Game Theoretic Models on Product Differentiation

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by
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This is to certify that this thesis entitled "Game Theoretic Models on Product Differentiation" submitted towards the partial fulfillment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research Pune, represents work carried out by Anup Mahesh Savale under the supervision of Dr. Sachin Jayaswal.

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Dedicated to my parents

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Abstract

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by Anup Mahesh Savale

Positioning and pricing of products are two very crucial decisions that a company needs to make in a competitive market. These decisions are in turn affected by a variety of factors, which include the intensity of competition; consumer demand; consumer behavior; and type of competing firms (incumbent or entrant; national brand or store brand). We, therefore, study product positioning/differentiation and pricing decisions of competing firms under three different settings described by different combinations of these factors. We use extensions of Hotelling's framework for horizontal product differentiation to model the three different settings, which are solved as multi-stage games using the concept of Sub-game Perfect Nash Equilibrium (SPNE). The first setting studies a duopoly market consisting of one national brand manufacturer and a retailer. The national brand manufacturer sells its product to the consumers through the retailer. The retailer also sells its own store brand, which is horizontally differentiated from the national brand. This study presents the following important insights: (i) firms differentiate less with the increase in retailer's margin on national brand;(ii) both the firms stand to loose while consumers benefit from any increases in the retailer's margin on the national brand. The second setting models the product differentiation for an entrant and incumbent firm in the presence of variety seeking consumers. We find that entrant differentiates less from the incumbent firm for markets with lower fractions of variety seekers, but differentiates more if higher fraction of variety seeking consumers is present in the market. Also, it is interesting to note that profits for the two firms initially decrease and then increase with increasing variety seeking fraction. Finally, the third setting models the competition between two symmetric firms based on delivery time in addition to product differentiation and pricing. This makes the model more interesting to study although more challenging to solve. The model turns out to be mathematically intractable. This is an ongoing research and we present certain directions for future research.

Contents

Chapter 1

Introduction

Game theory is a formal theory of interactive decision making involving two or more decision makers (players). In the corporate world, firms or companies are the players that need to take decisions, the outcome of which depends not only on their decisions but also on the decisions taken by their competitors. One such important decision that a company or a firm needs to take is how to position its product in a market or how to differentiate its product from the other competing firms in the market. Product differentiation can be at two levels:

- Horizontal Differentiation
- Vertical Differentiation

Two products are said to be differentiated horizontally if they are identical and if all the consumers cannot agree on which of them is the preferred product.For example, different flavours of ice-cream are horizontally differentiated since not all consumers may prefer the same flavour. Similarly, coffee with different caffeine content are horizontally differentiated. On the other hand, two products are said to be differentiated vertically if those products differ in some characteristic based on which all consumers agree that one is better than the other. For example, hygiene of food products, duration of flights etc.

Horizontal product differentiation has caught the attention of many researchers, mainly after the seminal contribution of Hotelling (1929). In the classical Hotelling's model of product differentiation, there are two firms and consumers, which are uniformly distributed on a main street. Consumers make choices based on the price of the product plus the linear transportation cost, which depends on their distances

from the firm. In this model, firms first simultaneously position themselves, and then simultaneously set the prices of their product so as to maximize their individual profits. Hotelling's model predicts that the two firms approach each other as close as possible and share the market equally. D'Aspremont et al. (1979) criticized this result by showing that, in the model, neither the minimum differentiation strategy nor any other location choices are sub-game perfect since they fail to imply equilibrium in prices for each sub-game. To address this problem, they modified the transportation cost structure from linear to quadratic. This established the Principle of Maximum Differentiation where firms maximally differentiate from each other.
The classic Hotelling's model and its correction stated above generated a lot of interest in the research community. Subsequent research works made numerous extensions of the basic Hotelling's model to make it more applicable to the real world scenarios. These are mentioned below:

Consumer Preferences The standard Hotelling's model assumes consumer preferences are uniformly distributed, which is often not true of real markets. For example, consumers in a city are not always uniformly distributed but are sometimes clustered around some specific locations. Cintio (2007) and Anderson and Neven(1986) extend the Hotelling's model for non-uniform consumer preferences, viz. clustered or triangular distributions, respectively. These two papers show that how preference of firms' spatial localization change with the change in distribution of consumers. For example, Anderson and Neven (1986) shows that there are distributions of consumers for which firms will locate at the extremes of the market, and as the distribution becomes more concentrated inwards, the firms will also tend to move inwards.

Cost structures The second aspect on which researchers extend Hotelling's standard model is cost structure, which includes both transportation and production costs (D'Aspremont et al.,1979; Stahl, 1982). D'Aspremont et al.(1979) replace the linear transportation cost structure with a quadratic structure in the Hotelling's framework. In addition to this, Li (2012) highlights the effects of exponential transportation cost structure in product differentiation. Also, Hamoudi (2002, 2005 and 2007) extensively uses concave transportation cost in the Hotelling's framework, and compares this with other convex transportation costs. Moreover, Egli (2007) and Sajeesh and Raju (2010) investigate the effects of combined linear and quadratic transport cost structure on Hotelling's predicted equilibrium. Recently, Lai and Tabuchi (2011) have used Webers transport cost minimization model (Weber, 1909) in the Hotelling's framework to see the correlation between the two and their subsequent effects on firms' equilibrium and optimal locations.

Number of firms The standard Hotelling model assumes a duopoly market. Dominique (1982) and Economides (1993) introduced oligopolistic competition between the firms located on a Hotelling's main street. Economides (1993) shows that there exists a non-cooperative equilibrium for prices when multiple firms are located symmetrically. Further, Brenner (2001) states that for more than three players, neither the D'Aspremont's principle of maximum differentiation nor the Hotelling's principle of minimum differentiation exists for a quadratic cost structure. On the contrary, the firms are located in the interior of the product space and price structures are convex in shape.

Demand functions Hotelling's model assumes that each consumer buys one unit of the product from one of the firms, thus the total demand is inelastic. Puu (2002) extends the Hotelling's model to an elastic demand function. He assumes a linear cost structure for transportation in the model and also a linear demand varying with price over a fixed interval. With this, he derives a stable solution to the location game and hence improvises the basic model.
In this thesis, we look at three models of product differentiation, which are extensions of the Hotelling's model. In chapter 1, we study an asymmetric duopoly market consisting of one national brand manufacturer and a retailer. The national brand manufacturer sells its product to the consumers through the retailer. The retailer also sells its own store brand, which is horizontally differentiated from the national brand. We model this situation using an extension of Hotelling's framework to study the product positioning and pricing decisions of two firms that are at different echelons of a supply chain.
In chapter 2, we model a market consisting of an incumbent firm. Using an extension of the Hotelling's framework, we study the product positioning and pricing strategy of a new entrant in the presence of variety seeking consumers in the market.
In chapter 3, we describe an extension of Hotelling's framework to study competition

between two firms based on three factors: positioning, pricing and delivery time. This is an ongoing research, and we present certain directions for future research.

The different models of product differentiation presented in three chapters of this thesis are studied as multi-stage games, which are solved using the concept of Subgame Perfect Nash Equilibrium (SPNE).

Chapter 2

Product Differentiation: National Brand vs. Store Brand

2.1 Introduction

National brands are the brands owned by a producer or a distributor, and have a nationwide presence. Examples include Maggie by Nestle, and Bingo by ITC. Store brands or private labels, on the other hand, are brands created by retailers, which are sold exclusively at their own retail outlets. For example, Feasters by Aditya Birla group, and Tasty treats by Future group. Store brands have traditionally been considered to be of inferior quality, cheap and no-name alternatives for national brands. There have been many studies explaining the superiority of national brands over the store brands (Blattberg and Wisniewski, 1989; Allenby and Rossi, 1991; Hardie, Johnson, and Fader, 1993). Blatberg and Wisniewski (1989) suggest that any price deal done by a national brand affects the store brands but the reverse is not true. This essentially suggests a clear advantage of a national brand over a store brand. Further, Allenby and Rossi (1991) report that for a given price reduction, a national brand attracts more customers than a store brand. Moreover, Bronnberg (1996) states that effectiveness of a promotional effort depends on the gap between the qualities of products of two competing firms. National brands, perceived to be of better quality, are thus more effective in their promotional efforts.

There is, however, a gradual shift in the way store brands are perceived vis-a-vis national brands, as corroborated by some of the recent research works. Hoch (1996), for example, argues that quality of private labels is a key to their success. In fact, Sethuraman (1992) identifies twelve deciding factors that can make private labels a huge success. Several other research works highlight competition between national brands and store brands on different dimensions (Cotterill et al., 2000; Hoch, 1996; Sayman et al., 2002; Verhoef et al., 2002; Bhardwaj et al., 2010). Coterill et al. (2000) highlight pricing as one of the important dimensions of competition between national and store brands. In addition to pricing, positioning decision of the store brand is equally important. Sayman (2002) focuses on the competition between a store brand and two national brands by considering the positioning decision of the store brand. Through his empirical studies, he further highlights that along with positioning of store brands, its comparable quality with the national brand adds to the competition as well as to the profits of the store brand. However, the extant literature has not considered horizontal product differentiation to model competition between store and national brands. The objective of our study in this chapter is to fill this gap.

2.2 Model

We study a market consisting of heterogeneous consumers whose preferences are uniformly distributed over a straight line. In this market, there are two players: a national brand manufacturer, denoted by A, and a retailer, denoted by B. National brand is the first to enter the market and the retailer follows it with its own store brand. Both the national and the store brands are sold to the consumers through the same retailer. The sequence of decisions in this model is as follows:

1. Firm A positions its product.
2. Firm B positions its product.
3. Both A and B simultaneously take their pricing decisions. A decides on wholesale price of the national brand and B decides on retail price of store brand.
4. Each consumer purchases the product that maximizes her individual utility.

Assumptions

1. Consumers preferences are distributed uniformly over the unit interval [-0.5, 0.5].

2. There is no constraint on the firms to position within the interval of consumers' ideal points. Thus, firms can even position themselves outside the range of [-0.5, 0.5]. This is consistent with Lilien et al. (1992), Tabuchi and Thisse (1995) and Tyagi (2000).

3. Every consumer buys only one unit of the product from either firm which maximizes her individual utility.

4. Like Lilien et al.(1992) and Tyagi (2000), the net utility a consumer gets by buying a product positioned at a distance d away from its ideal point and priced at p_i is $R - p_i - td^2$, where $t > 0$ and R is the consumer reservation price. R is assumed to be a constant and sufficiently large so that all consumers buy one of the two products. Moreover, from preposition 2.4.4, $R > 325/12$, so that all consumers have a positive utility after purchasing a product.

5. The marginal cost of production for both A and B is assumed to be identical and constant, and is taken to be zero without the loss of generality.

6. The price at which the retailer sells the national brand is $k * w$, where k is the markup percentage and w is the retailer's cost. $p_a = kw \, \forall \, k > 1$. Sequential entry Hotelling's model is a special case corresponding to $k = 1$.

2.2.1 Notations

A = National brand firm
B = Store brand firm
k = Markup percentage for the retailer on the national brand
d = Distance between consumer's position and product
a = Location of firm A
b = Location of firm B
R = Reservation price of a consumer
$\tilde{x}$ = Location of marginal consumer
w = Wholesale price of national brand
p_i = Price of the product i, where $i \in \{A, B\}$
q_i = Sales of firm i, where $i \in \{A, B\}$
Π_i = Profits of firm i, where $i \in \{A, B\}$

U_{total} = Consumer's total utility
Br_i= Best response function of firm i, where $i \in \{A, B\}$

2.3 Analysis

In the model described in section 2.2, either A can be to the left of B or vice-versa. As either of the assumption yields the same results, we take A's location to the left of B without the loss of generality. This model is a multi-stage game and thus, we use the concept of sub-game perfect equilibrium to analyze the decisions of firms and consumers by the method of backward induction.

2.3.1 Level 3: Marginal consumer's position

The marginal consumer is the one who is indifferent between the two firms since she receives the same utility from either of the firms. Let $\tilde{x}$ be the location of the marginal consumer.

$$R - kw - t(\tilde{x} - a)^2 = R - p_b - t(b - \tilde{x})^2 \tag{2.1}$$

$$\tilde{x} = \frac{(-a + b)(a + b)t - kw + p_b}{2(-a + b)t} \tag{2.2}$$

where $\tilde{x} \in [-0.5, 0.5]$. Now, all consumers to the left of the marginal consumer buy product A and all the consumers to the right of it buy product B.
Now sales of firm A is

$$q_a = (1/2) + \frac{(-a + b)(a + b)t - kw + p_b}{2(-a + b)t} \tag{2.3}$$

And sales of firm B is

$$q_b = (1/2) - \frac{(-a + b)(a + b)t - kw + p_b}{2(-a + b)t} \tag{2.4}$$

Also the profits of the firms are as

$$\Pi_a = w\left(1/2 + \frac{(-a + b)(a + b)t - kw + p_b}{2(-a + b)t}\right) \tag{2.5}$$

$$\Pi_b = p_b\left(1/2 - \frac{(-a+b)(a+b)t - kw + p_b}{2(-a+b)t}\right)$$
$$+(k-1)w\left(1/2 + \frac{(-a+b)(a+b)t - kw + p_b}{2(-a+b)t}\right) \tag{2.6}$$

2.3.2 Level 2: Pricing decisions

Each firm chooses a price that maximizes its own profit. Firm A decides the wholesale price of the store brand, while B decides the retail price of the store brand. But, as each firm's profit depends on the other firm's pricing decision, we find the best response (in terms of price) of a firm, given the price chosen by the other firm. Best responses are given by

$$Br_A(p_b) = \frac{(a-b)t + (b^2 - a^2)t + 2p_b}{-1+2k} \tag{2.7}$$

$$Br_B(w) = \frac{(a-b)t + (a^2 - b^2)t + 2kw}{-1+2k} \tag{2.8}$$

For equilibrium prices, we simultaneously solve the two best response equations and we get,

$$w = \frac{(b-a)(3+a+b)t}{1+2k} \tag{2.9}$$

$$p_b = \frac{(b-a)(4k-1-a-b)t}{1+2k} \tag{2.10}$$

Substituting these in the expressions for each firms sales and profit, we have

$$q_a = \frac{(3+a+b)k}{2+4k} \tag{2.11}$$

$$q_b = \frac{2-k(a+b-1)}{2+4k} \tag{2.12}$$

$$\Pi_a = \frac{(b-a)(3+a+b)^2 kt}{2(1+2k)^2} \tag{2.13}$$

$$\Pi_b = \frac{1}{2(1+2k)^2}\left[(b-a)\left\{-2-2k+13k^2+a^2k^2+b^2k^2\right.\right.$$
$$\left.\left.+2b(-1-3k+k^2)+2a(-1-3k+(1+b)k^2)\right\}t\right] \tag{2.14}$$

2.3.3 Level 1: Positioning decisions

Each firm positions its respective product so as to maximize its net profit. B's best position is obtained using $\partial\Pi_b/\partial b = 0$ and we have

$$b = \frac{1}{6k^2}\Big(4 + 12k - 4k^2 - 2ak^2 \pm \sqrt{(-4 - 12k + 4k^2 + 2ak^2)^2 - 12k^2(-2 - 2k + 13k^2 - a^2k^2)}\Big) \quad (2.15)$$

Now we take one value of b at a time and substitute it into the equation no. (2.13). With this we have two values of Π_a

$$\Pi_a = \frac{1}{54k^5(1+2k)^2}\Big(2 + 6k - 2(1+2a)k^2 \pm \sqrt{4 + 24k + (34-4a)k^2 - 6(3+2a)k^3 + (-35 + 4a + 4a^2)k^4}\Big) (2 + 6k + (7+2a)k^2 \pm \sqrt{(4 + 24k + (34-4a)k^2 - 6(3+2a)k^3 + (-35 + 4a + 4a^2)k^4)^2}t \quad (2.16)$$

Now, we take these two values of Π_a and solve for $\partial\Pi_a/\partial a = 0$
And we get the following solution which maximizes the profit function of firm A and satisfies all the conditions specified for this model.

$$a = \frac{-(5+7k)}{2(1+k)} \quad (2.17)$$

And $\forall\, k > 1$, we have

$$\partial^2\Pi_a/\partial a^2 = \frac{1}{27k^5(1+2k)^2}$$

$$\left(2k^2 + \frac{-4k^2 - 12k^3 + (4+8a)k^4}{2\sqrt{4} + 24k + (34-4a)k^2 - 6(3+2a)k^3 + (-35 + 4a + 4a^2)k^4}\right)^2 \Big(2 + 6k - 2(1+2a)k^2 + \sqrt{4 + 24k + (34-4a)k^2 - 6(3+2a)k^3 + (-35 + 4a + 4a^2)k^4}\Big) + \frac{1}{27k^5(1+2k)^2}$$

$$2\left(-4k^2+\frac{-4k^2-12k^3+(4+8a)k^4}{2\sqrt{4+24k+(34-4a)k^2-6(3+2a)k^3+(-35+4a+4a^2)k^4}}\right)$$
$$\left(2k^2\frac{-4k^2-12k^3+(4+8a)k^4}{2\sqrt{4+24k+(34-4a)k^2-6(3+2a)k^3+(-35+4a+4a^2)k^4}}\right)$$
$$\left(2+6k+(7+2a)k^2+\sqrt{4+24k+(34-4a)k^2-6(3+2a)k^3+(-35+4a+4a^2)k^4}\right)$$
$$+\frac{1}{27k^5(1+2k)^2}$$
$$\left(-\frac{(-4k^2-12k^3+(4+8a)k^4)^2}{4(4+24k+(34-4a)k^2-6(3+2a)k^3+(-35+4a+4a^2)k^4)^{(3/2)}}\right.$$
$$\left.+\frac{4k^4}{\sqrt{4+24k+(34-4a)k^2-6(3+2a)k^3+(-35+4a+4a^2)k^4}}\right)(2+6k-2$$
$$\left.1+2a)k^2)+\sqrt{(4+24k+(34-4a)k^2-6(3+2a)k^3+(-35+4a+4a^2)k^4)}\right)$$
$$\left(2+6k+(7+2a)k^2+\sqrt{4+24k+(34-4a)k^2-6(3+2a)k^3+(-35+4a+4a^2)k^4}\right)$$
$$+\frac{1}{54k^5(1+2k)^2}\left(-\frac{(-4k^2-12k^3+(4+8a)k^4)^2}{4(4+24k+(34-4a)k^2-6(3+2a)k^3+(-35+4a+4a^2)k^4)^{(3/2)}}\right.$$
$$\left.\frac{4k^4}{\sqrt{4+24k+(34-4a)k^2-6(3+2a)k^3+(-35+4a+4a^2)k^4}}\right)$$
$$\left(2+6k+(7+2a)k^2\sqrt{4+24k+(34-4a)k^2-6(3+2a)k^3+(-35+4a+4a^2)k^4}\right)<0 \tag{2.18}$$

Thus, at equilibrium, the position of B is,

$$\begin{aligned} b &= \frac{1}{6k^2}\left(4+12k-4k^2-2ak^2+\right.\\ &\quad\left.\sqrt{(-4-12k+4k^2+2ak^2)^2-12k^2(-2-2k+13k^2-a^2k^2)}\right) \end{aligned} \tag{2.19}$$

Substituting value of "a" from equation (2.17) into equation (2.19), we get

$$b=\frac{8+32k+29k^2+3k^3}{6k^2(1+k)} \tag{2.20}$$

Notations	Expression
q_a	$\frac{(3+a+b)k}{2+4k}$
q_b	$\frac{2-k(a+b-1)}{2+4k}$
p_a	$\frac{8k(1+2k)^2(2+4k+3k^2)t}{9k^4(1+k)^2}$
p_b	$\frac{4(2+4k+3k^2)(-2-8k-5k^2+9k^3+6k^4)t}{9k^4(1+k)^2}$
Π_a	$\frac{16(1+2k)^3(2+4k+3k^2)t}{27k^5(1+k)^3}$
Π_b	$\frac{4(1+2k)(-1-2k+3k^2)(2+4k+3k^2)^2t}{27k^4(1+k)^3}$
a	$\frac{-(5+7k)}{2(1+k)}$
b	$\frac{(8+32+29k^2+3k^3)k}{6k^2(1+k)}$

2.4 Results

2.4.1 Preposition 1

With increasing retailer's margin on national brand:

- National brand moves away from the most attractive location.
- Store brand moves towards the most attractive location.
- Product differentiation decreases.

Proof

$$a = \frac{-(5+7k)}{2(1+k)} \tag{2.21}$$

$$\partial a/\partial k = -\frac{1}{(1+k)^2} \tag{2.22}$$

As a is negative for $k = 1$ and $\partial a/\partial k < 0 \,\forall\, k > 1$. Thus, firm A moves away from the most attractive location in the market i.e. 0. Now,

$$b = \frac{8+32k+29k^2+3k^3}{6k^2(1+k)} \tag{2.23}$$

Now, b is positive for $k = 1$ and $\partial b/\partial k < 0 \,\forall\, k > 1$. This shows that, b decreases with increase in k. Also,

$$\lim_{k\to\infty}\left(\frac{8+32k+29k^2+3k^3}{6k^2(1+k)}\right) = \frac{1}{2} \tag{2.24}$$

Thus, firm B moves towards the most attractive location with increase in the value of k.

$$(b-a) = \frac{2(2+8k+11k^2+6k^3)}{3k^2(1+k)} \tag{2.25}$$

$$\partial(b-a)/\partial k = -\left(\frac{2(4+14k+16k^2+5k^3)}{3k^3(1+k)^2}\right) \tag{2.26}$$

Thus, we have $\partial(b-a)/\partial k < 0 \,\forall\, k > 1$
This result is interesting as we see less product differentiation, and hence more price competition with increasing k.

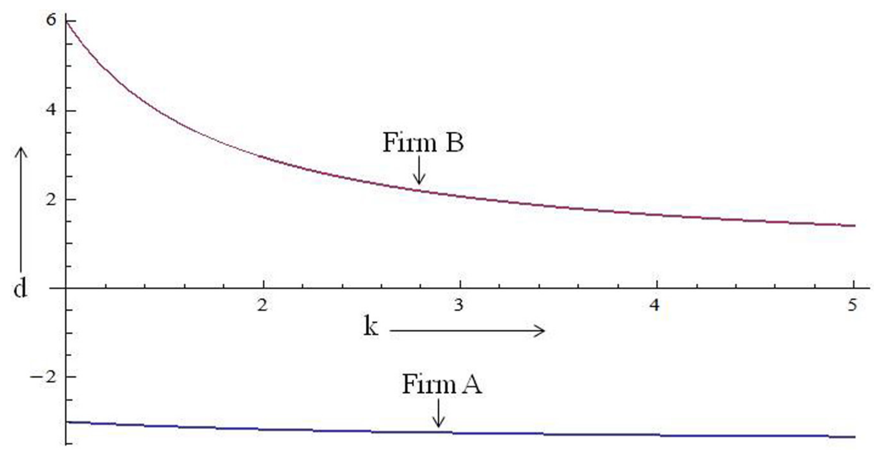

Figure 2.1: Position of Firm A and Firm B with respect to retailer's margin (k)

We observe that higher the margin retailer gets on the national brand, farther the firm A positions its product from the best possible location. But on the contrary, the store brand aggressively positions its product nearer to the best possible location in the market.
We observe this trend because of the double marginalization problem faced by the national brand manufacturer (firm A). These double margins constraint the national brand manufacturer's ability to compete in intense price wars with the retailer. As both the brands, national and store brands, are channelized to the consumers by the retailer, and with no retailer's cost associated with the store brand, it is in its interest to give a head-on price competition to the national brand while closing in on the same horizontal attributes. Thus, firm A wants to avoid the price competition and moves away from the best possible location in the market. But by moving away, firm A loses

on its demand and this trade-off makes the national brand positioning less sensitive to the retailers margin. On the other hand, retailer positions the store brand closer to the consumer ideal point in the product space to acquire a greater demand.

2.4.2 Preposition 2

With increasing retailer's margin on national brand:

- wholesale price of the national brand decreases
- retail price of the store brand increases.

Proof

$$p_a = kw = \frac{8k(1+2k)^2(2+4k+3k^2)t}{9k^4(1+k)^2} \tag{2.27}$$

$$p_b = \frac{4(2+4k+3k^2)(-2-8k-5k^2+9k^3+6k^4)t}{9k^4(1+k)^2} \tag{2.28}$$

Now,

$$\partial(p_a)/\partial k = -\left(\frac{16(4+24k+57k^2+68k^3+42k^4+12k^5)t}{9k^5(1+k)^3}\right) < 0\,\forall\, k > 1 \tag{2.29}$$

Thus, we say that price of the national brand decreases as k increases and the wholesale price decreases even more (by a factor of $1/k$).
Now, we have

$$\begin{aligned}\partial(p_b)/\partial k \quad &= \quad \frac{1}{9k^5(1+k)^3}\left(4\left(16+96k+216k^2+\right.\right.\\ &\quad\quad \left.\left.218k^3+78k^4-15k^5-15k^6\right)\right)\end{aligned} \tag{2.30}$$

such that

$$\partial(p_b)/\partial k = \begin{cases} > 0\,\forall\, k \in (1, 2.989]\,, \\ < 0\,\forall\, k > 2.989 \end{cases} \tag{2.31}$$

The value of $k = 2.989$ means a profit margin of 298.9 percent for the retailer on the national brand. Such a high margin doesn't exist in the real market scenarios. So we focus our attention on somewhat lower values of the k. When we have lower value of k in the market, the national brand is priced very high by the Firm A. But when k increases, the interest shifts from charging high prices to increasing the demand base.

Firm A wants to stay in the competition and as national brand's price is dependent upon the invoice price charged by the national firm A, it reduces the wholesale price "w" drastically. This steep decrease is made just to be able to compete in prices with the store brand. However, with increasing k, the retailer gets a large price band to price the store brand. And thus, even though the store brand is coming closer to the national brand on the product space, it is still beneficial for the retailer to charge a higher price from the consumers of the store brand.

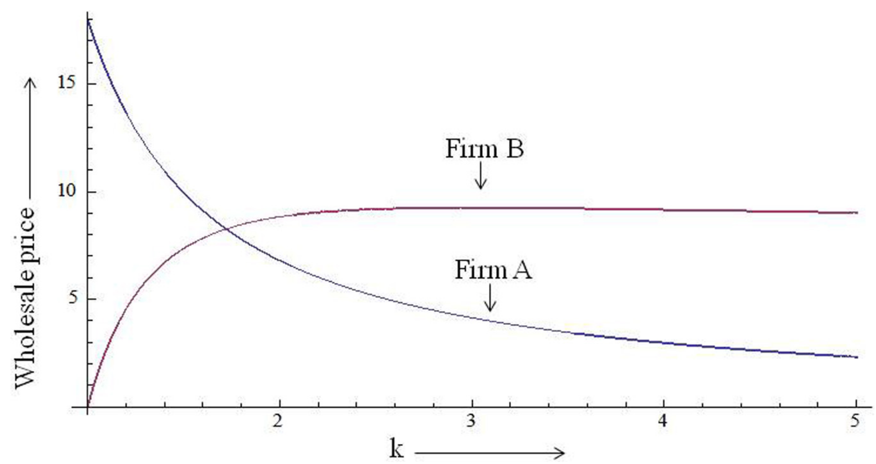

Figure 2.2: Wholesale price of national brand, p_a and store brand, p_b with respect to change in retailer's margin (k)

As presented in the graph, our model shows a very interesting result. We generally observe that the store brands are priced low as compared to the national brand (Allenby and Rossi, 1991). But here we say that, if the retailer gets a higher margin on the national brand i.e. $k \geq (2 + \sqrt{10})/3$ or 1.721, then the store brand can advantageously make its price to exceed the price of the national brand.

2.4.3 Preposition 3

With increasing retailer's margin on national brand:

- National brand manufacturer's profit decreases
- Retailer's profit increases

Proof Decisions of the firms are based upon the maximization of their respective profits. Profit of firm A is:

$$\Pi_a = \frac{16(1+2k)^3(2+4k+3k^2)t}{27k^5(1+k)^3} \tag{2.32}$$

Now,

$$\partial\Pi_a/\partial k = -\left(\frac{16(1+2k)^2(10+40k+65k^2+50k^3+18k^4)}{27k^6(1+k)^4}\right) \tag{2.33}$$

So we have, $\partial\Pi_a/\partial k < 0\,\forall\, k > 1$ and this means that profit of the firm A decreases with the increase in the retailer's margin. It can be easily understood by following the trend in Figure 2.3.

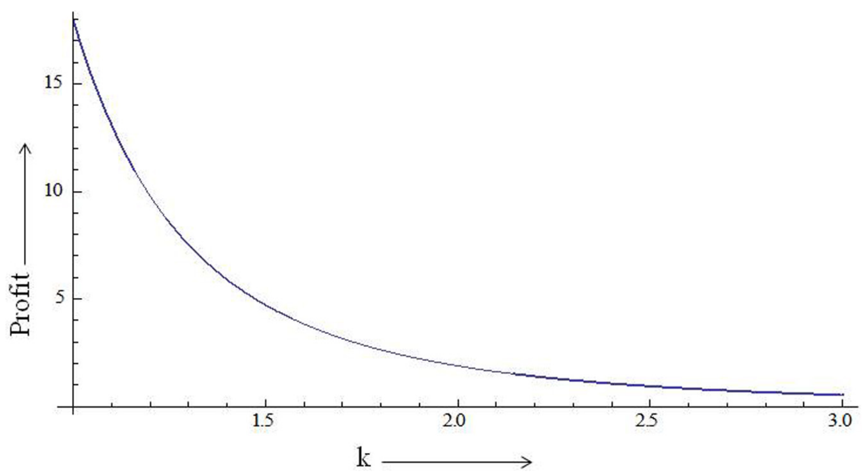

Figure 2.3: Reduction in profits of firm A with increase in retailer's margin, k

And Profit of firm B is:

$$\Pi_b = \frac{4(1+2k)(-1-2k+3k^2)(2+4k+3k^2)^2t}{27k^4(1+k)^3} \tag{2.34}$$

If we differentiate Π_b with respect to k, we get,

$$\partial\Pi_b/\partial k = \frac{1}{27k^5(1+k)^4}\left(4\left(16+124k+384k^2+ 608k^3+512k^4+219k^5+54k^6+27k^7\right)\right) \tag{2.35}$$

Thus, $\partial\Pi_b/\partial k < 0\,\forall\, k > 1$

This result is important because profits of firm B are increasing due to the increase in the market share as well as the price of the store brand. Thus, the retailer obviously benefits with the increase in the margins received from the sales of national brand, but the benefit is not a direct one. This is evident from the graph below:

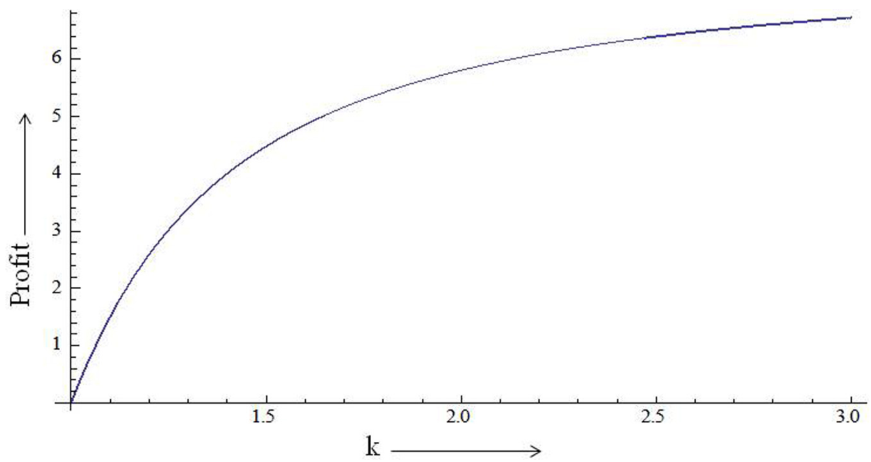

Figure 2.4: Profit gained by Firm B with increase in retailer's margin, k

Now, as the manufacturer and retailer of the national brand have conflicting interests over the value k. We say that, if the retailer margins are very low, then it is very difficult for the retailer to survive in the market, even if it is the owner of a store brand. And likewise, if the retailer margins are very high then it is in the interest of the national brand manufacturer to step out of the market. Another possibility is that the national brand manufacturer can actually try to avoid the downward decentralisation and actually form a direct link with the consumers by bypassing the retailers. This is a costly affair for the firm, but it indeed helps to survive in the market with a decent amount of profit, as the game transforms into an asymmetric manufacturing cost Hotelling duopoly model (Tyagi 2000).

2.4.4 Preposition 4

With increase in retailer's margin on national brand, consumers are better off.

Proof To prove this, we are first interested in knowing the location of the marginal consumer. So, substituting the values of a, b, w and p_b in the equation (2.2), we get

$$\tilde{x} = \frac{4 + 5k - 3k^2}{6k + 6k^2} \tag{2.36}$$

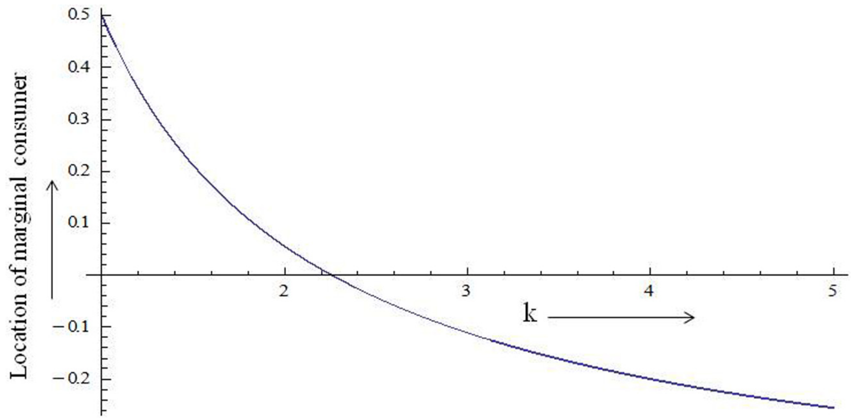

Figure 2.5: Location of marginal consumer $\tilde{x}$ with respect to k

Now, benefit of the consumers can be calculated by summing up the utility of the consumers available in market. So the total utility is the addition of:
Consumers buying the national brand product given by:

$$U_A = \int_{-1/2}^{\frac{4+5k-3k^2}{6k+6k^2}} \left(R - \left(k\frac{8(1+2k)^2(2+4k+3k^2)t}{9k^4(1+k)^2}\right) - \left((x - \frac{-5-7k}{2(1+k)})(x - \frac{-5-7k}{2(1+k)})\right)\right) \partial x \tag{2.37}$$

And
Consumers buying the store brand product

$$U_B = \int_{\frac{4+5k-3k^2}{6k+6k^2}}^{1/2} \left[R - \left(\frac{4(2+4k+3k^2)(-2-8k-5k^2+9k^3+6k^4)}{9k^4(1+k)^2}\right) - \left\{\left(\frac{8+32k+29k^2+3k^3}{6k^2(1+k)} - x\right)\left(\frac{8+32k+29k^2+3k^3}{6k^2(1+k)} - x\right)\right\}\right] \partial x \tag{2.38}$$

We thus solve the above two expressions and add them to get the total utility as:

$$U_{total} = U_A + U_B$$

$$U_{total} = \frac{1}{27k^4(1+k)^3}\left[48 + 81k^6(-12+R)\right.$$

$$+9k^7(-25+3R)+k^4(-657+27R-1088t)$$
$$+k^2(588-816t)-32t-32k(-9+8t)-8k^3(-39+164t)$$
$$+3k^5(27R-64(7+2t))\Big] \tag{2.39}$$

Now, we differentiate U_{total} with respect to k, and we have,

$$\partial U_{total}/\partial k = \frac{1}{27k^5(1+k)^4}\big(-64-208k+264k^2+ 2140k^3+4000k^4+3507k^5+1512k^6+297k^7\big) \tag{2.40}$$

Now $k > 1$ and hence, $264k^2 - 208k > 0 \,\forall\, k > 1$ and every other term containing higher powers of k is higher than -64. Hence, $\partial U_{total}/\partial k > 0 \,\forall\, k > 1$.
Thus we say that, consumer utility increases with increasing k. This is also seen in the following figure.

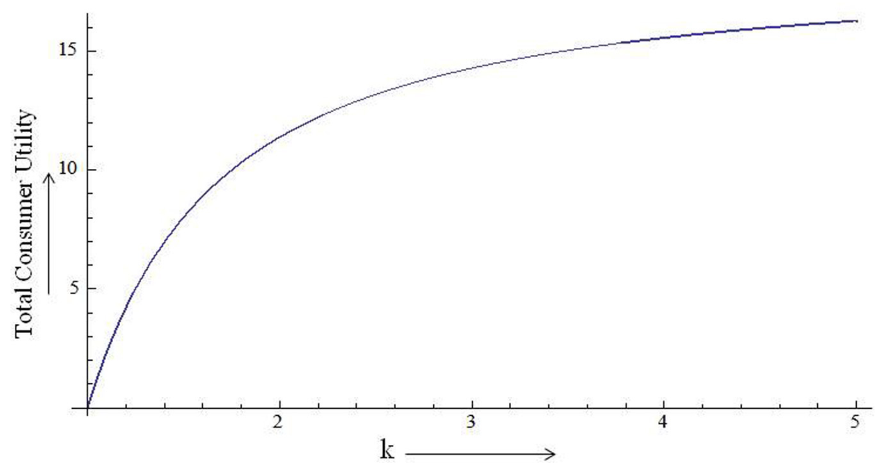

Figure 2.6: Change in consumer's utility, U_{total} with change in retailer's margin,k

For value of R less than 325/12, consumers get a negative utility and hence consumers will not buy from any of the two firms, which in turn violates our set of assumptions. Therefore, we put a constraint on the reservation price as $R \geq 325/12$.
The argument that increase in the retailer's margin on national brand benefits the consumers seems to be a counter-intuitive result, but we support this argument with the following two points. Firstly, we say that the store brand is coming near to the consumer ideal choice locations. This allows consumers to deviate lesser from their ideal location and hence reduces the negative utility caused by transportation. Secondly, we stress that with decreasing differentiation, both firms are actually indulging

into a price war. This reduction in prices of both the products actually decreases the negative utility caused to the consumers. In addition to this, we can in fact say that, this price war reduces the combined profits of the firms.

$$\Pi_{A+B} = \frac{1}{108k^5(1+k)^3}\big(128 + 1024k + 2880k^2 + 2800k^3 \\ -1696k^4 - 3111k^5 + 3591k^6 + 7047k^7 + 2889k^8\big) \quad (2.41)$$

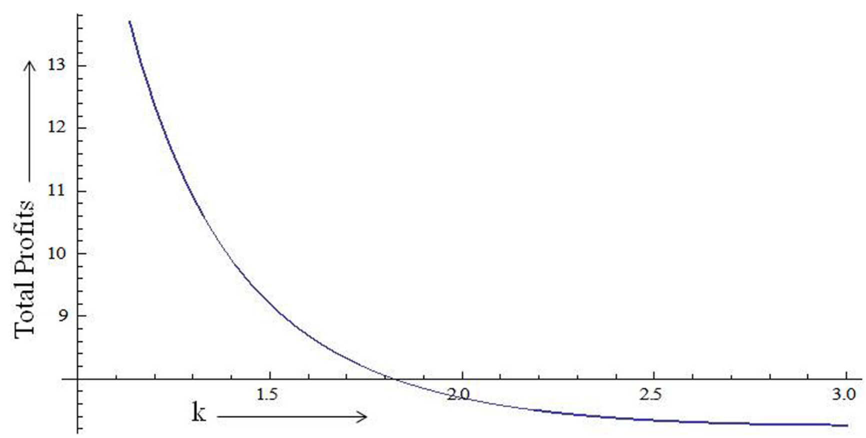

Figure 2.7: Combined profits of Firm A and Firm B, Π_{A+B} with respect to k

And as the increase in the retailer's margin for the national brand is hampering the combined profits, it actually benefits the consumers.

2.5 Conclusion

In this study, we look at the pricing and positioning decisions of both the national brand manufacturer and the retailer with respect to the changes in retail margins. We get some interesting insights from our study. It suggests that if the retailer's margin for the national brand is very high initially, then the store brand aggressively comes closer towards the national brand on the product space. Mills (1995) predicts that the higher the market share of the store brand, the higher is the margin that the retailer can obtain on national brands. It is interesting that we find the converse to be true as well. Moreover, we observe that the consumer utility increases with increase in retail margins on national brand.

In our model, we have taken retail margin to be exogenous. Our model is a sequential non-cooperative game and we haven't focussed on collusions or cooperative game theory. Thus, allowing collusion and cooperation in the model and endogenizing the retail margin can be a possible extension to our model. In addition to this, introducing multiple retailers with their respective store brands competing with the same national brand can also be an interesting extension. Further, it would be interesting to see the effects of advertisement; a tool national brands use to allure consumers from their competitors, in our model.

Chapter 3

Product Differentiation in a Variety Seeking Market

3.1 Introduction

Variety seeking behavior is the tendency of consumers to shift their preferences from one product to another during the course of time, irrespective of promotional influences. There are several theoretical and empirical studies that highlight the effects of consumers' variety seeking behavior on firms' product positioning and pricing decisions (Klemperer, 1987; McAlister, 1982; Seetharaman and Che, 2009; Sajeesh and Raju, 2010, Trivedi et al. 1994). Klemperer's paper (1987) presents a pioneer study in this regard. He argues that there is an associated switching cost whenever consumers switch their preference from one product to another. He uses this idea in a two-period model in which preferences of a fraction of consumers in period 2 are independent of their preferences in period 1. But rest of the consumers incur a cost in switching their preferences from period 1 to period 2. In contrast to Klemperer, Seetharaman and Che (2009) incorporate a staying cost for those consumers who stick to the same preference in period 2 as in period 1. Moreover, they model variety seeking tendency of a consumer by shifting her preference position from period 1 to its mirror image in period 2. On the other hand, changing the reservation price for some consumers from one period to another is also a method of modelling variety seeking (Sajeesh and Raju, 2010). This can be done in two ways:

- Boredom oriented: Decreasing the consumer's reservation price in period 2 for the product already bought by her in period 1

- Novelty oriented: Increasing the consumer's reservation price in period 2 for a product not brought by her in period 1.

It is surprising that, in the variety seeking literature, only Sajeesh and Raju (2010) have focussed on the positioning decision of the firms in variety seeking market. Moreover, they have incorporated simultaneous pricing and positioning decisions and completely overlooked the possibility of sequential entry of firms. We intend to fill in this gap in the literature by considering sequential rather than simultaneous positioning decisions. This preference for sequential positioning decision is due to the fact that, unlike pricing decisions, which can be easily modified and hence modelled by Nash equilibrium (Tabuchi and Thisse, 1995); positioning decisions are very sticky in nature. Even Anderson (1987); Tabuchi and Thisse (1995) agree that most location decisions are irreversible or irrevocable in nature and hence concept of sequential positioning decision is more realistic than having simultaneous positioning decision (Anderson, 1987). This is further strongly asserted by extensive use of sequential product positioning in Hotelling's duopoly in the numerous studies (Neven, 1987; Lilien et al., 1992; Tabuchi and Thisse, 1995; and Tyagi, 2000). In addition to this, Lilien et al. (1992) and Tyagi (2000), show that if the firm entering first doesn't have a cost disadvantage over the second entrant, then the first entrant positions at the best possible location in the market. This observation can be explained by stating that both firms want to avoid price competition and as the first firm enters the market, it is aware that the second firm will try to avoid the price competition and thus will position away from the first firm. This enables the first firm to locate at the best possible location in the market.

In this paper, we present a two firm sequential game model based on the Hotelling's framework in a variety seeking market. When the second mover enters the market, the first mover is already well established in the market. Using this entrant-incumbent model, we, in this paper, come up with some interesting insights for both the incumbent firm as well as the entrant firm about their pricing and positioning decisions in the presence of variety seeking consumers. The following section explains the model in detail.

3.2 Model

This model has a market consisting of heterogeneous consumers and their preferences are uniformly distributed over a straight line. A fraction α of the consumer population is variety seeker by nature. To observe the effect of variety seeking, the model consists of two different periods of pricing decisions and consumer allocations. Like Sajeesh and Raju (2010), our model also incorporates the idea of variety seeking in relation to decrease in the reservation price of variety seeking consumers from the purchase in first period to the purchase in second period. Furthermore, the market consists of two players, one of them is an incumbent firm and the other is a new entrant. The incumbent firm denoted by A, was a monopolist when it entered and because of the first mover advantage, it positioned itself at the best possible location on the Hotelling's line (Hotelling, 1929). The other firm is a new entrant in the market and we denote it by B. The sequence in this model is as follows:

1. A positions its product at the best possible location.

2. B positions its product.

3. In period 1, both firms A and B simultaneously decide their prices.

4. In period 1, each consumer decides to buy the product that maximizes their individual utility.

5. In period 2, the firms again simultaneously decide on their prices.

6. In period 2, each consumer decides to buy the product that maximizes their individual utility.

Assumptions

1. Consumers' preferences are distributed uniformly over the unit interval [-0.5, 0.5].

2. We fix the position of the incumbent firm at 0 for both the periods. But there is no constraint on the new entrant to position itself within the interval of consumers' ideal points. Thus like (Tyagi 2000), the new entrant firm can position itself outside the range of [-0.5, 0.5].

3. As this model has a concept of sequential entry, B enters the market after the positioning of firm A. The location is symmetric and hence without the loss of generality we assume that B enters the market and places itself to the right of A.

4. In both the periods, every consumer buys only one unit of the product from either firm which maximizes their individual utility.

5. Like Lilien et al.(1992) and Tyagi (2000), the net utility a consumer gets by buying a product positioned at a distance d away from its ideal point and priced at p_i is $R - p_i - td^2$, where $t > 0$ and R is the consumer reservation price. R is assumed to be a constant and sufficiently large so that all consumers buy one of two products.

6. The marginal cost of production for both A and B is assumed to be identical and constant, and is taken to be zero without the loss of generality.

7. The fraction α of consumers is of the variety seeking nature. For this fraction of consumers, second period reservation price decreases by L for the product they purchased in Period 1.

3.2.1 Notations

A = Incumbent firm
B = Entrant firm
a = Location of firm A
b = Location of firm B
R = Reservation price of consumer
p_i^k = Price of the product of firm k in period i, where $k \in \{A, B\}$ and $i \in \{1, 2\}$
q_i^k = Sales of firm k in period i, where $k \in \{A, B\}$ and $i \in \{1, 2\}$
Π_i^k = Profit function of firm k in period i, where $k \in \{A, B\}$ and $i \in \{1, 2\}$
Π_{total}^k = Total profit of firm k, where $k \in \{A, B\}$
α = Fraction of variety seeking consumers
L = Decrease in consumer reservation price in Period 2 due to boredom
$\tilde{x}$ = location of marginal consumer
Br_i= Best response function of firm i, where where $i \in \{A, B\}$

3.3 Analysis

The model described in section 3.2 is a multi-stage game; therefore we use the concept of sub-game perfect equilibrium and analyze the decisions of firms and consumers by the method of backward induction. Thus, we start with the last stage of the game, i.e. consumers' choice in period 2.

3.3.1 Stage 4: Consumers' choice in Period 2

To find the consumers' choices in period 2, we first locate the marginal consumer in the first period.

$$R - p_1^A - t(\tilde{x} - 0)^2 = R - p_1^B - t(\tilde{b} - x)^2 \tag{3.1}$$

$$\tilde{x} = \frac{b^2 t - p_1^A + p_1^B}{2bt} \tag{3.2}$$

Now, as B is to the right of A, all consumers from $[-0.5, \tilde{x}]$ buy from firm A and all consumers from $[\tilde{x}, 0.5]$ buy from firm B in the Period 1. Therefore, sales of firm A in Period 1 is

$$q_1^A = (1/2) + \frac{b^2 t - p_1^A + p_1^B}{2bt} \tag{3.3}$$

And that of firm B is

$$q_1^B = (1/2) - \frac{b^2 t - p_1^A + p_1^B}{2bt} \tag{3.4}$$

Now, sales of the firms in Period 2 will be different due to switching of some of the variety seeking consumers from one product to another. Sales of firm A in period 2 is thus given by the expression

$$q_2^A = (1/2) + \frac{b^2 t - p_2^A + p_2^B}{2bt} + \frac{\alpha(-p_2^A + p_2^B + p_1^A - p_1^B)}{2bt} \tag{3.5}$$

And sales of firm B in period 2 is given by the expression

$$q_2^B = (1/2) - \frac{b^2 t - p_2^A + p_2^B}{2bt} - \frac{\alpha(-p_2^A + p_2^B + p_1^A - p_1^B)}{2bt} \tag{3.6}$$

Using $\Pi = (p * q)$, profit function for the two firms in period 2 are

$$\Pi_2^A = p_2^A \left(\frac{1}{2} + \frac{b^2 t - p_2^A + p_2^B}{2bt} + \frac{\alpha(-p_2^A + p_2^B + p_1^A - p_1^B)}{2bt} \right) \tag{3.7}$$

$$\Pi_2^B = p_2^B \left(\frac{1}{2} - \frac{b^2 t - p_2^A + p_2^B}{2bt} - \frac{\alpha(-p_2^A + p_2^B + p_1^A - p_1^B)}{2bt} \right) \tag{3.8}$$

3.3.2 Stage 3: Pricing Decisions in Period 2

Each firm chooses a price that maximizes its own profit. But, as each firm's profit depends on the other firm's pricing decision, we find the best response (in terms of price) of a firm, given the price chosen by the other firm. Best responses are given by

$$Br_A(p_2^B) = \frac{bt + b^2 t + (1+\alpha)p_2^B + \alpha(p_1^A - p_1^B)}{2(1+\alpha)} \tag{3.9}$$

$$Br_B(p_2^A) = \frac{bt - b^2 t + (1+\alpha)p_2^A - \alpha(p_1^A - p_1^B)}{2(1+\alpha)} \tag{3.10}$$

For equilibrium prices, we simultaneously solve the two best response equations and we get,

$$p_2^A = \frac{3bt + b^2 t + \alpha(p_1^A - p_1^B)}{3(1+\alpha)} \tag{3.11}$$

$$p_2^B = \frac{3bt - b^2 t - \alpha(p_1^A - p_1^B)}{3(1+\alpha)} \tag{3.12}$$

Substituting these in equations (3.7) and (3.8) respectively, we have

$$\Pi_2^A = \frac{\left((b(3+b))\,t - \alpha(p_1^A - p_1^B)\right)^2}{18bt(1+\alpha)} \tag{3.13}$$

$$\Pi_2^B = \frac{\left((-b(3+b))\,t - \alpha(p_1^A - p_1^B)\right)^2}{18bt(1+\alpha)} \tag{3.14}$$

3.3.3 Stage 2: Pricing Decisions in Period 1

Each firm, in period 1, takes its pricing decision to maximize its total profit.

$$\Pi_{total}^i = \Pi_1^i + \Pi_2^i \; \forall \, i \in \{A, B\} \tag{3.15}$$

Thus we have,

$$\begin{aligned}\Pi_{total}^A &= p_1^A\left(\frac{1}{2}+\frac{b^2t-p_1^A+p_1^B}{2bt}\right)+\\ &\quad \frac{(b(3+b)t+\alpha(p_1^A-p_1^B))^2}{18bt(1+\alpha)}\end{aligned} \tag{3.16}$$

$$\begin{aligned}\Pi_{total}^B &= p_1^B\left(\frac{1}{2}-\frac{b^2t-p_1^A+p_1^B}{2bt}\right)+\\ &\quad \frac{(b(-3+b)t+\alpha(p_1^A-p_1^B))^2}{18bt(1+\alpha)}\end{aligned} \tag{3.17}$$

Now we solve $\partial\Pi_{total}^A/\partial p_1^A = 0$ and $\partial\Pi_{total}^B/\partial p_1^B = 0$ to obtain the best responses for each firm,

$$Br_A(p_1^B) = \frac{bt(9+9b+15\alpha+11b\alpha)-(-9-9\alpha+2\alpha^2)p_1^B}{2(9+9\alpha-\alpha^2)} \tag{3.18}$$

$$Br_B(p_1^A) = \frac{bt(9-9b+15\alpha-11b\alpha)-(-9-9\alpha+2\alpha^2)p_1^A}{2(9+9\alpha-\alpha^2)} \tag{3.19}$$

For equilibrium prices of period 1, we simultaneously solve the two best response equations and we get,

$$\begin{aligned}p_1^A &= \frac{1}{3(1+\alpha)(27+27\alpha-4\alpha^2)}\big(bt(81+27b+216\alpha\\ &\quad +60b\alpha+123\alpha^2+33b\alpha^2-20\alpha^3)\big)\end{aligned} \tag{3.20}$$

$$\begin{aligned}p_1^B &= \frac{1}{3(1+\alpha)(-27-27\alpha+4\alpha^2)}\big(bt(-81+27b-216\alpha\\ &\quad +60b\alpha-123\alpha^2+33b\alpha^2+20\alpha^3)\big)\end{aligned} \tag{3.21}$$

3.3.4 Stage 1: Positioning Decision for the entrant firm

Now analysing rearwards, firm B has to position itself into the market by maximising its total profit and hence we solve $\partial\Pi^B_{total}/\partial b = 0$ and get two values of b.

$$b = \frac{1}{9(1+\alpha)^2(162+171\alpha-8\alpha^2)}\Big[2916 + 8586\alpha + 7452\alpha^2$$
$$834\alpha^3 - 868\alpha^4 + 80\alpha^5 \pm \big((27+27\alpha-4\alpha^2)$$
$$(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)\big)^{1/2}\Big] \tag{3.22}$$

Out of these, for one value $\Pi^B_{total} = 0$ and therefore that solution is rejected. Hence we have a single solution which is the best response.
Thus, the value of b is given by the expression:

$$b = \frac{1}{9(1+\alpha)^2(162+171\alpha-8\alpha^2)}\Big[2916 + 8586\alpha + 7452\alpha^2$$
$$834\alpha^3 - 868\alpha^4 + 80\alpha^5 - \big((27+27\alpha-4\alpha^2)$$
$$(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)\big)^{1/2}\Big] \tag{3.23}$$

3.4 Results

3.4.1 Preposition 1

- When α is small; increase in α decreases differentiation, but for high α; increase in α increases the product differentiation.

- For very low α; entrant locates closer to the incumbent firm, but for very high α; entrant locates farther from the incumbent firm.

Proof Since $a = 0$, we have $b - a = b$. Therefore,

$$b - a = \frac{1}{9(1+\alpha)^2(-162-171\alpha+8\alpha^2)}((-2916 - 8586\alpha$$
$$-7452\alpha^2 - 834\alpha^3 + 868\alpha^4 - 80\alpha^5 +$$
$$\sqrt{(1+\alpha)^2(27+27\alpha-4\alpha^2)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big)\Big) \tag{3.24}$$

And we have

$$\partial(b-a)/\partial\alpha = \frac{1}{b}\left\{x\,(z-q) + y\,(z-q) + \frac{1}{q}\,(w\,(r+(s)))\right\} \tag{3.25}$$

where
$b = 9(1+\alpha)^3(162+171\alpha-8\alpha^2)^2$
$x = (1+\alpha)(-171+16\alpha)$
$z = 2916+8586\alpha+7452\alpha^2+834\alpha^3-868\alpha^4+80\alpha^5$
$q = \sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}$
$y = 2(-162-171\alpha+8\alpha^2)$
$w = (1+\alpha)(-162-171\alpha+8\alpha^2)$ $r = -8586-14904\alpha-2502\alpha^2+3472\alpha^3-400\alpha^4$
$s = 6259194+27326565\alpha+35048862\alpha^2-5366412\alpha^3-42444000\alpha^4$
$-23564817\alpha^5+2979984\alpha^6+2817664\alpha^7-599040\alpha^8+32000\alpha^9$

Now, as this equation is very large, we try to solve $\partial(b-a)/\partial\alpha = 0$ numerically and as $\alpha \in (0,1]$, we get only one solution of α in this range.
We have $\alpha_1 = 0.246832$
such that,

$$\partial(b-a)/\partial\alpha\,\text{is}\begin{cases}<0\,\forall\,\alpha<\alpha_1,\\>0\,\forall\,\alpha>\alpha_1\end{cases} \tag{3.26}$$

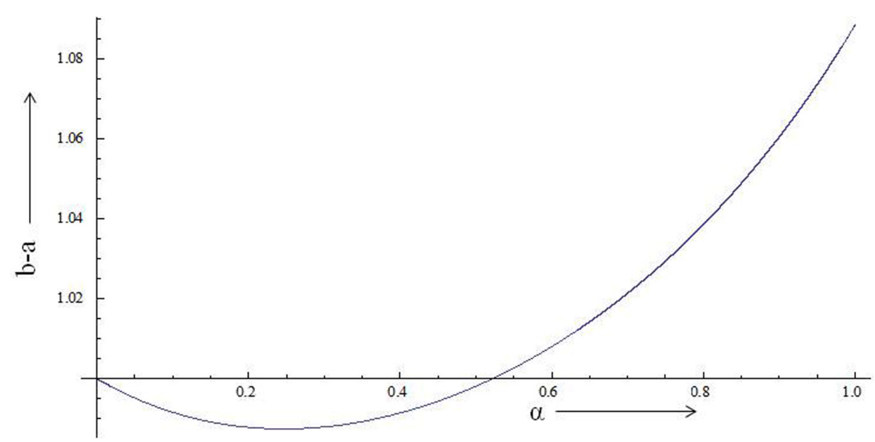

Figure 3.1: Product differentiation with change in fraction of variety seeking consumers, α

This proves that, if the fraction of variety seeking consumers in the market is less than α_1, then with the increase in variety seeking i.e. α , the entrant prefers to come closer to the incumbent product on the product space and if the fraction is more than α_1, then the entrant wishes to move apart with the increase in α. We explain this observation in the following way.

The presence of lesser variety seeking consumers in a market, gives an indication to the entrant that consumers will be loyal to the incumbent firm and will not switch easily. This makes the new entrant to differentiate less from the already existing product and enter a very high price competition with it. The basic idea is to try and reach closer to the consumer preferences and grab a section of the market by pricing lower than the incumbent firm. But, when majority of the consumers are variety seekers by nature, there will be lot of switching and hence the entrant will get consumers for his product that are bored by the incumbent product. This knowledge allows the entrant to safely enter the market without indulging into an intense price war with the existing firm. Thus, we see entrant differentiating more and trying to create a niche for itself.

3.4.2 Preposition 2

With the increase in the variety seeking consumers:

- Both firms charge lower prices in period 2 as compared to period 1.
- Entrant firm linearly decreases its average price, but the incumbent firm first decreases and then increases its average price over the two periods.

Proof Now we have,

$$
\begin{aligned}
p_1^A = -\frac{1}{81(1+\alpha)^4(162+171\alpha-8\alpha^2)^2(-27-27\alpha+4\alpha^2)} \Big[\big(2916+8586\alpha+7452\alpha^2 \\
+834\alpha^3-868\alpha^4+80\alpha^5 \\
-\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\big) \\
\big\{65610+295245\alpha+476685\alpha^2+306315\alpha^3+36345\alpha^4-21560\alpha^5+1360\alpha^6 \\
-9\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)} \\
-11\alpha\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\big\}\Big]
\end{aligned}
\tag{3.27}
$$

And,

$$p_2^A = -\frac{1}{27(1+\alpha)^4(162+171\alpha-8\alpha^2)^2(-27-27\alpha+4\alpha^2)}\Big[\Big(2916+8586\alpha+7452\alpha^2+ 834\alpha^3-868\alpha^4+80\alpha^5 -\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big) \Big\{21870+71685\alpha+77760\alpha^2+25965\alpha^3-3540\alpha^4-1400\alpha^5+160\alpha^6 -3\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)} -2\alpha\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big\}\Big] \tag{3.28}$$

So we have,

$$p_1^A - p_2^A = \frac{1}{81(1+\alpha)^4(162+171\alpha-8\alpha^2)^2(27+27\alpha-4\alpha^2)}\Big[5\alpha\Big(2916+8586\alpha+7452\alpha^2 +834\alpha^3-868\alpha^4+80\alpha^5 -\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big) \Big\{16038+48681\alpha+45684\alpha^2+9393\alpha^3-3472\alpha^4+176\alpha^5 -\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big\}\Big] \tag{3.29}$$

Here we have $(27+27\alpha-4\alpha^2) > 0\,\forall\,\alpha \in (0,1]$. Thus the denominator is greater than $0\,\forall\,\alpha \in (0,1]$. Now, it is easy to see that,
$2916+8586\alpha \gg \sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}$ $\forall\,\alpha \in (0,1]$
Hence,
$16038+48681\alpha \gg \sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}$ $\forall\,\alpha \in (0,1]$
Thus, we have the numerator also greater than $0\,\forall\,\alpha \in (0,1]$.
Therefore, we have $p_1^A - p_2^A > 0\,\forall\,\alpha \in (0,1]$

Similarly, we have,

$$p_1^B - p_2^B = \frac{1}{81(1+\alpha)^4(162+171\alpha-8\alpha^2)^2(27+27\alpha-4\alpha^2)}\left[5\alpha\left(2916+8586\alpha+7452\alpha^2 +834\alpha^3-868\alpha^4+80\alpha^5 +\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\right)\left\{10206+31509\alpha+30780\alpha^2+7725\alpha^3-1736\alpha^4+16\alpha^5+ \sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\right\}\right] > \forall\alpha\in(0,1] \tag{3.30}$$

Hence we say that both the firms charge lower prices in the 2^{nd} period as compared to 1^{st} period. This is further illustrated in Figure (3.2) and Figure (3.3).

Incumbent firm

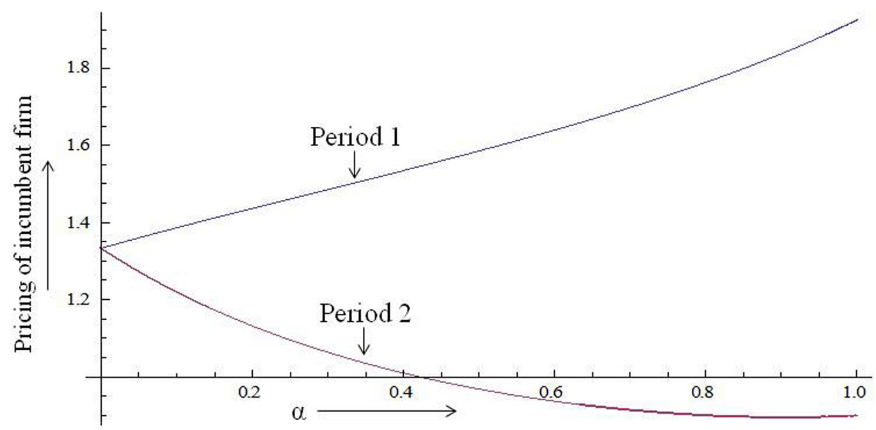

Figure 3.2: Pricing of the incumbent firm, p_1^A and p_2^A with respect to variety seeking consumers' fraction, α

These observations are consistent with the results of Sajeesh and Raju (2010) and with the findings of Zhang et al. (2000). The firms know that variety seeking consumers are satiated by the product they buy in the first period and hence wish to switch to other product. Thus, the competing firm tries to attract this pool of consumers. The decrease in price in the second period by both the firms is mainly for two purposes. Firstly, each firm wants to attract the consumers of the competing firm and secondly, each firm wants to stop its previous consumers from switching brands in the second period

Entrant firm

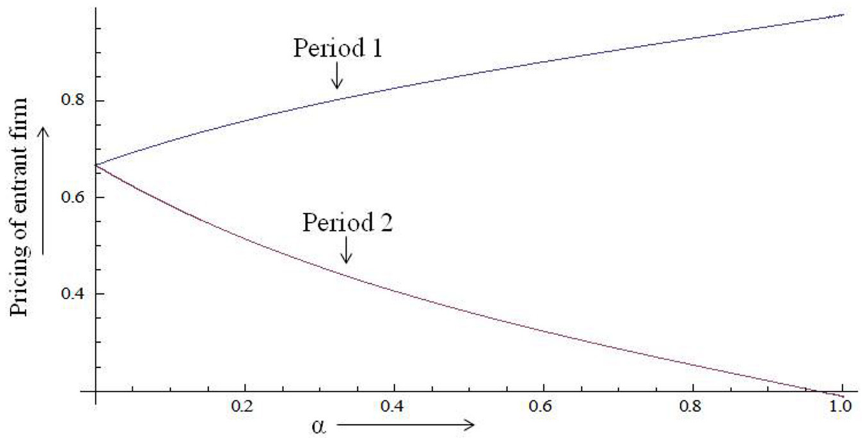

Figure 3.3: Pricing of the entrant firm, p_1^B and p_2^B with respect to variety seeking consumers' fraction, α

Now, the average price of the entrant firm is given by the expression,

$$(1/2)(p_1^B + p_2^B) = \frac{1}{162(1+\alpha)^4(162+171\alpha-8\alpha^2)^2(-27-27\alpha+4\alpha^2)} \Big[\Big(-2916 - 8586\alpha - 7452\alpha^2 - 834\alpha^3 + 868\alpha^4 - 80\alpha^5 + \sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big) \Big\{149769\alpha^2 + 100818\alpha^3 + 28617\alpha^4 + 872\alpha^5 - 880\alpha^6 + 18\,(1458 \sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big) + \alpha\,(10 +17\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big)\Big\}\Big] \tag{3.31}$$

The expression is pictorially represented for better understanding in Figure(3.4). The average price of the incumbent firm is given by the expression,

$$(1/2)(p_1^A + p_2^A) = \frac{1}{162(1+\alpha)^4(162+171\alpha-8\alpha^2)^2(-27-27\alpha+4\alpha^2)} [(2916 +8586\alpha + 7452\alpha^2 + 834\alpha^3 - 868\alpha^4 + 80\alpha^5 -\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big)$$

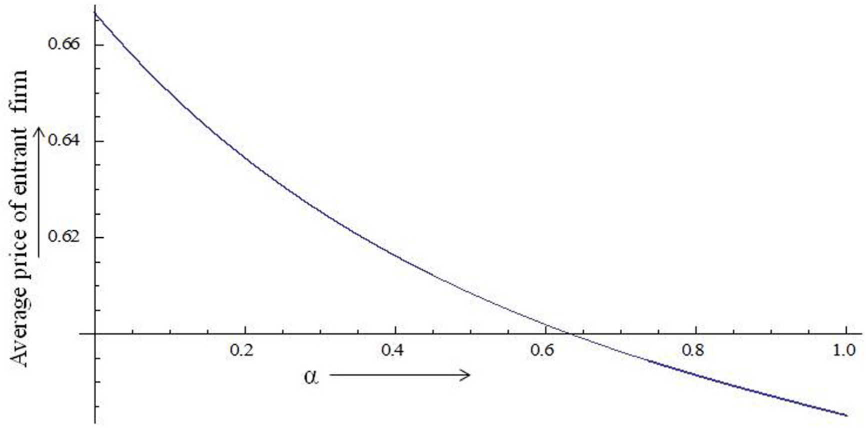

Figure 3.4: Changes in average price of entrant firm, $(1/2)(p_1^B + p_2^A)$ with respect to variety seeking fraction, α

$$-18\sqrt{(27 + 54\alpha + 23\alpha^2 - 4\alpha^3)^2(2916 + 5508\alpha - 1179\alpha^2 - 3720\alpha^3 + 400\alpha^4)}$$
$$-17\alpha\sqrt{(27 + 54\alpha + 23\alpha^2 - 4\alpha^3)^2(2916 + 5508\alpha - 1179\alpha^2 - 3720\alpha^3 + 400\alpha^4)}\Big\}\Big] \tag{3.32}$$

The corresponding graph is given in Figure (3.5):

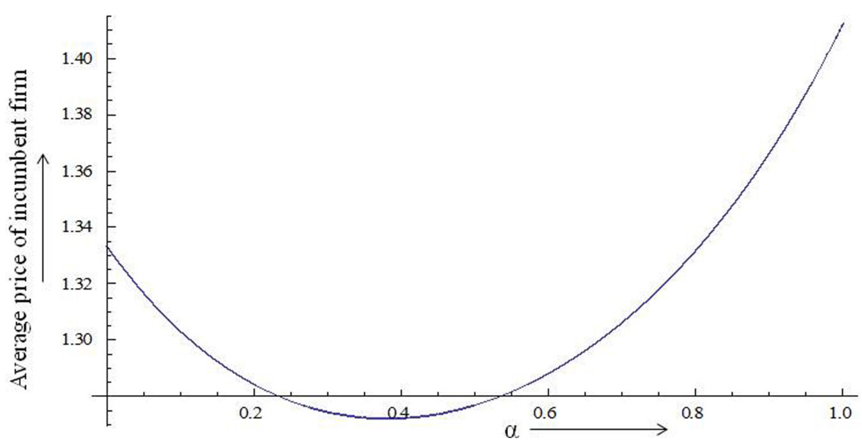

Figure 3.5: Average price of the incumbent firm with respect to fraction of variety seeking consumers, α

Interpreting the expressions and the graphs, we notice that the incumbent firm, unlike the entrant, doesn't linearly reduce its average price in the presence of variety seekers. In fact, in the presence of very high population of variety seekers in the market, the incumbent firm charges a higher average price than charged earlier. Since, with the increase in the population of variety seekers, the entrant differentiates more from the incumbent firm, there is a relaxation in the price competition. This existence of a niche and the advantage of being placed at the most attractive position in the market, allows the incumbent firm to reach a stage of quasi-monopolistic state and hence charge higher average price from the consumers. However, the average price of the entrant firm decreases with the increase in fraction of variety seekers in the market, since it has to attract more consumers to buy its product in order to establish itself in the market.

3.4.3 Preposition 3

The incumbent firm prefers a market with very high variety seekers, but the entrant firm prefers a market with very low variety seeking consumers. In addition to this, both the firms have aversion for a market with average variety seekers.

Proof Preference of a firm is known by its profit function as all firms have the intention of maximising their respective profits.

$$\begin{aligned}\Pi^A_{total} = \frac{1}{(729(1+\alpha)^4(162+171\alpha-8\alpha^2)^2(-27-27\alpha+4\alpha^2)}\,[t\,(-\\
2916-8586\alpha-7452\alpha^2-834\alpha^3+868\alpha^4-80\alpha^5\\
+\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big)\\
\big\{2423682\alpha^3-312777\alpha^4-278772\alpha^5+65240\alpha^6-4000\alpha^7-54\,(-18954\\
+5\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big)\\
-15\alpha\,(-265356+17\\
\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big)\\
+\alpha^2\,(5345271+50\\
\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big)\Big\}\Big]\end{aligned} \quad (3.33)$$

We now solve for $\partial\Pi^A_{total}/\partial\alpha = 0$ numerically and as $\alpha \in (0,1]$, we get only one solution of α in this range i.e. α_A

We have $\alpha_A = 0.365835$
such that

$$\partial\Pi^A_{total}/\partial\alpha \text{ is } \begin{cases} < 0\, \forall\, \alpha < \alpha_A, \\ > 0\, \forall\, \alpha > \alpha_A \end{cases} \tag{3.34}$$

This is further validated from the graph shown below.

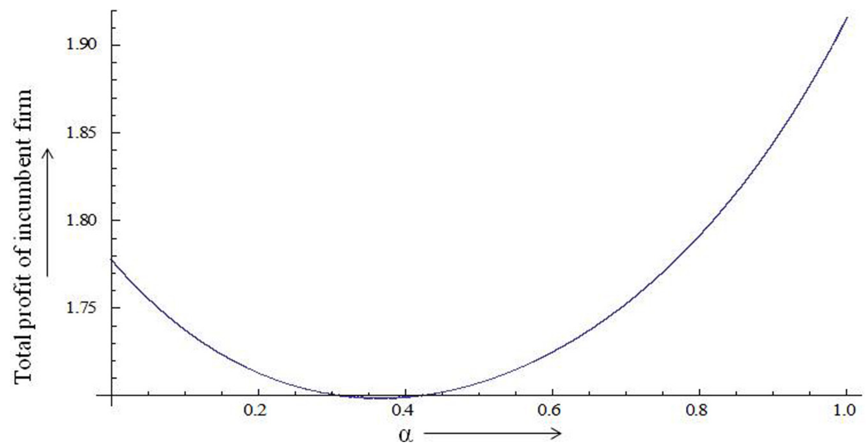

Figure 3.6: Changes in total profit of the incumbent firm Π^A_{total} with increasing variety seeking fraction α

And similarly we have,

$$\begin{aligned} \Pi^B_{total} = \frac{1}{729(1+\alpha)^4(162+171\alpha-8\alpha^2)^2(-27-27\alpha+4\alpha^2)} \Big[t\,\big(2916+8586\alpha+7452\alpha^2 \\ +834\alpha^3-868\alpha^4+80\alpha^5 \\ -\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big) \\ \big\{78732+306180\alpha+478467\alpha^2+388314\alpha^3+160371\alpha^4+10956\alpha^5-11320\alpha^6+800\alpha^7 \\ +54\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)} \\ +51\alpha\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)} \\ -10\alpha^2\sqrt{(27+54\alpha+23\alpha^2-4\alpha^3)^2(2916+5508\alpha-1179\alpha^2-3720\alpha^3+400\alpha^4)}\Big\}\Big] \end{aligned} \tag{3.35}$$

For this also we solve $\partial\Pi^B_{total}/\partial\alpha = 0$ numerically and as $\alpha \in (0,1]$, we get only one solution of α in this range.
Thus we have $\alpha_B = 0.701822$

Now

$$\partial\Pi^B_{total}/\partial\alpha \; is \begin{cases} < 0 \, \forall \, \alpha < \alpha_B, \\ > 0 \, \forall \, \alpha > \alpha_B, \end{cases} \tag{3.36}$$

These expressions are well illustrated by the following graph

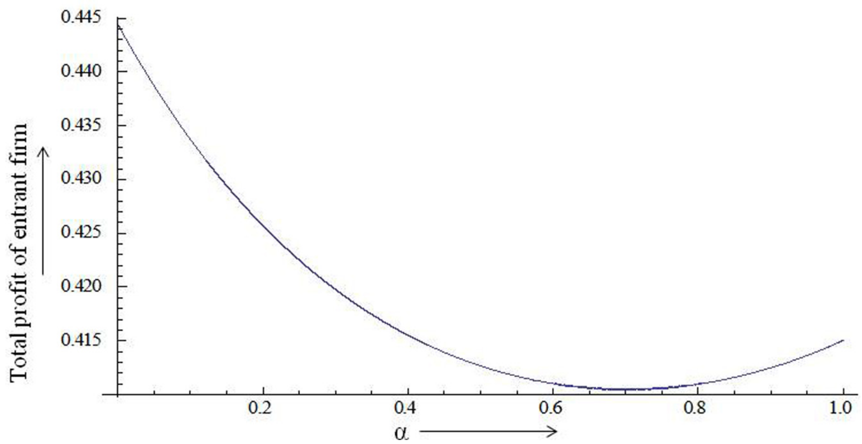

Figure 3.7: Total profit of the entrant firm Π^B_{total} with respect to variety seeking fraction α

We say that in the market with low variety seekers, firms can exploit the consumers as the firms are aware that very few people will switch to the other firm. But as the variety seekers increase in the population, firms get lesser profits. But when a very high fraction of the population is switchers, the firms differentiate more and hence have their own niches with relaxation in the price competition. All this helps the firms to earn high profits. Thus, this result shows that, variety seekers are not that disadvantageous for the firms. But, this directly contrasts with many empirical observations (Kahn et al, 1986; McAlister, 1982; Trivedi et al, 1994).
In addition to this , we say that there have been several empirical studies on variety seeking and the population of variety seekers in the population surveyed has never been very high (Hoyer and Ridgway, 1983; McAlister and Pessemier, 1982; Woratschek and Horbel, 2003). Che et.al (2007) found only 27 percent variety seekers in the population of breakfast cereal consumers. Woratschek and Horbel (2003,2006) have suggested in their study that consumer utility decreases with increase in variety seekers. Hence, if we constraint the range of variety seekers in the population to a medium level, then this preposition is consistent with the empirical findings.

3.5 Conclusion

In this paper, we incorporate the incumbent-entrant system into the Hotelling's framework which is applied in a market with a fraction of population of the consumers being variety seekers by nature. In this model, we come up with an insight that the entrant differentiates more from the incumbent firm for very low as well as for very high fractions of variety seekers in the market. But, for medium level variety seeking, entrant comes closer to the incumbent firm and enters into a relatively intense price competition. In addition to this, we also discover that the profit function of both the firms first decrease and then increase with the increase of variety seekers in the consumer population.

Our model has the incumbent-entrant system but, we haven't included any entry deterring strategies into the framework. Fixed entry costs, advertising and brand proliferation are a few entry deterring strategies which have been modelled extensively (Hay, 1976; Judd, 1985; Schmalensee, 1978; Shaked and Sutton, 1990). Incorporation of these strategies in our framework can be used to further extend our model.

Chapter 4

Product Differentiation under Time Based Competition

4.1 Introduction

In a real world market scenario, there are a number of factors which competing firms have to keep in mind to acquire profit margins. These include goods' quality, pricing, availability, delivery, firm's position, associated brands and customer's satisfaction. There are quite a few models which try to quantify the effects of each or combination of factors to predict profit margins of competing firms in duopolistic or oligopolistic settings. For example, Hotelling's model and its extension over last few decades try to establish a correlation between firm's position and pricing with firm's profitability. On the other hand, queuing theory from the literature of operations management brings in the effect of service time on a firm's profitability. Thus, it is clear that customer preferences not only include price and positioning but also time of delivery. Hence, a firm with better delivery time definitely has an upper hand on the other firm as indicated by the studies done by Li et al (1994).
There are ample studies, both empirical and theoretical, that strongly emphasize on customer preferences in terms of service time as a deciding factor in customer's inclination towards a particular firm. This in turn effects the pricing decisions of the firm as stated by Reitman (1994) and Allon (2008). Reitman, in his empirical studies on a gasoline filling station, found out that on an average, customers will prefer a slight increase in the price they pay for purchase than staying for a longer time for acquiring

the services. He empirically calculated that customers for a 6 percent reduction in congestion would pay about 1 percent more on actual charges. Similarly, Philip et al. (1995) also modelled similar results while considering time sensitive consumers and looking at their effects on firm's prices, operation policies, sales and profits in a competitive environment. Allon (2003 and 2008) has further modelled the competition between firms in terms of prices, service level and a combination of both while considering queuing systems from M/M/1 to G/GI/S. They established that service time variability can affect the relative market share and profit share of individual firms.

From the discussion above, it is evident that service time is an important factor that has to be quantified in relation to other major contributing factors to a firm's profitability. Hence, we propose to work it out in Hotelling's framework. As already stated before, there are quite a number of extensions in Hotelling's basic model which include cost structures, demand functions, number of firms, endogenous and exogenous controls. But so far, inclusion of the time domain in the Hotelling's framework has been ignored largely in the literature. Therefore, we aim to make a model more realistic by bringing in delivery time as an attribute for consumers' evaluation along with pricing and positioning. We, in this model, say that the firms along with their pricing and positioning decisions also take decisions about the delivery time they want to offer to the consumers. As the delivery time decisions are stickier than the pricing decisions (Allon and Federgruen, 2007; Jayaswal, 2009), we allow the firms to first simultaneously take their delivery time decision and then to simultaneously take their pricing decisions.

This model is a unique attempt to derive insights for the firms that are horizontally differentiated from each other and are competing on pricing, delivery time and positioning decisions with each other. This will give a unique perspective to the model with added robustness with spatial and temporal dimensions taken together.

4.2 Model

In this model, there are two firms, each offering a product horizontally differentiated from the other on a Hotelling's line. Both the firms A and B, in addition to pricing and positioning, are competing on an extra attribute i.e. the promised delivery time

(PDT). In this model, we assume demand in the system is defined by a Poisson process with rate $\lambda = 1$. The whole demand splits and goes to queue of each firm. Now, since splitting of a Poisson process is also a Poisson process, the demand for each firm arrives according to a Poisson process. As in the model, each consumer buys one and only one quantity of product, the demand arrival can be supposed to be the arrival of consumers in the queue of the respective firms. It is also assumed that the time, both firms A and B, take to serve a single consumer is exponentially distributed with rate $\mu_i, i \in \{A, B\}$ respectively. The sequence in this model is as follows:

Assumptions

1. Consumers preferences are distributed uniformly over the unit interval [-0.5, 0.5].

2. Both the firms simultaneously entered in the market and we assume that A is to the left of B without the loss of generality.

3. Every consumer buys one and only one unit of the product from either firm which maximizes its individual utility.

4. There is no constraint on the new entrant to position itself within the interval of consumers' ideal points. Thus like Lilien et al. (1992)and Tyagi (2000), the new entrant firm can position itself outside the range of [-0.5, 0.5].

5. Like (Tyagi 2000), in our model, a consumer gets a disutility by buying a product positioned at a distance d away from its ideal point and is quadratic in nature, td^2.

6. The satisfaction a consumer gets by buying any product is determined by the consumer reservation price denoted by R. R is assumed to be a constant and sufficiently large so that all consumers buy one of two products.

7. The disutility a consumer gets by buying a product with price, p_i, increases linearly with increase in p_i.

8. Similar to the disutility by pricing, the disutility by Promised Delivery Time, L_i, increases linearly with increasing L_i.

9. The marginal cost of production for both A and B is assumed to be identical and constant, and is taken to be zero without the loss of generality.

In addition to this, we also put a service level constraint that the actual delivery time (W_i) at steady state for a consumer should always be lower than the promised delivery time i.e. L_i with a probability of at least α.

$$P(W_i \leq L_i) \geq \alpha,\ i \in \{A, B\} \text{ and } \alpha \in (0, 1)$$

Now this means we have,

$$1 - \exp^{-(\mu-\lambda)L} \geq \alpha$$

This means, we have,

$$\mu \geq \lambda - \frac{\ln(1-\alpha)}{L}$$

Now $\mu_i, i \in \{A, B\}$ is the mean processing rate for the consumers at queue of firm A and B respectively. And as the firms want to maximize their profits, the processing rates should be at their minimum level that guarantees the desired processing level α. Hence at optimality, the equation (4.1) must be binding. Thus, each firm will have their mean processing rate as

$$\mu_i = \lambda_i - \frac{\ln(1-\alpha)}{L_i}\ \forall i \in \{A, B\}$$

Before explaining the analysis we plot the sequence, which this model follows.

1. Both firms A and B simultaneously take their positioning decision.

2. Both firms simultaneously decide on their respective promised delivery time (PDT).

3. Both firms simultaneously take their pricing decisions.

4. Consumers choose the product they want to buy depending upon the utility they get from buying that product. All the consumers are rational and their main intention is to maximize their utility.

4.2.1 Notations

R = Consumer Reservation Price
α = Target Processing Level
μ_i = Mean Processing Rate
W_i = Actual Delivery Time
L_i = Expected Delivery Time
Π_i = Profits of firm per unit time, where $i \in \{A, B\}$
q_a = Demand of firm A per unit time
q_b = Sales of firm B per unit time
U = Utility of a consumer
Br_i= Best response function, where $i \in \{A, B\}$
p_i = Equilibrium prices of firm i, where $i \in \{A, B\}$
$\tilde{x}$ = Location of marginal consumer

4.3 Analysis

This model is a multi-stage game and hence we use the concept of sub-game perfect equilibrium and analyze the decisions of firms and consumers by the method of backward induction.

4.3.1 Level 4: Marginal consumer's position

The marginal consumer is the consumer who is indifferent between both the firms and receives the same utility from each of the firms. The utility of any consumer buying from firm i is defined by

$$U = R - p_i - sL_i - td^2 \tag{4.1}$$

here s,t are constants
And for the marginal consumer

$$R - p_a - sL_a - t(\tilde{x} - a)^2 = R - p_b - sL_b - t(b - \tilde{x})^2 \tag{4.2}$$

$$\tilde{x} = \frac{(b^2 - a^2)t + s(-L_a + L_b) - p_a + p_b}{2(b - a)t} \tag{4.3}$$

here $\tilde{x} \in [-0.5, 0.5]$. Now, all consumers to the left of the marginal consumer buy product A and all the consumers to the right of it buy product B.
Now demand for firm A per unit time is

$$q_a = \frac{1}{2} + \frac{(b^2 - a^2)t + s(-L_a + L_b) - p_a + p_b}{2(b-a)t} \tag{4.4}$$

And sales of firm B per unit time is

$$q_b = \frac{1}{2} - \frac{(b^2 - a^2)t + s(-L_a + L_b) - p_a + p_b}{2(b-a)t} \tag{4.5}$$

Now, the profits of the firms per unit time are

$$\Pi_i = p_i q_i - k\mu_i \;\; \forall \, i \in \{A, B\}$$

Now we say that

$$\Pi_a = p_a \left(\frac{1}{2} + \frac{(b^2 - a^2)t + s(-L_a + L_b) - p_a + p_b}{2(b-a)t} \right) - k \left(\left(\frac{1}{2} \frac{(b^2 - a^2)t + s(-L_a + L_b) - p_a + p_b}{2(b-a)t} \right) - \left(\frac{\log[1-\alpha]}{L_a} \right) \right) \tag{4.6}$$

$$\Pi_b = p_b \left(\frac{1}{2} - \frac{(b^2 - a^2)t + s(-L_a + L_b) - p_a + p_b}{2(b-a)t} \right) - k \left(\left(\frac{1}{2} \frac{(b^2 - a^2)t + s(-L_a + L_b) - p_a + p_b}{2(b-a)t} \right) - \left(\frac{\log[1-\alpha]}{L_b} \right) \right) \tag{4.7}$$

4.3.2 Level 3: Pricing Decisions

Each firm chooses a price that maximizes its own profit. But, as each firm's profit depends on the other firm's pricing decision, we find the best response (in terms of price) of a firm, given the price chosen by the other firm. Best responses are given by

$$Br_A(p_b) = \frac{1}{2}(k + (b-a)t + (b^2 - a^2)t + s(L_b - L_a) + p_b) \tag{4.8}$$

$$Br_B(p_a) = \frac{1}{2}(k + (b-a)t - (b^2 - a^2)t - s(L_b - L_a) + p_a) \tag{4.9}$$

For equilibrium prices, we simultaneously solve the two best response equations and we get,

$$p_a = \frac{1}{3}(3k + 3(b-a)t + (b^2 - a^2)t + s(L_b - L_a)) \tag{4.10}$$

$$p_b = \frac{1}{3}(3k + 3(b-a)t - (b^2 - a^2)t - s(L_b - L_a)) \tag{4.11}$$

Substituting these in equation (4.6)and equation (4.7) respectively, we have

$$\Pi_a = \frac{1}{18(b-a)L_a}\big(18(b-a)k\log 1 - \alpha + s^2L_a^3 + L_a$$
$$(3a + a^2 - b(3+b) - sL_b)^2 - 2sL_a^2(-3a - a^2 + b(3+b) + sL_b)\big) \tag{4.12}$$

$$\Pi_b = \frac{1}{18(b-a)L_b}\big(k\log 1 - \alpha + (-3a + a^2 -$$
$$(-3+b)b + sL_a)^2L_b - 2s(-3a + a^2 - (-3+b)b + sL_a)L_b^2 + s^2L_b^3\big) \tag{4.13}$$

4.3.3 Level 2: Delivery Time Decisions

This stage of the game becomes mathematically intractable.

4.4 Conclusion

The model developed for product positioning under time based competition becomes mathematically intractable at stage 2 of the game. In future, we aim to develop a different model that captures the essence of product positioning under time based competition and at the same time is mathematically tractable.

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