Calculation of Higher Order Corrections to Drell-Yan Pair Production from some Higher Dimensional Operators

A Thesis

submitted to
Indian Institute of Science Education and Research Pune
in partial fulfillment of the requirements for the
BS-MS Dual Degree Programme

by

Aniruddha Vidyadhar Shirsat

IISER PUNE

Indian Institute of Science Education and Research Pune
Dr. Homi Bhabha Road,
Pashan, Pune 411008, INDIA.

April, 2019

Supervisor: Rohini Godbole
Co-supervisor: V. Ravindran

Certificate

This is to certify that this dissertation entitled Calculation of Higher Order Corrections to Drell-Yan Pair Production from some Higher Dimensional Operatorstowards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by Aniruddha Vidyadhar Shirsat at Indian Institute of Science Education and Research under the supervision of Rohini Godbole, Professor Center For Hgh Enegry Physics, IISc Bangalore and,V. Ravindran, Professor, Department of Physics IMSc Chennai during the academic year 2018-2019.

Rohini Godbole
IISc Bangalore

V. Ravindran
IMSc Chennai

Committee:

Rohini Godbole

V. Ravindran

Rajeev Bhalerao

This thesis is dedicated to my mother who has always supported me unconditionally

Declaration

I hereby declare that the matter embodied in the report entitled Calculation of Higher Order Corrections to Drell-Yan Pair Production from some Higher Dimensional Operators are the results of the work carried out by me at the Department of Physics, Indian Institute of Science Education and Research, Pune, under the supervision of Rohini Godbole and V. Ravindran and the same has not been submitted elsewhere for any other degree.

Aniruddha Vidyadhar Shirsat

Acknowledgments

Past 10 months of my dissertation project at Indian of Institute Sciences, Bangalore and Institute of Mathematical sciences, Chennai has been a fantastic journey. I have learnt a lot, developed as a scientist and as a person. I have several people to thank for this. Firstly, I am immensely grateful to Dr. Rohini Godbole for being such an amazing mentor. Her approach towards physics and positive attitude are just a few among many of her qualities that I have tried to learn and wish to inculcate in my future research. I would like to thank Prof. V. Ravindran for sharing his valuable advice and critical review of the work that has helped immensely for conducting this work. I would like to acknowledge Dr Swapan majhi for his suggestions and constant encouragement. I would like to extend my gratitude to Prof. Dr Rajeev Bhalerao, my project advisor, for sparing his valuable suggestions and his critical review of the work. Also, I thank my fellow classmates who made my stay enjoyable . Lastly, but most importantly, I thank my family for their constant motivation and support throughout this journey. I am immensely grateful for their confidence in me and for always boosting me to achieve higher!

Abstract

The standard model of particle physics is able to explain the structure of the universe very successfully. It has been proven very effective in understanding the processes of elementary particles. But there are scenarios which lie beyond the scope of SM. This project considers Lepton-Quark contact interaction in addition to the existing interaction in the SM. This project required learning QED and QCD as prerequisites for the later work which involved the analytical calculations of the differential cross-section including this LLQQ interactions which are parameterized in terms of six-dimensional operators. The project focused on appending these operators to the existing codes and comparing the output with the existing experimental data.

Contents

List of Figures

Chapter 1

Introduction

The Standard Model of Particle Physics has been proven extremely successful in understanding the dynamics of elementary particles. There are still many more phenomena which are out of the scope of the standard model. Many models have been proposed to explain such phenomena in the last few decades. Our attempt is to provide a framework to study physics Beyond Standard Model(BSM). The project is intended towards using available data DY process to investigate new physics scenarios.

Drell-Yan Process or Drell-Yan Pair production a standard process to probe physics at higher energies at Large Hadron Collider(LHC). It occurs in high energy Hadron-Hadron scattering. Quark and anti-quark from either of hadrons annihilate to form a boson which then forms a pair of lepton and anti-lepton. In 1970 Sidney Drell and Tung-Mow Yan first discussed this process and was later observed by J.H. Christenson.

The quark-lepton compositeness [4,5,6] implies that quarks and leptons share some common constituents. This leads to the interaction between quark and leptons which is beyond the scope of the existing standard model. But the four-fermion contact interaction is not just limited to the quark-lepton compositeness but also the four-fermion interactions mediated by the massive particle having a mass much greater than the energy transfer is approximated by such contact interaction terms. The effective interaction term in the Lagrangian, well below the scale Λ given in [4],

$$L = \frac{4\pi}{\Lambda}\left[\eta_{ij}(\bar{q}\gamma^{\mu}P_i q)(\bar{l}\gamma_{\mu}P_i l) + \zeta_{ij}(\bar{q}\gamma^{\mu}q)(\bar{l}\gamma_{\mu}l)\right]$$

where i, j = L,R andP_i are Chirality projection operators. QCD corrections can affect cross-section significantly. Even for simple processes like Drell-Yan including these interaction terms can play a crucial rule in increasing accuracy of the experiment. Correction terms of these contact interaction up to the next-to-leading order are discussed in Ref. [4]. This inspires us to include next-next-to-leading order corrections for the Hadronic processes.

Chapter 2

Methodology

The theory of quantum electrodynamics and quantum chromodynamics(QCD) is an integral part of the Standard Model of elementary particles as it successfully explains hadron properties, nuclear structure and various phenomena. Learning the elementary processes of quantum electrodynamics have crucial contributions to the processes which occur in QCD thus at the beginning of this project I studied these processes their helicity structures and significance and the basic analytic structures of scattering amplitudes.

The Project commenced by revising Quantum Electrodynamics. Various topics were studied including Elementary processes such as $e^+e^- \to \mu^+\mu^-$, Radiative Corrections, Divergences occurring in various processes, cancellation of divergences.

Later reading involved introduction to the Non-Abelian gauge theories and their quantization. In this spell of the project, I learned Lie algebras, Yang-Mills Lagrangian, Fadeev-Poppov Lagrangian and one loop divergences of Non-Abelian gauge theories. For an understanding of the Hadron structure and their dynamical properties learning QCD was crucial. I read Parton model, Deep inelastic scattering and Hard scattering processes in Hadron collisions.

The Standard Model Effective Field theory predicts the general form of the cross-section to be,

$$\sigma^i = \sigma^i_{SM} + \sum_n a^i_n c_n + \sum_{n \le m} b^{nm} c_n c_m \tag{2.1}$$

Our goal is to express such experimental observable as the function which depend on the wilson coefficients c_n of the relevant four-fermion operators. One of the challenge is to determine the coefficients a_n^i and b_{nm}^i which are discussed analytically in Ref. [2].

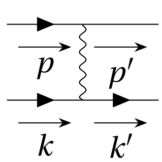
The next to leading order corrections to lepton pair production with the introduction of relevant operators for four-fermion contact interaction is discussed in Ref. [4]. The code for these calculations was developed by Dr Debajyoti Choudhury, Dr Swapan Majhi and Dr V. Ravindran. The aim of the project is to calculate next next to leading order correction to the Drell-Yan process. To get the result up to the one more order of correction the plan was to use the code developed by Dr Swapan Majhi and Dr V. Ravindran for the QCD corrections up to the next-next-to leading order corrections to the slepton production through annihilation of quarks.

The project focused on learning the analytic structures of these corrections and the structure of the higher dimensional operators which play a major role and developing code to include these corrections to the existing model.

Chapter 3

Theory

Calculations of cross-sections for elementary process in QED is straight forward. One of the easiest method is to use Feynman rules of QED. One of the Most discussed process is



$$e^+e^- \to \mu^+\mu^- \tag{3.1}$$

$$\equiv |M| = \bar{v}^{s'}(p')(-ie\gamma^{\mu})u^{s}(p)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\bar{u}^{r}(k)(-ie\gamma^{\nu})v^{r'}(k')$$

The differential cross-section is proportional to the square of matrix square element M.

$$\frac{d\sigma}{d\Omega} = const.|M|^2 \tag{3.2}$$

But, The divergence arises when we take into account first order corrections. To present an example it can be thought of the QED vertex. The first order correction leads to the cross section given by eq.(4)

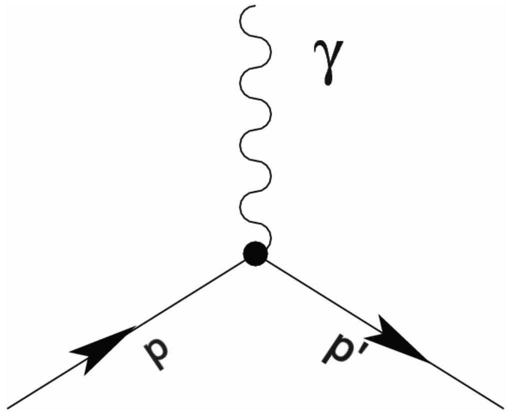


Figure 3.1: Image Source: The Standard Model of Electroweak Interactions - Pich, Antonio arXiv:1201.0537 [hep-ph] IFIC-11-73, FTUV-12-0102

$$\frac{d\sigma}{d\Omega}(p \to p') = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 - \frac{\alpha}{\pi}\log\left(\frac{-q^2}{m^2}\right)\log\left(\frac{-q^2}{\mu^2}\right) + O(\alpha^2)\right] \tag{3.3}$$

And, from the cross section for the process of emitting photon, we obtain that

$$\frac{d\sigma}{d\Omega}(p \to p' + \gamma) = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[\frac{\alpha}{\pi}\log\left(\frac{-q^2}{m^2}\right)\log\left(\frac{-q^2}{\mu^2}\right) + O(\alpha^2)\right] \tag{3.4}$$

$$\frac{d\sigma}{d\Omega}(p \to p') + \frac{d\sigma}{d\Omega}(p \to p' + \gamma) \equiv \left(\frac{d\sigma}{d\Omega}\right)_{measured} \tag{3.5}$$

Neither the elastic cross section nor the so0ft bremsstrahlung cross-sections measured individually. The separate cross-sections are divergent, but their sum is finite which is physically observable. In any real experiment if the sensitivity of the detector is E_l then, We obtain the result

$$\left(\frac{d\sigma}{d\Omega}\right)_{measured} \approx \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 - \frac{\alpha}{\pi}\log\left(\frac{-q^2}{m^2}\right)\log\left(\frac{-q^2}{E_l^2}\right) + O(\alpha^2)\right] \tag{3.6}$$

More examples of loop divergences can be discussed. I will discuss Vacuum polarization or photon self-energy. The observable is a sum of all possible diagrams of all orders. But here I will show calculation till first order correction. Defining $i\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi(q^2)$ to be sum of all 1-Particle-irreducible insertions into the photon propagator. Then exact photon two-point function is $\frac{-ig^{\mu\nu}}{q^2(1-\Pi(q^2))}$ The propagator has pole at $q^2 = 0$ and residue of the pole is $\frac{1}{1-\Pi(0)} \equiv Z_3$

The amplitude for scattering process will be shifted by this factor, relative to tree-level approximation.

$$\frac{e^2 g^{\mu\nu}}{q^2} \to \frac{Z_3 e^2 g^{\mu\nu}}{q^2} \tag{3.7}$$

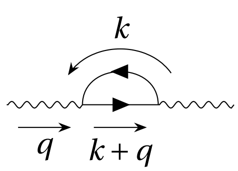
we can account for this shift by replacing $e \to \sqrt{Z_3}e$. This replacement is called charge renormalization. (physical charge)= $e = \sqrt{Z_3}e_0 = \sqrt{Z_3}$(bare charge) the effective electromagnetic coupling constant is

$$\alpha_{eff}(q^2) = \frac{e_0^2/4\pi}{1-\Pi(q^2)} \tag{3.8}$$

Restricting ourselves to first order

$$\alpha_{eff}(q^2) = \frac{\alpha}{1-[\Pi_2(q^2)-\Pi_2(0)]} \tag{3.9}$$

Computation of $\Pi_2(q^2) - \Pi_2(0) = \hat{\Pi}_2(q^2)$:



$$i\Pi_2^{\mu\nu}(q) = (-ie)^2(-1)\int \frac{d^4k}{(2\pi)^4} tr\left[\gamma^\mu \frac{i(\not{k}+m)}{k^2-m^2}\gamma^\nu \frac{i(\not{k}+\not{q}+m)}{(k+q)^2-m^2}\right] \tag{3.10}$$

Using Feynman Parameters, substituting $l = k + xq$ and performing Wick rotation we find,

$$i\Pi_2^{\mu\nu}(q) = -4ie^2\int dx \int \frac{d^4 l_E}{(2\pi)^4}\frac{-\frac{1}{2}g^{\mu\nu}l_E^2 + g^{\mu\nu}l_E^2 - 2x(1-x)q^\mu q^\nu + g^{\mu\nu}(m^2+x(1-x)q^2)}{(l_E^2+\Delta)^2} \tag{3.11}$$

where $\Delta = m^2 - x(1-x)q^2$. The ultra-violate divergences in this equation are removed by dimensional regularization. From the calculations we obtain $i\Pi^{\mu\nu}$ is proportional to the factor$(q^2 g^{\mu\nu} - q^\mu q^\nu)$. This is in agreement with ward identity.

$$i\Pi_2^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu) i\Pi_2^{\mu\nu}(q) \tag{3.12}$$

where,

$$i\Pi_2^{\mu\nu}(q) = \frac{-8e^2}{(4\pi)^2}\int_0^1 dx x(1-x)\frac{\Gamma\left(2-\frac{d}{2}\right)}{\Delta^{2-d/2}} \tag{3.13}$$

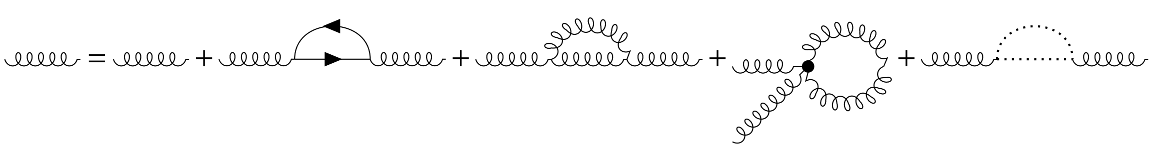
hence,

$$\hat{\Pi}_2(q^2) = \Pi_2(q^2) - \Pi_2(0) = \frac{-2\alpha}{\pi}\int_0^1 dx x(1-x)\log\left(\frac{m^2}{m^2 - x(1-x)q^2}\right) \tag{3.14}$$

Last three decades of 20^{th} century, it has been believed that fermions the elementary particles that make up all the matter we experience and interact through the exchange of bosons. Elementary fermions consist of quark and leptons(e^-, μ, τ and neutrinos corresponding to all three particles). Quarks bound states are responsible for the formation of Hadrons(mesons and baryons). Fermions interact with each other through strong, weak and electromagnetic forces. Electromagnetic interaction is understood in QED. While strong interaction is responsible for nucleon binding and interaction between constituents of nuclei. To explain the nature of Hadrons Bjorken and Feynman developed a model called The Parton Model. Which claims Hadrons are bound states of partons which are quarks and antiquarks which interact weakly at short distances. This peculiar property was called asymptotic freedom. To explain such behaviours we have to include the notion of non-Abelian gauge theories.

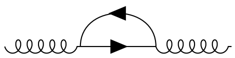
The symmetry group of QED is an Abelian group. But, this might not be the case all the time. The theories having non-Abelian symmetry group are more general. Thus we need to study similar calculations mathematical tools for such theories as well.

Calculation of gauge boson self-energy was studied. The g^2 order contributions come from four diagrams.



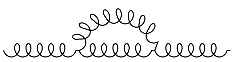
(3.15)

First let us start with fermion loop diagram. The color factors are already taken care of, and the amplitudes of these diagrams are shown.

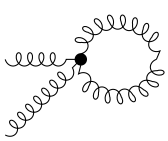


$$\equiv i(q^2 g^{\mu\nu} - q^\mu q^\nu)\delta^{ab}\left(\frac{-g^2}{(4\pi)^{d/2}}.\frac{4}{3}n_f C(r)\Gamma\left(2-\frac{d}{2}\right)\right) \tag{3.16}$$

The next two diagrams are from pure gauge sector. We will work in Feynman-'t Hooft gauge.

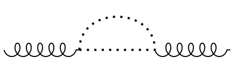


$$\begin{aligned}\equiv i\frac{g^2}{(4\pi)^{d/2}}C_2(G)\delta^{ab}\int dx\frac{1}{\Delta^{2-d/2}}.\Bigg(&\Gamma\left(1-\frac{d}{2}\right)g^{\mu\nu}q^2\left[\frac{3}{2}(d-1)x(1-x)\right]\\ &+\Gamma\left(2-\frac{d}{2}\right)g^{\mu\nu}q^2\left[\frac{1}{2}(2-x)^2+\frac{1}{2}(1+x)^2\right]\\ &-\Gamma\left(2-\frac{d}{2}\right)q^\mu q^\nu\left[(1-d/2)(1-2x)+(1+x)(2-x)\right]\Bigg)\end{aligned} \tag{3.17}$$



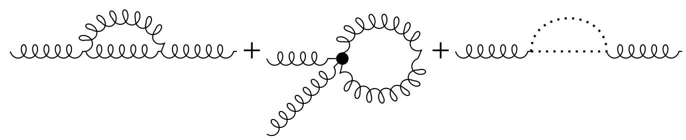
$$\begin{aligned}\equiv i\frac{g^2}{(4\pi)^{d/2}}C_2(G)\delta^{ab}\int dx\frac{1}{\Delta^{2-d/2}}.\Bigg(&-\Gamma\left(1-\frac{d}{2}\right)g^{\mu\nu}q^2\left[\frac{1}{2}d(d-1)x(1-x)\right]\\ &+\Gamma\left(2-\frac{d}{2}\right)g^{\mu\nu}q^2\left[(d-1)(1-x)^2\right]\Bigg)\end{aligned} \tag{3.18}$$

And the last diagram consists of ghost loop.



$$\equiv i\frac{g^2}{(4\pi)^{d/2}}C_2(G)\delta^{ab}\int dx\frac{1}{\Delta^{2-d/2}}.\left(-\Gamma\left(1-\frac{d}{2}\right)g^{\mu\nu}q^2\left[\frac{1}{2}x(1-x)\right]+\Gamma\left(2-\frac{d}{2}\right)g^\mu q^\nu\left[x(1-x)\right]\right) \tag{3.19}$$

The Last three diagrams sum up to be



$$\equiv i(q^2 g^{\mu\nu} - q^\mu q^\nu)\delta^{ab}\left(\frac{-g^2}{(4\pi)^{d/2}}.\frac{-5}{3}C(r)\Gamma\left((2-\frac{d}{2}\right)\right) \tag{3.20}$$

Further calculations leads us to Beta function$\big(\beta(g)\big)$ One loop beta function for QCD is given by

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3}C_2(G) - \frac{4}{3}n_f C(r)\right] \tag{3.21}$$

The β-function gives the rate at which the renormalized coupling constant changes. For small values of n_f the β-function is negative agreeing with our requirement of asymptotic freedom.

The most important concept required for future project was to learn Kinematics of Deep inelastic scattering. This included calculation of partonic cross section and their relations with total Hadronic cross section. This being understood, I calculated cross-sections for various hard scattering processes in Hadron collisions.

$$P(p_1) + P(p_2) \to l(l_1) + \bar{l}(l_2) + X \tag{3.22}$$

The total cross-section for Proton-Proton collision involving quark-antiquark coming from either of the protons and carrying x_1, x_2 fractions of total momentum of proton i.e. the parton momentum k_i is $k_i = x_i p_i$; scattering into leptonic final state($l\bar{l}$) takes the form

$$\sigma\Big(P(p_1) + P(p_2) \to l\bar{l} + X\Big) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_f f_f(x_1) f_{\bar{f}}(x_2).\sigma\Big(q(x_1 P) + \bar{q}(x_2 P) \to l\bar{l}\Big) \tag{3.23}$$

where X is any Hadronic final state, $f(x)$ are parton distribution functions and the sum runs over all flavours of quarks and antiquarks. The proton level process is shown in the diagram and partonic cross-section calculations are similar to the cross-sections of $e^+e^- \to \mu^+\mu^-$ process.

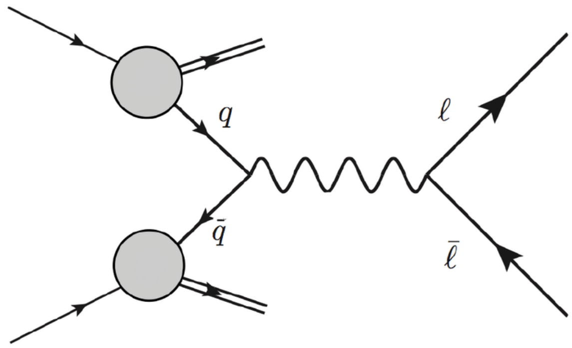


Figure 3.2: Image Source: www.resarchgate.net

The differetal cross-sections are expressed by the relation Ref. [4],

$$2S\frac{d\sigma^{P_1P_2}}{dQ^2} = \sum_{ab=q,\bar{q},g}\int_0^1 dx_1\int_0^1 dx_2 f_a^{P_1}(x_1) f_b^{P_2}(x_2)\int_0^1 dz 2\hat{s}\frac{d\sigma^{ab}}{dQ^2}\delta(\tau - zx_1x_2) \tag{3.24}$$

The variables denote the following quantities.

$$S \equiv (p_1+p_2)^2 \qquad \hat{s} \equiv (k_1+k_2)^2 \qquad Q^2 \equiv (l_1+l_2)^2$$

$$\tau \equiv \frac{Q^2}{S} \qquad z \equiv \frac{Q^2}{\hat{s}} \qquad \tau \equiv zx_1x_2 \tag{3.25}$$

$$2S\frac{d\sigma^{P_1P_2}}{dQ^2}(\tau, Q^2) = \frac{1}{2\pi}\sum_{j,j'=\gamma,Z,VA,SP}\tilde{P}_j(Q^2)\tilde{P}^*_{j'}(Q^2)L_{jj'}(Q^2)W_{jj'}^{P_1P_2}(\tau, Q^2) \tag{3.26}$$

Where W is Hadronic structure function given by,

$$W_{jj'}^{P_1P_2}(\tau, Q^2) = \sum_{j,j',a,b}\int_0^1 dx_1\int_0^1 dx_2 f_a^{P_1}(x_1) f_b^{P_2}(x_2)\int_0^1 dz\delta(\tau - zx_1x_2)\bar{\Delta}_{ab}^{jj'}(z, Q^2, \epsilon) \tag{3.27}$$

The bare partonic coefficient function $\bar{\Delta}$ is,

$$\bar{\Delta}_{ab}^{jj'}(z, Q^2, \epsilon) = \int dPS_{m+1}\left|M^{a\to jj'}\right|^2 T_{jj'}(q). \tag{3.28}$$

With dependence on the spin of the current through $T_{jj'}$ as discussed in Ref. [4].

We encounter singularities in the bare partonic coefficient function $\bar{\Delta}$ with ultraviolet, collinear and soft divergences. To deal with this problem we use the dimensional regularization. We add all the contributions fro gluon emission diagrams for renormalization and collinear divergences are later removed by the mass factorization. Thus, the bare partonic coefficient function $\bar{\Delta}$ is expressed in terms of Drell-Yan coefficient function after the mass factorization as,

$$\bar{\Delta}_{ab}^{jj'}(z,Q^2,\epsilon) = \sum_{c,d} \Gamma_{ca}(z,\mu_F,1/\epsilon) \bigotimes \Gamma_{db}(z,\mu_F,1/\epsilon) \bigotimes \Delta_{ab}^{jj'}(z,Q^2,\epsilon) \tag{3.29}$$

The μ_F is the factorization scale and the $\otimes$ stands for the operation

$$f \bigotimes g(x) = \int_x^1 \frac{dy}{y} f(y) g(x/y) \tag{3.30}$$

and $\Gamma_{ca}(z,\mu_F,1/\epsilon$'s are the singular transition functions called Altarelli-Parisi kernels. These functions have perturbative expansion in powers of $a_s s = \alpha_s/4\pi$.

$$\Gamma_{ab}(z,\mu_F,1/\epsilon = \sum_{i=0}^{\infty} a_s^i(\mu_F^2) \Gamma_{ab}^i(z,\mu) \tag{3.31}$$

The $\bar{M}S$ mass factorization scheme we find these functions to be

$$\Gamma_{ab}^0(z,\mu) = \delta_{ab}\delta(1-z)\Gamma_{ab}^1(z,\mu) = -\frac{1}{\epsilon} P_{ab}^{(0)}(z) \tag{3.32}$$

with P^i's being the Altarelli-Parisi splitting functions. Writing eq. 28 up to the order $a_s s = \alpha_s/4\pi$ we can see the relations,

$$\bar{\Delta}_{q\bar{q}}^{jj'} = \Delta_{q\bar{q}}^{(o),jj'} + a_s \frac{2}{\epsilon} \Gamma_{q\bar{q}}^{(1)} \bigotimes \Delta_{q\bar{q}}^{(0),jj'} + \Delta_{q\bar{q}}^{(1),jj'}$$

$$\bar{\Delta}_{qg}^{jj'} = a_s \frac{2}{\epsilon} \Gamma_{qg}^{(1)} \bigotimes \Delta_{qg}^{(0),jj'} + \Delta_{qg}^{(1),jj'} \tag{3.33}$$

which is finite coefficient function. These coefficient functions helps us in finding the physical hadronic cross-section when integrted with parton distribution functions. According to Ref. [4] the renormalized parton-parton fluxes are given by the relations,

$$H_{q\bar{q}}(x_1,x_2,\mu_F^2) = f_q^{P_1}(x_1,\mu_F^2) f_{\bar{q}}^{P_2}(x_2,\mu_F^2) + f_{\bar{q}}^{P_1}(x_1,\mu_F^2) f_q^{P_2}(x_2,\mu_F^2)$$

$$H_{gq}(x_1,x_2,\mu_F^2) = f_g^{P_1}(x_1,\mu_F^2)(f_q^{P_2}(x_2,\mu_F^2) + f_{\bar{q}}^{P^2}(x_2,\mu_F^2)) \tag{3.34}$$

$$H_{qg}(x_1,x_2,\mu_F^2) = H_{gq}(x_1,x_2,\mu_F^2)$$

This makes the inclusive differential cross-section to be

$$2S\frac{d\sigma^{P_1P_2}}{dQ^2}(\tau,Q^2) = \sum_q \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dz\delta(\tau - zx_1x_2)\left[F_{SM+VA,q}G_{SM+VA,q} + F_{SP,q}G_{SP,q}\right]$$

$$G_{SM+VA,q} = H_{q\bar{q}}(x_1,x_2,\mu_F^2)\left[\Delta_{q\bar{q}}^{(0),SM}(z,Q^2,\mu_F^2) + a_s\Delta_{q\bar{q}}^{(1),SM}(z,Q^2,\mu_F^2)\right]$$

$$+\left[H_{qg}(x_1,x_2,\mu_F^2) + H_{gq}(x_1,x_2,\mu_F^2)\right]a_s\Delta_{q\bar{q}}^{(1),SM}(z,\mu_F^2) \tag{3.35}$$

$$G_{SP,q} = H_{q\bar{q}}(x_1,x_2,\mu_F^2)\left[\Delta_{q\bar{q}}^{(0),SP}(z,Q^2,\mu_F^2) + a_s\Delta_{q\bar{q}}^{(1),SP}(z,Q^2,\mu_F^2)\right]$$

$$+\left[H_{qg}(x_1,x_2,\mu_F^2) + H_{gq}(x_1,x_2,\mu_F^2)\right]a_s\Delta_{q\bar{q}}^{(1),SP}(z,\mu_F^2)$$

and constants $F_{SP,q}$ and $F_{SM+VA,q}$ are completely dependent on the coupling constants and propagators,

$$F_{SM+VA,q} = \frac{4\alpha^2}{3}\Bigg[\Big[\frac{e_q^2}{Q^2} - 2e_q g_l^V g_q^V Z_Q \frac{Q^2 - M_Z^2}{Q^2} + \frac{1}{4}\big((g_l^R)^2 + (g_l^L)^2\big)\big((g_q^R)^2 + (g_q^L)^2\big)Z_Q\Big]$$

$$+\frac{2}{\alpha\Lambda^2}\Big(-e_q \sum_{i,j=L,R}\eta_{ij} + Z_Q(Q^2 - M_Z^2)\sum_{i,j=L,R}\eta_{ij} g_q^i g_l^j + \frac{Q^2}{\alpha^2\Lambda^4}\sum_{i,j=L,R}|\eta_{ij}|^2\Big)\Bigg] \tag{3.36}$$

$$F_{SP,q} = \frac{Q^2}{\Lambda^4}\sum_{i,j=L,R}|\zeta_{ij}|^2$$

where, $Z \equiv \frac{Q^2}{(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}$

We need the expressions for the terms$\Delta_{q\bar{q}}^{(0),VA}$, $\Delta_{q\bar{q}}^{(1),VA}$, $\Delta_{q(\bar{q})g}^{(1),VA}$ which are coefficient functions in the case vector-axial vector couplings. Simultaneously, we need coefficient functions for Scalar-pseudoscalar couplings which are $\Delta_{q\bar{q}}^{(0),SP}$, $\Delta_{q\bar{q}}^{(1),SP}$, $\Delta_{q(\bar{q})g}^{(1),SP}$. These terms are discussed in details in Ref.[4].

Till now we have discussed only till the oder next to leadg orders. But, the calculation of next next to leading order QCD correction requires the highers orders for Altarelli-Parisi kernels and its expression can foung in the Ref. [3].

$$\Gamma_{ab}^2(z,\mu) = \frac{1}{2\epsilon^2}\sum_c\Big(P_{ac}^{(0)}(z)\bigotimes P_{cb}^{(0)}(z) + 2\beta_0 P_{ab}^{(0)}(z)\Big) + \frac{1}{\epsilon}P_{ab}^{(1)}(z) \tag{3.37}$$

and we determine the coefficient functions of the order a_s^2 (suppressing the argument z and ϵ) to be,

$$\Delta_q^{\bar{(2)}}q = \Delta_{q\bar{q}}^{(2)} + 2\Gamma_{q\bar{q}}^{(1)}\bigotimes\Delta_{q\bar{q}}^{(0)} + \frac{2}{\epsilon}P_{qq}^0\bigotimes\Delta_{q\bar{q}}^{(1)} + \frac{2}{\epsilon}P_{qg}^0\bigotimes\Delta^{(2)} + \frac{1}{\epsilon^2}P_{qq}^0\bigotimes P_{qq}^0\bigotimes\Delta_{q\bar{q}}^{(0)}$$

$$\Delta_{qg}^{\bar{(2)}} = \Delta_{qg}^{(2)} + \Gamma_{\bar{q}g}^{(1)}\bigotimes\Delta_{q\bar{q}}^{(2)} + \frac{1}{\epsilon}P_{qq}^0\bigotimes\Delta_{qg}^{(0)} + \frac{1}{\epsilon}P_{qg}^0\bigotimes\Delta_{q\bar{q}}^{(1)} + \frac{1}{\epsilon}P_{gg}^0\bigotimes\Delta_{qg}^{(1)} + \frac{1}{\epsilon^2}P_{qq}^0\bigotimes P_{qg}^0\bigotimes\Delta_{q\bar{q}}^{(0)}$$

$$\Delta_{gg}^{\bar{(2)}} = \Delta_{gg}^{(2)} + \frac{4}{\epsilon}P_{qg}^0\bigotimes\Delta_{qg}^{(1)} + \frac{1}{\epsilon^2}P_{qg}^0\bigotimes P_{qg}^0\bigotimes\Delta_{q\bar{q}}^{(0)} \tag{3.38}$$

To get the full expressions for the coefficient functions of order a_s^2 we convolute the above equations by substituting the expressions for the lower order coefficients. The complete expressions can be found in Ref. [3].

The corrections beyond the Standard Model can be systematically calculated by effective Lagrangians. As discussed in Ref. [2], "Making few assumptions like particle content and symmetries and neglecting lepton number violation", significant correction terms are given by dimension six-operators.

$$L_{eff} = L_{SM} + \sum_i \frac{c_i}{v^2} O_i \tag{3.39}$$

The four-fermion contact interactions of the form LLQQ where Leptons(electron, muon, tau) are denoted by L and Quarks(up, down, charm, strange, top, bottom) are denoted by Q, which are absent in SM are taken into account in this effective Lagrangian as dimension six operators. The most general parameterization of lepton-quark four-fermion is achieved as mentioned in the Ref. [2].

Chirality Conserving,

$$O_{lq}^{(1)} = (\bar{l}\gamma^\mu l)(\bar{q}\gamma_\mu q) \qquad O_{lq}^{(3)} = (\bar{l}\sigma_I\gamma^\mu l)(\bar{q}\sigma_I\gamma_\mu q)$$

$$O_{lu} = (\bar{l}\gamma^\mu l)(\bar{u}\gamma_\mu u) \qquad O_{ld} = (\bar{l}\gamma^\mu l)(\bar{d}\gamma_\mu d)$$

$$O_{eq} = (\bar{e}\gamma^\mu e)(\bar{q}\gamma_\mu q) \qquad O_{eu} = (\bar{e}\gamma^\mu e)(\bar{u}\gamma_\mu u)$$

$$O_{ed} = (\bar{e}\gamma^\mu e)(\bar{d}\gamma_\mu d) \tag{3.40}$$

and Chirality violating,

$$O_{qd\epsilon} = (\bar{l}e)(\bar{d}q) \qquad O_{lq\epsilon} = (\bar{l}e)\epsilon(\bar{q}^T u) \qquad O_{ql\epsilon} = (\bar{q}e)\epsilon(\bar{l}^T u) \tag{3.41}$$

Where e,u and d are SM lepton and quark singletes; σ_I are Pauli matrices; l denotes SM lepton doublet and q denotes quark doublets andd $\epsilon = i\sigma_2$.

These operators contribute to the differential cross-section of Drell-Yan process. The partonic level cross-sections are,

$$48\pi\frac{d\hat{\sigma}}{d\hat{t}}(\bar{u}u\to\bar{l}l) = \left[\left|A^{SM}_{u_Ll_R}+\frac{c_{qe}}{v^2}\right|^2+\left|A^{SM}_{u_Rl_L}+\frac{c_{lu}}{v^2}\right|^2+\frac{1}{2v^4}\left[|c_{ql\epsilon}|^2+Re(c_{lq\epsilon}c^*_{ql\epsilon})\right]\right]\frac{\hat{t}^2}{\hat{s}^2}$$

$$+\left[\left|A^{SM}_{u_Ll_L}+\frac{c^(1)_{lq}-c^(3)_{lq}}{v^2}\right|^2+\left|A^{SM}_{u_Rl_R}+\frac{c_{eu}}{v^2}\right|^2-\frac{1}{2v^4}Re(c_{lq\epsilon}c^*_{ql\epsilon})\right]\frac{\hat{u}^2}{\hat{s}^2}$$

$$+\frac{1}{2v^4}\left[|c_{ql\epsilon}|^2+Re(c_{lq\epsilon}c^*_{ql\epsilon})\right], \tag{3.42}$$

$$48\pi\frac{d\hat{\sigma}}{d\hat{t}}(\bar{d}d\to\bar{l}l) = \left[\left|A^{SM}_{d_Ll_R}+\frac{c_{qe}}{v^2}\right|^2+\left|A^{SM}_{d_Rl_L}+\frac{c_{ld}}{v^2}\right|^2\right]\frac{\hat{t}^2}{\hat{s}^2}+\left[\left|A^{SM}_{d_Ll_L}+\frac{c^(1)_{lq}-c^(3)_{lq}}{v^2}\right|^2+\left|A^{SM}_{d_Rl_R}+\frac{c_{ed}}{v^2}\right|^2\right]\frac{\hat{u}^2}{\hat{s}^2}$$

$$+\frac{|c_{qd\epsilon}|^2}{2v^4}, \tag{3.43}$$

where, $A^{SM}_{d_\psi l_\phi} = \frac{e^2Q_\psi Q_\phi}{\hat{s}}+\frac{g_\psi g_\phi}{\hat{s}-m_Z^2+im_Z\Gamma_Z}$

with $g_\psi = \frac{g}{c_w}[T^3_\psi - s^2_W Q_\psi]$, Q is electric charge, T^3 weak isosopin component and $g, m_Z, \Gamma_Z$are the $SU(2)_L$ coupling, Z-boson mass and width respectively. $s_W(c_W)$ are trignometric sine(cosine) function of the weak angle. Neglecting corrections of order $m_Z^2/\hat{s} << 1$

$$\sigma = \sigma^{SM}+\frac{1}{\Lambda^2}\sum_{q=u,d}[F_1^qA_1^q+F_2^qA_2^q]+\frac{1}{\Lambda^4}\sum_{q=u,d}[G_1^qB_1^q+G_2^qB_2^q+G_3^qB_3^q], \tag{3.44}$$

With the coefficients $A^{u,d}_{1,2,3}$ and $B^{u,d}_{1,2,3}$ expressed as,

$$A_1^u = [e^2Q_uQe+g_{u_L}g_{e_L}](c^{(1)}_{lq}-c^{(3)}_{lq})+[e^2Q_uQe+g_{u_R}g_{eR}](c_{eu})$$

$$A_2^u = [e^2 Q_u Qe + g_{u_L} g_{e_R}](c_{qe}) + [e^2 Q_u Qe + g_{u_R} g_{e_L}](c_{lu})$$

$$A_1^d = [e^2 Q_d Qe + g_{d_L} g_{e_L}](c_{lq}^{(1)} + c_{lq}^{(3)}) + [e^2 Q_d Qe + g_{d_R} g_{e_R}](c_{ed})$$

$$A_2^d = [e^2 Q_d Qe + g_{d_L} g_{e_R}](c_{qe}) + [e^2 Q_d Qe + g_{d_R} g_{e_L}](c_{ld}) \tag{3.45}$$

$$B_1^u = 4(c_{lq}^{(1)} - c_{lq}^{(3)})^2 + c_{eu}^2 - 2Re(c_{lq\epsilon} c_{ql\epsilon}^*) \qquad B_1^d = (c_{lq}^{(1)} - + c_{lq}^{(3)})^2 + 4c_{qd}^2$$

$$B_2^u = 4c_{qe}^2 + 4c_{lu}^2 + |c_{ql\epsilon}|^2 + 2Re(c_{lq\epsilon} c_{ql\epsilon}^*) \qquad B_2^d = 4c_{qe}^2 + 4c_{ld}^2$$

$$B_3^u = 2|c_{ql\epsilon}|^2 + 2Re(c_{lq\epsilon} c_{ql\epsilon}^*) \qquad B_3^d = 2|c_{qde}|^2$$

From these equation it is clear that the coefficients $A_{1,2,3}^{u,d}$ and $B_{1,2,3}^{u,d}$ are dependent on four-fermion operators.

Chapter 4

Discussion

As discussed earlier we have achieved in 3.44, to express the cross-section as per the form $\sigma^i = \sigma^i_{SM} + \sum_n a^i_n c_n + \sum_{n\leq m} b^{nm} c_n c_m$.Having obtained the analytic structure of six-dimensional operators corresponding to the Lepton-Quark contact interaction we can proceed to developing code which can calculate the differential cross-section for the Drell-Yan pair production. The NLO corrections for Drell-Yan pair production are performed in the Ref. [4] and NNLO corrections for Slepton production are performed in Ref. [3]. Modifying existing code for NLO correction of Lepton-Quark with the referring these NNLO QCD corrections we hope to obtain desired result.

Chapter 5

Summary and Future Plan

In the first half, the project was focused on learning the non-Abelian gauge theories, studying loop divergences in QCD and calculation of cross section for different processes involving electromagnetic and strong interactions. Latter half involved the review of the literature in the field of Standard Model Effective Field theory and NLO and NNLO QCD corrections to the processes at hadron colliders and the analytical modifications to existing model and development of the FORTRAN code to include the six-dimensional operators. In the future, we plan to find the matrix square elements for the process which involves with the help of the developed code including these beyond standard model interactions.

Bibliography

[1] Peskin, Michael E. An introduction to quantum field theory. CRC Press, 2018.

[2] de Blas, Jorge, Mikael Chala, and José Santiago. "Global constraints on lepton-quark contact interactions." Physical Review D 88.9 (2013): 095011.

[3] Majhi, Swapan, Prakash Mathews, and V. Ravindran. "NNLO QCD corrections to the resonant sneutrino/slepton production at hadron colliders." Nuclear Physics B 850.2 (2011): 287-320.

[4] Choudhury, Debajyoti, Swapan Majhi, and Vajravelu Ravindran. "NLO corrections to lepton pair production beyond the standard model at hadron colliders." Journal of High Energy Physics 2006.01 (2006): 027.

[5] Jogesh C. Pati, Abdus Salam and J.A. Strathdee Phys.Lett. B59 (1975) 265; H. Fritzsch and G. Mandelbaum, Phys.Lett. B102 (1981) 319; W. Buchmuller, R.D. Peccei and T. Yanagida, Phys.Lett. B124 (1983) 67; Nucl.Phys.B227 (1983) 503; Nucl.Phys.B237 (1984) 53; U. Baur and H. Fritzsch, Phys.Lett. B134 (1984) 105; Xiaoyuan Li and R.E. Marshak, Nucl.Phys.B268 (1986) 383; I. Bars, J.F. Gunion and M. Kwan Nucl.Phys.B269 (1986) 421; G. Domokos and S. Kovesi-Domokos, Phys.Lett.B266 (1991) 87; Jonathan L. Rosner and Davison E. Soper Phys.Rev.D45 (1992) 3206; Markus A. Luty and Rabindra N. Mohapatra, Phys.Lett.B396 (1997) 161 [hep-ph/9611343]; K. Hagiwara, K. Hikasa and M. Tanabashi, Phys.Rev.D66 (2002) 010001; Phys.Lett.B592 (2004) 1

[6] For a review and additional references, see R.R. Volkas and G.C. Joshi, Phys. Rep. 159 (1988) 303.