Numerical Studies on disordered interacting fermion chain

A Thesis

submitted to
Indian Institute of Science Education and Research Pune
in partial fulfillment of the requirements for the
BS-MS Dual Degree Programme

by

Rohit Bhagchandani

Indian Institute of Science Education and Research Pune
Dr. Homi Bhabha Road,
Pashan, Pune 411008, INDIA.

April, 2018

Supervisor: Dr. M.S. Santhanam

Certificate

This is to certify that this dissertation entitled Numerical Studies on disordered interacting fermion chaintowards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by Rohit Bhagchandaniat Indian Institute of Science Education and Research under the supervision of Dr. M.S. Santhanam , Associate Professor, Department of Physics , during the academic year 2017-2018.

Dr. M.S. Santhanam

Committee:

Dr. M.S. Santhanam

Dr. T.S. Mahesh

Declaration

I hereby declare that the matter embodied in the report entitled Numerical Studies on disordered interacting fermion chain are the results of the work carried out by me at the Department of Physics, Indian Institute of Science Education and Research, Pune, under the supervision of Dr. M.S. Santhanam and the same has not been submitted elsewhere for any other degree.

Rohit Bhagchandani

Acknowledgments

I would like to thank my advisors, Dr. Santhanam and Dr. Sreejith for helping and guiding me, my friends and my family for their support and belief in me. I would also like to thank IISER for providing me this facility to learn and DST for providing me the INSPIRE scholarship.

Abstract

Here we study the behaviour of a fermionic system in a linear tight binding lattice in the presence of interaction and disorder, and whether the system displays Many-Body Localization. We also observe the phase transitions in a driven Floquet MBL system and how its behaviour and phases differ from a static system.

Contents

Abstract **ix**

1 Introduction **1**

2 Preliminaries **3**

2.1 Time evolution 3

2.2 Quantum Thermalization 4

2.3 Eigenstate Thermalization Hypothesis 4

2.4 Single Particle Localization 5

2.5 Many Body Localization 6

3 Disordered interacting tight binding model **7**

3.1 IPR and disorder 8

3.2 Large disorder 8

3.3 IPR for Ground state 10

3.4 Next nearest neighbour interaction 11

4 Periodically driven transverse field model **13**

4.1 Transverse Field Model 13

4.2 Analogous 2 spin system 17

5 Summary **19**

5.1 Results and Discussion 19

5.2 Future work 20

Chapter 1

Introduction

In closed and isolated quantum systems, two phases of dynamics are observed - namely eigenstate thermalization and localization. The discovery of localization of single particle states in quantum systems is accredited to P.W.Anderson [1], after whom this phenomenon was named as Anderson Localization. Anderson studied a single particle in a tight binding Hamiltonian model with disorder in the form of random potentials at each site. He deduced that for a sufficiently large disorder, the particle's wavefunction stops diffusing and as a result the particle remains localized close to the state it was initially in.

Following Anderson's lead, recently others have started exploring localization in multi-particle systems in the presence of interactions as well as disorder. Thus using various techniques such as perturbation theory [2] , exact diagonalisation [3] and DMRG [4] techniques it has been shown that even in isolated multi-particle systems with strong interaction, localization can occur. In such systems, the occurrence of Many-Body Localization phase depends on various factors [6], such as the form of interaction, its strength, the strength of disorder, initial state etc.

Furthermore, localization has been observed even in periodically driven Floquet many-body systems [8]. In such systems, the periodic driving constantly pumps energy into a closed system. Thus, one may expect that such a system would absorb energy indefinitely and approach an infinite temperature featureless ETH state. However, in the presence of a strong disorder, such a situation can be avoided [7]. Under these specific conditions, instead of the system thermalizing, we observe that the dynamics is analogous to localization, which

we call Floquet-Many Body Localization.

So while an ETH system acts like a bath for its subsystems, a localized system does not and therefore none of its subsystems thermalize. In other words, there are emergent local integrals of motion whose values do not change under the dynamics and thus do not reach thermalized equilibrium values [9].

In this thesis, we study regimes in which ETH is broken. We also study the phase transitions that these MBL systems display. However, unlike the conventional approach of studying phase transitions using statistical ensembles of the system, here we study the properties of its eigenstates and eigenvalues. This is because the difference between an ETH system and an MBL system is only apparent when observing the dynamics and disappears when evaluating thermodynamic quantities by averaging over a canonical ensemble.

We study localization in a linear tight binding model acting on a fermionic system, in the presence of interaction and disorder. Then by changing the strengths of the interaction and disorder we observe the changes in its properties. We also study phase transitions in a Floquet-MBL system in a periodically driven transverse field model. Here instead of studying dynamics directly, we use properties of eigenstates as a proxy for observing.

Chapter 2

Preliminaries

2.1 Time evolution

The time evolution of a wavefunction $|\psi(t)\rangle$ is defined by the Schrodinger's equation

$$i\hbar \frac{\mathrm{d}\,|\psi(t)\rangle}{\mathrm{d}t} = H\psi(t)$$

Thus the time evolution of a state $\psi(t)$ can be determined by a unitary time evolution operator $U(t)$

$$U(t) = \mathcal{T} e^{-i(\int_0^t H(t')dt')}$$

where $\mathcal{T}$ denotes time ordering. such that

$$|\psi(t)\rangle = U(t)\,|\psi(0)\rangle$$

2.2 Quantum Thermalization

Earlier we defined thermalization as the ability of a system to act as a reservoir for one of its subsystems. Here we present a formal definition of thermalization.

Consider a system A which can be divided into a subsystem S and a bath B, s.t. $H_A = H_S \otimes H_B$. For this subsystem, the reduced density is obtained by tracing the density matrix over the bath, i.e. $\rho_S = Tr_B(\rho)$. For thermal equilibrium at temperature T, the system is described by

$$\rho^{eq}(T) = \frac{1}{z(T)} e^{-\frac{H}{k_B T}}$$

where $z(T)$ is the partition function. For such a system, we say subsystem S thermalizes if

$$\rho_S = Tr_B(\rho^{eq}(T)).$$

For quantum systems at thermal equilibrium, it is sufficient to have one parameter each per conserved quantity to uniquely identify that system. This introduces an apparent paradox, wherein unitary time evolution of a state to its thermal equilibrium results in the system losing information about its initial state. However, the information about the initial state is not actually lost. Upon time evolution the information spreads into the system, and after sufficiently long timescales becomes inaccessible to local measurements. To recover this spread out information would require one to make global measurements.

2.3 Eigenstate Thermalization Hypothesis

The Eigenstate Thermalization Hypothesis assumes that for any given initial state, the system eventually thermalizes. As a consequence, it implies that all many-body eigenstates of the Hamiltonian are at thermalized too. Consider an eigenstate $|n\rangle$ of a local non-trivial Hamiltonian H such that

$$H\,|n\rangle = E_n\,|n\rangle \tag{2.1}$$

with E_n corresponding to the temperature at thermal equilibrium. In the limit that $dim(\rho_S) \to \infty$, $\dim(\rho_B) \to \infty$ and $\frac{dim(\rho_B)}{dim(\rho_S)} \to 0$, ETH claims that the ρ_S approaches Gibbs ensemble with constant energy constraints [11] [12]. That is to say,

$$\rho_S^{eq}(T) = \frac{1}{z(T)} e^{-\beta H}$$

where β is determined by the energy density of sub-system at equilibrium.

For periodically driven ETH systems, since the drive is constantly pumping energy into the system, it keeps absorbing and heating. For a system with a bounded Hilbert space, it thermalizes to the infinite temperature state [13]. Such a system continues acting as a reservoir for its subsystems, but not just for the exchange of energy, instead the system provides more degrees of freedom to the subsystem. This results in greater entanglement between the system and the subsystem, thereby diffusing the information about the initial state at the global scale.

2.4 Single Particle Localization

ETH fails for localized systems, wherein a particle stays localized close its initial state. Here we take a look at a tight binding hamiltonian that displays single particle localization.

$$H = J \sum_{ij} (c_i^\dagger c_j + c_j^\dagger c_i) + \sum_i U_i c_i^\dagger c_i$$

Anderson showed that the eigenstates for this system are localized in 1 and 2 dimensions, and given strong enough disorder localization holds for higher dimensions too [**?**]. In such cases the Hamiltonian has a localized spectrum, given by

$$\psi_i(\vec{r}) \sim e^{-\frac{\vec{r}-\vec{R_i}}{\epsilon}} \tag{2.2}$$

where $\vec{R_i}$ is the location of the particle and ϵ is the localization length, which depends on factors like energy of the particle and the disorder strength of the system.

2.5 Many Body Localization

Many body Localization is essentially Anderson localization for multiple particles interacting with each other, where different dynamics are observed because of the interaction between particles. For a strong enough disorder ETH can be avoided even in interacting multi particle systems. It has also been observed that MBL phase displayed by fermions and bosons are largely the same[6], since MBL occurs at high energy. In this thesis we study how MBL depends on various parameters, by analyzing the properties of eigenspectrum.

Chapter 3

Disordered interacting tight binding model

The model we use here is a fermion system with the tight binding Hamiltonian along with interaction and disorder:

$$H = J\sum_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + J_z \sum_i n_i n_{i+1} + \sum_i h_i n_i$$

where $c_i^\dagger$ is a creation operator, c_i is an annihilation operator and n_i is a number operator all acting on the ith site. Here the disorder(h_i) at each site is chosen randomly over a range of values $[-\delta, \delta]$. We study this model for 14 sites and 7 particles.

Using a Jordan-Wigner transform this Hamiltonian can be mapped to a Hubbard Hamiltonian with an interaction. The transform is given by:

$$\sigma_j^+ = e^{i\pi \sum_{k<j} n_k} c_j^\dagger$$

$$\sigma_j^- = e^{-i\pi \sum_{k<j} n_k} c_j$$

$$\sigma_j^z = 2n_j - 1$$

3.1 IPR and disorder

Inverse Participation Ratio for a state $|\psi\rangle = \sum_i a_i |\psi_i\rangle$ is defined as $(\sum_i \mid a_i^4 \mid)^{-1}$. For a localized state, contribution from a single basis i would be a_i 1 and all rest would be close to 0. As a result, IPR for localized state is close to 1. For a completely delocalized state, all $a_i \sim \frac{1}{\sqrt{D}}$ where D is the dimension of the Hilbert space of the system.

Thus, IPR for a delocalized state scales with the dimension of its Hilbert space and is larger than the IPR for localized state. Precisely, IPR for localized systems scales as D^0 whereas IPR for delocalised systems scales as D^1. Further, IPR for states that scale as fractals scales as D^d where d is a fraction.

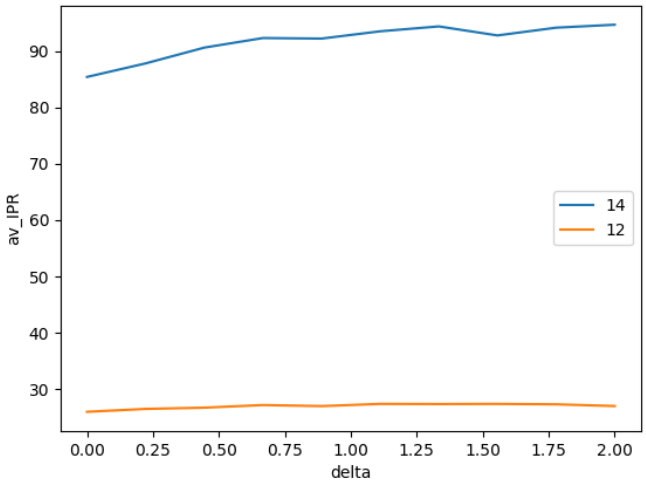
In Figure 3.1 we plot the IPR vs disorder for different interaction strengths. We do it for system size 12 and 14. We observe that the N = 12 system is more localized that N=14. Because of computational constraints, here instead of looking at scaling of IPR, we try to get some insight by calculating IPR for 2 different system sizes.

3.2 Large disorder

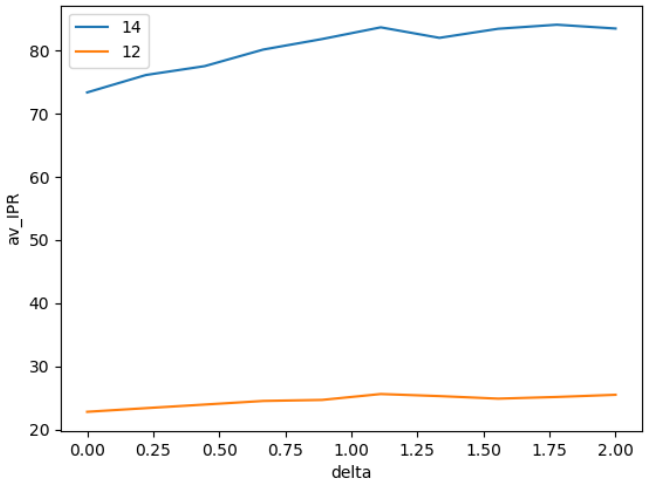
Here we observe the behaviour of the system at a very large disorder. We plot the value of IPR vs Interaction Strength (J_z) for disorder strength $\delta = 20$ in Figure 3.2

In the presence of a large disorder at large interaction the system behaves like a mott insulator [14]. This happens because at low energy, at half filling, the system has a charge ordered state. This charge density wave results in a higher delocalization for lower interaction strength, and this order disappears as we increase interaction strength. Thus, the system becomes more localized as a result.

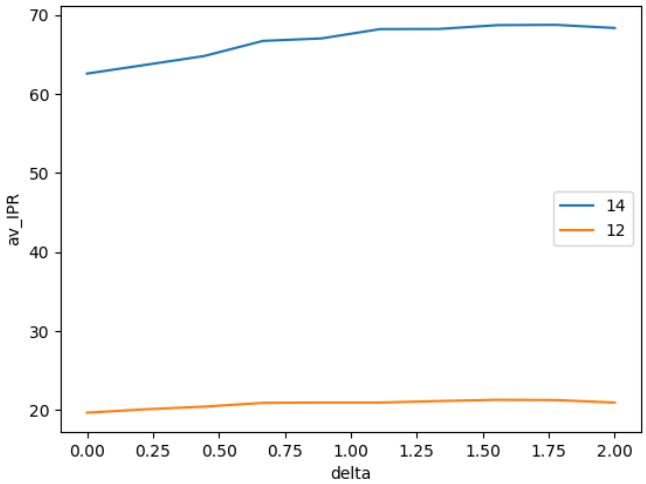
For a charge density wave, the nearest neighbour interaction doesn't matter, thus it isn't affected by a change in interaction strength. However, introducing strong disorder changes



(a) IPR v/s disorder for $J_z = 0.1$



(b) IPR v/s disorder for $J_z = 1$



(c) IPR v/s disorder for $J_z = 2$

Figure 3.1: IPR v/s disorder

that. Since the results obtained in the previous sections were for small range of disorder, we note from Figure 3.2 that the charge density wave doesn't melt in that range.

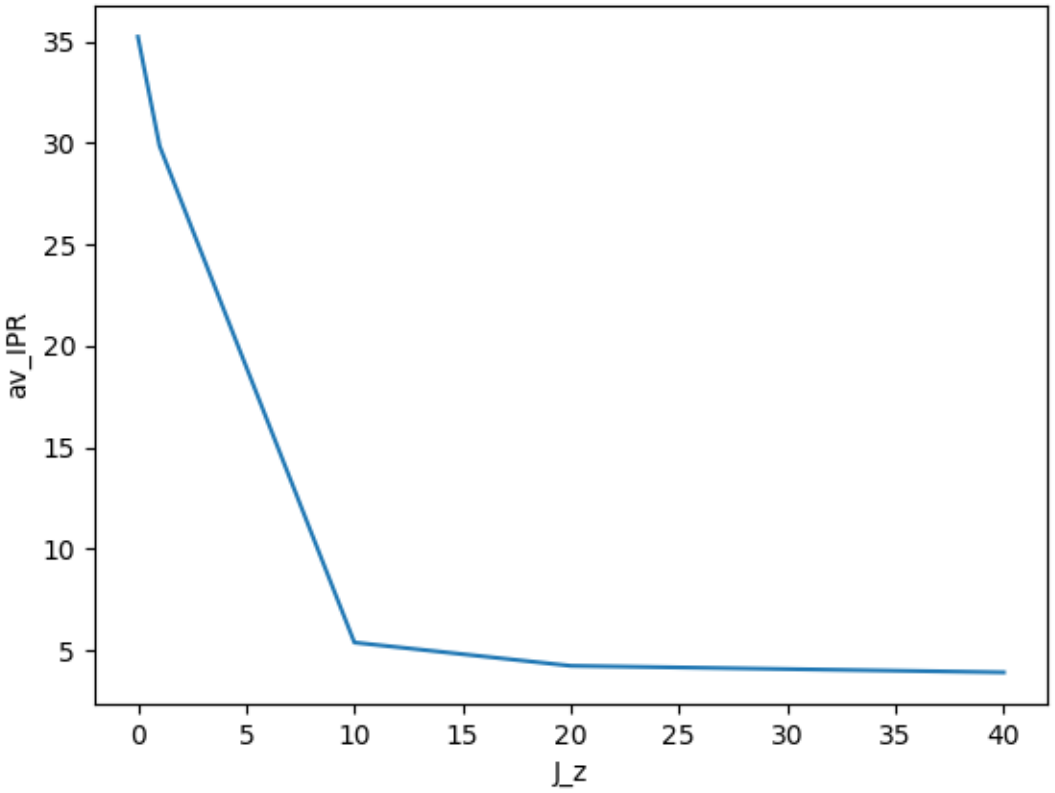
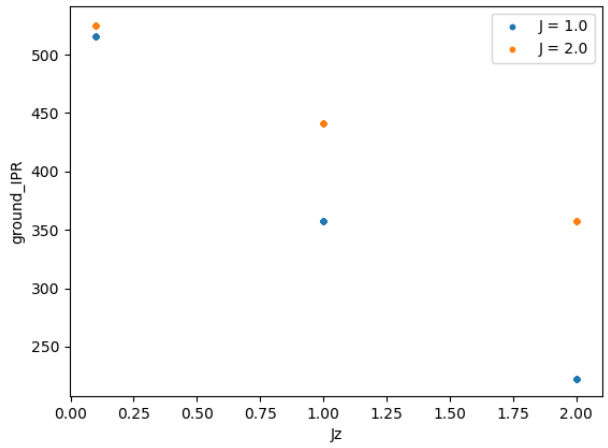


Figure 3.2: IPR vs Interaction strength for large disorder

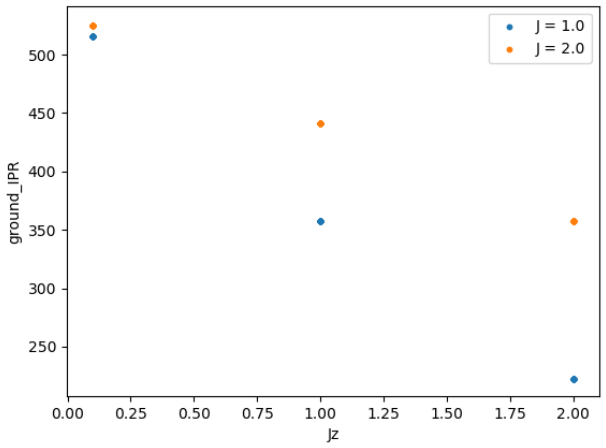
3.3 IPR for Ground state

The ground state for the Hubbard model for low interaction strength we have here behaves like a charge density wave. To observe how the localization for the ground state change we plot the the IPR for ground state for different values of interaction strength in Figure 3.3.

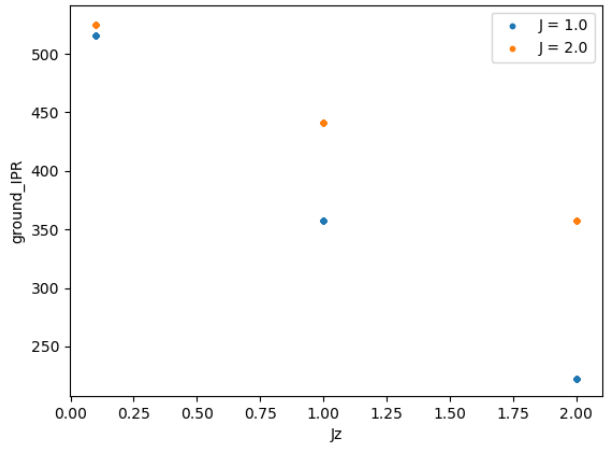
We observe that there is a decrease in the value of IPR as interaction strength increases. We also observe that localization for the ground state is roughly the same for different disorder strengths.



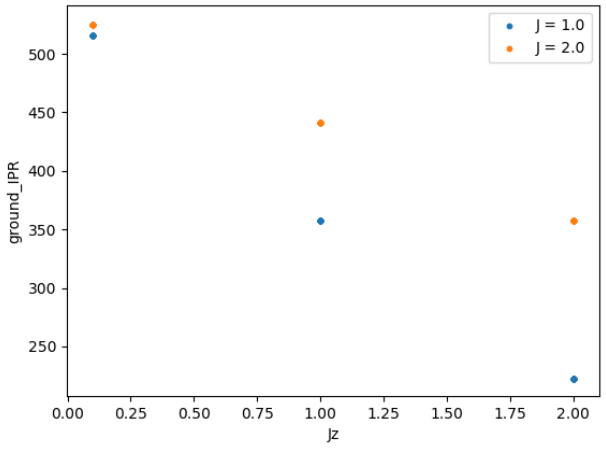
(a) Ground state IPR v/s interaction for $\delta = 0.44$



(b) Ground state IPR v/s interaction for $\delta = 0.88$



(c) Ground state IPR v/s interaction for $\delta = 1.33$



(d) Ground state IPR v/s interaction for $\delta = 1.77$

Figure 3.3

3.4 Next nearest neighbour interaction

As we saw earlier, since in the low interaction limit at half filling, the charge density wave is observed. As a result, due to the nearest neighbour interaction, this system behaves like a non interacting system. To avoid that situation, we introduce next-nearest neighbour interaction.

In this above, we change the interaction from nearest neighbour to next hearest neighbour.

The Hamiltonian thus becomes

$$H = J\sum_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + J_z \sum_i n_i n_{i+1} n_{i+2} + \sum_i h_i n_i$$

Plotting IPR v against interaction strength for a large disorder ($\delta = 20$) in Figure 3.4 and comparing it to the previous result, we observe that the localization has decreased by a factor of 2 for next-nearest neighbour interaction. This is because a longer interaction range makes a charge ordered density wave less favourable.

We notice here a

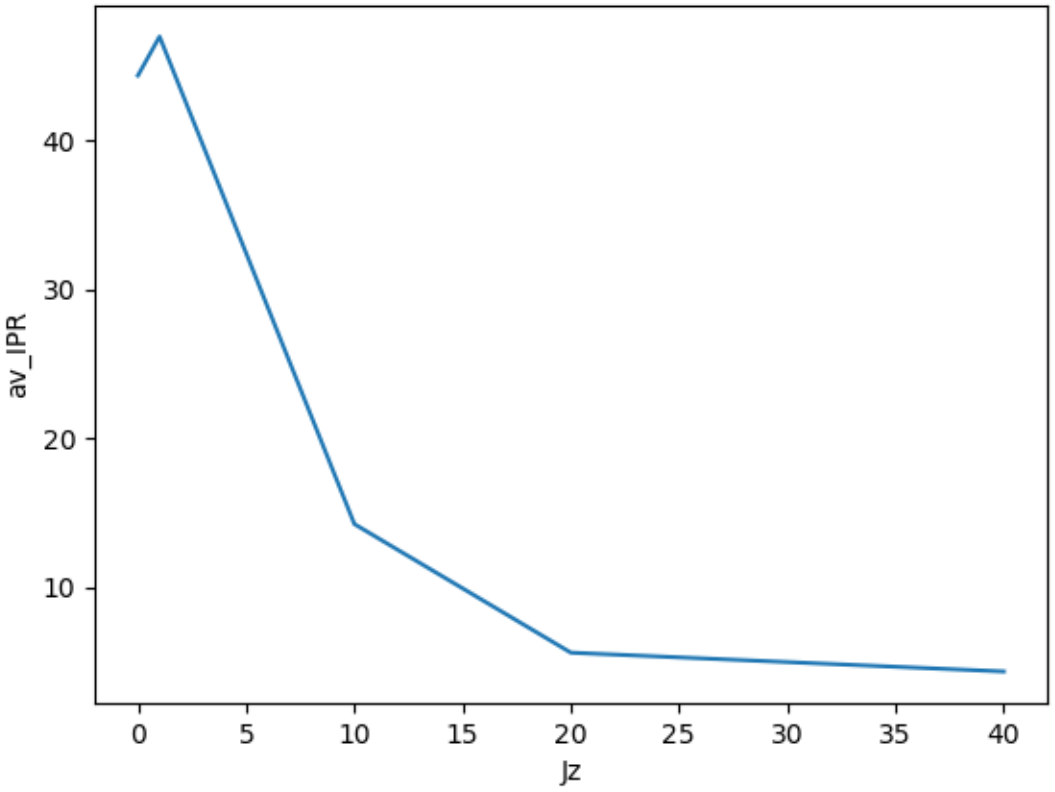


Figure 3.4: IPR vs Interaction strength for large disorder for next-nearest neighbour

Chapter 4

Periodically driven transverse field model

Floquet spin systems are periodically driven spin chains wherein the Hamiltonian varies periodically with a fixed time period T, i.e. $H(t) = H(t+T)$. For such a system the time evolution is defined by the unitary floquet operator.

$$U_f(t) = \mathcal{T} e^{-i(\int_0^t H(t')dt')} \tag{4.1}$$

One might expect a driven many-body system to absorb energy indefinitely and approach an infinite temperature. However, such a situation can avoided for floquet systems with the presence of a disorder interaction in the system. As a result such systems give rise to interesting dynamical behaviour and allow phases to be defined. One such phase is the π Spin Glass, also referred to as a time crystal.

4.1 Transverse Field Model

The following model is characterised by the Hamiltonian

$$H(t) = \sum_{i=0}^{L-1} J_z \sigma_i^z \sigma_{i+1}^z + \sum_{n \in \mathbb{Z}^+} \sum_{i=0}^{L} h \sigma_i^x \delta(t - nT) \tag{4.2}$$

Here the first term is an interaction term between neighbouring spin and the second term corresponds to a field along the x direction applied at integral intervals of a fixed time period T. As a result $H(t) = H(t+T)$ and the hamiltonian describes a floquet system with a period T. The unitary time evolution operator for this system is

$$U_f(T) = e^{-i\sum_{i=0}^{L-1} J_z \sigma_i^z \sigma_{i+1}^z T} e^{-i\sum_{i=0}^{L} h\sigma_i^x} \tag{4.3}$$

4.1.1 Spontaneous symmetry braking

Time crystals display spontaneous Time Translation Symmetry Braking(TTSB). The definition for TTSB is that it occurs if the eigenstates of Floquet unitary operator are not short range correlated [15]. To illustrate TTSB, consider the Floquet unitary operator

$$U_f = e^{-itH_{HBL}} e^{i\pi/2\sum_i \sigma_i^x} \tag{4.4}$$

$$H_{MBL} = \sum_i (J^z \sigma_i^z \sigma_{i+1}^z + h^z \sigma_i^z + h^x \sigma_i^x \tag{4.5}$$

The second term in the Floquet operator $e^{i\pi/2\sum_i \sigma_i^x} = \prod_i i\sigma_i^x$ flips all the spins. For $h_i^x = 0$ the eigenstates of H_{MBL} are simply the eigenstates of σ_i^z. For such eigenstates $|\{s_i\}\rangle$ we have $s_i = \pm 1$ and $\sigma_k^z |\{s_i\}\rangle = s_k |\{s_i\}\rangle$.

Applying H to $|\{s_i\}\rangle$ we obtain $H_{MBL} |\{s_i\}\rangle = (E^+(\{s_i\}) + E^-(\{s_i\})) |\{s_i\}\rangle$, where $E^+(\{s_i\}) = \sum_i J^z \sigma_i^z \sigma_{i+1}^z$ and $E^-(\{s_i\}) = \sum_i (h^z s_i)$.

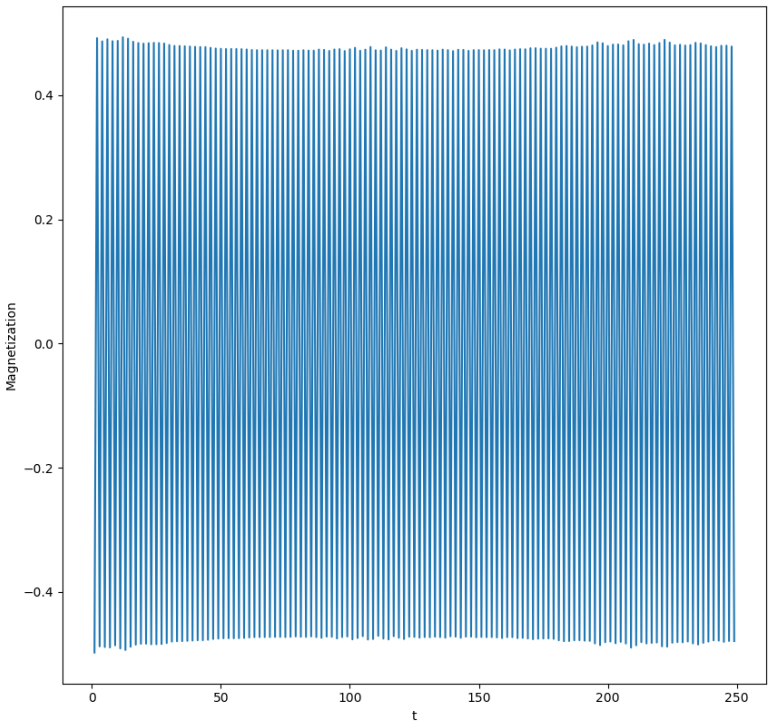
Thus the eigenstates for the Floquet operator are $(e^{itE^-(\{s_i\})/2} |\{s_i\}\rangle \pm e^{-itE^-(\{s_i\})/2} |\{-s_i\}\rangle)/\sqrt{2}$ with eigenvalues $\pm e^{itE^+(\{s_i\})}$.

Since the eigenstates of the Floquet unitary operator has long range correlations, TTSB occurs for $h_i^x = 0$. Furthermore, it has been shown using perturbation theory that TTSB holds upto weak perturbations of the Floquet operator. Therefore the eigenstates of the perturbed Floquet operator with $h_i^x \neq 0$ will also satisfy TTSB.

4.1.2 Error correction

In the previous hamiltonian, setting the value of the parameter h to π would correspond to applying a pi pulse to the system at periodic intervals. As a result we expect the spins to rotate by an angle π along the x-axis, and thus the spins flip along the z axis . However, if we now introduce a small error in h, we observe that the neighbouring interaction term between the spins acts as correction. This effect can be observed for small deviations from π in the magnitude of h.

Figure 4.1: Magnetization plot for system for h = $\pi - 0.1$ and J = 1

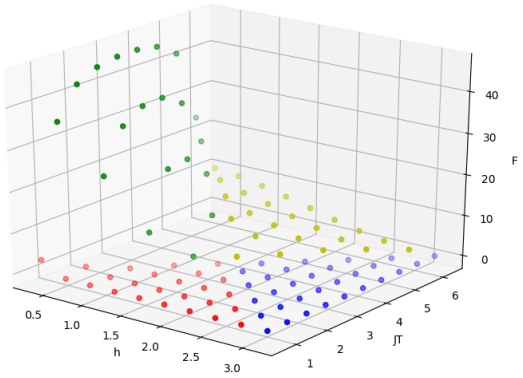


From the numerical simulation we also observe that for the driving period of the system T, the period of oscillation in the magnetization is 2T.

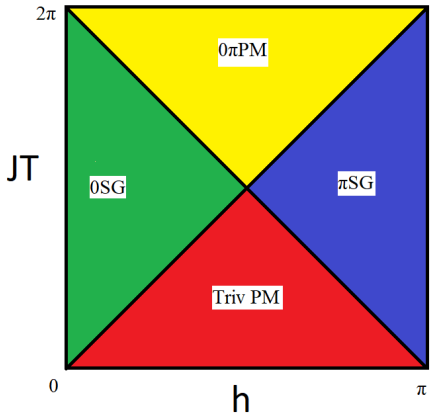
4.1.3 Phase diagram

We noted earlier that the dynamics of the system are different for different magnitudes of the term h. Thus the behaviour of the system is dependent of the strength of the different interactions in the hamiltonian. We thus can characterize the different phases of matter by observing the dynamics for different magnitudes of the various parameters.

We simulate the system for the values for $J_z \in [0, 2\pi]$, $h \in [0, \pi]$ and $T = 1$. As a result we would expect oscillations in magnetization to be of period $2T$. We therefore characterise the different phases by performing a fourier transform for frequency $1/2T$, i.e. $\omega = 0.5$. We can thus obtain a phase diagram and observe the various phase transitions for the above model.



(a) Phase diagram for L=10 Transverse field model



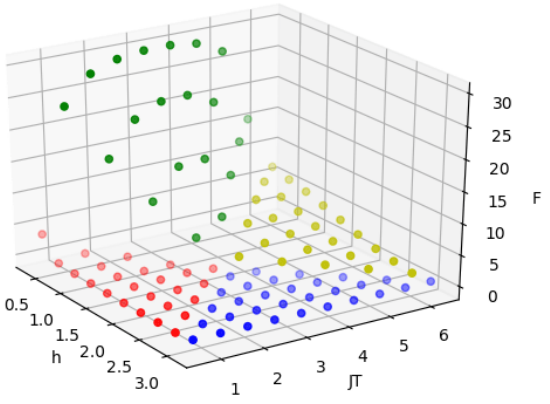
(b) 2d representation of the phase diagram showing the different phases

The π Spin Glass phase is be characterized by it's correlations $\mathcal{C}_{ij} = \langle \sigma_i^z \sigma_j^z \rangle \neq 0$ for all eigenstates and the spectrum of its unitary operator contains pairs of cat states with splitting exponentially close to π/T [17].

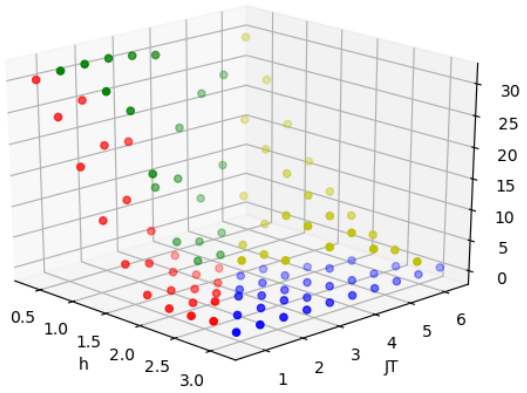
4.1.4 Stability

To check the robustness of the model, we perturb the system by adding a disorder term to the hamiltonian of the form

$$H(t) = \sum_{i=0}^{L-1} J_z \sigma_i^z \sigma_{i+1}^z + \sum_{n\in\mathbb{Z}^+} \sum_{i=0}^{L} h\sigma_i^x \delta(t - nT) + v \sum_{i=0}^{L} \sigma_i^z \tag{4.6}$$



(a) Phase diagram weak perturbation for v = 0.01



(b) Phase diagram weak perturbation for v = 1

We observe that the phase is stable for weak perturbation. As a result the phase diagram is unchanged for small values of the parameter v. However, upon increasing the strength of disorder the phase diagram changes and only the πSG is absolutely stable [16].

4.2 Analogous 2 spin system

Consider an operator O, with eigenstates $\{|i\rangle\}$, that commutes with the hamiltonian H. Since $[O, H] = 0$, we must have $\langle i|[O, H]|j|i|[O, H]|j\rangle = 0$.

$$\implies \langle i|(OH - HO)|j|i|(OH - HO)|j\rangle = 0 \implies (O_i - O_j)\langle i|H|j|i|H|j\rangle = 0 \implies H_{ij} \neq 0 \, if \, O_i = O_j$$

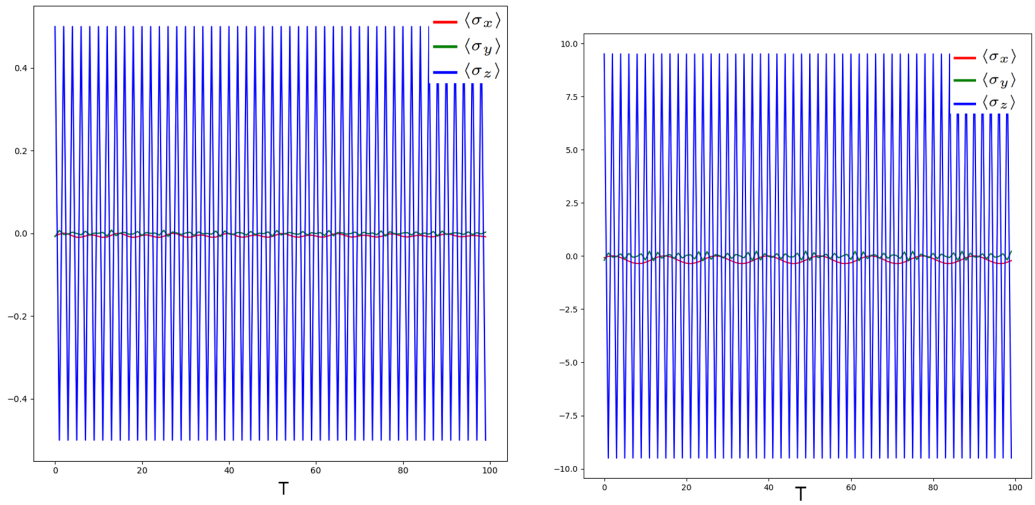
Since H_{ij} can be non zero only for states $|i\rangle$, $|j\rangle$ corresponding to the same eigenvalues of operator O, we claim that there exists a basis for which H is of block diagonal form; where non-zero blocks of the Hamiltonian correspond to the same eigenvalue of O.

The operator $L^2 = \sum_i S_i^2$ commutes with the hamiltonian given in eq.(2.1), i.e. $[L^2, H] = 0$. As a result we can transform to a basis where H is of block diagonal form. Since H is

of block diagonal form, so is the Floquet unitary operator e^{-iHt}. Thus, starting with a state $|\psi\rangle$ belonging to the hilbert subspace of H, the time evolution with Floquet unitary operator maps it within the same subspace. This property of the system allows us focus on the dynamics of subset of spins, while treating the rest of the spin system as a single entity.

With this motivation, we build a system analogous to our system of L=10 spin 1/2 chain with the transverse field Hamiltonian. This system consists of a s=1/2 spin coupled to a s=9 spin. The Hamiltonian describing the system is

$$H = JS^z M^z + \sum_{n\in\mathbb{Z}^+} h(S^x + M^x)\delta(t - nT) \tag{4.7}$$



(a) Time dependence of σ^μ for L = 10 spin 1/2 system

(b) Time dependence of σ^μ for spin 1/2 - spin 9 system

Here S^z, S^x are spin operators acting on s=1/2 spin and M^z, M^x are spin operators acting on s=9 spin.

From the numerical simulations we observe that the dynamics of this 2 spin system is similar to that of L=10 spin 1/2 system.

Chapter 5

Summary

5.1 Results and Discussion

We studied Many Body Localization in a fermionic system in the presence of interaction and disorder for system size 14 with 7 particles in a half filling configuration.

For this system we observed the typical IPR values to be in the range of 10 - 100. This means that the system is fairly localized, since for a delocalized system the value of IPR scales with the dimension of its Hilbert space, which is much larger than the values we obtained.

For this system we observed the increase in IPR with system size. However, the slight increase in average IPR with interaction strength was unexpected. We do note however, that since this since the system is half-filled, it is likely displaying a charge ordered state, called a charge density wave, in which alternate sites are occupied by particles. In such a scenario, nearest neighbour interaction doesn't affect the system by a significant amount, and the system esentially behaves like a non-intercating disordered system. This would likely explain our results since increasing the interaction strength doesn't have much effect on the system then.

We also observe that in the presence of a large disorder, the system is slightly delocalized for low interaction strength. On increasing the disorder strength we find that the system

localizes. This indicates the competing effects at play in the system, where upon increasing the interaction strength melts a disorder stabilised state.

We also study MBL in a periodically driven transverse field Ising model. We note an interesting property of this system called Time Translation Symmetry Braking.

We also found that the system auto corrects small errors in the periodic drive. We observed this by introducing a small error in the pi pulse driving the system. However, because of the presence of interaction in the Hamiltonian, the system corrected the small error in the drive and still maintained a regular behaviour. We also observed the various phase transitions this system displays and their stability against weak perturbations.

5.2 Future work

In the context of disordered interacting tight binding model, we would like to study the system for larger and more system sizes to observe how the the properties of eigenstates scale with length.

We also obtained some results that were unexpected, such as the increase in IPR with interaction strength and disorder. We believe that these results could be because of the system displaying Mott insulator phase and charge density phase. Hence, this requires some further analysis of the properties of system.

For the Floquet transverse field model, we would like to study the dynamics of the system in the presence of an external bath.

Bibliography

[1] P. W. Anderson, *Absence of Diffusion in Certain Random Lattices* PhysRev.109.1492

[2] D.M. Basko, I.L. Aleiner, B.L. Altshuler, *Metal-insulator transition in a weakly interacting many-electron system with localized single-particle states* , arXiv:cond-mat/0506617

[3] Arijeet Pal, David A. Huse, *The many-body localization phase transition* , arXiv:1010.1992

[4] Vedika Khemani, Achilleas Lazarides, Roderich Moessner, S. L. Sondhi, *Phase Structure of Driven Quantum Systems* Phys. Rev. Lett. 116, 250401

[5] Jonas A. Kjll, Jens H. Bardarson, Frank Pollmann, *Many-body localization in a disordered quantum Ising chain*, arXiv:1403.1568

[6] Rahul Nandkishore, David A. Huse, *Many body localization and thermalization in quantum statistical mechanics*, arXiv:1404.0686

[7] Achilleas Lazarides, Arnab Das, and Roderich Moessner, *Equilibrium states of generic quantum systems subject to periodic driving*, arXiv:1403.2946

[8] Achilleas Lazarides, Arnab Das, and Roderich Moessner, *Periodic Thermodynamics of Isolated Quantum Systems*, arXiv:1401.0164

[9] J. Z. Imbrie, V. Ros, A. Scardicchio, *Review: Local Integrals of Motion in Many-Body Localized systems*, arXiv:1609.08076

[10] Marcos Rigol, Vanja Dunjko, Maxim Olshanii, *Thermalization and its mechanism for generic isolated quantum systems* arXiv:0708.1324

[11] J. M. Deutsch, *Quantum statistical mechanics in a closed system* Phys. Rev. A 43, 2046

[12] Mark Srednicki, *Chaos and quantum thermalization* Phys. Rev. E 50, 888

[13] Pedro Ponte, Anushya Chandran, Z. Papi, Dmitry A. Abanin, *Periodically driven ergodic and many-body localized quantum systems* arXiv:1410.3455

[14] Andrey E. Antipov, Younes Javanmard, Pedro Ribeiro, Stefan Kirchner, *Interaction-tuned Anderson versus Mott localization* arXiv:1605.01390

[15] Dominic V. Else, Bela Bauer, Chetan Nayak, *Floquet Time Crystals*, arxiv:1603.08001.

[16] C.W. von Keyserlingk, Vedika Khemani, S. L. Sondhi, *Absolute Stability and Spatiotemporal Long-Range Order in Floquet systems*, arxiv:1605.00639.

[17] R. Moessner, s. L. Sondhi, *Equilibration and Order in Quantum Floquet Matter*, arxiv:1701.08056.

[18] Jens H. Bardarson, Frank Pollmann, Joel E. Moore, *Unbounded growth of entanglement in models of many-body localization*, arXiv:1202.5532

[19] Angelo Russomanno, Alessandro Silva, Giuseppe E. Santoro, *Periodic steady state and interference in a periodically driven quantum system* arXiv:1204.5084

[20] D'Alessio, Marcos Rigol, *Long-time behavior of periodically driven isolated interacting lattice systems* arXiv:1402.5141

[21] Achilleas Lazarides, Arnab Das, Roderich Moessner, *Many-body localization in periodically driven systems* arXiv:1410.8518

[22] Mark Srednicki, *The approach to thermal equilibrium in quantized chaotic systems* arXiv:cond-mat/9809360