Quantum Quench and Thermalization in Free Scalar Field Theory and its Gravity Dual (Arbitrary Dimensions)

A thesis submitted towards partial fulfilment of
BS-MS Dual Degree Programme

by

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under the guidance of

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Certificate

This is to certify that this thesis entitled "Thermalization in Free Scalar Field theory in Higher Dimensions" submitted towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research Pune represents original research carried out by Parijat Banerjee at Tata Institute of Fundamental Research, under the supervision of Gautam Mandal during the academic year 2017-2018.

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Declaration

I hereby declare that the matter embodied in the report entitled "Thermalization in Free Scalar Field Theory in Higher Dimensions" are the results of the investigations carried out by me at the Department of Theoretical Physics, Tata Institute of Fundamental Research, under the supervision of Gautam Mandal and the same has not been submitted elsewhere for any other degree.

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Acknowledgements

I would sincerely like to thank my supervisor, Professor Gautam Mandal, for his insight and guidance at every step of the project. I would like to thank my collaborators, Anurag Kaushal and Adwait Gaikwad, who are graduate students at TIFR. I acknowledge the contribution of Naveen Prabhakar, a post-doc at TIFR, to my learning of very important physical concepts. I would like to thank the company I have had at TIFR for making my fifth year project a memorable experience. I am grateful to TIFR for allowing me to pursue my Master's Thesis at one of their esteemed departments. I thank Nabamita Banerjee for introducing me to the research group at TIFR and enabling me to work with them. Finally, I thank IISER for the opportunity to work and grow academically through the pursuit of research.

Abstract

In this work we explore the thermalization of two point functions of free scalars in $d+1$ dimensions after a quantum quench. The mass is time dependent and is taken to zero. A detailed analysis of this problem has been done in 1+1 dimensions, in [1]. My goal here is to extend this understanding to higher dimensions. In previous works, [2] [1], we have seen that the post quench system equilibriates to a generalised Gibb's Ensemble. The post-quench observables retain a memory of the quench by having signatures set by the scale of the pre-quench state. We restrict our calculations to two kinds of pre-quench states: the ground state of the pre-quench Hamiltonian, and specific excited states called the squeezed states (CC and GCC).

All observables seem to reach an equilibrium. The correlation functions in the CC approach a thermal ensemble and those in the GCC approach a GGE, at late times, and are related by, $\beta = 4\kappa_2$ and $\mu = 4\kappa_4$. We observe a distinction between odd and even dimensions. The approach to equilibrium is exponentially decaying in time for odd d and fall off as a power law for even d. We also calculate the time dependent thermal correlator and observe a similar difference in odd and even dimensions.

In the second last section, we discuss the possibility of UV-IR mixing in the GCC and GGE. The post-quench state is characterized by infinite number of conserved charges which may correspond to irrelevant operators in the theory. Observables seem to be affected by all such operators even at low energies, which is interesting given our intuitions set by Wilsonian renormalization.

Finally, we wish to understand the implications of this result in the context of holography, with the knowledge that thermalization in the gauge theory corresponds to a quasi-normal decay to a black-hole. A quench itself can correspond to an excitation in the bulk.

Contents

Chapter 1

Introduction

This is where the preliminaries of the project is introduced.

1.1 Ergodicity and Subsystem Thermalization

Thermalization is the process of attaining thermal equilibrium. We are interested in knowing what class of operators thermalize. The quantum ergodic theorem seems to answer our question. Ergodic systems, classically, are those systems which are uniformly distributed in the entire phase space. When we describe something as being ergodic, its time average is same as its average over phase space. Quantum mechanically, the latter is equivalent to taking an ensemble average.

- What are the conditions that need to be satisfied for us to say that a class of observables (say $\{\hat{A}\}$) have reached thermal equilibrium? We should find that when we allow the states to evolve unitarily, for most times in the long run, $|\langle \hat{A}(t)\rangle - A_{mc}| < \epsilon$, where A_{mc} is given by the value of the observable in the microcanonical ensemble and ϵ is set by the experimental tolerance [3].

- Classically, if a system starts from a phase point X_o and has constant energy E_o, ergodicity demands that

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T dt\delta(X - X(t)) = \rho_{mc}(E) \tag{1.1}$$

  i.e., X(t) should cover the entire phase space for every initial condition X_o. In quantum mechanics, given a Hamiltonian of eigenstates $|\Psi_\alpha(t)\rangle$ of energy E_α, a microcanonical ensemble can be defined by taking an energy shell of energy width δE. We define $\rho_{mc}(E) = \sum_{E<E_\alpha<E+\delta E} \frac{1}{\Omega}|\Psi_\alpha\rangle\langle\Psi_\alpha|$ where Ω is the number of states in the microcanonical shell. Say we start with an initial state,$|\Psi_o\rangle = \sum_{E<E_\alpha<E+\delta E} c_\alpha|\Psi_\alpha\rangle$. $|\Psi(t)\rangle = \sum_\alpha c_\alpha e^{-iE_\alpha t}|\Psi_\alpha\rangle$

$$\therefore |\Psi(t)\rangle\langle\Psi(t)| = \sum_\alpha |c_\alpha|^2|\Psi_\alpha\rangle\langle\Psi_\alpha| + \sum_\alpha\sum_\beta c_\alpha c_\beta^* e^{-i(E_\alpha - E_\beta)t}|\Psi_\alpha\rangle\langle\Psi_\beta| \tag{1.2}$$

Now when we take $\overline{|\Psi(t)\rangle\langle\Psi(t)|}$, the second term gives zero since it is fluctuating. At this stage it is important, that the system is non-degenerate such that, $E_m \neq E_n$ for $n \neq m$. Thus,

$$\overline{|\Psi(t)\rangle\langle\Psi(t)|} = \sum_{\alpha} |c_\alpha|^2 |\Psi_\alpha\rangle\langle\Psi_\alpha| = \hat{\rho}_{diag} \tag{1.3}$$

$\hat{\rho}_{diag} = \hat{\rho}_{mc}$ when $|c_\alpha|^2 = \frac{1}{\Omega}$, which is true only for a very special class of states. A better way to look at ergodicity is in terms of operators rather than states. A sensible expectation in a quantum ergodic system is to expect, that given a set of macroscopic observables $\{\hat{A}\}$,

$$\langle\Psi(t)|\hat{A}(t)|\Psi\rangle \to^{t\to+\infty} Tr[\hat{A}\rho_{mc}] \equiv \langle\hat{A}\rangle_{mc} \tag{1.4}$$

irrespective of the initial state $|\Psi_o\rangle$. This should hold for most times at long times. This means that the mean square difference between LHS and RHS in Eq. (2) is vanishingly small when averaged over long times. Parallely, ergodicity can be defined as,

$$\overline{\langle\Psi(t)|\hat{A}(t)|\Psi\rangle} = Tr[\hat{A}\rho_{mc}] \equiv \langle\hat{A}\rangle_{mc} \tag{1.5}$$

If the expectation value relaxes to a well defined state , then the two definitions (Eq.2 and Eq.3) coincide.This loosely means that $\hat{A}\rho_{mc}$ can be considered equivalent to $\hat{A}\rho_{diag}$

- Thermalization in the present case means asymptotic approach to an equilibrium ensemble which is called the Generalized Gibbs Ensemble (GGE) because it is characterized by chemical potentials μ's completely with possibly an infinite number of conserved charges W's.

$$\rho_{GGE} = \frac{1}{Z} e^{-\sum_\alpha \mu_\alpha W_\alpha} \tag{1.6}$$

- If we look at a closed quantum system as a whole in a pure state, then its density matrix as the system evolves remains pure. It does not seem possible for the *whole* system to thermalize. But there is a sense in which part of the system can still thermalize. This is known as **Subsystem Thermalization**. Even though the full quantum state is pure, local observables within a subsystem can be a mixed state and therefore expected to show thermalization. More quantitatively this means that

$$\langle O_1 O_2 \cdots\rangle = \langle\psi(t)|O_1 O_2 \cdots |\psi(t)\rangle \to \text{Trace}\,(\rho_{sub} O_1 O_2 \cdots) \tag{1.7}$$

where ρ_{sub} is the reduced density matrix for the subsystem of interest attained by tracing over the remaining part. This is not so hard to expect. A a simple example consider two qubits (spins) in a Bell pair $|bp\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ with the density matrix

$$\rho_{bp} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1.8}$$

Evidently $\rho_{bp}^2 = \rho_{bp}$. Tracing over the 2nd qubit we get

$$\rho_1 = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \tag{1.9}$$

But this represents a mixed state. This also shows that there is a relationship between thermalization and entanglement between subsystems.

Thermalisation demands that any system, irrespective of the initial conditions reaches a specific state asymptotically. In case of integrable systems, the trajectories in phase space are confined by constraint equations to region of the phase space. For unitary theories, the integrable systems will return to their initial state in finite time due to the periodic nature of their orbits. Consider an integrable system, with initial conditions such that it excites only one normal mode of the system. The system forever stays in that state, since there is not exchange of energy between the orthogonal normal modes, thus it never reaches the thermal state asymptotically. However, if degrees of freedom are more than the number of constraints, the system might still be free to span a sub manifold of the phase space. Thus, integrable systems can also experience subsystem thermalization. For a review on thermalization in quantum systems, see references [4] [5].

Chapter 2

Quantum Quench

2.1 The Lagrangian

Our choice of the physical system here is a scalar field with no interactions. This is simple and convenient for our purpose. Quenching in general is a material science term which means rapid cooling of a metal piece to obtain desired properties. In our context it means varying one or more couplings and parameters in our theory by making them explicit functions of time. This may seem too ambitious at first but there are physically realizable systems where we can achieve this. For example in ultra-cold gases, a sharp change in the background magnetic field is a perfect example of a quantum quench. Since we are dealing with a free theory, the only parameter at our disposal is the mass of the scalar. So we consider the following theory in general d dimensions

$$S = -\frac{1}{2}\int d^d x \left(\partial_\mu \phi \partial^\mu \phi + m^2(t)\phi^2\right) \tag{2.1}$$

with the equation of motion (EOM) being $(\partial^2 + m^2(t))\phi = 0$. Here we study this theory for a specific mass function $m(t)$ which starts with some finite mass m_0 and drops to $m = 0$ later (see fig.2.1). [6] But even before going into that, by a very simple mapping to a quantum mechanical scattering problem in 1-D, we can solve for $\phi(t)$.

2.2 Scattering in Quantum Mechanics

We can perform a partial Fourier Transform (because of translational symmetry in spatial dimensions)

$$\phi(\vec{x}, t) = \int_{-\infty}^{+\infty} \frac{d^{d-1}k}{(2\pi)^{d-1}}\, \phi(\vec{k}, t) e^{i\vec{k}\cdot\vec{x}} \tag{2.2}$$

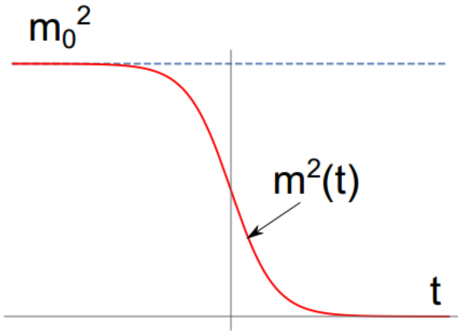


Figure 2.1: Mass quench from m_0 to 0(taken from [1])

QM	QFT
x	t
E	$\vec{k}^2$
$V(x)$	$-m^2(t)$
$\psi(E,x)$	$\phi(\vec{k},t)$

Table 2.1: Analogy between quench and scattering problem.

with $\phi^*(\vec{k},t) = \phi(-\vec{k},t)$. Now the action becomes [1]

$$S = \frac{1}{2}\int d^{d-1}k\, dt \left(|\dot{\phi}(\vec{k},t)|^2 - (k^2 + m^2(t))|\phi(\vec{k},t)|^2\right) \tag{2.3}$$

the EOM decouples for different Fourier modes

$$-\frac{d^2}{dt^2}\phi(\vec{k},t) = (k^2 + m^2(t))\phi(\vec{k},t) \tag{2.4}$$

This looks like a Schroedinger-type equation in one spatial dimension, if we identify these things (see table 2.1). Then following Landau [6], we can solve this exactly for a certain types of mass functions. Here we give a brief summary of how to go about it for a specific mass function of the type in the figure above.

2.3 A Specific Quench protocol

We work with the mass function

$$m^2(t) = m_0^2(1 - \tanh(\rho t))/2 \tag{2.5}$$

In the far past and the far future since the mass function/potential approaches a constant value, $\phi(\vec{k},t)$ must look like plane wave. If we call $u_{in}(\vec{k},t)$ the solution

[1] Here metric signature is taken to be (-+++)

which approaches positive frequency in the far past and similarly $u_{out}(\vec{k},t)$ in the far future, i.e.

$$u_{in}(\vec{k},t) \xrightarrow{t\to-\infty} \frac{e^{-i\omega_{in}t}}{\sqrt{2\omega_{in}}}, \qquad u_{out}(\vec{k},t) \xrightarrow{t\to\infty} \frac{e^{-i\omega_{out}t}}{\sqrt{2\omega_{out}}} \tag{2.6}$$

where $\omega_{in} = \sqrt{\vec{k}^2 + m_o^2}$ and $\omega_{out} = |\vec{k}|$. Since this a second order linear ordinary differential equation, we must have 2 independent solutions which are $u_{in}(\vec{k},t)$ and $u^*_{in}(\vec{k},t)$ or $u_{out}(\vec{k},t)$ and $u^*_{out}(\vec{k},t)$, which is just a different basis. The two therefore must be linearly related through a Bogoliubov transformation

$$u_{in}(\vec{k}) = \alpha(\vec{k})u_{out}(\vec{k}) + \beta(\vec{k})u^*_{out}(-\vec{k}) \tag{2.7}$$

$$u_{out}(\vec{k}) = \alpha^*(\vec{k})u_{in}(\vec{k}) - \beta(\vec{k})u^*_{in}(-\vec{k}) \tag{2.8}$$

As this was already done in [1] and also done in Birrell and Davies, we merely state the following solutions for u_{in} and u_{out}

$$\begin{aligned} u_{in} &= \frac{e^{-i\omega_{in}t}}{\sqrt{2\omega_{in}}} 2F_1\left(\frac{i\omega_-}{\rho}, -\frac{i\omega_+}{\rho}; 1 - \frac{i\omega_{in}}{\rho}; e^{-2\rho t}\right) \\ u_{out} &= \frac{e^{-i\omega_{out}t}}{\sqrt{2\omega_{out}}} 2F_1\left(\frac{i\omega_-}{\rho}, \frac{i\omega_+}{\rho}; \frac{i\omega_{out}}{\rho} + 1; -e^{-2\rho t}\right) \end{aligned} \tag{2.9}$$

and the Bogoliubov coefficients

$$\begin{aligned} \alpha(\vec{k}) &= \sqrt{\frac{\omega_{out}}{\omega_{in}}} \frac{\Gamma(-\frac{i\omega_{out}}{\rho})\Gamma(1 - \frac{i\omega_{in}}{\rho})}{\Gamma(-\frac{i\omega_+}{2\rho})\Gamma(1 - \frac{i\omega_+}{2\rho})} \\ \beta(\vec{k}) &= \sqrt{\frac{\omega_{out}}{\omega_{in}}} \frac{\Gamma(\frac{i\omega_{out}}{\rho})\Gamma(1 - \frac{i\omega_{in}}{\rho})}{\Gamma(\frac{i\omega_-}{2\rho})\Gamma(1 - \frac{i\omega_-}{2\rho})} \end{aligned} \tag{2.10}$$

And so

$$\phi(\vec{k},t) = a_{in}(\vec{k})u_{in}(\vec{k},t) + a^*_{in}(-\vec{k})u^*_{in}(-\vec{k},t) = a_{out}(\vec{k})u_{out}(\vec{k},t) + a^*_{out}(-\vec{k})u^*_{out}(-\vec{k},t) \tag{2.11}$$

Moreover the in and out vacua are also related by a Bogoliubov transform [1]

$$|0_{in}\rangle = \exp\left[\frac{1}{2}\sum_k \gamma(\vec{k}) a^\dagger_{\vec{k}} a^\dagger_{-\vec{k}}\right] |0_{out}\rangle \tag{2.12}$$

Sudden limit

If the quench is done suddenly, i.e.

$$\rho \to \infty \tag{2.13}$$

which gives a step function drop

$$m^2(t) = m_0^2\Theta(-t) \tag{2.14}$$

Now the Bogoliubov coefficients become

$$\alpha(\vec{k}) = \frac{1}{2}\frac{|\vec{k}|+\omega_{in}}{\sqrt{|\vec{k}|\omega_{in}}}, \qquad \beta(\vec{k}) = \frac{1}{2}\frac{|\vec{k}|-\omega_{in}}{\sqrt{|\vec{k}|\omega_{in}}} \tag{2.15}$$

and the in and out waves

$$u_{in} = \frac{e^{-i\omega_{in}t}}{\sqrt{2\omega_{in}}}, \qquad u_{out} = \frac{e^{-i\omega_{out}t}}{\sqrt{2\omega_{out}}} \tag{2.16}$$

With everything defined above, we can write our field ϕ as

$$\begin{aligned}\phi(\vec{x},t) &= \int_{-\infty}^{+\infty} \frac{d^{d-1}k}{(2\pi)^{d-1}} \left(a_{in}(\vec{k})u_{in}(\vec{k},t) + a_{in}^*(-\vec{k})u_{in}^*(-\vec{k},t)\right) e^{i\vec{k}\cdot\vec{x}} \\ &= \int_{-\infty}^{+\infty} \frac{d^{d-1}k}{(2\pi)^{d-1}} \left(a_{out}(\vec{k})u_{out}(\vec{k},t) + a_{out}^*(-\vec{k})u_{out}^*(-\vec{k},t)\right) e^{i\vec{k}\cdot\vec{x}}\end{aligned} \tag{2.17}$$

We will do most of our calculations in this limit.

2.3.1 The Propagator

$|0,in\rangle$ being the ground state of the initial Hamiltonian, the propagator in $|0,in\rangle$ is just the quantity

$$\begin{aligned}&\langle 0_{in}|\phi(\vec{x_1},t_1)\phi(\vec{x_2},t_2)|0_{in}\rangle \\ =&\langle 0_{in}|\int_{-\infty}^{+\infty} \frac{d^{d-1}k}{(2\pi)^{d-1}} \left(a_{in}(\vec{k})u_{in}(\vec{k},t_1) + a_{in}^\dagger(-\vec{k})u_{in}^*(-\vec{k},t_1)\right) e^{i\vec{k}\cdot\vec{x_1}} \\ &\int_{-\infty}^{+\infty} \frac{d^{d-1}q}{(2\pi)^{d-1}} \left(a_{in}(\vec{q})u_{in}(\vec{k},t_2) + a_{in}^\dagger(-\vec{q})u_{in}^*(-\vec{q},t_2)\right) e^{i\vec{q}\cdot\vec{x_2}}|0_{in}\rangle \\ =&\int \frac{d^{d-1}k}{(2\pi)^{d-1}} u_{in}(\vec{k},t_1)u_{in}^*(\vec{k},t_2)e^{i\vec{k}\cdot(\vec{x_1}-\vec{x_2})}\end{aligned} \tag{2.18}$$

Out of the 4 terms only $aa^\dagger$ survives and we use the commutation relation $[a(\vec{k}), a^\dagger(\vec{q})] = (2\pi)^{d-1}\delta^{d-1}(\vec{k}-\vec{q})$ and then integrate over the delta function to get the last line. We can also write this in terms of the 'out' variables

$$\begin{aligned}&\int \frac{d^{d-1}k}{(2\pi)^{d-1}} u_{in}(\vec{k},t_1)u_{in}^*(\vec{k},t_2)e^{i\vec{k}\cdot(\vec{x_1}-\vec{x_2})} \\ =&\int \frac{d^{d-1}k}{(2\pi)^{d-1}} \Big[|\alpha(\vec{k})|^2 u_{out}(\vec{k},t_1)u_{out}^*(\vec{k},t_2) + \alpha(\vec{k})\beta^*(\vec{k})u_{out}(\vec{k},t_1)u_{out}(-\vec{k},t_2) \\ &+ \alpha^*(\vec{k})\beta(\vec{k})u_{out}^*(-\vec{k},t_1)u_{out}^*(\vec{k},t_2) + |\beta(\vec{k})|^2 u_{out}^*(-\vec{k},t_1)u_{out}(-\vec{k},t_2)\Big]e^{i\vec{k}\cdot(\vec{x_1}-\vec{x_2})}\end{aligned} \tag{2.19}$$

We can also express the Bogoliubov coefficients as

$$
\begin{aligned}
|\alpha(\vec{k})|^2 &= \frac{1}{1-|\gamma(\vec{k})|^2}, \quad |\beta(\vec{k})|^2 = \frac{|\gamma(\vec{k})|^2}{1-|\gamma(\vec{k})|^2} \\
\alpha(\vec{k})\beta^*(\vec{k}) &= \frac{\gamma(\vec{k})}{1-|\gamma(\vec{k})|^2}, \quad \alpha^*(\vec{k})\beta(\vec{k}) = \frac{\gamma^*(\vec{k})}{1-|\gamma(\vec{k})|^2}
\end{aligned}
\tag{2.20}
$$

where

$$
\gamma(\vec{k}) = \frac{\beta^*(\vec{k})}{\alpha^*(\vec{k})} \tag{2.21}
$$

2.3.2 The Cardy-Calabrese(CC) Ansatz and the CC state

The Cardy-Calabrese ansatz states that the post-quench state immediately after the the sudden quench is given by the CC state

$$
|CC\rangle = e^{-\kappa_2 H}|Bd\rangle \tag{2.22}
$$

that is a Euclidean time evolution acting on the Drichlet boundary state $|Bd\rangle$. But as MPS argued, this state is not correct at least in 2-D. The correct post-quench state is given by a generalized Cardy-Calabrese state (gCC)

$$
|gCC\rangle = \exp\left[-\kappa_2 H - \sum_{n>2} \kappa_n W_n\right]|Bd\rangle \tag{2.23}
$$

where W_n are additional conserved charges other than the Hamiltonian.

2.3.3 Preparing gCC states (by quenching squeezed states)

The gCC states can be obtained from excited states of the initial Hamiltonian. Instead of the initial state being the ground state of the pre-quench Hamiltonian, we can choose it be any arbitrary squeezed state.

$$
|\psi_{in}\rangle = |f\rangle = \exp\left[\frac{1}{2}\sum_k f(\vec{k}) a^\dagger_{in}(\vec{k}) a^\dagger_{in}(-\vec{k})\right]|0_{in}\rangle \tag{2.24}
$$

This is just the Bogoliubov transformation of $|0_{in}\rangle$. As the $|0_{in}\rangle$ state can itself be written as a Bogoliubov transform of $|0_{out}\rangle$, the post quench state mathematically looks like the $|0_{in}\rangle$ state with $\gamma' s$ replaced by 'effective' $\gamma' s$

$$
|f\rangle = \exp\left[\frac{1}{2}\sum_{\vec{k}} \gamma_{eff}(\vec{k}) a^\dagger_{out}(\vec{k}) a^\dagger_{out}(-\vec{k})\right]|0_{out}\rangle, \quad \gamma_{eff} = \frac{\beta^*(\vec{k}) + f(\vec{k})\alpha(\vec{k})}{\alpha^*(\vec{k}) + f(\vec{k})\beta(\vec{k})} \tag{2.25}
$$

By tailoring the choice of the squeezing function $f(k)$ (or equivalently $\gamma_{eff}(k)$) can lead to a particular gCC state. For example choosing

$$f(k) = 1 - \frac{2|\vec{k}|}{\sqrt{\vec{k}^2 + m^2} \tanh\Big(\kappa_2|\vec{k}|+\kappa_4|\vec{k}|^3\Big) + |\vec{k}|} \tag{2.26}$$

leads to a gCC state with $\kappa_n = 0$ for $n > 4$

$$|\psi_{in}\rangle = |f_4\rangle = \exp[-\kappa_2 H - \kappa_4 W_4]|D\rangle \tag{2.27}$$

In particular if $\kappa_4 = 0$, then we end up with the CC state $|CC\rangle = \exp[-\kappa_2 H]|D\rangle$, which is the same state as in the Cardy-Calabrese ansatz.

κ_2 and κ_4 are tunable parameters independent of the mass (the scales in the pre quench state). However, when the pre quench state is the ground state $|0_{in}\rangle$, the parameters are set by mass, which is the only scale in the theory. Then the post quench state is given by gCC, where κ_2 is of the order of $1/m$ and κ_4 is of the order $1/m^3$, and so on.

The propagator in squeezed state

The propagator in the squeezed state is just given by replacing γ by γ_{eff} (op α and β byα_{eff} and β_{eff}) in the ground state propagator

$$\begin{aligned}
&\langle f|\phi(\vec{x_1},t_1)\phi(\vec{x_1},t_1)|f\rangle \\
&= \int \frac{dk}{2\pi}\Big[|\alpha_{eff}(\vec{k})|^2 u_{out}(\vec{k},t_1)u^*_{out}(\vec{k},t_2) + \alpha_{eff}(\vec{k})\beta^*_{eff}(\vec{k})u_{out}(\vec{k},t_1)u_{out}(-\vec{k},t_2) \\
&+ \alpha^*_{eff}(\vec{k})\beta_{eff}(\vec{k})u^*_{out}(-\vec{k},t_1)u^*_{out}(\vec{k},t_2) + |\beta_{eff}(\vec{k})|^2 u^*_{out}(-\vec{k},t_1)u_{out}(-\vec{k},t_2)\Big]e^{i\vec{k}\cdot(\vec{x_1}-\vec{x_2})}
\end{aligned} \tag{2.28}$$

Squeezed states are important because they are experimentally realizable.

2.4 Thermal and GGE

The GGE 2 point function is defined as

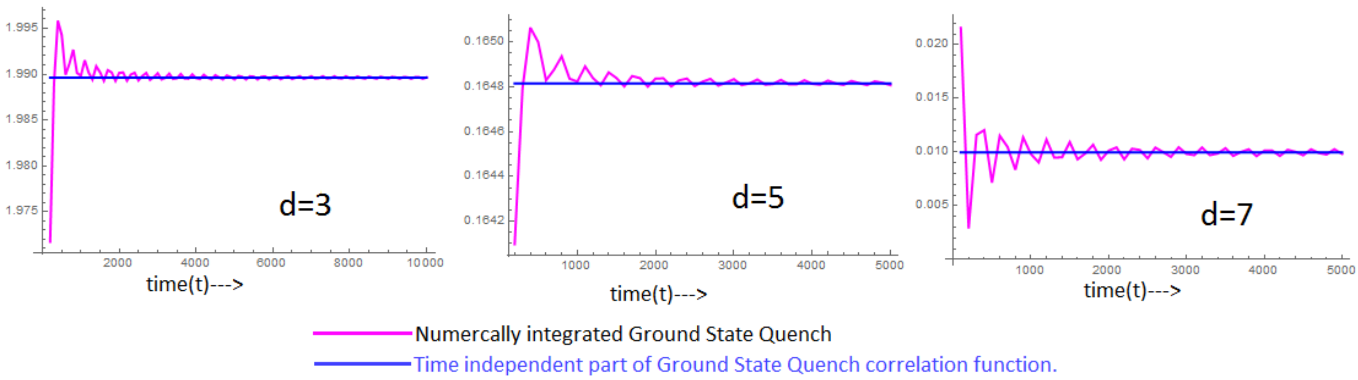
$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} = \frac{1}{Z}Tr(e^{-\beta H-\mu W_4}\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)) \tag{2.29}$$

Defining $\vec{x} = \vec{x1} - \vec{x2}$, $t = t1 - t2$ and $G_\pm = \frac{1}{|\vec{k}|(\pm e^{\pm(\beta k+\mu k^3)}\mp 1)}$, the thermal correlator becomes:

$$\langle\phi(\vec{x_1},t_1)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} = \frac{1}{2}\int\frac{d^dk}{(2\pi)^d}\Big[G_- e^{\iota\vec{k}\cdot\vec{x}}e^{-\iota|\vec{k}|t} + G_+ e^{-\iota\vec{k}\cdot\vec{x}}e^{\iota|\vec{k}|t}\Big] \tag{2.30}$$

The detailed derivation is given in appendix A.

2.5 Results for the Ground State Quench



From the above figure we see that the two point correlation function of ϕ in the ground state of the pre-quench Hamiltonian 2.18 which looks like

$$\int \frac{d^dk}{(2\pi)^d} e^{i\vec{k}.\vec{x}} \frac{2k^2 - m^2\cos(2kt) + m^2}{4k^2\sqrt{k^2+m^2}} \tag{2.31}$$

does thermalize to some equilibrium valus at large times. The numerics are not very steady and exhibit oscillations that decay with time, but do not go to zero. While performing the numerical integral, I have always taken m greater than k_{max}. This has to do with the subtleties in the **sudden limit**. In the quench profile 2.5, we took $\rho \to 0$. However, in practice $\rho \approx \frac{1}{m}$. Thus, m is the largest scale in the theory and thus sets an upper limit on the momentum. This is discussed in detail in Appendix E of [1].

Chapter 3

The Slab Propagator

The following calculation of propagators using the method of images has been inspired by the works of Sotiriadis and Cardy in [7].

3.1 The CC state ($\kappa_4 = 0$)

The Calabrese Cardy state is given b:

$$|CC\rangle = \exp[-\kappa H]|D\rangle \tag{3.1}$$

where $|D\rangle$ is a Dirichlet boundary state with the boundary condition that $\phi_{|D\rangle} = 0$. The 2-point correlator of ϕ in the CC state can be written as $\langle D|e^{-\kappa H}\phi(\tau_1)\phi(\tau_2)e^{-\kappa H}|D\rangle$, here τ is Euclidean time. Now in the Heisenberg picture, the real time dependent operator can be written in terms of the operator at time $t = 0$ as, $\phi(t) = e^{iHt}\phi(0)e^{-iHt}$, where the -iHt term corresponds to positive evolution in time. Now in the Euclidean picture, we take $\tau = it$. Then we have the corresponding relation $\phi(\tau) = e^{H\tau}\phi(0)e^{-H\tau}$, where the $e^{-H\tau}$ corresponds to evolution in positive Euclidean time. Our problem is a translationally invariant problem, so I have left the $\vec{x}$ dependence implicit. Then the propagator can be mapped to a slab propagator problem as:

$$\langle D|e^{-\kappa H}\phi(\tau_1)\phi(\tau_2)e^{-\kappa H}|D\rangle = \langle D|e^{-\kappa H}e^{\tau_1 H}\phi(0)e^{-\tau_1 H}e^{\tau_2 H}\phi(0)e^{-\tau_2 H}e^{-\kappa H}|D\rangle \tag{3.2}$$

This corresponds to the following picture:

Now the boundary condition implies that the Green's function takes the value zero, whenever any of the charges lies on a boundary. From electrodynamics we know that this problem can be solved by the method of images where the boundaries are replaced by an infinite number of positive and negative charges, such that the Green's function vanishes at the position of the boundaries. One of the operators is treated as a probe, and infinite images of the other are taken (in this case, $\phi(\tau_1)$. We are left to solve the Laplace's equation: $\partial^2\phi = \rho(x,\tau)$, where $\partial^2 = \partial_\tau^2 + \partial_{\vec{x}}^2$. Note that the CC state corresponds to a post quench state which, according to the quench problem described before follows the dispersion relation $\omega = |\vec{k}|$, and has zero mass. The Green's function for a single point source satisfies,

$$(\partial_\tau^2 + \partial_{\vec{x}}^2)G(\vec{x} - \vec{x'}, \tau) = \delta^d(\vec{x} - \vec{x'})\delta(\tau) \tag{3.3}$$

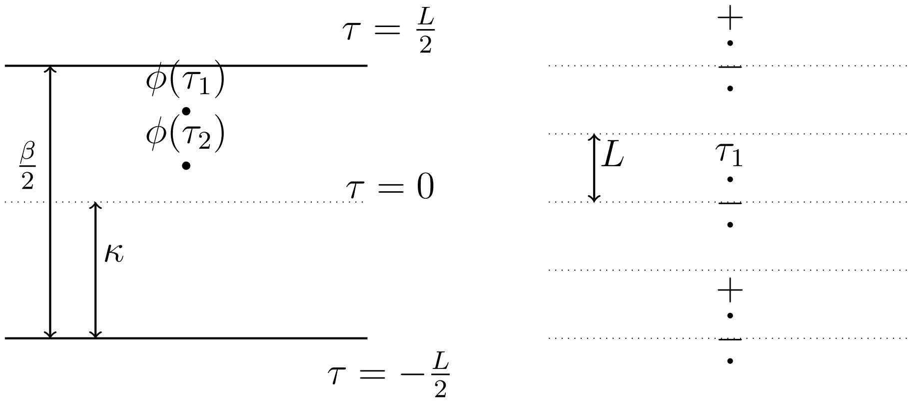


Figure 3.1: slab for CC state correlator

For a translationally invariant problem, upon taking the Fourier transform this becomes (in Euclidean d+1 dimensions),

$$\int (\partial_\tau^2 + \partial_{\vec{k}}^2) G(\vec{k}, \tau) e^{i\vec{k}.\vec{x}} \frac{d^d\vec{k}}{2\pi^d} = \int \delta(\tau) e^{i\vec{k}.\vec{x}} \frac{d^d\vec{k}}{2\pi^d} \tag{3.4}$$

$$\Rightarrow (\partial_\tau^2 + \partial_{\vec{k}}^2) G(\vec{k}, \tau) = \delta(\tau) \tag{3.5}$$

$$\Rightarrow \int (\omega^2 + \vec{k}^2) G(\omega, \vec{k}) e^{i\omega\tau} d\omega = \int e^{i\omega\tau} d\omega \tag{3.6}$$

$$\Rightarrow G(\omega, \vec{k}) = \frac{1}{\omega^2 + \vec{k}^2} \tag{3.7}$$

Therefore,

$$\begin{aligned} \therefore G(\vec{k}, \tau) =& \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{e^{i\omega\tau}}{\omega^2 + \vec{k}^2} \\ =& \frac{1}{2k} e^{-k|\tau|} \end{aligned} \tag{3.8}$$

where, the last step is arrived at by doing a contour integral. In our case the boundaries are replaced by an infinite number of positive and negative charges. Thus the Laplace's equation looks like:

$$\begin{aligned} (\partial_\tau^2 + \partial_{\vec{x}}^2) G(\vec{x} - \vec{x'}, \tau_1, \tau_2) =& \delta^d(\vec{x} - \vec{x'}) \Big(\sum_{n=0}^{\infty} \delta(|\tau_1 - \tau_2| + 2nL) + \sum_{n=1}^{\infty} \delta(-|\tau_1 - \tau_2| + 2nL) \\ & - \sum_{n=0}^{\infty} \delta(\tau_1 - \tau_2 + (2n+1)L) - \sum_{n=1}^{\infty} \delta(-\tau_1 - \tau_2 + (2n-1)L) \Big) \end{aligned} \tag{3.9}$$

Thus,

$$\begin{aligned} G(\vec{k}, \tau) =& \frac{1}{2|\vec{k}|} \Big(\sum_{n=0}^{\infty} e^{-|\vec{k}|(|\tau_1 - \tau_2| + 2nL)} + \sum_{n=1}^{\infty} e^{-|\vec{k}|(-|\tau_1 - \tau_2| + 2nL)} \\ & - \sum_{n=0}^{\infty} e^{-|\vec{k}|(\tau_1 + \tau_2 + (2n+1)L)} - \sum_{n=1}^{\infty} e^{-|\vec{k}|(-\tau_1 - \tau_2 + (2n-1)L)} \Big) \end{aligned} \tag{3.10}$$

Performing the k integral and analytically continuing τ to t we would get $G(\vec{x}, t_1, t_2)$, which we know, from Path integrals, corresponds to the two point correlation function of an operator at t_1 with an operator at t_2 and $\vec{x}$ is their spatial separation. One can perform the n sum first and then do the k integral. This gives rise to the same integral as obtained from calculating expectation value in the squeezed state with $\kappa_2 \neq 0$ and $\kappa_4 = 0$. We might also choose to perform the k-integral first and then perform the n sum. It so appears. For even d, we do not have a very good analytical handle over these problems. In these situations the second approach seems to give us a better numerical control over our results.

$$\langle \phi(\vec{x_1}, t_1)\phi(\vec{x_2}, t_2)\rangle_{slab} = G(\vec{x} = \vec{x_1} - \vec{x_2}, t_1, t_2) \tag{3.11}$$

$$= \int \frac{d^d k}{(2\pi)^d} \frac{e^{i\vec{k}\cdot\vec{x}}}{2|\vec{k}|} \Big(\sum_{n=0}^{\infty} e^{-|\vec{k}|(|\tau_1 - \tau_2| + 2nL)} + \sum_{n=1}^{\infty} e^{-|\vec{k}|(-|\tau_1 - \tau_2| + 2nL)} - \sum_{n=0}^{\infty} e^{-|\vec{k}|(\tau_1 + \tau_2 + (2n+1)L)} - \sum_{n=1}^{\infty} e^{-|\vec{k}|(-\tau_1 - \tau_2 + (2n-1)L)} \Big) \tag{3.12}$$

d	Autocorrelation function in CC state $\langle\phi(t_1,0)\phi(t_2,0)\rangle_{CC}$
2	$\frac{1}{4\pi\beta}\left(H_{\frac{i(t_1+t_2)}{\beta}+\frac{1}{2}}+H_{-\frac{i(t_1+t_2)}{\beta}-\frac{1}{2}}-H_{\frac{i(t_2-t_1)}{\beta}}-H_{-\frac{i(t_2-t_1)}{\beta}}\right)-\frac{2}{4\pi(\beta+2i(t_1+t_2))}-\frac{i}{4\pi(t_2-t_1)}$
3	$-\frac{1}{4\beta^2}\left(\operatorname{csch}^2\left(\frac{\pi(t_2-t_1)}{\beta}\right)+\operatorname{sech}^2\left(\frac{\pi(t_1+t_2)}{\beta}\right)\right)$
4	$\frac{1}{16\pi^2}\left(\frac{-\psi^{(2)}\left(\frac{i(t_2-t_1)}{\beta}+1\right)+\psi^{(2)}\left(\frac{i(t_1+t_2)}{\beta}+\frac{3}{2}\right)-\psi^{(2)}\left(1-\frac{i(t_2-t_1)}{\beta}\right)+\psi^{(2)}\left(\frac{1}{2}-\frac{i(t_1+t_2)}{\beta}\right)}{\beta^3}-\frac{16}{(\beta+2i(t_1+t_2))^3}+\frac{2i}{(t_2-t_1)^3}\right)$
5	$\frac{\pi}{12\beta^4}\left(\left(\left(\cosh\left(\frac{2\pi(t_2-t_1)}{\beta}\right)+2\right)\operatorname{csch}^4\left(\frac{\pi(t_2-t_1)}{\beta}\right)+\left(\cosh\left(\frac{2\pi(t_1+t_2)}{\beta}\right)-2\right)\operatorname{sech}^4\left(\frac{\pi(t_1+t_2)}{\beta}\right)\right)\right)$
6	$-\frac{1}{128\pi^3}\left(\frac{\psi^{(4)}\left(\frac{i(t_2-t_1)}{\beta}+1\right)-\psi^{(4)}\left(\frac{i(t_1+t_2)}{\beta}+\frac{3}{2}\right)+\psi^{(4)}\left(1-\frac{i(t_2-t_1)}{\beta}\right)-\psi^{(4)}\left(\frac{1}{2}-\frac{i(t_1+t_2)}{\beta}\right)}{\beta^5}+\frac{768}{(\beta+2i(t_1+t_2))^5}+\frac{24i}{(t_2-t_1)^5}\right)$
7	$-\frac{\pi^2}{120\beta^6}\left((26\cosh\left(\frac{2\pi(t_1-t_2)}{\beta}\right)+\cosh\left(\frac{4\pi(t_1-t_2)}{\beta}\right)+33)\operatorname{csch}^6(\frac{\pi(t_1-t_2)}{\beta})+(-26\cosh\left(\frac{2\pi(t_1+t_2)}{\beta}\right)+\cosh\left(\frac{4\pi(t_1+t_2)}{\beta}\right)+33)\operatorname{sech}^6(\frac{\pi(t_1+t_2)}{\beta})\right)$
8	$\frac{1}{1536\pi^4\beta^7}\left(-\psi^{(6)}\left(\frac{i(t_2-t_1)}{\beta}+1\right)+\psi^{(6)}\left(\frac{i(t_1+t_2)}{\beta}+\frac{3}{2}\right)-\psi^{(6)}\left(1-\frac{i(t_2-t_1)}{\beta}\right)+\psi^{(6)}\left(\frac{1}{2}-\frac{i(t_1+t_2)}{\beta}\right)\right)-\frac{60}{\pi^4(\beta+2i(t_1+t_2))^7}+\frac{15i}{32\pi^4(t_2-t_1)^7}$
9	$\frac{\pi^3}{1680\beta^8}\left(\left(1191\cosh\left(\frac{2\pi(t_1-t_2)}{\beta}\right)+120\cosh\left(\frac{4\pi(t_1-t_2)}{\beta}\right)+\cosh\left(\frac{6\pi(t_1-t_2)}{\beta}\right)+1208\right)\operatorname{csch}^8\left(\frac{\pi(t_1-t_2)}{\beta}\right)+\left(1191\cosh\left(\frac{2\pi(t_1+t_2)}{\beta}\right)-120\cosh\left(\frac{4\pi(t_1+t_2)}{\beta}\right)+\cosh\left(\frac{6\pi(t_1+t_2)}{\beta}\right)-1208\right)\operatorname{sech}^8\left(\frac{\pi(t_1+t_2)}{\beta}\right)\right)$
10	$-\frac{\psi^{(8)}\left(\frac{i(t_2-t_1)}{\beta}+1\right)-\psi^{(8)}\left(\frac{i(t_1+t_2)}{\beta}+\frac{3}{2}\right)+\psi^{(8)}\left(1-\frac{i(t_2-t_1)}{\beta}\right)-\psi^{(8)}\left(\frac{1}{2}-\frac{i(t_1+t_2)}{\beta}\right)}{24576\pi^5\beta^9}+\frac{8}{\pi^5(\beta+2i(t_1+t_2))^9}+\frac{105i}{64\pi^5(t_2-t_1)^9}$

Clearly, when I take $t_2=t_1=t$, we get a divergence which is because of the coincidence of the operators. Now I subtract out the divergence and I get the one point function at spacetime point (t,0). Then in the $t\to\infty$ limit, we get:

d	One point function in CC state $\langle \phi\phi(t,0)\rangle_{CC}$
2	$\left(\frac{2\left(\log\left(\frac{2t}{\beta}\right)+\gamma\right)}{4\pi\beta} - \frac{7\beta^3}{1920\pi(2t)^4} - \frac{\beta}{48\pi(2t)^2} + O\left(\left[\frac{1}{2t}\right]^5\right)\right) + \frac{\frac{1}{16\pi\beta} - \frac{(2t)^2}{4(\pi\beta^3)} + \frac{2it}{4\pi\beta^2} + \cdots}{N^2} + O([\frac{1}{N}]^3)$
3	$\lim_{t\to\infty} -\frac{\text{sech}^2\left(\frac{2\pi t}{\beta}\right)}{4\beta^2} \longrightarrow \frac{1}{12\beta^2} + \frac{\text{e}^{-\frac{4\pi t}{\beta}}}{4\beta^2} + \cdots$
4	$\lim_{t\to\infty} \frac{\psi^{(2)}\left(\frac{2it}{\beta}+\frac{3}{2}\right)+\psi^{(2)}\left(\frac{1}{2}-\frac{2it}{\beta}\right)+4\zeta(3)}{16\pi^2\beta^3} - \frac{1}{\pi^2(\beta+4it)^3} \longrightarrow \frac{\zeta(3)}{4\pi^2\beta^3} + \frac{1}{32\pi^2\beta t^2} + \cdots$
5	$\lim_{t\to\infty} \frac{\pi\left(\cosh\left(\frac{4\pi t}{\beta}\right)-2\right)\text{sech}^4\left(\frac{2\pi t}{\beta}\right)}{12\beta^4} \longrightarrow \frac{\pi}{180\beta^4} + \frac{\text{e}^{-\frac{4\pi t}{\beta}}}{12\beta^4} + \cdots$
6	$\lim_{t\to\infty} \frac{\psi^{(4)}\left(\frac{2it}{\beta}+\frac{3}{2}\right)+\psi^{(4)}\left(\frac{1}{2}-\frac{2it}{\beta}\right)+48\zeta(5)}{128\pi^3\beta^5} - \frac{6}{\pi^3(\beta+4it)^5} \longrightarrow \frac{3\zeta(5)}{8\pi^3\beta^5} - \frac{3}{512(\pi^3\beta)t^4} + \cdots$
7	$\lim_{t\to\infty} -\frac{\pi^2\left(-26\cosh\left(\frac{4\pi t}{\beta}\right)+\cosh\left(\frac{8\pi t}{\beta}\right)+33\right)\text{sech}^6\left(\frac{2\pi t}{\beta}\right)}{120\beta^6} \longrightarrow \frac{\pi^2}{945\beta^6} + \frac{\text{e}^{-\frac{4\pi t}{\beta}}}{120\beta^6} + \cdots$
8	$\lim_{t\to\infty} -\frac{\psi^{(6)}(1)}{768\pi^4\beta^7} + \frac{\psi^{(6)}\left(\frac{2it}{\beta}+\frac{3}{2}\right)+\psi^{(6)}\left(\frac{1}{2}-\frac{2it}{\beta}\right)}{1536\pi^4\beta^7} - \frac{60}{\pi^4(\beta+4it)^7} \longrightarrow -\frac{\psi^{(6)}(1)}{768(\pi^4\beta^7)} + \frac{5}{2048\pi^4\beta t^6} + \cdots$
9	$\lim_{t\to\infty} \frac{\pi^3\left(1191\cosh\left(\frac{4\pi t}{\beta}\right)-120\cosh\left(\frac{8\pi t}{\beta}\right)+\cosh\left(\frac{12\pi t}{\beta}\right)-1208\right)\text{sech}^8\left(\frac{2\pi t}{\beta}\right)}{1680\beta^8} \longrightarrow \frac{\pi^3}{3150\beta^8} + \frac{\text{e}^{-\frac{4\pi t}{\beta}}}{1680\beta^8} + \cdots$
10	$\lim_{t\to\infty} \frac{\psi^{(8)}\left(\frac{2it}{\beta}+\frac{3}{2}\right)+\psi^{(8)}\left(\frac{1}{2}-\frac{2it}{\beta}\right)}{24576\pi^5\beta^9} - \frac{840}{\pi^5(\beta+4it)^9} + \frac{105\zeta(9)}{32\pi^5\beta^9} \longrightarrow \frac{105\zeta(9)}{32\pi^5\beta^9} - \frac{105}{65536(\pi^5\beta)t^8} + \cdots$

There is a distinct difference between odd and even dimensions. The autocorrelation function and the one point function go as power law in time in even d, which is odd spacetime dimension D. The decay to equilibrium is exponential in case of odd d, which is even spacetime dimension D. Also, the one point function decays to a constant, which is what we expect in a translationally invariant system. But, this constant is not zero. The CC state decays to a thermal equilibrium, which is an excited state of the CFT. An excited state has a scale in the theory, which is its energy. In this case, the energy depends on temperature of the system, which is reflected in the appearance of β in the constant.

3.2 The Thermal State ($\mu = 0$)

As encountered earlier, the correlations function in a thermal equilibrium is given by

$$\begin{aligned} \langle \phi(\vec{x_1}, t_1,)\phi(\vec{x_2}, t_2)\rangle_\beta &= \frac{1}{Z} Tr(e^{-\beta H}\phi(\vec{x_1}, t_1,)\phi(\vec{x_2}, t_2)) \\ &= \frac{1}{Z}\sum_{\{N_k\}} \langle \{N_k\}| e^{-\beta H}\phi(\vec{x_1}, t_1)\phi(\vec{x_2}, t_2)|\{N_k\}\rangle \end{aligned} \tag{3.13}$$

Now this can be mapped to a cylinder, which opens up to a plane with periodic boundary conditions, repeated after an interval β. Upon going to Euclidean space and keeping $\vec{x}$ dependence implicit, we get

$$= \frac{1}{Z}\langle\{N_k\}|e^{-\beta H}e^{\tau_1 H}\phi(0)e^{-\tau_1 H}e^{\tau_2 H}\phi(0)e^{-\tau_2 H}|\{N_k\}\rangle \tag{3.14}$$

This corresponds to the following picture:

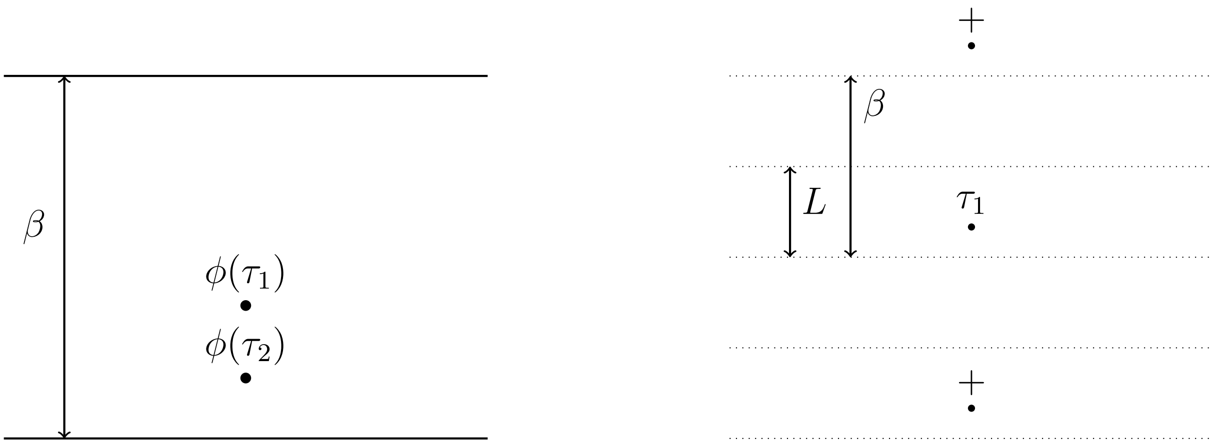


Figure 3.2: slab for Thermal state correlator

From the figure, it is clear that this is the same as removing the negative charges from the CC slab and summing over only the positive charges. Also, comparing the two figures one finds that, $\beta = 2L = 4\kappa$. Therefore, the Euclidean Green's function in momentum space, now looks like:

$$G(\vec{k},\tau) = \frac{1}{2|\vec{k}|}\Big(\sum_{n=0}^{\infty} e^{-|\vec{k}|(|\tau_1-\tau_2|+2nL)} + \sum_{n=1}^{\infty} e^{-|\vec{k}|(-|\tau_1-\tau_2|+2nL)}\Big) \tag{3.15}$$

$$\begin{aligned}\langle\phi(\vec{x_1},t_1)\phi(\vec{x_2},t_2)\rangle_\beta &= G(\vec{x}=\vec{x_1}-\vec{x_2},t_1,t_2)\\ &= \int \frac{d^dk}{(2\pi)^d}\frac{e^{i\vec{k}\cdot\vec{x}}}{2|\vec{k}|}\Big(\sum_{n=0}^{\infty} e^{-|\vec{k}|(|\tau_1-\tau_2|+2nL)} + \sum_{n=1}^{\infty} e^{-|\vec{k}|(-|\tau_1-\tau_2|+2nL)}\Big)\end{aligned} \tag{3.16}$$

We can double check that, performing the n sum first gives us the same integrand as obtained in the previous derivation of the thermal correlator. For this section, however, we perform the k integral first on every term and then sum the n terms up.

Autocorrelation Function in Thermal State		
d	$\langle\phi(0,0)\phi(t,0)\rangle_\beta$ (Recursion relation: $\langle\phi\phi\rangle_{d+2} = \frac{\Omega_{d+2-1}}{4\pi^2\Omega_{d-1}}(-\frac{\partial^2}{\partial t^2})\langle\phi\phi\rangle_d$	Large time limit (Real Leading Order term)
2	$\lim_{N\to\infty}\left(-\frac{i}{4\pi t}-\frac{\psi^{(0)}(1-\frac{it}{\beta})+\psi^{(0)}(1+\frac{it}{\beta})}{4\pi\beta}+\frac{\psi^{(0)}(1+N-\frac{it}{\beta})+\psi^{(0)}(1+N+\frac{it}{\beta})}{4\pi\beta}\right)$	$-\frac{1}{4\pi\beta}\log\left(\frac{t^2}{N^2\beta^2}\right)$
3	$-\frac{1}{4\beta^2}\text{csch}^2[\frac{\pi t}{\beta}]$	$-\frac{1}{4\beta^2}e^{-\frac{2\pi t}{\beta}}$
4	$\frac{i}{8\pi^2 t^3} - \frac{\psi^{(2)}(1-\frac{it}{\beta})+\psi^{(2)}(1+\frac{it}{\beta})}{16\pi^2\beta^3}$	$\frac{1}{8\pi^2\beta t^2}$
5	$\frac{\pi}{12\beta^4}\text{csch}^4(\frac{\pi t}{\beta})\left(2+\cosh\left(\frac{2\pi t}{\beta}\right)\right)$	$\frac{\pi}{12\beta^4}e^{-\frac{2\pi t}{\beta}}$
6	$-\frac{3i}{16\pi^3 t^5} - \frac{\psi^{(4)}(1-\frac{it}{\beta})+\psi^{(4)}(1+\frac{it}{\beta})}{128 pi^3\beta^5}$	$\frac{3}{32\pi^3\beta t^4}$
7	$-\frac{\pi^2}{120\beta^6}\text{csch}^6(\frac{\pi t}{\beta})\left(33+26\cosh\left(\frac{2\pi t}{\beta}\right)+\cosh\left(\frac{4\pi t}{\beta}\right)\right)$	$-\frac{\pi^2}{120\beta^6}e^{-\frac{2\pi t}{\beta}}$
8	$\frac{15i}{32\pi^4 t^7} - \frac{\psi^{(6)}(1-\frac{it}{\beta})+\psi^{(6)}(1+\frac{it}{\beta})}{1536\pi^4\beta^7}$	$-\frac{5}{32\pi^4\beta t^6}+$
9	$\frac{\pi^3}{1680\beta^8}\text{csch}^8(\frac{\pi t}{\beta})\left(1208 + 1191\cosh\left(\frac{2\pi t}{\beta}\right) + 120\cosh\left(\frac{4\pi t}{\beta}\right) + \cosh\left(\frac{6\pi t}{\beta}\right)\right)$	$\frac{\pi^3}{1680\beta^8}e^{-\frac{2\pi t}{\beta}}$
10	$-\frac{105i}{64\pi^5 t^9} - \frac{\psi^{(8)}(1-\frac{it}{\beta})+\psi^{(8)}(1+\frac{it}{\beta})}{24576\pi^5\beta^9}$	$\frac{105}{256\pi^5\beta t^8}$

The autocorrelation functions in the thermal state decay to zero at large times. This can be understood physically as follows. Say, a system is already in thermal equilibrium. Then, it is perturbed at one of the sites and the effect of this perturbation is measured at the same site as a function of time. Gradually as the system relaxes back to thermal equilibrium once again, the effect of the perturbation completely dies out, as though it had never happened. This is seen in the autocorrelation function. However, the two point correlation function for two spatially separated points should be interpreted slightly differently. When you slightly perturb a site in a system at thermal equilibrium the disturbance propagates spatially and is dissipated out at large times. Thus, when the effect of the disturbance is measured at a point separated by a finite distance, the effect seems to be a decaying function of time(either exponential or a power law). However, when you start with a thermal state with a prescribed energy given by its temperature, the correlation between two spatially separated points is a function of the energy of the system. Thus, at large times, it is not zero but a constant characterized by its temperature. For example at d=3,

$$\langle \phi(r,t)\phi(0,0)\rangle_\beta = \frac{\coth\left(\frac{\pi(r-t)}{\beta}\right) + \coth\left(\frac{\pi(r+t)}{\beta}\right)}{8\pi\beta r} \tag{3.17}$$

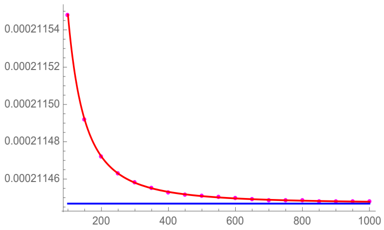
3.2.1 Odd d

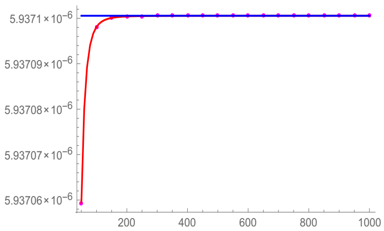
Approach to thermal equilibrium of CC correlator		
d	$\langle \mathrm{CC}\vert\phi(0,t)\phi(r,t)\vert\mathrm{CC}\rangle$	$\langle\phi(0,0)\phi(r,0)\rangle_\beta$
3	$\frac{2\coth\left(\frac{\pi r}{\beta}\right)-\tanh\left(\frac{\pi(r+2t)}{\beta}\right)-\tanh\left(\frac{\pi(r-2t)}{\beta}\right)}{8\pi\beta r}$	$\frac{\coth\left(\frac{\pi r}{\beta}\right)}{4\pi\beta r}$
5	$\frac{\beta\coth\left(\frac{\pi r}{\beta}\right)+\pi r\,\mathrm{csch}^2\left(\frac{\pi r}{\beta}\right)}{8\pi^2\beta^2r^3} - \frac{\beta(\tanh\left(\frac{\pi(r+2t)}{\beta}\right)+\tanh\left(\frac{\pi(r-2t)}{\beta}\right))}{16\pi^2\beta^2r^3} + \frac{\pi r(\mathrm{sech}^2(\frac{\pi(r+2t)}{\beta})+\mathrm{sech}^2(\frac{\pi(r-2t)}{\beta}))}{16\pi^2\beta^2r^3}$	$\frac{\beta\coth\left(\frac{\pi r}{\beta}\right)+\pi r\,\mathrm{csch}^2\left(\frac{\pi r}{\beta}\right)}{8\pi^2\beta^2r^3}$
7	$-\frac{3\left(\tanh\left(\frac{\pi(r+2t)}{\beta}\right)+\tanh\left(\frac{\pi(r-2t)}{\beta}\right)\right)}{32\pi^3\beta r^5} + \frac{3\left(\mathrm{sech}^2\left(\frac{\pi(r+2t)}{\beta}\right)+\mathrm{sech}^2\left(\frac{\pi(r-2t)}{\beta}\right)\right)}{32\pi^2\beta^2r^4} + \frac{\tanh\left(\frac{\pi(r+2t)}{\beta}\right)\mathrm{sech}^2\left(\frac{\pi(r+2t)}{\beta}\right)}{16\pi\beta^3r^3} + \frac{\tanh\left(\frac{\pi(r-2t)}{\beta}\right)\mathrm{sech}^2\left(\frac{\pi(r-2t)}{\beta}\right)}{16\pi\beta^3r^3} + \frac{3\beta^2\coth\left(\frac{\pi r}{\beta}\right)+\pi r\left(3\beta+2\pi r\coth\left(\frac{\pi r}{\beta}\right)\right)\mathrm{csch}^2\left(\frac{\pi r}{\beta}\right)}{16\pi^3\beta^3r^5}$	$\frac{3\beta^2\coth\left(\frac{\pi r}{\beta}\right)+\pi r\left(3\beta+2\pi r\coth\left(\frac{\pi r}{\beta}\right)\right)\mathrm{csch}^2\left(\frac{\pi r}{\beta}\right)}{16\pi^3\beta^3r^5}$
9	$\frac{15\coth\left(\frac{\pi r}{\beta}\right)}{32\pi^4\beta r^7} + \frac{\mathrm{csch}^4\left(\frac{\pi r}{\beta}\right)}{16\pi\beta^4r^4} + \frac{\left(15\beta^2+4\pi^2r^2\coth^2\left(\frac{\pi r}{\beta}\right)+12\pi\beta r\coth\left(\frac{\pi r}{\beta}\right)\right)\mathrm{csch}^2\left(\frac{\pi r}{\beta}\right)}{32\pi^3\beta^4r^6} + \frac{3\tanh\left(\frac{\pi(r+2t)}{\beta}\right)\left(4\pi^2r^2\mathrm{sech}^2\left(\frac{\pi(r+2t)}{\beta}\right)-5\beta^2\right)}{64\pi^4\beta^3r^7} + \frac{3\tanh\left(\frac{\pi(r-2t)}{\beta}\right)\left(4\pi^2r^2\mathrm{sech}^2\left(\frac{\pi(r-2t)}{\beta}\right)-5\beta^2\right)}{64\pi^4\beta^3r^7} + \frac{\mathrm{sech}^2\left(\frac{\pi(r+2t)}{\beta}\right)\left(15\beta^2-6\pi^2r^2\mathrm{sech}^2\left(\frac{\pi(r+2t)}{\beta}\right)+4\pi^2r^2\right)}{64\pi^3\beta^4r^6} + \frac{\mathrm{sech}^2\left(\frac{\pi(r-2t)}{\beta}\right)\left(15\beta^2-6\pi^2r^2\mathrm{sech}^2\left(\frac{\pi(r-2t)}{\beta}\right)+4\pi^2r^2\right)}{64\pi^3\beta^4r^6}$	$\frac{15\coth\left(\frac{\pi r}{\beta}\right)}{32\pi^4\beta r^7} + \frac{\mathrm{csch}^4\left(\frac{\pi r}{\beta}\right)}{16\pi\beta^4r^4} + \frac{\left(15\beta^2+4\pi r\coth\left(\frac{\pi r}{\beta}\right)\left(3\beta+\pi r\coth\left(\frac{\pi r}{\beta}\right)\right)\right)\mathrm{csch}^2\left(\frac{\pi r}{\beta}\right)}{32\pi^3\beta^4r^6}$

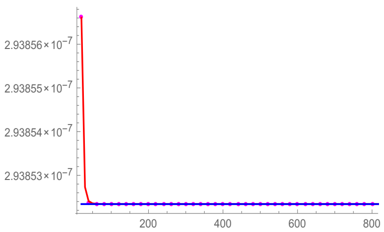
Clearly from 3.2.1 we see that for **odd** d, the decay in time is **exponential**, with the leading order exponent being $\frac{2\pi}{\beta}$

3.2.2 Even d

Approach to thermal equilibrium of the CC correlator (Plotted against time)		
d	$r = 4,\ \beta = 4\pi$	Legends and values

4		• Pink dotted: $\langle \text{CC}\lvert\phi(t,0)\phi(t,\vec{x})\rvert\text{CC}\rangle$
		• Blue Line:$\langle\phi(0,0)\phi(0,\vec{x})\rangle_\beta$
		  – value: 0.000211447
		• Red line: Fit of $\langle\phi\phi\rangle_{\text{CC}}$ with $a + bt^{-c}$
		  – a=0.000211447
		  – c=2.00622
		• Range of t:[100, 1000]

6		• Pink dotted: $\langle \text{CC}\lvert\phi(t,0)\phi(t,\vec{x})\rvert\text{CC}\rangle$
		• Blue Line:$\langle\phi(0,0)\phi(0,\vec{x})\rangle_\beta$
		  – value: 5.9371×10^{-6}
		• Red line: Fit of $\langle\phi\phi\rangle_{\text{CC}}$ with $a + bt^{-c}$
		  – a=5.9371×10^{-6}
		  – c=4.07256
		• Range of t:[50,1000]

8		• Pink dotted: $\langle \text{CC}\lvert\phi(t,0)\phi(t,\vec{x})\rvert\text{CC}\rangle$
		• Blue Line:$\langle\phi(0,0)\phi(0,\vec{x})\rangle_\beta$
		  – value: 2.93852×10^{-7}
		• Red line: Fit of $\langle\phi\phi\rangle_{\text{CC}}$ with $a + bt^{-c}$
		  – a=2.93852×10^{-7}
		  – c=6.09988
		• Range of t:[20,800]

Approach to thermal equilibrium of the CC correlator $\langle\phi(0,0)\phi(t,0)\rangle_\beta$			
Value of r	$d=4$, Range of t: [100,1000]	$d=6$ Range of t: [100,1000]	$d=8$ Range of t: [20,800]
1	2.0048	4.0176	6.0897
2	2.0051	4.0183	6.0919
4	2.0062	4.0210	6.0998
10	2.0139	4.0398	6.0555

From 3.2.2 and 3.2.2 we see that for **even** d the correlation functions fall off as **power laws** in time.

3.2.3 Explanation for the Odd-Even difference

The Thermal Auto-Correlator

The auto-correlation function in the thermal state is defined as (Euclidean ordered i.e. $\tau_1 > \tau_2$)

$$\begin{aligned} G_{th}(\tau_1,\vec{r};\tau_2,\vec{r}) &= \langle\phi(\tau_1,\vec{r})\phi(\tau_2,\vec{r})\rangle_\beta \\ &= \frac{1}{Z}Tr(e^{-\beta H}\phi(\tau_1,\vec{r})\phi(\tau_2,\vec{r})) \end{aligned} \tag{3.18}$$

We can unfold the cylinder by using the method of images. Working in the partial Fourier Transform variables, $G_{th}(\tau_1,r;\tau_2,r)$ becomes

$$\begin{aligned} G_{th}(\tau_1,\vec{r};\tau_2,\vec{r}) &= \int\frac{d^dk}{(2\pi)^d}G_{th}(\tau_1,\tau_2;k) \\ &= \int\frac{d^dk}{(2\pi)^d}\frac{1}{2\omega_k}S_{\rm th}(\tau_1-\tau_2) \\ &= \frac{\Omega_{d-1}}{2(2\pi)^d}\int_0^\infty dk\,k^{d-2}S_{\rm th}(\tau_1-\tau_2) \end{aligned} \tag{3.19}$$

where

$$S_{\rm th}(\tau) = \sum_{n=0}^{\infty}e^{-k(n\beta+\tau)} + \sum_{n=1}^{\infty}e^{-k(n\beta-\tau)} \tag{3.20}$$

Ω_{d-1} is the solid angle of a d-1 dimensional spherical surface in d space dimensions and we have used the dispersion relation $\omega_k = |k|$. This is easily seem from the figure below.

Now we can do the following jugglery

$$
\begin{aligned}
G_{th}(\tau_1-\tau_2=\tau;0)_\beta &= \frac{\Omega_{d-1}}{2(2\pi)^d}\int_0^\infty dk\, k^{d-2}\left[\sum_{n=0}^{\infty} e^{-k(n\beta+\tau)} + \sum_{n=1}^{\infty} e^{-k(n\beta-\tau)}\right] \\
&= \frac{\Omega_{d-1}}{2(2\pi)^d}\left[\sum_{n=0}^{\infty}\left(-\frac{\partial}{\partial\tau}\right)^{d-2}\int_0^\infty dk\, e^{-k(n\beta+\tau)} + \sum_{n=1}^{\infty}\left(\frac{\partial}{\partial\tau}\right)^{d-2}\int_0^\infty dk\, e^{-k(n\beta-\tau)}\right] \\
&= \frac{\Omega_{d-1}}{2(2\pi)^d}\left[\sum_{n=0}^{\infty}\left(-\frac{\partial}{\partial\tau}\right)^{d-2}\left(\frac{1}{n\beta+\tau}\right) + \sum_{n=1}^{\infty}\left(\frac{\partial}{\partial\tau}\right)^{d-2}\left(\frac{1}{n\beta-\tau}\right)\right] \\
&= \frac{\Omega_{d-1}(d-2)!}{2(2\pi)^d}\left[\sum_{n=0}^{\infty}\left(\frac{1}{n\beta+\tau}\right)^{d-1} + \sum_{n=1}^{\infty}\left(\frac{1}{n\beta-\tau}\right)^{d-1}\right]
\end{aligned}
\tag{3.21}
$$

In the second step we can get the powers of k by action of derivatives with respect to τ. In the third line we have performed the k integral. Finally in the last step we evaluate the derivatives. These sums can be evaluated exactly in Mathematica. The result for a few small dimensions ($d > 2$) is tabulated below.

To see the origin of this contrasting behavior between odd and even dimensions, one can do the following. First we analytically continue $\tau \to \tau + it$. Then in the limit $\tau \to 0$ and $t \gg \beta$, we can approximate the sum by an integral using the Euler-Maclaurin formula. For $d > 2$

$$
\begin{aligned}
\sum_{n=0}^{\infty}\left(\frac{1}{n\beta+it}\right)^{d-2} &\approx \frac{1}{\beta^{d-1}}\int_0^\infty \frac{dn}{(n+it/\beta)^{d-1}} \\
&\approx \frac{1}{(d-2)\beta}\frac{1}{(it)^{d-2}}
\end{aligned}
\tag{3.22}
$$

The term with $n\beta - \tau$ can be approximated similarly

$$
\begin{aligned}
\sum_{n=1}^{\infty}\left(\frac{1}{n\beta-it}\right)^{d-2} &\approx \frac{1}{\beta^{d-1}}\int_1^\infty \frac{dn}{(n-it/\beta)^{d-1}} \\
&= \frac{1}{(d-2)\beta}\frac{1}{(1-it)^{d-2}} \\
&\approx \frac{1}{(d-2)\beta}\frac{1}{(-it)^{d-2}}
\end{aligned}
\tag{3.23}
$$

In the large t limit, $n = 0$ or $n = 1$ does not really matter. Therefore we have found that this naive approximation of the sum by an integral gives us

$$
\begin{aligned}
G_{th}(\tau+it;r=0)_\beta &= \frac{\Omega_{d-1}(d-2)!}{2(2\pi)^d}\left[\frac{1}{(it)^{d-2}} + \frac{1}{(-it)^{d-2}}\right] \\
&\approx \begin{cases} \frac{\Omega_{d-1}(d-3)!}{(2\pi)^d}\frac{1}{(it)^{d-2}} & \text{d is even} \\ 0 & \text{d is odd} \end{cases}
\end{aligned}
\tag{3.24}
$$

This gives us power law decay in even dimensions. Of course in odd dimensions the cancellation is not exact and we get an exponential decay instead.

Auto-Correlator in the CC state

The auto-correlation function in the CC state is defined as (Euclidean ordered i.e. $\tau_1 > \tau_2$)

$$
\begin{aligned}
G_{CC}(\tau_1, \vec{r}; \tau_2, \vec{r}) &= \langle \phi(\tau_1, \vec{r}) \phi(\tau_2, \vec{r}) \rangle \\
&= \int \frac{d^d k}{(2\pi)^d} G_{CC}(\tau_1, \tau_2; k) \\
&= \int \frac{d^d k}{(2\pi)^d} \frac{1}{2\omega_k} \left(S_{\text{th}}(\tau_1 - \tau_2) + S_{\text{CC}}(\tau_1 + \tau_2) \right) \\
&= \frac{\Omega_{d-1}}{2(2\pi)^d} \int_0^\infty dk\, k^{d-2} \left(S_{\text{th}}(\tau_1 - \tau_2) + S_{\text{CC}}(\tau_1 + \tau_2) \right)
\end{aligned}
\tag{3.25}
$$

where

$$
\begin{aligned}
S_{\text{th}}(\tau) &= \sum_{n=0}^{\infty} e^{-k(2nL+\tau)} + \sum_{n=1}^{\infty} e^{-k(2nL-\tau)} \\
S_{\text{CC}}(\tau') &= -\sum_{n=0}^{\infty} e^{-k((2n+1)L+\tau')} - \sum_{n=1}^{\infty} e^{-k((2n-1)L-\tau')}
\end{aligned}
\tag{3.26}
$$

We have already evaluated the thermal part (S_{th}). The S_{CC} term is no different. The role of $\tau = (\tau_1 - \tau_2)$ is now played by $\tau' = (\tau_1 + \tau_2)$. Repeating the same exercise as in the thermal case above gives us

$$
\begin{aligned}
G_{CC}(\tau_1, \vec{r}; \tau_2, \vec{r}) - G_{th}(\tau_1, \vec{r}; \tau_2, \vec{r})_\beta &= \frac{\Omega_{d-1}}{2(2\pi)^d} \int_0^\infty dk\, k^{d-2} S_{\text{CC}}(\tau_1 + \tau_2) \\
= -\frac{\Omega_{d-1}(d-2)!}{2(2\pi)^d} &\left[\sum_{n=0}^{\infty} \left(\frac{1}{(2n+1)L + \tau'} \right)^{d-2} + \sum_{n=1}^{\infty} \left(\frac{1}{(2n-1)L - \tau'} \right)^{d-2} \right]
\end{aligned}
\tag{3.27}
$$

Analytically continuing to real time ($\tau' \to \tau' + it'$) and once again approximating the sum by an Euler-Maclaurin integral in the large time limit gives us the same odd-even behavior

$$
\begin{aligned}
G_{CC}(\tau_1, \vec{r}; \tau_2, \vec{r}) - G_{th}(\tau_1, \vec{r}; \tau_2, \vec{r})_\beta &= \frac{\Omega_{d-1}(d-2)!}{2(2\pi)^d} \left[\sum_{n=0}^{\infty} \frac{1}{(it')^{d-2}} + \sum_{n=1}^{\infty} \frac{1}{(-it')^{d-2}} \right] \\
&\approx \begin{cases} \frac{\Omega_{d-1}(d-3)!}{(2\pi)^d} \frac{1}{(it')^{d-2}} & \text{d is even} \\ 0 & \text{d is odd} \end{cases}
\end{aligned}
\tag{3.28}
$$

3.2.4 r dependence after thermalization

Our critical quench takes us to a massless theory, this theory has no scale to begin with. Our Post quench Hamiltonian is a critical Hamiltonian at zero temperature.

When we start an excited state however, the the energy of the excited state introduces a scale into the system. Thus, as observables in the excited state decay to thermal state, the equilibrium is characterized by a temperature that reflects this scale. For correlations functions in a thermal state, the system already has a scale (the temperature). We expect that correlation functions in d spatial dimensions should look like, $\frac{e^{-r/\xi}}{r^p}$, where p is some rational number and ξ is some correlation length set by the temperature, the inverse of which can be perceived as a thermal mass. At quantum criticality, the correlation length becomes infinite and the correlation function becomes just dependent on the geometry of the system, hence falls off as a power law in terms of spatial separation r between the operators. The free energy cost for transmitting the response to a perturbation in the system (measured by the free energy) is zero. In our case, we are calculating the time independent correlation function at a generic temperature set by $\frac{1}{\beta}$. So we expect a Yukawa-like behaviour. As mentioned earlier the thermal state can be represented as a cylinder, where the Euclidean time has been compactified. We can use the **Kaluza Klein reduction** to obtain the correlator in the thermalized state.

$$\phi(\vec{x},\tau) = \sum_{n\in\mathbb{Z}} \phi_n(\vec{x}) e^{i\frac{2\pi n}{\beta}\tau} \tag{3.29}$$

For the free scalar field he Klein Gordon equation looks like:

$$\Box\phi = \partial^M\partial_M\phi = 0, \quad \text{where} \quad M = 1, ...d+1 \tag{3.30}$$

Now using the form of ϕ after implementing periodic boundary conditions, we get:

$$\partial^\mu\partial_\mu\phi_n - (\frac{2\pi n}{\beta})^2\phi_n = 0, \quad \forall \quad \text{n} = 0, \pm1... \pm\infty \quad \text{where} \quad \mu = 1, ...d \tag{3.31}$$

which looks like the massive Klein Gordon equation in one lower dimension with thermal mass $m_{th} = \frac{2\pi|n|}{\beta}$

Now in $d+1 = 4$ dimensions, for fields satisfying $(\partial^2 - m_n^2)\phi_n = 0$,

$$\langle\phi_n(0,0)\phi_n(r,0)\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{e^{ikx}}{k^2+m_n^2} = \frac{e^{-m_n r}}{r} \tag{3.32}$$

Since now each ϕ can be decomposed into n modes, we get:

$$\langle\phi(0,0)\phi(r,0)\rangle_\beta = \frac{1}{4\pi\beta r} + 2\sum_{n=1}^{\infty}\frac{e^{-\frac{2\pi n r}{\beta}}}{4\pi\beta r} \tag{3.33}$$

The first term corresponds to the zeroeth mode and the we see that as $\beta \to 0$ (or temperature goes to infinity), only the zeroeth mode contribution remains. This limit also corresponds to the large r limit, since in the exponentials, r and β appear together as $\frac{r}{\beta}$ which is a dimensionless quantity. At large r, the leading order correction is an exponentially decaying term with coefficient, $\frac{2\pi}{\beta}$. The results are slightly puzzling to me, since even at an arbitrary temperature, the correlation function seems to be predominated by a power law decay. Also at infinite temperature, the thermal fluctuations are so high that there should be a rapid drop in the correlation function as a function of separation. However, that is clearly not the case in the results above, since it becomes solely a power law decay in r at that limit. The thermal correlation functions in odd spatial dimensions are tabulated in the second column of Table 3.2.1. Clearly, upon doing a large r expansion, the leading terms are Greens functions in d spatial dimensions. We also found the same thing for Even d, numerically. The numerical results have been tabulated below 3.2.4. When the correlators are just spatially separated and then β which is the circumference of the cylinder, is taken to zero, the correlators don't see the Euclidean time direction at all. Thus, the correlators exactly match the Green's function in one less spacetime dimension.The value of a is much much smaller than the values of the correlation function at finite values of r. This is important because, if that were not the case, then that would mean the correlation function between two infinitely separated points is non-zero, which is inadmissible due to **cluster decomposition**.

leading behaviour in r of $\phi\phi$ correlator in Thermal equilibrium		
$d = D - 1$	Numerical value of c in the fit of $\langle\phi(0,0)\phi(r,0)\rangle_\beta$ with $a + br^{-c}$	Greens function in d dimensions
3	$0.9999 \approx 1$	r^{-1}
4	$2.0000 \approx 2$	r^{-2}
5	$2.9999 \approx 3$	r^{-3}
6	$4.0000 \approx 4$	r^{-4}
7	5.0090	r^{-5}
8	6.0097	r^{-6}
9	7.0252	r^{-7}

The Greens function in 2 dimensions is Logarithmic with respect to spatial separation. The fit of the $d + 1 = 3$ function also gave a Log fit:

At least with respect to behavior at large spatial separations in thermal equilibrium, we observe no difference between Odd and Even dimensions. The general formula for spatially separated thermal correlation function is, in the high temperature/ large distance limit is (for $d > 2$) :

$$\frac{1}{r^{d-2}} + \frac{e^{-\frac{2\pi r}{\beta}}}{r^{d-2}} \tag{3.34}$$

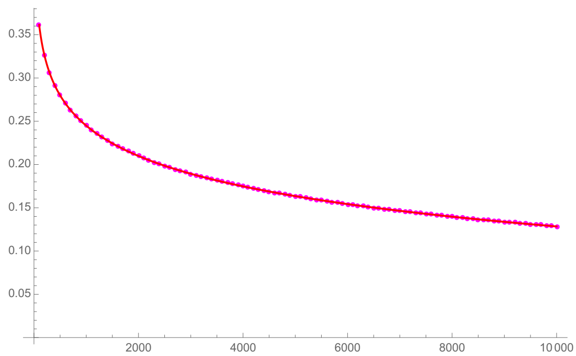


Figure 3.3: Time independent thermal correlator in 3 spacteime dimensions (d=2) fitted with $0.5943 - 0.0506 \log r$

Chapter 4

The GCC and the GGE

4.1 The gCC State (κ_4 != 0)

The 2 point functions in the gCC state cannot be calculated in the slab picture. We therefore use the expressions for the equal time correlator and evaluate them numerically. The integral to be evaluated is

$$
\begin{aligned}
&\langle gCC|\phi(0,t)\phi(\vec{r},t)|gCC\rangle \\
&= \int_{-\infty}^{\infty} \frac{d^d k}{(2\pi)^d} e^{i\vec{k}\cdot\vec{r}} \frac{1}{2|\vec{k}|} \left[\coth\left(2|\vec{k}|(\kappa_2 + \kappa_4|\vec{k}|^2)\right) - \cos\left(2|\vec{k}|t\right) \operatorname{csch}\left(2|\vec{k}|(\kappa_2 + \kappa_4|\vec{k}|^2)\right)\right] \\
&= \Omega(d-2) \int_0^{\infty} \frac{dk\, k^{d-2}}{2(2\pi)^d} \int_0^{\pi} \mathrm{d}\theta e^{ikr\cos\theta} (\sin\theta)^{d-2} \left[\coth\left(2k(\kappa_2 + \kappa_4 k^2)\right) - \cos(2kt) \operatorname{csch}\left(2k(\kappa_2 + \kappa_4 k^2)\right)\right] \\
&= \Omega(d-2) \int_0^{\infty} \frac{dk\, k^{d-2}}{2(2\pi)^d} \int_{-1}^{1} dx e^{ikrx} \left(1-x^2\right)^{\frac{d-3}{2}} \left[\coth\left(2k(\kappa_2 + \kappa_4 k^2)\right) - \cos(2kt) \operatorname{csch}\left(2k(\kappa_2 + \kappa_4 k^2)\right)\right]
\end{aligned} \tag{4.1}
$$

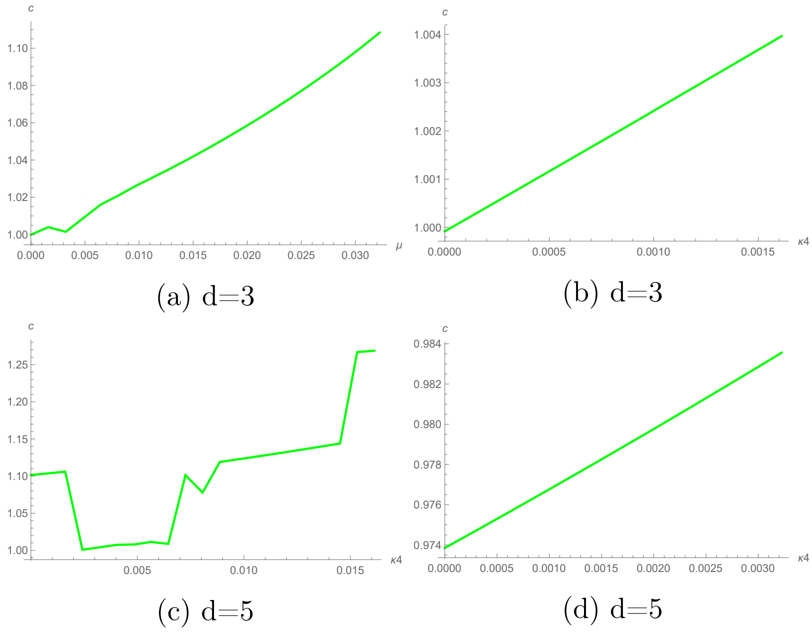
Here $\Omega(d)$ is the solid angle of S^d, the sphere in d dimensions. The angular factor can be calculated in general for $d > 1$ as

$$
\int_{-1}^{1} \left(1-x^2\right)^{\frac{d-3}{2}} e^{ikrx}\, dx = \sqrt{\pi}\, \Gamma\left(\frac{d-1}{2}\right) {}_0\tilde{F}_1\left(;\frac{d}{2};-\frac{1}{4}k^2 r^2\right) \tag{4.2}
$$

where ${}_0\tilde{F}_1(;;)$ is the Hypergeometric0F1Regularized function. With the angular integral in hand we can now evaluate the the 2-point function in the gCC state and study its variation with κ_4 in various dimensions. For $\kappa_4 = 0$, we reproduce the same results as in the slab. The leading correction to thermalization is then expected to be linear when $\bar{\kappa_4}$ is small ($\bar{\kappa_4} = \kappa_4/\kappa_2^3$ is defined to be dimensionless). See figure 4.1

4.1.1 Odd d

For odd d and for small κ_4, we qualitatively find an exponentially decaying approach to thermalization with thermal relaxation rate given by $\exp\left[-\frac{\pi}{\kappa_2}(1 + \alpha_d\bar{\kappa_4} + \mathcal{O}(\bar{\kappa_4}^2))t\right]$, with α_d being a number which depends on the dimension d.



(a) d=3 (b) d=3

(c) d=5 (d) d=5

Figure 4.1: GCC 2 point function of ϕ: The top row is $d = 3$ while the bottom is $d = 5$ for values $\kappa_2 = \pi$, $r = 2$, $t \in [10, 50]$ integrated up to $k_{max} = 100$. The thermalization exponent is written in units of π/κ_2 for $r = 2$. In general the modification is non-linear (left) but for sufficiently small $\bar{\kappa_4}$, it is linear (right).

4.1.2 Even d

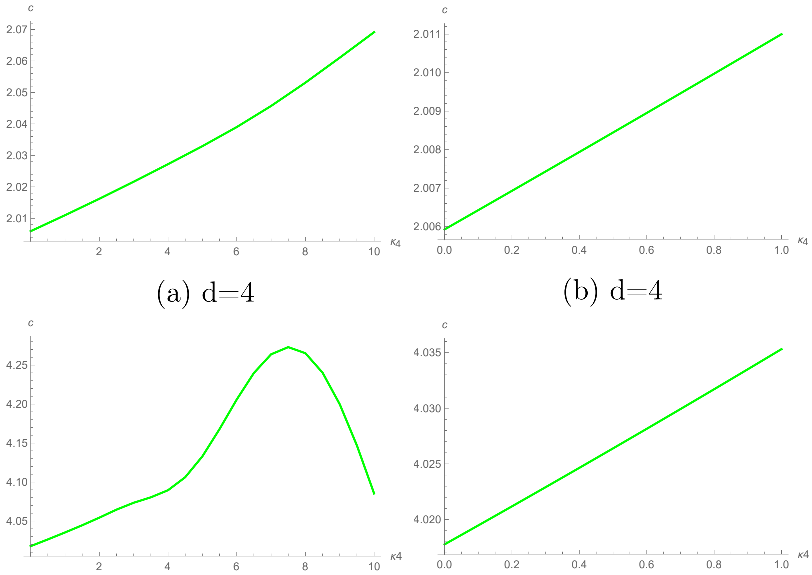
For even d and for small κ_4, we qualitatively find a power law approach to equilibrium $t^{-(d-2)[1+\alpha_d\kappa_4+\mathcal{O}(\bar{\kappa_4}^2)]}$ with α_d being a number which depends on the dimension d. This is seen from the figure 4.2.

4.2 The GGE Correlator (μ!=0)

The 2-point function in the GGE ensemble with non-zero μ is given by

$$\begin{aligned}\langle\phi(\vec{r},t)\phi(0,0)\rangle_{\beta,\mu} &= \int_{-\infty}^{\infty}\frac{d^dk}{(2\pi)^d}e^{i\vec{k}\cdot\vec{r}}\frac{1}{2|\vec{k}|}\left[\coth\left(\frac{\beta|\vec{k}|+\mu|\vec{k}|^3}{2}\right)\cos\left(|\vec{k}|t\right)-i\sin\left(|\vec{k}|t\right)\right]\\ &= \Omega(d-2)\int_0^{\infty}\frac{dk\,k^{d-2}}{2(2\pi)^d}\int_0^{\pi}d\theta e^{ikr\cos\theta}(\sin\theta)^{d-2}\left[\coth\left(\frac{\beta k+\mu k^3}{2}\right)\cos(kt)-i\sin(kt)\right]\\ &= \Omega(d-2)\int_0^{\infty}\frac{dk\,k^{d-2}}{2(2\pi)^d}\int_{-1}^{1}dx e^{ikrx}\left(1-x^2\right)^{\frac{d-3}{2}}\left[\coth\left(\frac{\beta k+\mu k^3}{2}\right)\cos(kt)-i\sin(kt)\right]\end{aligned} \tag{4.3}$$

Here $\Omega(d)$ is the solid angle of S^d, the sphere in d dimensions. The time 't' appearing here is the time separation of the 2 operator insertions as compared to gCC equal-time correlator, where 't' is the time after the sudden quench. We can also study



(a) d=4 (b) d=4

(c) d=6 (d) d=6

Figure 4.2: GCC 2 point function of ϕ: The top row is $d = 4$ while the bottom is $d = 6$ for values $\kappa_2 = 1$, $r = 2$, $t \in [20, 100]$ integrated up to $k_{max} = 100$. In general the modification to the thermalization coefficient is non-linear (left) but for sufficiently small $\bar{\kappa_4}$, it is linear (right).

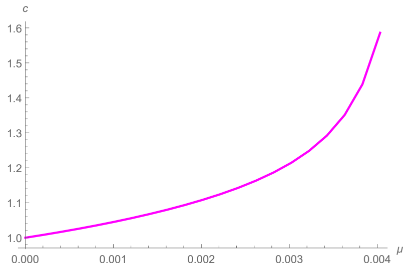
the autocorrelation function ($r = 0$) which is given by

$$\begin{aligned} \langle \phi(0,0)\phi(0,t)\rangle_{\beta,\mu} &= \int_{-\infty}^{\infty} \frac{d^d k}{(2\pi)^d} \frac{1}{2|\vec{k}|} \left[\coth\left(\frac{\beta|\vec{k}|+\mu|\vec{k}|^3}{2} \right) \cos\Big(|\vec{k}|t\Big) - i \sin\Big(|\vec{k}|t\Big) \right] \\ &= \Omega(d-1) \int_0^{\infty} \frac{dk}{2(2\pi)^d} k^{d-2} \left[\coth\left(\frac{\beta k + \mu k^3}{2} \right) \cos(kt) - i \sin(kt) \right] \end{aligned} \tag{4.4}$$

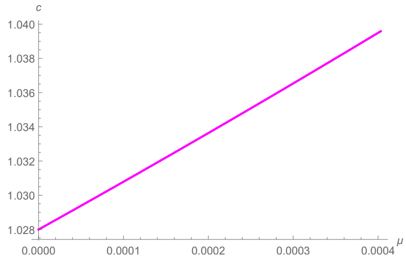
In this case the $\kappa_4 = 0$ integral can be done exactly and agrees with the cylinder calculation done in the section above. Its the same exponential decay for all odd space dimensions and some power law for even dimensions. And as we will see qualitatively that adding a small κ_4 slightly increases the exponent in both the cases. The Sin term (imaginary part) is the same even with $\kappa_4 \neq 0$ and is calculated to be (in d space dimensions)

$$-\frac{1}{4}\pi^{-\frac{d}{2}-\frac{1}{2}} t^{1-d} \cos\left(\frac{\pi d}{2}\right) \Gamma\left(\frac{d-1}{2}\right) \tag{4.5}$$

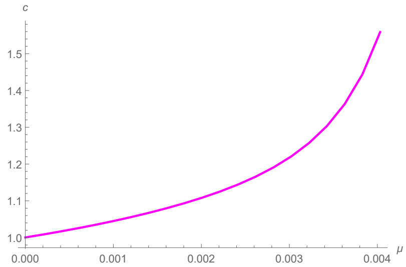
which vanishes for odd d but not for even d. The real part in each case however is now more complicated and has to be handled numerically. We however study the case when $r \neq 0$ as it is more generic and



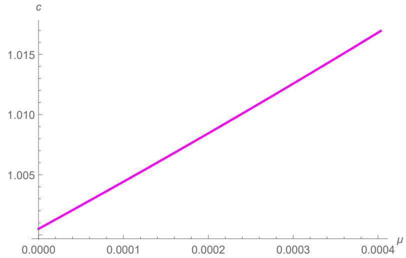
(a) d=3



(b) d=3



(c) d=5



(d) d=5

Figure 4.3: GGE 2 point function of ϕ:The top row is $d = 3$ while the bottom is $d = 5$ for values $\beta = 2\pi$, $r = 2$, $t \in [10, 50]$ integrated up to $k_{max} = 100$. The thermalization exponent is written in units of $2\pi/\beta$. In general the modification to the thermalization coefficient is non-linear (left) but for sufficiently small $\bar{\mu}$, it is linear (right).

4.2.1 Odd d

We vary μ and note the effect it has on the thermalization rate. In odd dimensions, for small $\bar{\mu}$ it is always given by $\exp\left[-\frac{2\pi}{\beta}(1 + \alpha_d\,\bar{\mu} + \mathcal{O}(\bar{\mu}^2))t\right]$ where α_d is a number which depends on the dimension d. This can be seen from the figure 4.3.

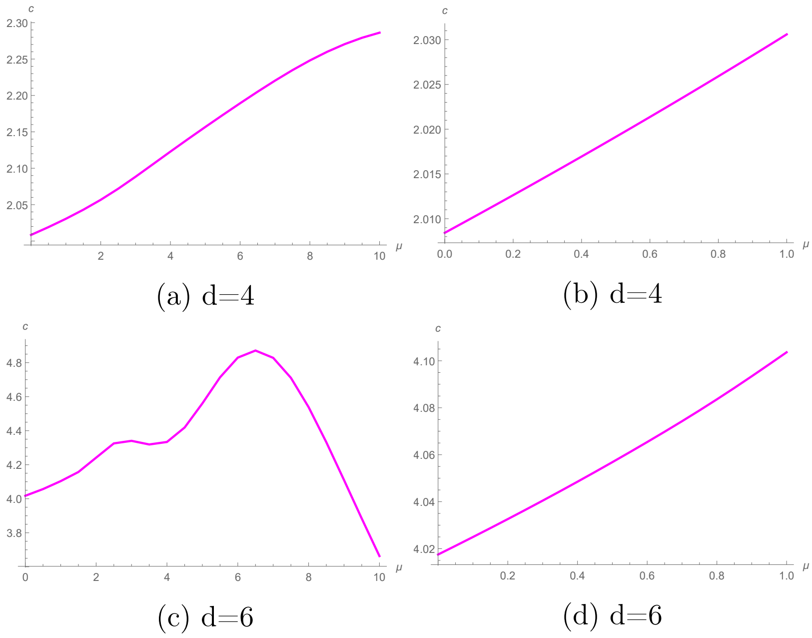
4.2.2 Even d

Again we vary $\bar{\mu}$ and note the effect it has on the approach to thermalization in even dimensions. Here, for small $\bar{\mu}$ it is given by $t^{-(d-2)[1+\alpha_d\bar{\mu}+\mathcal{O}(\bar{\mu}^2)]}$ with α_d depending on the dimension d. In general though, for large values of $\bar{\mu}$ the nature of the function might something completely different as is shown in figure 4.4

4.2.3 Recursion Relation for the Correlator in the Thermal/GGE Ensemble

The GGE two-point function in arbitrary dimensions is given by

$$\langle\phi(\vec{x_1}, t_1,)\phi(\vec{x_2}, t_2)\rangle^d_{\beta,\mu} = \frac{1}{2}\int \frac{d^dk}{(2\pi)^d}\left[G_- e^{\iota\vec{k}\cdot\vec{x}}e^{-\iota|\vec{k}|t} + G_+ e^{-\iota\vec{k}\cdot\vec{x}}e^{\iota|\vec{k}|t}\right] \tag{4.6}$$



(a) d=4 (b) d=4

(c) d=6 (d) d=6

Figure 4.4: GGC 2 point function of ϕ:The top row is $d = 4$ while the bottom is $d = 6$ for values $\beta = 1$, $r = 2$, $t \in [20, 100]$ integrated up to $k_{max} = 100$. In general the modification to the thermalization coefficient is non-linear (left) but for sufficiently small $\bar{\mu}$, it is linear (right).

where $\vec{x} = \vec{x1} - \vec{x2}$, $t = t1 - t2$ and

$$G_{\pm} = \frac{1}{|\vec{k}|(\pm e^{\pm(\beta|k|+\mu|k|^3)} \mp 1)} \tag{4.7}$$

This can also be written as

$$\begin{aligned}\langle\phi(\vec{r},t)\phi(0,0)\rangle^d_{\beta,\mu} &= \int_{-\infty}^{\infty} \frac{d^dk}{(2\pi)^d} e^{i\vec{k}\cdot\vec{r}} \frac{1}{2|\vec{k}|}\left[\coth\left(\frac{\beta|\vec{k}|}{2}\right)\cos\left(|\vec{k}|t\right) - i\sin\left(|\vec{k}|t\right)\right] \\ &= \frac{\Omega_{d-2}}{2(2\pi)^d}\int_0^\infty dk\, k^{d-2}\int_0^\pi d\theta e^{ikr\cos\theta}(\sin\theta)^{d-2}\left[\coth\left(\frac{\beta k}{2}\right)\cos(kt) - i\sin(kt)\right]\end{aligned} \tag{4.8}$$

Ω_{d-1} is the solid angle of a d-1 dimensional spherical surface in d space dimensions. The time 't' appearing here is the time separation of the 2 operator insertions. Now we pull out a $k^2(\sin\theta)^2 = k^2 - k^2(\cos\theta)^2$ and write it as a derivative operator acting on the correlation function in $d-2$ dimensional space as follows

$$\begin{aligned}\langle\phi(\vec{r},t)\phi(0,0)\rangle^d_{\beta,\mu} &= \frac{\Omega_{d-2}}{2(2\pi)^d}\int_0^\infty dk\, k^{d-4}\int_0^\pi \mathrm{d}\theta e^{ikr\cos\theta}(\sin\theta)^{d-4}\left[\coth\left(\frac{\beta k}{2}\right)\cos(kt)-i\sin(kt)\right]\times k^2(\\
&= \frac{\Omega_{d-2}}{2(2\pi)^d}\int_0^\infty dk\, k^{d-4}\int_0^\pi \mathrm{d}\theta e^{ikr\cos\theta}(\sin\theta)^{d-4}\left[\coth\left(\frac{\beta k}{2}\right)\cos(kt)-i\sin(kt)\right]\times (k^2\\
&= \frac{\Omega_{d-2}}{2(2\pi)^d}\left(-\partial_t^2+\partial_r^2\right)\int_0^\infty dk\, k^{d-4}\int_0^\pi \mathrm{d}\theta e^{ikr\cos\theta}(\sin\theta)^{d-4}\left[\coth\left(\frac{\beta k}{2}\right)\cos(kt)-i\sin\right.\\
&= \frac{\Omega_{d-2}}{4\pi^2\Omega_{d-4}}\left(-\partial_t^2+\partial_r^2\right)\frac{\Omega_{d-4}}{2(2\pi)^{d-2}}\int_0^\infty dk\, k^{d-4}\int_0^\pi \mathrm{d}\theta e^{ikr\cos\theta}(\sin\theta)^{d-4}\left[\coth\left(\frac{\beta k}{2}\right)\mathrm{co}\right.\\
&= \frac{\Omega_{d-2}}{4\pi^2\Omega_{d-4}}\left(-\partial_t^2+\partial_r^2\right)\langle\phi(\vec{r},t)\phi(0,0)\rangle^{d-2}_{\beta,\mu}\end{aligned} \tag{4.9}$$

Thus we see that the recursion relation connects the correlator in d dimensions to correlators in d-2 dimensions. This is consistent with our results. If in odd d there is an exponential falloff then we would have an exponential falloff in all odd dimensions. Similarly if we have a power law decay in even d, then we would have power law decays with increasing degrees in higher even dimensions.

4.3 UV/IR Mixing

Consider a lattice at a certain temperature with a spin degree of freedom at each site. The temperature specifies energy and therefore the (macro) state of our lattice system. A 2-point function (g_2 say) would correspond to for example how the spin at site number i responds to perturbing the spin at site 0. In general g_2 would be a function of the distance (index) i and other scales present in the problem. At zero temperature when all the spins aligned together, lets say that g_2 has a certain behavior. Now consider the same 2-point function at very high temperature T, the spins are all disordered and the same 2-point function g_2 now is expected to be different. g_2 is a function of the temperature and if there is some other quantity characterizing the state of our system (a chemical potential), it should also show up in the n-point functions.

From the above example we see that the 2-point function depends on the state in which it is calculated. Now lets link this with the our understanding of Renormalization Group (RG) and RG flow. In the Wilsonian way of thinking, we can integrate the momentum modes up to a certain cut-off Λ, but in the process the couplings in the Lagrangian must also change and become functions of Λ so that the path integral and therefore all n-point functions remain unchanged. In the ground state, we can lower the cutoff up to $k\sim 1/r$ but in an excited state with very large energy E_s(or momenta k_s) we can only come down to $\sim k_s$ i.e. $\Lambda\sim k_s$. Due to this extra scale in the problem g_2 is expected to be a function of k_s. In general this information about the excited state must show up in the n-point functions. This is what is seen in our calculations above. The κ_4 characterizing the excited gCC state now appears in the 2-point function and similarly μ in the GGE ensemble. In chapter 6 we have discussed the concept of UV-IR mixing using a Toy example.

4.4 Interpretation of W's in higher D

Maldacena and Zhiboedov in their paper [8], showed that any CFT with higher spin conserved currents has correlation functions that exactly match that of a free field theory, which can constitute N number of free bosons or fermions. The presence of one single higher spin conserved current implies the presence of an infinite number of higher spin conserved current. Thus, in higher dimensions, apart from the global conformal symmetry, there exists a higher spin symmetry, given by the W_∞ algebra. The operators corresponding to this can be In $2D$ CFT, the Virasoro is a subset of the W_∞ algebra. For free scalars, the operators corresponding to such currents can be constructed as, $\phi \overleftrightarrow{\partial}_{\mu_1} \overleftrightarrow{\partial}_{\mu_2} \ldots \overleftrightarrow{\partial}_{\mu_n} \phi$. In the context of our problem, this is particularly relevant, since our post quench state is a CFT of free scalars characterized by these higher spin charges.

Chapter 5

Detailed Calculations in $d = 3$

5.1 Limits

We calculate two-point functions of different types- *Non-Equilibrium* and *Equilibrium* correlators. Non-Equilibrium correlators are calculated in different initial states, namely the ground state, the CC and gCC state with Hamiltonian and one extra conserved charge. Equilibrium correlators comprise of the thermal and GGE. We are mostly interested in the equal-time correlators (ETC's). If the Subsystem Thermalization is correct, then the non-equilibrium ETC's must agree with the equilibrium ETC's in certain limits which are describe in the figures below. The first

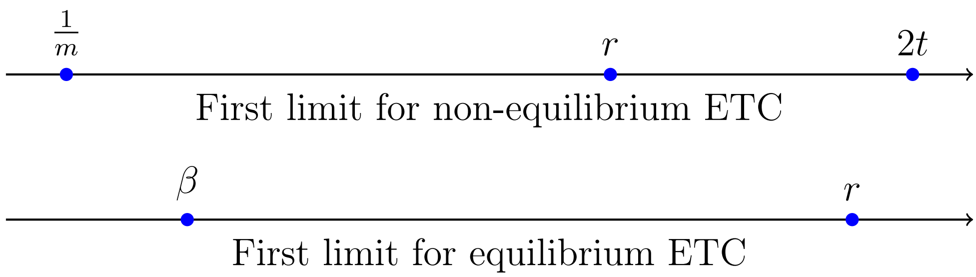


Figure 5.1: First Limit

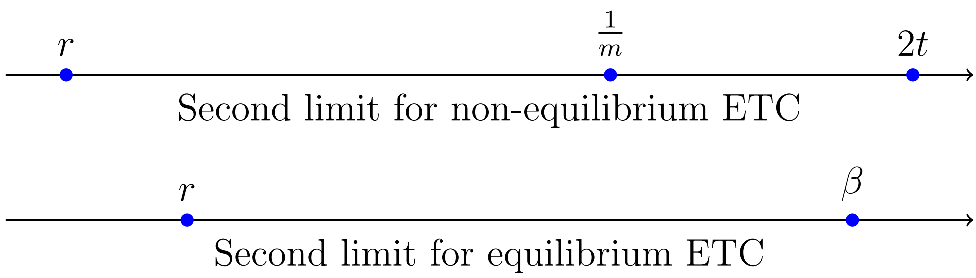


Figure 5.2: Second Limit

limit is defined as

$$\begin{array}{cc} 2mt \gg mr \gg 1 & \text{Non-equilibrium ETC} \\ r/\beta \gg 1 & \text{Equilibrium ETC} \end{array} \tag{5.1}$$

The second limit is defined as

$$\begin{array}{cc} 2mt \gg 1, mr \ll 1 & \text{Non-equilibrium ETC} \\ r/\beta \ll 1 & \text{Equilibrium ETC} \end{array} \tag{5.2}$$

Hence forth we refer to them just as first limit and the second limit respectively.

5.2 Ground state quench

In the sudden limit, the ground state $\phi\phi$ ETC is

$$\langle 0_{in}|\phi(\vec{x},t)\phi(0,t)|0_{in}\rangle = \int \frac{d^3k}{(2\pi)^3}\left(\frac{2|\vec{k}|^2{+}m^2 - m^2Cos(2|\vec{k}|t)}{4(|\vec{k}|^2{+}m^2)^{\frac{1}{2}}\ |\vec{k}|^2}\right)e^{i\vec{k}.\vec{x}} \tag{5.3}$$

We get this expression by plugging in the expression for the in-out waves u_{in} ,u_{out} and the Bogoliubov coefficients in the sudden limit. Details of the calculations are in the appendix B. Performing this complicated integral by techniques of complex analysis gives

$$\langle 0_{in}|\phi(r,t)\phi(0,t)|0_{in}\rangle = \begin{cases} \frac{m}{16\pi r} + \frac{m^2}{16\sqrt{2}\ \pi^{\frac{3}{2}}}\left(\frac{2e^{-mr}}{mr\sqrt{mr}} + \frac{e^{-m(2t+r)}}{mr\ \sqrt{m(r+2t)}} + \frac{e^{-m(2t-r)}}{mr\ \sqrt{m(2t-r)}}\right) + \cdots & r < 2t \\ \\ \frac{m^2}{16\sqrt{2}\pi^{\frac{3}{2}}}\left(\frac{2e^{-mr}}{mr\sqrt{mr}} + \frac{e^{-m(r+2t)}}{mr\ \sqrt{m(r+2t)}} + \frac{e^{-m(r-2t)}}{mr\ \sqrt{m(r-2t)}}\right) + \cdots & r > 2t \end{cases} \tag{5.4}$$

The ground state $\partial_i\phi\partial_i\phi$ ETC is

$$\langle 0_{in}|\partial_i\phi(\vec{r},t)\partial_i\phi(0,t)|0_{in}\rangle = \int \frac{d^3k}{(2\pi)^3}\left(\frac{2|\vec{k}|^2{+}m^2 - m^2Cos(2|\vec{k}|t)}{4(|\vec{k}|^2{+}m^2)^{\frac{1}{2}}\ |\vec{k}|^2}\right)e^{i\vec{k}.\vec{x}}k^2 \tag{5.5}$$

Notice that this is just the same as $\langle\phi\phi\rangle$ except for a factor of k^2. This time the integral is done exactly

$$\langle 0_{in}|\partial_i\phi(\vec{r},t)\partial_i\phi(0,t)|0_{in}\rangle = \begin{cases} -\frac{(m^3r^2+4m)K_1(mr)+2m^2rK_0(mr)}{8\pi^2r^3} - \frac{m^3K_1(m(r+2t))}{16\pi^2r} - \frac{m^3K_1(m(r-2t))}{16\pi^2r} & ,r > 2t \\ -\frac{(m^3r^2+4m)K_1(mr)+2m^2rK_0(mr)}{8\pi^2r^3} - \frac{m^3K_1(m(r+2t))}{16\pi^2r} + \frac{m^3K_1(m(2t-r))}{16\pi^2r} & ,r < 2t \end{cases} \tag{5.6}$$

The leading order contribution at late times in the first limit is

$$\langle 0_{in}|\partial_i\phi(\vec{r},t)\partial_i\phi(0,t)|0_{in}\rangle = -\frac{m^4}{16\sqrt{2}\pi^{3/2}}\left(\frac{2e^{-mr}}{mr\sqrt{mr}} + \frac{e^{-m(r+2t)}}{mr\sqrt{m(r+2t)}} - \frac{e^{-m(2t-r)}}{mr\sqrt{m(2t-r)}}\right) + \cdots \tag{5.7}$$

and the second limit (with $t \to \infty$ and small r) is

$$\langle 0_{in}|\partial_i\phi(\vec{r},t)\partial_i\phi(0,t)|0_{in}\rangle = -\frac{1}{2\pi^2\ r^4} + \cdots \tag{5.8}$$

5.3 Squeezed state quench

The $\phi\phi$ ETC in the quenched state $|f_4\rangle$ (see eq. 2.27) is

$$\langle f_4|\phi(0,t)\phi(\vec{r},t)|f_4\rangle = \frac{1}{8\pi^2\iota r}\int_{-\infty}^{\infty} dk\, e^{ikr}\left(\coth\bigl(2k(\kappa_2+\kappa_4k^2)\bigr) - \cos(2kt)\,\mathrm{csch}\bigl(2k(\kappa_2+\kappa_4k^2)\bigr)\right) \tag{5.9}$$

while the $\partial_i\phi\partial_i\phi$ ETC in $|f_4\rangle$ is

$$\langle f_4|\partial_i\phi(0,t)\partial_i\phi(\vec{r},t)|f_4\rangle = \frac{1}{8\pi^2\iota r}\int_{-\infty}^{\infty} dk\, e^{ikr}\left(\coth\bigl(2k(\kappa_2+\kappa_4k^2)\bigr) - \cos(2kt)\,\mathrm{csch}\bigl(2k(\kappa_2+\kappa_4k^2)\bigr)\right)k^2 \tag{5.10}$$

same as $\phi\phi$ except for a factor of k^2 in the integral.

CC state

The $\phi\phi$ ETC in the CC state ($\kappa_4 = 0$ in $|f_4\rangle$) is (this is exact without any approximations)

$$\langle CC|\phi(0,t)\phi(\vec{r},t)|CC\rangle = \frac{m}{32\pi r}\left[2\coth\left(\frac{\pi m r}{4}\right) - \tanh\left(\frac{\pi m(r+2t)}{4}\right) - \tanh\left(\frac{\pi m(r-2t)}{4}\right)\right] \tag{5.11}$$

which in the first limit is

$$\langle CC|\phi(0,t)\phi(\vec{r},t)|CC\rangle = \frac{m}{16\pi r} + \frac{m}{8\pi r}\left(e^{-\frac{\pi m r}{2}} + e^{-\frac{\pi m(2t+r)}{2}} - e^{-\frac{\pi m(2t-r)}{2}}\right) + \cdots \tag{5.12}$$

and in the second limit

$$\langle CC|\phi(0,t)\phi(\vec{r},t)|CC\rangle = \frac{1}{4\pi^2r^2} + \cdots \tag{5.13}$$

The $\partial_i\phi\partial_i\phi$ ETC in the CC state ($\kappa_4 = 0$ in $|f_4\rangle$) is (again exact without any approximations)

$$\begin{aligned}\langle CC|\partial_i\phi(0,t)\partial_i\phi(\vec{r},t)|CC\rangle = -\frac{\pi m^3}{256 r}\Bigg[&2\coth\left(\frac{\pi m r}{4}\right)\mathrm{csch}^2\left(\frac{\pi m r}{4}\right) + \tanh\left(\frac{\pi m(r+2t)}{4}\right)\mathrm{sech}^2\left(\frac{\pi m(r+2t)}{4}\right)\\ &\tanh\left(\frac{\pi m(r-2t)}{4}\right)\mathrm{sech}^2\left(\frac{\pi m(r-2t)}{4}\right)\Bigg]\end{aligned} \tag{5.14}$$

which in the first limit is

$$\langle CC|\partial_i\phi(0,t)\partial_i\phi(\vec{r},t)|CC\rangle = -\frac{\pi m^3}{32 r}\left(e^{-\pi m r/2} + e^{-\pi m t}\right) + \cdots \tag{5.15}$$

and in the second limit is

$$\langle CC|\partial_i\phi(0,t)\partial_i\phi(\vec{r},t)|CC\rangle = -\frac{1}{2\pi^2r^4} - \frac{\pi m^3}{16 r}e^{-\pi m t} + \cdots \tag{5.16}$$

gCC state

The $\phi\phi$ ETC in $|f_4\rangle = |gCC\rangle$ is calculated as follows. We have to take care of contribution from poles occurring when the arguments of coth and csch functions become $i\,n\pi$. Introducing the dimensionless parameter $\bar{\kappa_4} = \kappa_4/\kappa_2^3$ and keeping track of the slowest decaying transient perurbatively in $\bar{\kappa_4}$, we get for $2t > r$ (first limit)

$$\langle gCC|\phi(0,t)\phi(\vec{r},t)|gCC\rangle \approx \frac{1}{16\pi r\kappa_2} + \frac{1}{16\pi r\kappa_2}\left(1+\frac{3\pi^2}{4}\bar{\kappa_4}+\cdots\right)\Big[2\exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\bar{\kappa_4}+\cdots\right)r\right)+ \\ \exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\bar{\kappa_4}+\cdots\right)(2t+r)\right)-\exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\bar{\kappa_4}+\cdots\right)(2t-r)\right)\Big] \tag{5.17}$$

while for $r > 2t$ it is

$$\langle gCC|\phi(0,t)\phi(\vec{r},t)|gCC\rangle \approx \frac{1}{16\pi r\kappa_2}\left(1+\frac{3\pi^2}{4}\bar{\kappa_4}+\cdots\right)\Big[2\exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\bar{\kappa_4}+\cdots\right)r\right)+ \\ \exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\bar{\kappa_4}+\cdots\right)(r+2t)\right)+exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\bar{\kappa_4}+\cdots\right)(r-2t)\right)\Big] \tag{5.18}$$

Similarly the $\partial_i\phi\partial_i\phi$ ETC in $|f_4\rangle = |gCC\rangle$ is calculated to be (for $2t > r$ i.e first limit)

$$\langle gCC|\partial_i\phi(r,t)\partial_i\phi(0,t)|gCC\rangle \approx -\frac{\pi}{64\kappa_2^3 r}\left(1+\frac{5\pi^2\bar{\kappa_4}}{4}+O\left(\bar{\kappa_4}^{3/2}\right)\right)\times\Big[2\exp\{-\frac{\pi}{2\kappa_2}r(1+\frac{\pi^2\bar{\kappa_4}}{4}+O\left(\bar{\kappa_4}^2\right))\}+ \\ \exp\{-\frac{\pi}{2\kappa_2}(2t+r)(1+\frac{\pi^2\bar{\kappa_4}}{4}+O\left(\bar{\kappa_4}^2\right))\}-\exp\{-\frac{\pi}{2\kappa_2}(2t-r)(1+\frac{\pi^2\bar{\kappa_4}}{4}+O\left(\bar{\kappa_4}^2\right))\}\Big] \tag{5.19}$$

while for $r > 2t$ its

$$\langle gCC|\partial_i\phi(r,t)\partial_i\phi(0,t)|gCC\rangle \approx -\frac{\pi}{64\kappa_2^3 r}\left(1+\frac{5\pi^2\bar{\kappa_4}}{4}+O\left(\bar{\kappa_4}^{3/2}\right)\right)\times\Big[2\exp\{-\frac{\pi}{2\kappa_2}r(1+\frac{\pi^2\bar{\kappa_4}}{4}+O\left(\bar{\kappa_4}^2\right))\}+ \\ \exp\{-\frac{\pi}{2\kappa_2}(r+2t)(1+\frac{\pi^2\bar{\kappa_4}}{4}+O\left(\bar{\kappa_4}^2\right))\}+\exp\{-\frac{\pi}{2\kappa_2}(r-2t)(1+\frac{\pi^2\bar{\kappa_4}}{4}+O\left(\bar{\kappa_4}^2\right))\}\Big] \tag{5.20}$$

5.4 Correlators in GGE and the Thermal Ensemble

We also calculate the correlation functions in the thermal and the generalized Gibbs ensemble. If the Subsystem Thermalization really holds true, then the correlation functions we have calculated must agree with these at least in appropriate limits.

GGE $\phi\phi$ Correlator

The real time propagator in GGE (with one other chemical potential) is defined as

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} = \frac{1}{Z}Tr\left(e^{-\beta H-\mu W_4}\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\right) \tag{5.21}$$

In $3+1$ dimensions this is simplified to (see Appendix for details)

$$\langle\phi(\vec{x_1},t_1)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} = \int_{-\infty}^{+\infty}\frac{dk}{8\pi^2\iota r}\left[\frac{e^{\iota k(r+t)}}{e^{\beta k+\mu k^3}-1}+\frac{e^{+\iota k(r-t)}}{-e^{-\beta k-\mu k^3}+1}\right] \tag{5.22}$$

where $\vec{r} = \vec{x1} - \vec{x2}$, $t = t1 - t2$ and $k(r) = |\vec{k}|(|\vec{r}|)$. The GGE calculation is very similar to the gCC calculation. Again we have simple poles when denominator hits $in\pi$. Defining a dimensionless parameter $\bar{\mu} = \mu/\beta^3$, after an involved calculation we get a perturbative solution in $\bar{\mu}$.

For $t > r$

$$\begin{aligned}\langle\phi(\vec{x_1},t_1)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} \approx & \frac{1}{4\pi r\beta}\left(1+12\pi^2\ \bar{\mu}+\cdots\right)\times \\ & \left[exp\left(-\frac{2\pi}{\beta}\left(1+4\pi^2\bar{\mu}+\cdots\right)(t+r)\right) - exp\left(-\frac{2\pi}{\beta}\left(1+4\pi^2\bar{\mu}+\cdots\right)(t-r)\right)\right]\end{aligned} \tag{5.23}$$

while for $t < r$, we have

$$\begin{aligned}\langle\phi(\vec{x_1},t_1)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} \approx & \frac{1}{4\pi r\beta} + \frac{1}{4\pi r\beta}\left(1+12\pi^2\ \bar{\mu}+\cdots\right)\times \\ & \left[exp\left(-\frac{2\pi}{\beta}\left(1+4\pi^2\bar{\mu}+\cdots\right)(t+r)\right) + exp\left(-\frac{2\pi}{\beta}\left(1+4\pi^2\bar{\mu}+\cdots\right)(r-t)\right)\right]\end{aligned} \tag{5.24}$$

The ETC here is

$$\langle\phi(\vec{x_1},t_1)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} \approx \frac{1}{4\beta\pi r} + \frac{1}{2\pi r\beta}(1+12\pi^2\bar{\mu}+\cdots)\exp\left(-\frac{2\pi r}{\beta}(1+4\pi^2\bar{\mu}+\cdots)\right) \tag{5.25}$$

Thermal $\phi\phi$ Correlator

The real time propagator in Thermal Ensemble is obtained by putting $\mu = 0$ in the GGE correlator

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_\beta = \frac{1}{Z}Tr\left(e^{-\beta H}\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\right) \tag{5.26}$$

which simplifies to

$$\begin{aligned}\langle\phi(\vec{x_1},t_1)\phi(\vec{x_2},t_2)\rangle_\beta &= \int_{-\infty}^{+\infty}\frac{dk}{8\pi^2\iota r}\left[\frac{e^{\iota k(r+t)}}{e^{\beta k}-1} + \frac{e^{+\iota k(r-t)}}{-e^{-\beta k}+1}\right] \\ &= \int_{-\infty}^{+\infty}\frac{dk}{4\pi^2 x}\sin kx\left[\frac{e^{\iota kt}}{e^{\beta k+\mu k^3}-1}\right]\end{aligned} \tag{5.27}$$

This is computed exactly

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_\beta = \frac{\coth\left(\frac{\pi(r+t)}{\beta}\right)}{8\pi\beta r} + \frac{\coth\left(\frac{\pi(r-t)}{\beta}\right)}{8\pi\beta r} \tag{5.28}$$

We are interested in the Equal Time Correlator, where $t = 0$

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_1)\rangle_\beta = \frac{\coth\left(\frac{\pi r}{\beta}\right)}{4\pi\beta r} \tag{5.29}$$

The first limit of which is

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_1)\rangle_\beta \approx \frac{1}{4\pi\beta r}\left(1+2e^{-\frac{2\pi r}{\beta}}+\cdots\right) \tag{5.30}$$

Note that this matches the GGE ETC when we put $\bar{\mu}=0$. The second limit is

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_1)\rangle_\beta \approx \frac{1}{4\pi^2 r^2}+\cdots \tag{5.31}$$

GGE $\partial_i\phi\partial_i\phi$ Correlator

The real time $\partial_i\phi\partial_i\phi$ propagator in GGE (with one other chemical potential) is defined as

$$\langle\partial_i\phi(\vec{x_1},t_1,)\partial_i\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} = \frac{1}{Z}Tr\left(e^{-\beta H-\mu W_4}\partial_i\phi(\vec{x_1},t_1,)\partial_i\phi(\vec{x_2},t_2)\right) \tag{5.32}$$

In $3+1$ dimensions this is simplified to (see AppendixD for details)

$$\langle\partial_i\phi(\vec{x_1},t_1)\partial_i\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} = \int_{-\infty}^{+\infty}\frac{dk}{8\pi^2\iota r}\left[\frac{e^{\iota k(r+t)}}{e^{\beta k+\mu k^3}-1}+\frac{e^{+\iota k(r-t)}}{-e^{-\beta k-\mu k^3}+1}\right]k^2 \tag{5.33}$$

where $\vec{r}=\vec{x1}-\vec{x2}$, $t=t1-t2$ and $k(r)=|\vec{k}|(|\vec{r}|)$. Again we solve this perturbatively in $\bar{\mu}$.

For $r>t$

$$\langle\partial_i\phi(\vec{x1},t1),\partial_i\phi(\vec{x2},t2)\rangle_\beta,\mu = -\frac{\pi}{\beta^3 r}\left(1+20\pi^2\bar{\mu}+O\left(\bar{\mu}^{3/2}\right)\right)\times \\ \left[\exp\left[-\frac{2\pi}{\beta}(r+t)(1+4\pi^2\bar{\mu}+O(\bar{\mu}^2))\right]+\exp\left[-\frac{2\pi}{\beta}(r-t)(1+4\pi^2\bar{\mu}+O(\bar{\mu}^2))\right]\right] \tag{5.34}$$

while for $t>r$

$$\langle\partial_i\phi(\vec{x1},t1),\partial_i\phi(\vec{x2},t2)\rangle_\beta,\mu = -\frac{\pi}{\beta^3 r}\left(1+20\pi^2\bar{\mu}+O\left(\bar{\mu}^{3/2}\right)\right)\times \\ \left[\exp\left[-\frac{2\pi}{\beta}(r+t)(1+4\pi^2\bar{\mu}+O(\bar{\mu}^2))\right]-\exp\left[-\frac{2\pi}{\beta}(t-r)(1+4\pi^2\bar{\mu}+O(\bar{\mu}^2))\right]\right] \tag{5.35}$$

The ETC just becomes ($t_2=t_1$)

$$\langle\partial_i\phi(\vec{x1},t1),\partial_i\phi(\vec{x2},t1)\rangle_{\beta,\mu} = -\frac{2\pi}{\beta^3 r}\left(1+20\pi^2\bar{\mu}+O\left(\bar{\mu}^{3/2}\right)\right)\exp\left[-\frac{2\pi}{\beta}r(1+4\pi^2\bar{\mu}+O(\bar{\mu}^2))\right] \tag{5.36}$$

Thermal $\partial_i\phi\partial_i\phi$ Correlator

Putting $\mu=0$ in the GGE correlator

$$\langle\partial_i\phi(\vec{x_1},t_1,)\partial_i\phi(\vec{x_2},t_2)\rangle_\beta = \frac{1}{Z}Tr\left(e^{-\beta H}\partial_i\phi(\vec{x_1},t_1,)\partial_i\phi(\vec{x_2},t_2)\right) \tag{5.37}$$

which simplifies to

$$\begin{aligned}\langle \partial_i\phi(\vec{x_1},t_1)\partial_i\phi(\vec{x_2},t_2)\rangle_\beta &= \int_{-\infty}^{+\infty}\frac{dk}{8\pi^2\iota r}\left[\frac{e^{\iota k(r+t)}}{e^{\beta k}-1}+\frac{e^{+\iota k(r-t)}}{-e^{-\beta k}+1}\right]k^2 \\ &= \int_{-\infty}^{+\infty}\frac{dk}{4\pi^2 x}\sin kx\left[\frac{e^{\iota kt}}{e^{\beta k+\mu k^3}-1}\right]k^2\end{aligned} \tag{5.38}$$

This is again calculated exactly

$$\langle \partial_i\phi(\vec{x_1},t_1)\partial_i\phi(\vec{x_2},t_2)\rangle_\beta = -\frac{\pi\coth\left(\frac{\pi(r-t)}{\beta}\right)\operatorname{csch}^2\left(\frac{\pi(r-t)}{\beta}\right)}{4\beta^3 r}-\frac{\pi\coth\left(\frac{\pi(r+t)}{\beta}\right)\operatorname{csch}^2\left(\frac{\pi(r+t)}{\beta}\right)}{4\beta^3 r} \tag{5.39}$$

The ETC ($t=0$) is

$$\langle \partial_i\phi(\vec{x_1},t_1)\partial_i\phi(\vec{x_2},t_2)\rangle_\beta = -\frac{\pi\coth(\pi r/\beta)\operatorname{csch}^2(\pi r/\beta)}{2\beta^3 r} \tag{5.40}$$

which in the first limit ($r \gg \beta$) is

$$= -\frac{2\pi}{r\beta^3}e^{-2\pi r/\beta}+\cdots \tag{5.41}$$

and in the second limit ($r \ll \beta$)

$$= -\frac{1}{2\pi^2 r^4}+O(r^0) \tag{5.42}$$

5.5 General comments

Comparing GGE with GCC we see that $\beta = 4\kappa_2$ and $\mu = 4\kappa_4$. Transients are obtained in the Ground State when we do the calculation without taking the deep quench limit (unlike [7]). $\partial\phi$ is a descendant field in 4 dimensions and ϕ is a primary field in 4 dimensions. Thus they have different conformal dimensions, however, they both exhibit the same decay coefficient in the exponential which is, $\frac{2\pi}{\beta}$. In the small r limit the correlator reduces to the spatial Greens function in $d+1$ dimensions.There are differences in the pole structures of the ground state and the GCC and GGE. The Ground State has a branch cut singularity, whereas GGE and GCC have discrete poles. In the exponents of GCC and GGE, we observe mixing of relevant operators with irrelevant operators. These can be treated as signatures of UV-IR mixing which is discussed in the following chapter 6.The correlation function in the GGE, however, is not obtained by doing a path integral from a partition function corresponding to a modified Hamiltonian, that follows the dispersion relation $\omega = |k|+\alpha|k|^3$. The time evolution of the GGE still happens according to the Hamiltonian H with $\omega = |k|$.

Chapter 6

Wilsonian Renormalization and UV-IR mixing

Consider the action,

$$S = \int dx^4 [(\partial\phi)^2 + m^2\phi^2 + g^2\phi^6] \tag{6.1}$$

In 4 dimensions the mass is a relevant coupling and g is an irrelevant coupling. The two point correlation function of ϕ sees a correction in the mass of the field due to the ϕ^6 interaction term. Thus instead of decay of the two point correlation function being $\approx e^{-mr}$ we see that it goes as $\approx e^{-\bar{m}r(1+\frac{\bar{g}^2}{\bar{m}^2})}$ where, $\bar{m} = \frac{m}{\Lambda}$ and $\bar{g} = g\Lambda$, where Λ is the energy cutoff of the theory.Taking m and g to be constants, we see that as we go to lower and lower energy scales, that is, $\Lambda \to 0$, the second term in the exponent goes to zero. Thus, preferentially the effect of g dies out at lower energies (or large distances). The exponent does not have an RG invariant. Thus, we do not see any UV-IR mixing. The above calculation has been inspired from chapter 12 of Peskin and Shroeder [9].

6.1 Toy Model

Now, consider a toy model,

$$S = \int d^4x \Big((\partial\phi)^2 + m^2\phi^2 + g^2(\partial^2\phi)^2 \Big) \tag{6.2}$$

Clearly in 4 spacetime dimensions, m is a relevant coupling and g is an irrelevant coupling.Now, mg would be an RG invariant. The two point function of ϕ in 4 dimensional Euclidean space is given by,

$$\langle 0|\phi(x_2)\phi(x_1)|0\rangle = \int d^4p \frac{e^{ipx}}{p^2 + m^2 + g^2p^4}, \quad x = x_2 - x_1 \tag{6.3}$$

The answer is a Bessel function which when expanded in the large r limit depends on r, m, g and the product $s = mg$. Since s is a dimensionless parameter, we can do a perturbative expansion in s, where up to leading order we get,

$$e^{-mr(1+\frac{s^2}{2}+\frac{7s^4}{8}+\cdots)}\Big(\sqrt{\frac{m}{2}}(\frac{\pi}{r})^{\frac{3}{2}} + O(s^2) + \cdots) + O(\frac{1}{r^{\frac{5}{2}}}) + \cdots\Big) \tag{6.4}$$

$$+\, e^{-\frac{r}{g}(1-\frac{s^2}{2}-\frac{5s^4}{8}+\cdots)}\Big(\sqrt{\frac{1}{2g}}(\frac{\pi}{r})^{\frac{3}{2}} + O(s^2) + \cdots) + O(\frac{1}{r^{\frac{5}{2}}}) + \cdots\Big) \tag{6.5}$$

Thus here we see that the dependence on g persists even at large r through the RG invariant mg. This is very similar to what we obtain for the GGE and GCC where irrelevant operators like μ and κ_4 seem to affect the two point correlation functions even at large by combining with relevant operators to form RG-invariants. This can be viewed as UV-IR mixing, as was also discussed in [1].

Chapter 7

Relation to Holography

There are two kinds of AdS/CFT dualities. Maldacena proposed a duality between a CFT(N=4, Super-Yang-Mills theory) and a string theory living on a 10 dimensional spacetime. The second kind was proposed by Polyakov and Klebanov and and gives a correspondence between certain CFTs and type Vasiliev theory, which is the theory of higher spin gravity. We discuss both the of the following concepts below and explain how both these dualities are of interest to us.

7.1 The Maldacena Type duality

The N=4 super Yang Mills Theory is a $3+1$ dimensional theory of Matrix/ Adjoint fields such as gauge fields $A^{\mu}_{ij}(\vec{x},t)$, spinor fields $\psi^{\alpha}_{ij}(\vec{x},t)$, or scalar fields ϕ^{a}_{ij}, where i and j take values from 1 to N. The Lagrangian has a coupling g_{YM}. At this stage we define the t'Hooft coupling as $\lambda = g^2_{\mathrm{YM}}N$. This theory is dual to a String theory on $\mathrm{AdS}_5 \times \mathrm{S}_5$. The defining parameters in the String theory are g_s, the string coupling, l_s, the string length, and R_{AdS}, the radius of the AdS geometry.The following relations connect the field theory with the string theory:

$$g^2_{\mathrm{YM}} = g_s \tag{7.1}$$

$$\lambda = \Big(\frac{R_{AdS}}{l_s}\Big)^4 \tag{7.2}$$

From the above two equations, it follows that, when I set $R_{AdS} = 1$,then

$$G_N = \frac{1}{N^2} \tag{7.3}$$

$$l_s = \lambda^{-\frac{1}{4}} \tag{7.4}$$

We are interested in the physics as we vary λ and N. It turns out we get four regimes.

The gravitational constant G_N for the AdS theory is proportional to $\frac{1}{N^2}$, and so as $N \to \infty$, $G_N \to 0$. Now we know that in gravity, G_N comes in the denominator of the action. For example, the Einstein Hilbert action looks like $S = \frac{1}{16\pi G_N c^{-4}} \int \sqrt{-g} R d^4x$.

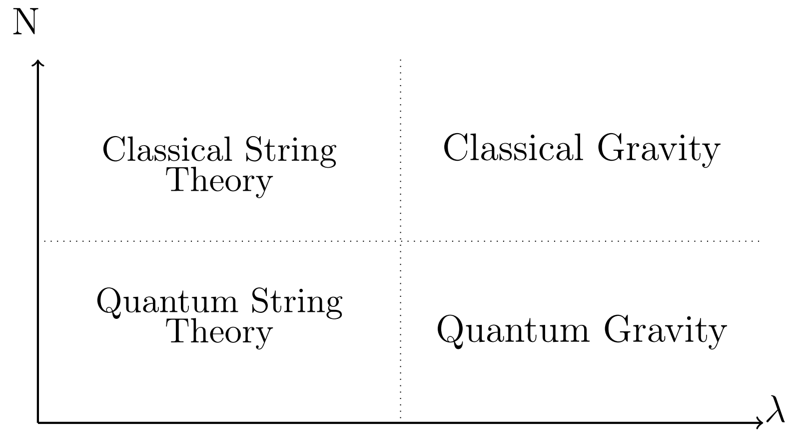


Figure 7.1: The regimes of the holographic dual of N=4 SYM

Thus while computing the path integral, when we exponentiate the action, the G_N appears like $\hbar$, in the quantum action. Thus, taking G_N to 0 is analogous to taking $\hbar$ to 0, which is the classical limit. We see that for a certain λ, as N decreases and G_N increases we traverse the quantum regime. As we make N larger and larger, making G_N smaller and smaller, we approach the classical regime.

Secondly, from 7.2 we see that the string length is comparable to the radius of the AdS in the small λ limit. At large values of λ when $l_s \ll R_{AdS}$, the string theory is not perceived and the geometry becomes like just AdS gravity.

Quasinormal decay

In the AdS/CFT correspondence, a thermal state in the CFT corresponds to a large static black hole in the AdS bulk. Perturbing the thermal state is similar to perturbing the black hole and the decay of an excitation in the AdS to a black hole is the return to thermal equilibrium in the CFT. Now, AdS space with a black hole can be viewed as a box, with boundaries at the AdS radius and the black hole horizon. Now say we excite a field in the spacetime and we wish to see how this excitation propagates. In the box picture, it has two reflecting surfaces at the two boundaries, thus we expect standing wave solutions, which can be expanded in terms of the normal modes characterized by discrete frequencies. An AdS with a black hole can be viewed as a box with a reflecting boundary condition at infinity and an absorbing boundary at the black hole horizon. In this case the solutions are given by quasinormal modes characterized by complex frequencies and are found to decay in time. Physically this means that any excitations will get absorbed by the black hole eventually.

In our case we study a free scalar field theory which is trivially a CFT. This would be similar to the $\lambda \to 0$ limit of N=4 SYM, which in the large N limit would correspond to Classical String theory. As of now, in our calculations we have dealt with a single scalar, but its possible to extend our study this case. We understand thermalization in the field theory end through the concept of subsystem thermalization as described in the Introduction. However, its interpretation in terms of quasinormal decay in the bulk is incomplete because it is not dual to Classical Gravity where such an interpretation is possible.

There are some quantities or observables that remain invariant from the string theory end to the gravity end. For example, the Log of the number of microstates in the weak λ limit matches exactly with the Bekenstein-Hawking entropy calculated at large λ. Thus, we hope to find a connection between the decay of correlation functions to thermal equilibrium in the free field theory and the quasinormal decay to an AdS black hole. A thermal state in 2D CFT is dual to a BTZ black hole in AdS_3. Given the differences that we find in odd and even dimensions in the field theory, an interesting question to study would be, what observables decay as a power law in time after exciting the bulk. Much of my understanding of the above concepts comes from studying [10], [11] and [12].

7.2 The Polyakov-Klebanov type duality

In this kind of duality, vector-valued (O(N)) conformal field theory in D dimensions is dual to a Vasiliev theory in AdS_{D+1}. Vasiliev theory is a theory of AdS gravity in which spin-2 gravitons are coupled with higher spins. Say, in $2+1$ dimensions we have a theory constructed out of vector-valued fields $\phi_i(\vec{x}, t)$, by which I mean they exhibit some internal symmetries which make them of the O(N) kind. They could have a simple Lagrangian such as $\mathcal{L} = (\partial_\mu \phi_i)^2 + g(\phi_i \phi_i)^2$. Now in $2+1$ dimensions, g is a relevant coupling. Thus, by arguments of Wilsonian RG flow, its effect increases at lower energies(IR). Thus, there are cases when this theory can become conformal. One is at $g = 0$, which is the UV CFT, and second, when $g = g*$, which is an RG fixed point (called the Wilson-Fischer fixed point) and is the IR CFT. In general, we can obtain CFTs which are RG fixed points of the "vector" theories which are dual to higher spin gravity in one higher spacetime dimension. When we talk of free field theory, we take the UV CFT, where the coupling is zero. In $D = 2$, this kind of duality is described by the Gaberdiel-Gopakumar duality [13]. Without going into too much detail, I would like to put forth that the CFT is labeled by another parameter. Let's call it $\hat{\lambda}$. It turns out that when $\hat{\lambda} = 0$, the theory consists of free fermions and and when $\hat{\lambda} = 1$, the theory consists of free bosons.

Now these CFTs live on the boundary of the holographic gravity theory. If these CFTs exhibit higher number of conserved charges, corresponding to higher number of conserved currents, then they couple to the higher spin gravity fields from the bulk. How do we see this? When we have a theory with conserved currents, the generating functional for the n-point functions of the conserved currents is given by, $Z[A_\mu] = \int \mathcal{D}\psi \exp\big\{(iS_o[\psi] + \int \bar{A}_\mu J^\mu\big\}$, where $\bar{A}_\mu$ can be thought of as a gauge field coupling to the conserved current. These functionals have the property, $z[A_\mu] = Z[A_\mu + \partial_\mu \lambda]$, which is easy to show given, $\partial_\mu J^\mu = 0$. If A^μ is a field in the bulk, then $\bar{A}_\mu$ is the value it takes on the boundary. Similarly, a conserved currents of the form $J^{\mu\nu}$, $J^{\mu\nu\rho}$, $J^{\mu\nu\rho\sigma}$ couple to higher spin gravity gauge fields like, $A_{\mu\nu}, A_{\mu\nu\rho}, A_{\mu\nu\rho\sigma}$. In

$2D$, these extra conserved currents in the field theory have corresponding conserved charges which satisfy the W_∞ algebra. As mentioned earlier, the presence of one higher spin conserved current demands the existence of infinitely many higher spin conserved currents. This is so because the algebra is not closed for a finite number of them. In case of free bosons, only the currents with even number of indices survive $(W_2, W_4, W_6, \cdots)$. The 2nd rank tensor $J^{\mu\nu}$ is actually $T^{\mu\nu}$. We see, in $2D$, that the OPE expansions of the energy momentum tensor with a W operator of rank n, depend on other W operators of other ranks. Naively,three point functions in the field theory correspond to three point vertices of higher spin gauge fields in the bulk. Thus, we see that the three point function of the stress energy tensor with other Ws would imply the coupling of spin-2 gauge field with higher spins.

Keeping the above in mind, we can pose similar questions about the relation of observables in the bulk and in the field theory. It was shown in [2] that a decay to a GGE in the field theory corresponds to a collapse to a higher spin black hole. In two dimensions the relaxation to GGE was exponential and the exponent exactly matched the imaginary part of the quasinormal frequency in AdS_3. As explored by both [2] and [1], the exponent was $\frac{2\pi}{\beta}(\Delta_k + \sum_n \tilde{\mu} Q_{n,k})$, where $\tilde{\mu}$ are like the chemical potentials associated with the higher conserved charges and Δ_k corresponds to the conformal dimension of the operator.

The conformal dimension of an operator charges with dimension, thus if these results were to hold true in higher dimensions we would expect the relaxation rate to change with dimension. However, quite surprisingly for all $d+1$ with d odd, we see the same decay rate $\frac{2\pi}{\beta}$. What is more surprising is that for even d, the decay is a power law and not an exponential. The power law however depends on the dimension and the relaxation in time is $\approx t^{-d+2}$. Clearly then, the holographic duality is not well understood. It is also true that Vasiliev theory has been studied in 2D and 3D. Thus in principle, we should be able to compare our $2+1$ dimesional results with calculations in AdS_4 Vasiliev theory.

Appendix A

Thermal and GGE

The GGE 2 point function is defined as

$$\begin{aligned}\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} &= \frac{1}{Z}Tr(e^{-\beta H-\mu W_4}\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)) \\ &= \frac{1}{Z}\sum_{\{N_k\}}\langle\{N_k\}|e^{-\beta H-\mu W_4}\phi(\vec{x_1},t_1)\phi(\vec{x_2},t_2)|\{N_k\}\rangle\end{aligned} \tag{A.1}$$

Using the partial Fourier transform

$$\phi(\vec{x},t) = \int e^{\iota\vec{k}\cdot\vec{x}}\phi(\vec{k},t)dk \tag{A.2}$$

where

$$\phi(\vec{k},t) = a(\vec{k})u(\vec{k},t) + a^\dagger(\vec{-k})u^*(\vec{-k},t) \tag{A.3}$$

$$\begin{aligned}\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} =& \frac{1}{Z}\sum_{\{N_k\}}\langle\{N_k\}|e^{-\beta H-\mu W_4}\int\int\frac{d^dk}{(2\pi)^d}\frac{d^dq}{(2\pi)^d}e^{\iota(\vec{k}\cdot\vec{x1}+\vec{q}\cdot\vec{x2})}\Big(a(\vec{k})u(\vec{k},t_1) \\ &+ a^\dagger(\vec{-k})u^*(\vec{-k},t_1))(a(\vec{q})u(\vec{q},t_2) + a^\dagger(\vec{-q})u^*(\vec{-q},t_2)\Big)|\{N_k\}\rangle\end{aligned} \tag{A.4}$$

Out of the resulting four terms only two terms give non-zero values. Since $H = \sum_k N_k|\vec{k}|$ and $W_4 = \sum_k N_k|\vec{k}|^3$

$$e^{-\beta H-\mu W_4}|\{N_k\}\rangle = e^{-\beta\sum_k N_k|\vec{k}|-\mu\sum_k N_k|\vec{k}|^3}|\{N_k\}\rangle \tag{A.5}$$

Thus we get

$$\begin{aligned}\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} =& \frac{1}{Z}\int\int\frac{d^dk}{(2\pi)^d}\frac{d^dq}{(2\pi)^d}e^{\iota(\vec{k}\cdot\vec{x1}+\vec{q}\cdot\vec{x2})}\sum_{\{N_k\}}e^{-\beta\sum_k N_k|\vec{k}|-\mu\sum_k N_k|\vec{k}|^3} \\ &\langle\{N_k\}|(a(\vec{k})u(\vec{k},t_1)a^\dagger(\vec{-q})u^*(\vec{-q},t_2) + a^\dagger(\vec{-k})u^*(\vec{-k},t_1)a(\vec{q})u(\vec{q},t_2))|\{N_k\}\rangle\end{aligned} \tag{A.6}$$

Using the commutation relation

$$[a(\vec{k}), a^\dagger(-\vec{q})] = (2\pi)^d \delta^d(\vec{k} + \vec{q}) \tag{A.7}$$

and the form of the number operator

$$a^\dagger(-\vec{q})a(\vec{k}) = N_k (2\pi)^d \delta^d(\vec{k} + \vec{q}) \tag{A.8}$$

Therefore

$$\langle \phi(\vec{x_1}, t_1,)\phi(\vec{x_2}, t_2)\rangle_{\beta,\mu} = \frac{1}{Z}\int\int \frac{d^dkd^dq}{(2\pi)^{2d}} e^{\iota(\vec{k}\cdot\vec{x1}+\vec{q}\cdot\vec{x2})} \sum_{\{N_k\}} e^{-\beta\sum_k N_k|\vec{k}|-\mu\sum_{\{N_k\}} N_k|\vec{k}|^3}(2\pi)^d\delta^d(\vec{k}+\vec{q})$$
$$\Big[\langle\{N_k\}|(N_k+1)|\{N_k\}\rangle u(\vec{k}, t_1)u^*(\vec{-q}, t_2) + \langle\{N_k\}|N_q|\{N_k\}\rangle u^*(\vec{-k}, t_1)u(\vec{q}, t_2)\Big] \tag{A.9}$$

Doing the q integral for the first term and k integral for the second and then writing it in terms of a single dummy variable:

$$\langle \phi(\vec{x_1}, t_1,)\phi(\vec{x_2}, t_2)\rangle_{\beta,\mu} = \frac{1}{Z}\int \frac{d^dk}{(2\pi)^d} \sum_{\{N_k\}} e^{-\beta\sum_k N_k|\vec{k}|-\mu\sum_{\{N_k\}} N_k|\vec{k}|^3}\Big[\langle\{N_k\}|(N_k+1)|\{N_k\}\rangle$$
$$u(\vec{k}, t_1)u^*(\vec{k}, t_2)e^{\iota\vec{k}\cdot(\vec{x1}-\vec{x2})} + \langle\{N_k\}|N_k|\{N_k\}\rangle u^*(\vec{k}, t_1)u(\vec{k}, t_2)e^{-\iota\vec{k}\cdot(\vec{x1}-\vec{x2})}\Big] \tag{A.10}$$

Since free scalar fields are bosons the number density is that of the Bose-Einstein distribution. Directly using $\langle N_k\rangle = \frac{1}{Z}\sum_{\{N_k\}} e^{-\beta\sum_k N_k|\vec{k}|-\mu\sum_{\{N_k\}} N_k|\vec{k}|^3}$,$\langle\{N_k\}|N_k|\{N_k\}\rangle = \frac{1}{e^{\beta|\vec{k}|+\mu|\vec{k}|^3}-1}$, we get

$$\langle \phi(\vec{x_1}, t_1,)\phi(\vec{x_2}, t_2)\rangle_{\beta,\mu} = \frac{1}{2}\int \frac{d^dk}{(2\pi)^d}\Big[\frac{e^{-\iota|\vec{k}|(t1-t2)}e^{\iota\vec{k}\cdot(\vec{x1}-\vec{x2})}}{|\vec{k}|(1-e^{-\beta|\vec{k}|+\mu|\vec{k}|^3})} + \frac{e^{\iota|\vec{k}|(t1-t2)}e^{-\iota\vec{k}\cdot(\vec{x1}-\vec{x2})}}{|\vec{k}|(e^{\beta|\vec{k}|+\mu|\vec{k}|^3}-1)}\Big] \tag{A.11}$$

Defining $\vec{x} = \vec{x1} - \vec{x2}$, $t = t1 - t2$ and $G_\pm = \frac{1}{|\vec{k}|(\pm e^{\pm(\beta k+\mu k^3)}\mp 1)}$

$$\langle \phi(\vec{x_1}, t_1)\phi(\vec{x_2}, t_2)\rangle_{\beta,\mu} = \frac{1}{2}\int \frac{d^dk}{(2\pi)^d}\Big[G_- e^{\iota\vec{k}\cdot\vec{x}}e^{-\iota|\vec{k}|t} + G_+ e^{-\iota\vec{k}\cdot\vec{x}}e^{\iota|\vec{k}|t}\Big] \tag{A.12}$$

Appendix B

Calculation Details of Ground state correlator

The two point function (at equal times) is calculated as

$$\begin{aligned}\langle 0_{in}|\phi(\vec{r},t)\phi(0,t)|0_{in}\rangle &= \int \frac{dk}{2\pi} u_{in}(\vec{k},t)u^*_{in}(\vec{k},t)e^{i\vec{k}\cdot\vec{r}} \\ &= \int \frac{d^3k}{(2\pi)^3}\left(\frac{2k^2+m^2-m^2Cos(2kt)}{4(k^2+m^2)^{\frac{1}{2}}\ k^2}\right)e^{i\vec{k}.\vec{r}}\end{aligned} \tag{B.1}$$

doing the angular integral[1], we get

$$= \int_{-\infty}^{\infty}\frac{dk}{16\pi^2}\left(\frac{2k^2+m^2-m^2\ Cos(2kt)}{(k^2+m^2)^{\frac{1}{2}}}\right)\left(\frac{e^{ikr}}{ikr}\right) \tag{B.2}$$

Lets look at the time independent part first

$$= \int_{-\infty}^{\infty}\frac{dk}{16\pi^2}\left(\frac{2k^2+m^2}{k(k^2+m^2)^{\frac{1}{2}}}\right)\left(\frac{e^{ikr}}{ir}\right) \tag{B.3}$$

Assuming $r > 0$ we close the contour always in the upper half plane so that integral along C_2 and along C_6 is 0. Integral along C_3 and C_5 is identical. Now integral along $C_1 + C_1' + 2\ C_3 = 0$ and integral along $C_1' = -\pi i \times Re(0) = \frac{-m}{16\pi r}$ from half residue theorem. The integral along C_3 after change variables, $k = im\ (1 + \ u/mr)$ then ($dk = \frac{i}{r}\ du$) is in the following form

$$-C_3 = \frac{e^{-mr}}{16\pi^2\ r^2}\int_0^{\infty} du\ e^{-u}\ \frac{\left(1+2\ \frac{u}{mr}\ \left(2+\frac{u}{mr}\right)\right)}{\left(1+\frac{u}{mr}\right)\sqrt{\frac{u}{mr}\left(2+\frac{u}{mr}\right)}} \tag{B.4}$$

[1]

$$\int \frac{d^3k}{(2\pi)^3}e^{i\vec{k}.\vec{r}}f(|k|) = \frac{1}{2\pi^2\ r}\int_0^{\infty} dk\ k\ f(k)\sin(kr) = \frac{1}{4\pi^2 r\ i}\left[\int_0^{\infty} e^{ikr}kf(k)dk + \int_{-\infty}^{0} e^{ikr}kf(-k)dk\right]$$

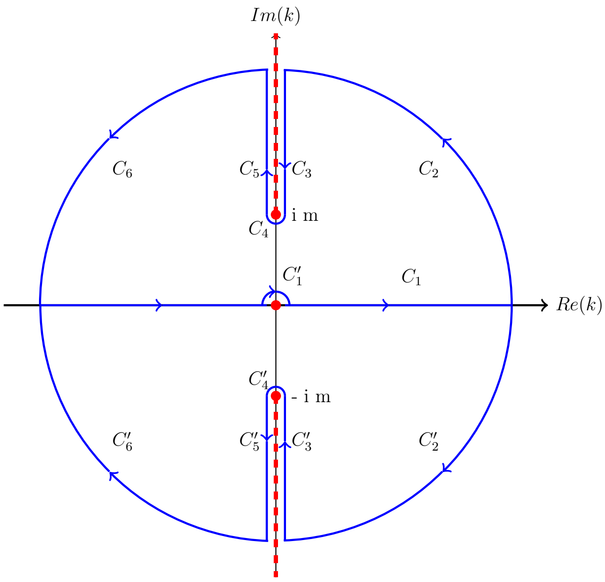


Figure B.1: Singularity Structure of Integrand in ground state Fourier integral

In the asymptotics of mr we expand above function around $\frac{u}{mr} = 0$

$$-C_3 = \frac{e^{-mr}}{16\pi^2\, r^2}\left[\int_0^\infty \frac{e^{-u}\sqrt{mr}}{\sqrt{2u}}\, du \; + \; \int_0^\infty \frac{e^{-u}\; 11\; \sqrt{u}}{4\sqrt{2}\; mr}\, du \; + \; \int_0^\infty \mathcal{O}\left((mr)^{\frac{-3}{2}}\right) e^{-u} u^{\frac{3}{2}} du\right] \tag{B.5}$$

$$-C_3 = \frac{e^{-mr}}{16\pi^2\, r^2}\left[\sqrt{\frac{mr}{2}}\;\Gamma\left(\frac{1}{2}\right) \; + \; \frac{11}{4\sqrt{2}\; mr}\;\Gamma\left(\frac{3}{2}\right) \; + \; \mathcal{O}\left(\frac{1}{(mr)^{\frac{3}{2}}}\right)\right] \tag{B.6}$$

In the asymptotic limit $mr \gg 1$ we only consider the first term

$$\int_{-\infty}^{\infty} \frac{dk}{16\pi^2}\left(\frac{2k^2+m^2}{k(k^2+m^2)^{\frac{1}{2}}}\right)\left(\frac{e^{ikr}}{ir}\right) = -\left(C_1' + 2C_3\right) = \frac{m}{16\;\pi\; r} + \left(\frac{m^2}{8\sqrt{2}\;\pi^{\frac{3}{2}}}\right)\; e^{-mr}\;(mr)^{-\frac{3}{2}} \tag{B.7}$$

now lets look at time dependent part, we follow the exact same procedure followed for the time independent part.

$$= \int_{-\infty}^{\infty} \frac{dk}{32\pi^2}\left(\frac{-m^2}{i\; r\; k\; (k^2+m^2)^{\frac{1}{2}}}\right) e^{ik(r+2t)} + \int_{-\infty}^{\infty} \frac{dk}{32\pi^2}\left(\frac{-m^2}{i\; r\; k\; (k^2+m^2)^{\frac{1}{2}}}\right) e^{ik(r-2t)} \tag{B.8}$$

We consider a general form of the above two integrals

$$\int_{-\infty}^{\infty} \frac{dk}{32\pi^2}\left(\frac{-m^2}{i\; r\; k\; (k^2+m^2)^{\frac{1}{2}}}\right) e^{ika}$$

We try to integrate it along branch cuts, we will have to choose the contour depending upon the Sign(a), when $a > 0$, $C_1 = (i\pi Re(0) - 2C_3)$ and when $a < 0$ $C_1 = -(i\pi Re(0) + 2C_3)$,

$i\pi \times Re(0) = -m/32\pi r$ and integral along C_3 is evaluated using the same techniques as before.

$$C_1 = \frac{-m\ Sign(a)}{32\pi\ r} + \frac{e^{-|a|m}}{16\pi^2|a|r}\left[\sqrt{\frac{|a|m}{2}}\ \Gamma\left(\frac{1}{2}\right) - \frac{5}{4\sqrt{2|a|m}}\ \Gamma\left(\frac{3}{2}\right) + \mathcal{O}\left(\frac{1}{(m|a|)^{\frac{3}{2}}}\right)\right] \tag{B.9}$$

Using above results, for $r > 2t$ and in the limit $m(r+2t) \gg 1$, $m(r-2t) \gg 1$

$$\int_{-\infty}^{\infty} \frac{dk}{16\pi^2}\left(\frac{-m^2\ Cos(2kt)}{(k^2+m^2)^{\frac{1}{2}}}\right)\left(\frac{e^{ikr}}{ikr}\right) \approx \frac{-m}{16\ \pi\ r} + \frac{m^2}{16\sqrt{2}\ \pi^{\frac{3}{2}}}\left(\frac{e^{-m(r+2t)}}{mr\ \sqrt{m(r+2t)}} + \frac{e^{-m(r-2t)}}{mr\ \sqrt{m(r-2t)}}\right) \tag{B.10}$$

for $r < 2t$ and in the limit $m(r+2t) \gg 1$, $m(2t-r) \gg 1$

$$\int_{-\infty}^{\infty} \frac{dk}{16\pi^2}\left(\frac{-m^2\ Cos(2kt)}{(k^2+m^2)^{\frac{1}{2}}}\right)\left(\frac{e^{ikr}}{ikr}\right) \approx \frac{m^2}{16\sqrt{2}\ \pi^{\frac{3}{2}}}\left(\frac{e^{-m(r+2t)}}{mr\ \sqrt{m(r+2t)}} + \frac{e^{-m(2t-r)}}{mr\ \sqrt{m(2t-r)}}\right) \tag{B.11}$$

Hence in (1+3) dimension 2 point function after critical quench in the asymptotic limit from Equation B.7, Equation B.10 and Equation B.11 following expression

$$\langle 0_{in}|\phi(r,t)\phi(0,t)|0_{in}\rangle = \begin{cases} \frac{m}{16\ \pi\ r} + \frac{m^2}{16\sqrt{2}\ \pi^{\frac{3}{2}}}\left(\frac{2e^{-mr}}{mr\sqrt{mr}} + \frac{e^{-m(r+2t)}}{mr\ \sqrt{m(r+2t)}} + \frac{e^{-m(2t-r)}}{mr\ \sqrt{m(2t-r)}}\right) + \cdots & r < 2t \\ \frac{m^2}{16\sqrt{2}\ \pi^{\frac{3}{2}}}\left(\frac{2e^{-mr}}{mr\sqrt{mr}} + \frac{e^{-m(r+2t)}}{mr\ \sqrt{m(r+2t)}} + \frac{e^{-m(r-2t)}}{mr\ \sqrt{m(r-2t)}}\right) + \cdots & r > 2t \end{cases} \tag{B.12}$$

We are interested in the long time behavior of the correlation function. Thus taking $t \to \infty$, in which case $r < 2t$.

$$= \frac{m}{16\pi r} + \frac{\sqrt{m}e^{-mr}}{8\sqrt{2}\pi^{\frac{3}{2}}r\sqrt{r}} + \cdots \tag{B.13}$$

Thus the decay rate as seen from the slowest spatial transient is $-mr$.
For the $\langle\partial_i\phi\partial_i\phi\rangle$ correlator, the derivatives pull an extra factor of k^2. This kills the double pole at the origin. Moreover we can do the integral directly in Mathematica to get the answer quoted in the main text.

Appendix C

Calculation Details GCC/CC correlator

We have

$$
\begin{aligned}
\langle f_4|\phi(\vec{0},t_1,)\phi(\vec{r},t_2)|f_4\rangle &= \int \frac{d^3k}{(2\pi)^3}\frac{e^{\iota\vec{k}\cdot\vec{r}}}{2|\vec{k}|}\Big(\coth\Big(2|\vec{k}|(\kappa_2+\kappa_4k^2)\Big)-\cos(2kt)\,\mathrm{csch}\Big(2|\vec{k}|(\kappa_2+\kappa_4k^2)\Big)\Big)\\
&= \int_0^\infty \frac{k^2dk}{(2\pi)^2}\frac{\sin(kr)}{k^2r}\Big(\coth\big(2k(\kappa_2+\kappa_4k^2)\big)-\cos(2kt)\,\mathrm{csch}\big(2k(\kappa_2+\kappa_4k^2)\big)\Big)\\
&= \frac{1}{8\pi^2\iota r}\int_\infty^\infty dke^{\iota kr}\Big(\coth\big(2k(\kappa_2+\kappa_4k^2)\big)-\cos(2kt)\,\mathrm{csch}\big(2k(\kappa_2+\kappa_4k^2)\big)\Big)\\
&= \frac{1}{8\pi^2\iota r}\int_\infty^\infty dk\Big(e^{\iota kr}\coth\big(2k(\kappa_2+\kappa_4k^2)\big)-\frac{1}{2}e^{\iota k(r+2t)}\,\mathrm{csch}\big(2k(\kappa_2+\kappa_4k^2)\big)\\
&-\frac{1}{2}e^{\iota k(r-2t)}\,\mathrm{csch}\big(2k(\kappa_2+\kappa_4k^2)\big)\Big)
\end{aligned}
\tag{C.1}
$$

For the CC case ($\kappa_4 = 0$), the Fourier Transform is evaluated directly in Mathematica. The harder case is gCC. For simplicity we break the integral as follows

$$
= \int_{-\infty}^{\infty}\frac{dk}{16\ \pi^2\ i\ r}\Bigg[\underbrace{2\ e^{ikr}\ coth(2k(\kappa_2+\kappa_4\ k^2))}_{T_1(k)}\underbrace{-e^{ik(r+2t)}\ cosech(2k(\kappa_2+\kappa_4\ k^2))}_{T_2(k)}\underbrace{-e^{ik(r-2t)}\ cosech(2k(\kappa_2+\kappa}_{T_3(k)}
\tag{C.2}
$$

The integrand has poles at the solutions of

$$2k\ \kappa_2 + 2\kappa_4\ k^3 = i\ n\ \pi \tag{C.3}$$

here we introduce a dimensionless parameter $\bar{\kappa_4} = \kappa_4/\kappa_2^3$, in the small expansion roots of the above equation are

$$
\begin{aligned}
k_1(n) &= \frac{i\pi}{2\kappa_2}\left(n+\frac{\pi^2n^3}{4}\bar{\kappa_4}+O(\bar{\kappa_4}^2)\right)\\
k_2(n) &= \frac{i}{\kappa_2}\left(\frac{1}{\sqrt{\bar{\kappa_4}}}-\frac{\pi n}{4}-\frac{3\pi^2n^2}{32}\sqrt{\bar{\kappa_4}}+O(\bar{\kappa_4})\right)\\
k_3(n) &= -\frac{i}{\kappa_2}\left(\frac{1}{\sqrt{\bar{\kappa_4}}}+\frac{\pi n}{4}-\frac{3\pi^2n^2}{32}\sqrt{\bar{\kappa_4}}+O(\bar{\kappa_4})\right)
\end{aligned}
\tag{C.4}
$$

The singularity structure for the integrand in Equation C.2 is given in Figure C.1 Consider

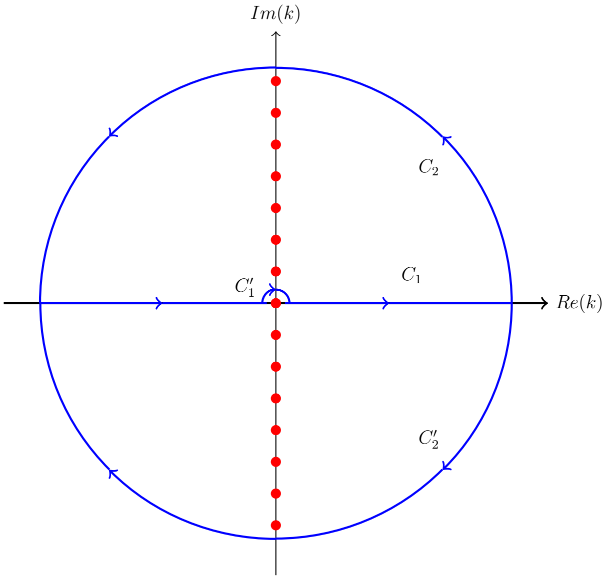


Figure C.1: Singularity Structure of Integrand in Equation C.2

general form of each term in the integral

$$= \int_{-\infty}^{\infty} \frac{dk}{16\ \pi^2\ i\ r} e^{ika} f(k) \tag{C.5}$$

for $a > 0$ we close the contour in the upper half of the complex plane and for $a < 0$ we close it in the lower half, then principle value of Equation C.5

$$\begin{aligned} C_1 + C_1' + C_2 &= [\text{Sum of all the poles sitting on positive Im(k) axis}] \qquad a > 0 \\ C_1 + C_1' + C_2' &= -\,[\text{Sum of all the poles sitting on non-positive Im(k) axis}] \qquad a < 0 \end{aligned} \tag{C.6}$$

Integral along $C_2 = C_2' = 0$ and integral along $C_1' = -\pi i \times Re(0)$ from half residue theorem.

$$\begin{aligned} C_1 &= \pi i \times Re(0) + [\text{Sum of all the poles sitting on positive Im(k) axis}] \qquad a > 0 \\ C_1 &= -\pi i \times Re(0) - [\text{Sum of all the poles sitting on negative Im(k) axis}] \qquad a < 0 \end{aligned} \tag{C.7}$$

We will try to obtain result perturbatively in $\bar{\kappa_4}$, for any value of n with $\bar{\kappa_4} \ll 1$, $k_2(n)$ and $k_3(n)$ will give us very fast decaying transients and $k_1(0) = 0$ will give us the slowest decaying transient, next immediate fast decaying transients will be given by poles at $k_1(1)$ and $k_1(-1)$. hence in the small $\bar{\kappa_4}$ limit we only consider contribution from poles at $k_1(0)$, $k_1(1)$ and $k_1(-1)$. Contribution of a pole at $k_1(n)$ for each term in Equation C.2 is

$$\begin{aligned} 2\pi i \times Re[T_1(k_1(n))] &= \frac{e^{ik_1(n)r}}{8\pi r \kappa_4\ (k_1(n) - k_2(n))\ (k_1(n) - k_3(n))} \\ 2\pi i \times Re[T_2(k_1(n))] &= \frac{(-1)^{n+1}\ e^{ik_1(n)(r+2t)}}{16\pi r \kappa_4\ (k_1(n) - k_2(n))\ (k_1(n) - k_3(n))} \end{aligned}$$

$$2\pi i \times Re[T_3(k_1(n))] = \frac{(-1)^{n+1}\ e^{ik_1(n)(r-2t)}}{16\pi r\kappa_4\ (k_1(n)-k_2(n))\ (k_1(n)-k_3(n))} \tag{C.8}$$

To evaluate Equation C.8 perturbatively in $\bar{\kappa_4}$ we consider expansion of the following quantity

$$\frac{1}{16\pi r\kappa_4(k_1(n)-k_2(n))(k_1(n)-k_3(n))} \approx \frac{1}{16\pi r\kappa_2}\left(1+\frac{3\pi^2\ n^2}{4}\ \bar{\kappa_4}+\cdots\right) \tag{C.9}$$

For $r > 2t$, we can see from Equation C.8 that contribution from pole at $k_1(0)$ from all three terms add up to 0 ($a > 0$ case for all three terms). We close the contour in the upper half of the complex plane and using Equation C.4,Equation C.7, Equation C.8 and Equation C.9, we obtain expression for leading order (contribution from $k_1(1)$) perturbatively in $\bar{\kappa_4}$,

$$\begin{aligned}\approx \frac{1}{16\pi r\ \kappa_2}\left(1+\frac{3\pi^2}{4}\bar{\kappa_4}+\cdots\right)\Bigg[2\ exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\ \bar{\kappa_4}+\cdots\right)r\right)+ \\ exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\ \bar{\kappa_4}+\cdots\right)(r+2t)\right)+exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\ \bar{\kappa_4}+\cdots\right)(r-2t)\right)\Bigg]\end{aligned} \tag{C.10}$$

now when $r < 2t$ we evaluate first 2 terms in Equation C.2 by using results for $a > 0$ case in Equation C.7 and for the third term we use results for $a < 0$ case. we get contributions from poles at $k_1(0)$ and $k_1(1)$ for first 2 terms and contribution from $k_1(0)$ and $k_1(-1)$ for the last term, again using Equation C.4,Equation C.7, Equation C.8 and Equation C.9, we obtain expression for leading order and first transient perturbatively in $\bar{\kappa_4}$.

$$\begin{aligned}\approx \frac{1}{16\pi r\ \kappa_2}+\frac{1}{16\pi r\ \kappa_2}\left(1+\frac{3\pi^2}{4}\bar{\kappa_4}+\cdots\right)\Bigg[2\ exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\bar{\kappa_4}+\cdots\right)r\right)+ \\ \exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\bar{\kappa_4}+\cdots\right)(r+2t)\right)-\exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\bar{\kappa_4}+\cdots\right)(2t-r\end{aligned} \tag{C.11}$$

Taking $t \to \infty$ limit

$$= \frac{1}{16\pi r\kappa_2}\left(1+\frac{3\pi^2}{4}\bar{\kappa_4}\right)[2\ exp\left(-\frac{\pi}{2\kappa_2}\left(1+\frac{\pi^2}{8\kappa_2}\bar{\kappa_4}+\cdots\right)r\right)] \tag{C.12}$$

When $\bar{\kappa_4} = 0$ this squeezed state calculation will be reduced to CC state calculation. Again the $\langle\partial_i\phi\partial_i\phi\rangle$ calculation is very similar and we do not include it in here.

Appendix D

Calculation Details of Thermal/GGE correlator

The GGE 2-point function is

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} = \frac{1}{Z}Tr(e^{-\beta H-\mu W_4}\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)) \tag{D.1}$$

$$= \frac{1}{Z}\sum_{\{N_k\}}\langle\{N_k\}|e^{-\beta H-\mu W_4}\phi(\vec{x_1},t_1)\phi(\vec{x_2},t_2)|\{N_k\}\rangle \tag{D.2}$$

Using the partial Fourier transform for the field and the 'mode expansion'

$$\phi(\vec{x},t) = \int e^{\iota\vec{k}\cdot\vec{x}}\phi(\vec{k},t)dk \tag{D.3}$$

where

$$\phi(\vec{k},t) = a(\vec{k})u(\vec{k},t) + a^\dagger(-\vec{k})u^*(-\vec{k},t) \tag{D.4}$$

We use the occupation number representation of the Hamiltonian to continue further.

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} = \frac{1}{Z}\sum_{\{N_k\}}\langle\{N_k\}|e^{-\beta H-\mu W_4}\int\int\frac{d^dk}{(2\pi)^d}\frac{d^dq}{(2\pi)^d}e^{\iota(\vec{k}\cdot\vec{x1}+\vec{q}\cdot\vec{x2})} \tag{D.5}$$

$$(a(\vec{k})u(\vec{k},t_1) + a^\dagger(-\vec{k})u^*(-\vec{k},t_1))(a(\vec{q})u(\vec{q},t_2) + a^\dagger(-\vec{q})u^*(-\vec{q},t_2))|\{N_k\}\rangle \tag{D.6}$$

Out of the resulting four terms only two terms give non-zero values. Since $H = \sum_k N_k|\vec{k}|$ and $W_4 = \sum_k N_k|\vec{k}|^3$

$$e^{-\beta H-\mu W_4}|\{N_k\}\rangle = e^{-\beta\sum_k N_k|\vec{k}|-\mu\sum_k N_k|\vec{k}|^3}|\{N_k\}\rangle \tag{D.7}$$

Thus we get

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu} = \frac{1}{Z}\int\int\frac{d^dk}{(2\pi)^d}\frac{d^dq}{(2\pi)^d}e^{\iota(\vec{k}\cdot\vec{x1}+\vec{q}\cdot\vec{x2})}\sum_{\{N_k\}}e^{-\beta\sum_k N_k|\vec{k}|-\mu\sum_k N_k|\vec{k}|^3} \tag{D.8}$$

$$\langle\{N_k\}|(a(\vec{k})u(\vec{k},t_1)a^\dagger(\vec{-q})u^*(\vec{-q},t_2)+a^\dagger(\vec{-k})u^*(\vec{-k},t_1)a(\vec{q})u(\vec{q},t_2))|\{N_k\}\rangle \tag{D.9}$$

Using the commutation relation

$$[a(\vec{k}),a^\dagger(-\vec{q})]=(2\pi)^d\delta^d(\vec{k}+\vec{q}) \tag{D.10}$$

and the form of the number operator

$$a^\dagger(-\vec{q})a(\vec{k})=N_k(2\pi)^d\delta^d(\vec{k}+\vec{q}) \tag{D.11}$$

Therefore

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu}=\frac{1}{Z}\int\int\frac{d^dk}{(2\pi)^d}\frac{d^dq}{(2\pi)^d}e^{\iota(\vec{k}\cdot\vec{x1}+\vec{q}\cdot\vec{x2})}\sum_{\{N_k\}}e^{-\beta\sum_k N_k|\vec{k}|-\mu\sum_{\{N_k\}}N_k|\vec{k}|^3}(2\pi)^d\delta^d(\vec{k}+\vec{q}) \tag{D.12}$$

$$\Big(\langle\{N_k\}|(N_k+1)|\{N_k\}\rangle u(\vec{k},t_1)u^*(\vec{-q},t_2)+\langle\{N_k\}|N_q|\{N_k\}\rangle u^*(\vec{-k},t_1)u(\vec{q},t_2)\Big) \tag{D.13}$$

Doing the q integral for the first term and k integral for the second and then writing it in terms of a single dummy variable:

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu}=\frac{1}{Z}\int\frac{d^dk}{(2\pi)^d}\sum_{\{N_k\}}e^{-\beta\sum_k N_k|\vec{k}|-\mu\sum_{\{N_k\}}N_k|\vec{k}|^3}\Big[\langle\{N_k\}|(N_k+1)|\{N_k\}\rangle \tag{D.14}$$

$$u(\vec{k},t_1)u^*(\vec{k},t_2)e^{\iota\vec{k}\cdot(\vec{x1}-\vec{x2})}+\langle\{N_k\}|N_k|\{N_k\}\rangle u^*(\vec{k},t_1)u(\vec{k},t_2)e^{-\iota\vec{k}\cdot(\vec{x1}-\vec{x2})}\Big] \tag{D.15}$$

Directly using $\langle N_k\rangle=\frac{1}{Z}\sum_{\{N_k\}}e^{-\beta\sum_k N_k|\vec{k}|-\mu\sum_{\{N_k\}}N_k|\vec{k}|^3}\langle\{N_k\}|N_k|\{N_k\}\rangle=\frac{1}{e^{\beta|\vec{k}|+\mu|\vec{k}|^3}-1}$, we get

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu}=\frac{1}{2}\int\frac{d^dk}{(2\pi)^d}\Big[\frac{e^{-\iota|\vec{k}|(t1-t2)}e^{\iota\vec{k}\cdot(\vec{x1}-\vec{x2})}}{|\vec{k}|(1-e^{-\beta|\vec{k}|+\mu|\vec{k}|^3})}+\frac{e^{\iota|\vec{k}|(t1-t2)}e^{-\iota\vec{k}\cdot(\vec{x1}-\vec{x2})}}{|\vec{k}|(e^{\beta|\vec{k}|+\mu|\vec{k}|^3}-1)}\Big] \tag{D.16}$$

Defining $\vec{x}=\vec{x1}-\vec{x2}$, $t=t1-t2$ and $G_\pm=\frac{1}{|\vec{k}|(\pm e^{\pm(\beta|k|+\mu|k|^3)}\mp1)}$

$$\langle\phi(\vec{x_1},t_1,)\phi(\vec{x_2},t_2)\rangle_{\beta,\mu}=\frac{1}{2}\int\frac{d^dk}{(2\pi)^d}\Big[G_-e^{\iota\vec{k}\cdot\vec{x}}e^{-\iota|\vec{k}|t}+G_+e^{-\iota\vec{k}\cdot\vec{x}}e^{\iota|\vec{k}|t}\Big] \tag{D.17}$$

For $d=3$, doing the angular integral

$$\int\frac{d^3k}{(2\pi)^3}e^{\pm\iota kr\cos\theta}=\int_0^\infty\frac{k^2dk}{(2\pi)^2}\frac{2\sin(kr)}{kr} \tag{D.18}$$

$$=\int_{-\infty}^\infty\frac{k^2dk}{4\pi^2}\frac{\sin(kr)}{kr} \tag{D.19}$$

$$=\frac{1}{8\pi^2\iota r}\int_{-\infty}^\infty kdk(e^{\iota kr}-e^{-\iota kr}) \tag{D.20}$$

$$= \frac{1}{4\pi^2 \iota r} \int_{-\infty}^{\infty} k dk e^{\iota k r} \tag{D.21}$$

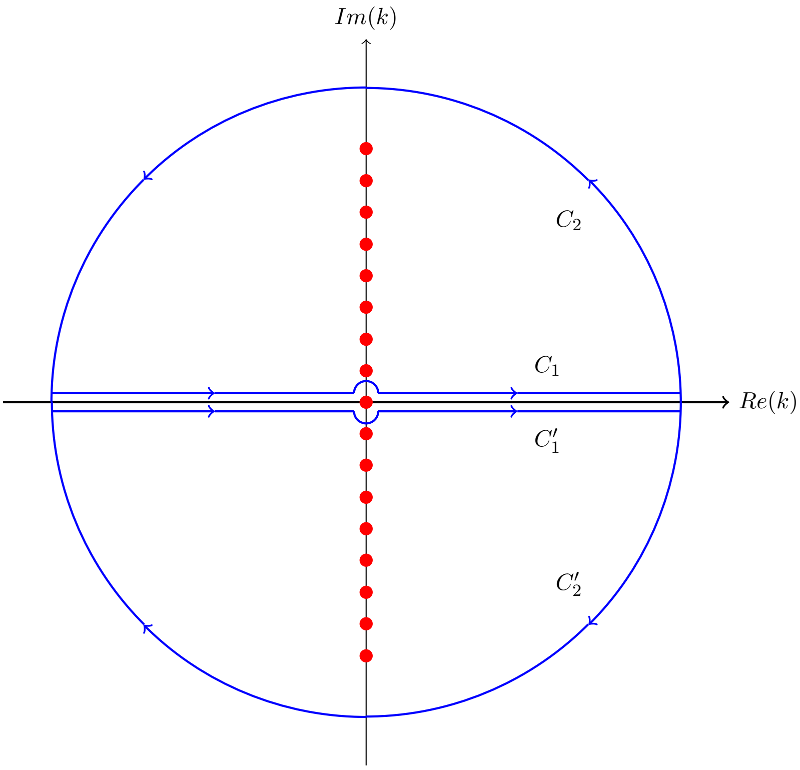
gives

$$\langle \phi(\vec{x_1}, t_1,)\phi(\vec{x_2}, t_2) \rangle_{\beta,\mu} = \int_{-\infty}^{+\infty} \frac{dk}{8\pi^2 \iota x} \Big[\frac{e^{\iota k(x+t)}}{e^{\beta k + \mu k^3} - 1} + \frac{e^{+\iota k(x-t)}}{-e^{-\beta k - \mu k^3} + 1} \Big] \tag{D.22}$$

which is what we reported in the main report. For the **thermal** case ($\mu = 0$), the 'Fourier transform' can be done directly in Mathematica, giving

$$\langle \phi(\vec{x_1}, t_1,)\phi(\vec{x_2}, t_2) \rangle_{\beta} = \frac{\coth\left(\frac{\pi(r+t)}{\beta}\right)}{8\pi\beta r} + \frac{\coth\left(\frac{\pi(r-t)}{\beta}\right)}{8\pi\beta r} \tag{D.23}$$

while for the GGE case we have to perform a careful contour integral. The singularity structure for the above integral is shown in the figure below



Singularity Structure of Integrand of (30) in 1+3 dimensions

The poles are at the solution of the equation

$$\frac{\beta}{2} k + \frac{\mu}{2} k^3 = i \ n \ \pi \tag{D.24}$$

This is the exact same singularity structure for squeezed states with $\beta = 4\kappa_2$ and $\mu = 4\kappa_4$ identification. So following the same procedure the smallest singularity is at $k = 0$, when $r > t$ for both the terms in the integrand we choose a contour which closes in the upper half of the complex plane. Both of them have the same contribution from the pole at $k = 0$

which is $\frac{1}{4\beta\pi r}$ when integrated along C_1' and along C_1 both terms get 0 contribution from $k = 0$ poles hence the average along C_1' and C_1 is $1/(4\beta\pi r)$.

In the case when $t > r$ for the 2nd term we have to choose C_2' which gives a -ve sign for the contribution from the second term and hence for $r > t$ contribution from pole at $k = 0$ is 0.

Defining $\bar{\mu} = \frac{\mu}{\beta^3}$. Lets look at the contribution from the next smallest poles which is $n = \pm 1$ we can write $\beta\ k + \mu\ k^3 = \mu(k - k_1)(k - k_2)(k - k_3) + I\ 2n\pi$ where k_1, k_2 and k_3 are same as before with the $\beta = 4\kappa_2$ and $\mu = 4\kappa_4$ identification.

Now when $t > r$ for the first term $k_1(1)$ is the closest pole and for the second term $k_1(-1)$ is the closest pole hence we have

$$\approx \frac{1}{4\pi r\beta}\left(1 + 12\pi^2\bar{\mu} + \cdots\right)\left[\exp\left(-\frac{2\pi}{\beta}\left(1 + 4\pi^2\bar{\mu} + \cdots\right)(t + r)\right) - \exp\left(-\frac{2\pi}{\beta}\left(1 + 4\pi^2\bar{\mu} + \cdots\right)(t - r)\right)\right] \tag{D.25}$$

when $t < r$ for both terms next smallest pole is at $k_1(1)$ and in that case we have

$$\approx \frac{1}{4\pi r\beta} + \frac{1}{4\pi r\beta}\left(1 + 12\pi^2\bar{\mu} + \cdots\right)\left[\exp\left(-\frac{2\pi}{\beta}\left(1 + 4\pi^2\bar{\mu} + \cdots\right)(t + r)\right) + \exp\left(-\frac{2\pi}{\beta}\left(1 + 4\pi^2\bar{\mu} + \cdots\right)(r - t)\right)\right] \tag{D.26}$$

The $\langle \partial_i\phi\partial_i\phi\rangle$ correlator is not very different. All it does is that the derivatives pull an extra factor of k^2. This kills the pole at the origin. and the above calculation is almost same so we do not repeat it here.

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