon^2 a\frac{\partial^2\gamma_{ij}}{2}\dot{\zeta}\partial_i\partial_j\partial^{-2}\dot{\zeta}\right) \tag{A.29}$$

$$\begin{aligned}\mathscr{O}_6 = \int \sqrt{-g}c_6{}^{(3)}R_{ij}\delta K_{jk}\delta K_{ki} = \int -\frac{1}{2}c_6 a\partial^2\gamma_{ij}\partial_j\partial_k\left(-\frac{\zeta}{H}a^{-2} + \varepsilon\partial^{-2}\dot{\zeta}\right)\partial_i\partial_k\left(-\frac{\zeta}{H}a^{-2} + \varepsilon\partial^{-2}\dot{\zeta}\right) \\ - c_6 a\dot{\gamma}_{ij}\partial_j\partial_k\zeta\partial_k\partial_i\left(-a^{-2}\frac{\zeta}{H} + \varepsilon\partial^{-2}\dot{\zeta}\right) - c_6 a\dot{\gamma}_{ij}\partial^2\zeta\partial_j\partial_i\left(-a^{-2}\frac{\zeta}{H} + \varepsilon\partial^{-2}\dot{\zeta}\right)\end{aligned} \tag{A.30}$$

$$\mathscr{O}_7 = \int \sqrt{-g}c_7{}^{(3)}R_{ij}\partial_i\delta g^{00}\partial_j\delta g^{00} = \int -2c_7 a^{-1}\frac{\partial^2\gamma_{ij}}{H^2}\partial_i\dot{\zeta}\partial_j\dot{\zeta} \tag{A.31}$$

$$\mathscr{O}_8 = \int \sqrt{-g}c_8{}^{(3)}R_{ij}{}^{(3)}R_{ij}\delta g^{00} = \int 2c_8 a^{-1}\partial^2\gamma_{ij}\partial_i\partial_j\zeta\frac{\dot{\zeta}}{H} \tag{A.32}$$

$$\mathscr{O}_9 = \int \sqrt{-g}\frac{1}{\Lambda_9}{}^{(3)}R_{ij}{}^{(3)}R_{jk}\delta K_{ki} = \int \frac{1}{\Lambda_9}\left(a^{-3}\frac{\partial^2\gamma_{ij}}{H}\partial_i\partial_k\zeta\partial_j\partial_k\zeta + a^{-3}\partial^2\frac{\gamma_{ij}}{H}\partial_i\partial_j\partial^2\zeta\right. \tag{A.33}$$

$$\left. + \frac{1}{2}a^{-1}\dot{\gamma}_{ij}\partial_i\partial_k\zeta\partial_j\partial_k\zeta + a^{-1}\dot{\gamma}_{ij}\partial_i\partial_j\zeta\partial^2\zeta - \varepsilon a^{-1}\partial^2\gamma_{ij}\partial_j\partial_k\zeta\partial_k\partial_i\partial^{-2}\dot{\zeta} - \varepsilon a^{-1}\partial^2\gamma_{ij}\partial^2\zeta\partial_i\partial_j\partial^{-2}\dot{\zeta}\right) \tag{A.34}$$

$$\mathscr{O}_{10} = \int \sqrt{-g}\frac{1}{\Lambda_{10}}{}^{(3)}R_{ij}{}^{(3)}R_{ij}\delta K = \int \frac{1}{\Lambda_{10}}\left(a^{-3}\frac{\partial^2\gamma_{ij}}{H}\partial_i\partial_j\zeta\partial^2\zeta - \varepsilon a^{-1}\partial^2\gamma_{ij}\partial_i\partial_j\zeta\dot{\zeta}\right) \tag{A.35}$$

$$\mathscr{O}_{11} = \int \frac{1}{\Lambda_{11}^2}{}^{(3)}R_{ij}{}^{(3)}R_{jk}{}^{(3)}R_{ki} = \int -\frac{3}{2}\frac{1}{\Lambda_{11}^2}a^{-3}\partial^2\gamma_{ij}\partial_i\partial_k\zeta\partial_j\partial_k\zeta - 3\frac{c_{11}}{\Lambda^2}a^{-3}\partial^2\gamma_{ij}\partial_i\partial_j\zeta\partial^2\zeta \tag{A.36}$$

$$\mathscr{O}_{12} = \int \sqrt{-g}\frac{1}{\Lambda_{12}^2}{}^{(3)}R_{ij}{}^{(3)}R_{ij}{}^{(3)}R = \int -4\frac{1}{\Lambda_{12}^2}a^{-3}\partial^2\gamma_{ij}\partial_i\partial_j\zeta\partial^2\zeta \tag{A.37}$$

$$\mathscr{O}_{13} = \int \sqrt{-g}\Lambda_{13}\delta K_{ij}\delta K_{jk}\delta K_{ki} = \int \frac{3}{2}a^3\Lambda_{13}\dot{\gamma}_{ij}\partial_j\partial_k\left(\frac{\zeta}{H}a^{-2} - \varepsilon\partial^{-2}\dot{\zeta}\right)\partial_k\partial_i\left(\frac{\zeta}{H}a^{-2} - \varepsilon\partial^{-2}\dot{\zeta}\right) \tag{A.38}$$

$$\mathscr{O}_{14} = \int \sqrt{-g}\Lambda_{14}\delta K_{ij}\delta K_{ji}\delta K = \int a^3\Lambda_{14}\dot{\gamma}_{ij}\partial_j\partial_i\left(\frac{\zeta}{H}a^{-2} - \varepsilon\partial^{-2}\dot{\zeta}\right)\left(\partial^2\frac{\zeta}{H}a^{-2} - \varepsilon\dot{\zeta}\right) \tag{A.39}$$

## A.5 Issue with Gauge

The EFT of inflation is written after fixing the time diffs, so naturally, the operators in the EFT do not respect time diffs. Therefore, they are NOT gauge-invariant. Consider the following operator

$$\sqrt{-g}\left(\delta g^{00}(t)\right)^2 {}^{(3)}R(t) \tag{A.40}$$

Naively if we just calculate the cubic $\zeta$ interactions coming from this operator, one can easily see that we get a non-zero answer in the unitary gauge, but zero in the flat gauge. Now, one can make this operator completely gauge-invariant by introducing the Stueckelberg field, $\pi$. The gauge invariant operator reads

$$\sqrt{-g}\left(\frac{\partial(t+\pi)}{\partial x^\mu}\frac{\partial(t+\pi)}{\partial x^\nu}g^{\mu\nu} + 1\right)^2 {}^{(3)}R(t+\pi) = \sqrt{-g}\Big[(1+\dot{\pi})^2 g^{00} + \partial_i\pi\partial_j\pi g^{ij} \tag{A.41}$$

$$+2g^{0i}\partial_i\pi(1+\dot{\pi}) + 1\Big]^2 {}^{(3)}R(t+\pi) \tag{A.42}$$

Since we are interested in the cubic vertex, we need the expression for ${}^{(3)}R$ only up to first order.

$${}^{(3)}R = \partial_k\Gamma^k_{ii} - \partial_i\Gamma^k_{ik} \tag{A.43}$$

$$= -a^{-2}\partial^2 g_{ii} \tag{A.44}$$

The operator becomes

$$= -\left[(1+\dot{\pi})^2 g^{00} + \partial_i\pi\partial_j\pi g^{ij} + 2g^{0i}\partial_i\pi(1+\dot{\pi}) + 1\right]^2 e^{-2\rho}\partial^2 g_{ii}(t+\pi) \tag{A.45}$$

Let us evaluate this operator in the two gauges [14]
**Flat gauge:** $\delta\phi \neq 0$

$$= -6(\delta N|_{\psi=0} - \dot{\pi})^2 H\partial^2\pi \tag{A.46}$$

$$= -6\left(\frac{\dot{\zeta}}{H}\right)^2 \partial^2\zeta \tag{A.47}$$

**Co-moving gauge:** $\delta\phi = 0$

$$= -6\frac{\dot{\zeta}^2}{H^2}\partial^2\zeta \tag{A.48}$$

The vertex for $\zeta$ matches in the two gauges as expected. Since, this operator just contains derivatives, at leading order, this gives a vanishing contribution to the local bispectrum.

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