RESOURCE LIMITED PLAYERS IN LARGE GAMES

A thesis submitted towards partial fulfilment of
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by

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Certificate

This is to certify that this thesis entitled *Resource Limited Players in Large Games* submitted towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research Pune represents original research carried out by *Tarun Ayitam* at *IMSc, Chennai*, under the supervision of *Prof R.Ramanujam* during the academic year 2013-2014.

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Abstract

General game theory assumes that a rational player would play an optimal strategy if one exists. When you consider instead programs as players (not only in the context of gaming, but also stock market, e-trade, web services etc), this is not tenable, since computing an optimal strategy may not be feasible.

Moreover, the notion of player type in game theory assumes complete knowledge of mutual beliefs of all players, which is a problem for resource limited players. there have been attempts from automata theory and complexity theory to address the question, where the player is assumed to be a finite state automaton or a probabilistic polynomial time turing machine (with bounded error, a BPP machine in short). However, many foundational questions need to be reworked.

The project involves modelling such games and looking at some interesting questions within that model.

Contents

Chapter 1

Introduction

Babichenko introduces a model and suggests Dynamics within that model, in his paper [3]. However, the model assumes that the players can only view their actions and their individual payoffs. A few negative results(that strategy entropy leading play to nash equilibrium doesn't exist) have been presented in his paper [3]. However, in a game with complete information, algorithms to compute nash equilibria are known. We explore a model in which players possess partial information. We try to optimize the Dynamics suggested by Babichenko, in this new model.

Players are limited by resources- space and computational resources. These limitations are captured by modelling the strategies with automata [2]. In a large game, number of players is sufficiently large to assume that the players cannot observe the number of players and their outcomes. We explore a model which can capture these limitations, which one usually encounters in a large game.

I had studied Neyman and Okada's papers on *Strategic entropy*[1] and two person repeated games with finite automata [2] to understand the concept of Strategic entropy and to be able to appreciate modelling of repeated games using Automata. Babichenko's paper on *Completely Uncoupled Dynamics and Nash Equilibrium* describes a model for large games and a few results exploring existence of strategy mapping under certain conditions [3]

We have suggested a model which allows players to see a probability distribution over the collection of action sets. This allows the players to have partial information about the state of the game. We have explored the dynamics in this model and presented a proof of correctness of the suggested dynamics.

The second chapter introduces relevant definitions and notation. Known results are listed and proved in the third chapter. Our model is introduced in the fourth chapter. Strategy mapping and a proof of convergence to a nash equilibrium(if one exists), is presented in the fifth chapter. In the sixth chapter, a few interesting questions and further directions for research are presented.

Chapter 2

Preliminaries

2.1 Notation

$\Delta(A)$ is the set of probability distributions on A.

We will use σ to represent a mixed strategy and s to represent a pure strategy.
If $a = (a_1, a_2 \ldots a_n)$ is an n-tuple, then $a_{-i} = (a_1, a_2 \ldots a_{i-1}, a_{i+1} \ldots a_n)$

For subset $\beta \subset \alpha$ and for a function f over α, let $f|_\beta$ be the function restricted to β.

2.2 Definitions

2.2.1 Game

A basic static(one-shot) finite game G is given in strategic form as follows: There are $n \geq 2$ players, denoted by $i = 1, 2, \ldots, n$. $N = \{1, 2, \ldots, n\}$ is the set of all the players.

For simplicity, we will assume that a countable set A_C induces induces all potential actions of the players. Thus an action set of players i is a finite subset $A^i \subset C$, where $|A^i| \geq 2$. Denote by $\mathfrak{B}$ all the action sets for a single player, $A = A^1 \times A^2 \times \ldots \times A^n$ is the set of action profiles.

The payoff function(or utility function) of player i is a real valued function $u^i : A \to \mathbb{R}$. We assume(for simplicity) that all the payoffs are bounded by a finite value K (i.e, $|u^i(a)| \leq K$ for every $i \in \mathbb{N}$ and every $a \in A$). $u = (u^1, u^2, \ldots, u^n)$ is the payoff function profile, we will call it a *payoff function* from now on. We will identify a game Γ through its payoff function u, for convenience.

We will denote set of all possible mixed actions of a player i by $\Delta(A^i)$, it is simply the probability simplex over A^i.

We can extend the payoff functions u^i multilinearly from A to $\Delta(A)$:

$u^i : \Delta(A^1) \times \Delta(A^2) \times \ldots \times \Delta(A^n) \rightarrow \mathbb{R}$

$A^{-i} = (a^1, \ldots, a^{i-1}, a^{i+1}, \ldots, a^n)$ is the set of actions of all players except i. Simmilarly, $A^{-i} = A^1 x \ldots \times A^{i-1} \times A^{i+1} \times \ldots \times A^n$ consists of actions of all the players but the player i.

We will call an action $a^i \in A^i$ as the best response to a^{-i} if $u^i(a^i, a^{-i}) \geq u^i(\bar{a^i}, a^{-i})$ for every $\bar{a^i} \in A^i$. An action profile $a = (a^1, a^2, \ldots a^n) \in A$, s.t. a^i is a best response to $a^{-i} \forall i$, is a pure Nash equilibrium. Simmilarly, when we have a mixed action profile $x = (x^1, x^2, \ldots x^n), x^i \in \Delta(A^i)$, x^i is said to be an ϵ-best reply to x^{-i} if $u^i|(x^i, x^{-i})| \geq |u^i(y^i, x^{-i})| - \epsilon$ for all $y^i \in \Delta(A^i)$. $x = (x^1, x^2, \ldots, x^n)$ is a Nash ϵ-equilibrium, if for all $i \in \mathbb{N}$, x^i is an ϵ-best reply to x^{-i}.

Dynamic setup

Repeated play, at discrete times $t = 1, 2, \ldots$ of the static game Γ is a dynamic play. $x^i(t) \in \Delta(A^i)$ denotes the mixed action played by player i at the time t, and $a^i(t) \in A^i$ is the action realized by the strategy. $x(t) = (x^i(t)_{i=1}^n) \in \Delta(A^i)$ and $a(t) = (a^i(t))_{i=1}^n \in A$ denote the profiles.

As per the 'completely uncoupled' restriction, we will assume that each player i, at the end of time t, observes the action that he played $a^i(t)$ and his own payoff $u^i(a(t))$. At time t, player i will know his previous actions and incentives $o^i(t) = ((a^i(t'))_{t'=1}^t), (u^i(a(t'))_{t'=1}^t)$, which we will call the observations sequence of that player. Consider the set $O^*_{A^i}$ of all potential observations sequences of a participant with action set A^i.

For a game with action set A, history of play is $h(t) = (a(1), a(2), \ldots, a(t))$, where $a(t') \in A$ for every $t' < t$. Let $H_{t,A}$ denote the set of all possible histories of the play till time t, and $H^*_A = \cup_{t=0}^{\infty} H_{t,A}$.

2.2.2 Strategies

A Borel measurable mapping $f_B : O_B^* \to |\Delta(B)|$ which assigns a mixed action to every potential observations sequence of player, is a completely uncoupled strategy of a player with action set B. For a player with action set B, we will denote by F_B the set of all completely uncoupled strategies. The set of all completely uncoupled strategies is denoted by $\mathfrak{F} = \cup_B F_B$[1].

The knowledge of each player is his action set, before the game starts. We will define a completely uncoupled strategy mapping $\phi : \mathfrak{B} \to \mathfrak{F}$ to be a mapping which assigns a completely uncoupled strategy $\phi(A^i) = f_{A^i} \in F_{A^i}$ for every actions set $A^i \in \mathfrak{B}$. For a strategy mapping ϕ, and a game having action profile $A = A^1 \times A^2 \times \ldots A^n$, players' strategies will be $(f_{A^1}, f_{A^2}, \ldots, f_{A^n})$.

Denote $f = (f_{A^1}, f_{A^2} \ldots, f_{A^n})$ to be the strategy profile. Probabilistic play of the game is defined by the stategy mapping. We will call a history of play $h(t)$ as realizable by a strategy profile f, if after time t, when strategy profile f is followed, the probability of history being equal to h(t) is greater than 0.

2.2.3 Types of Convergence

A probabilistic play of the game is included in every strategy profile f. Consider some equilibrium concept E, say a pure Nash equilibrium or a mixed Nash equilirbium or a Nash ϵ-equilibrium. f *leads to almost sure convergence of play to E* if there exists T such that $x(t) \in E$ for every $t > T$, almost surely.

Given $\epsilon \geq 0$, f *leads to play of E* with frequency $1 - \epsilon$ if almost surely (i.e. with probability 1)
$\lim_{t\to\infty} \inf \frac{|\{t|x(t)\in E\}|}{t} \geq 1 - \epsilon$

Note: Almost sure convergence of play is a strictly stronger than convergence with frequency 1. This is because an action not in E can be played infinitely many times, but with a frequency tending to zero.

A strategy mapping ϕ leads to a.s. convergence of the play to E if $f = (\phi(A^i))_{i=1}^n$ leads to a.s. convergence of the play to E in every finite game.

Similarly, a strategy mapping ϕ leads to play of E with frequency $1 - \epsilon$ if $f = (\phi(A^i))_{i=1}^n$ leads to play of E with frequency $1 - \epsilon$ in every finite game.

2.2.4 Genericity

For every game u with n-players and a set of actions B, we can consider u^i as an element of $[-K, K]^{|B|}$, and u as an element of $[-K, K]^{n|B|}$. Therefore, we can define Lebesgue measure $\mu(\alpha)$ of game set α as a measure in $(R)^{n|B|}$. In the same way, we define the measure of a set $\alpha \subset U^i$ or $\alpha \subset U^{-i}$.

A certain property is said to be valid in every generic game with action profile set B, if the property holds in all the games with action profile set B, but for a subset of games having measure zero. A certain property is said to be valid in every generic game if $\forall B \in \mathfrak{B}$ the property is valid in almost every game with action profile set B.

By generic game, we mean a collection of all games but a set of games which has a measure zero.

2.2.5 Root node

Let $P = \{x | (a, x) \in O$ for some $a \in A^n\}$
Let $\beta = \{A_0 \in A| \quad \exists p \in P \quad$ s.t. $p_i(a) \neq p \quad \forall a \in A_0 \times A^{n-1}\}$
If $|\beta| = |A|^{n-1} - 1$ then the play observed by player i, is in its root node.

2.2.6 Automata

Definition 1. A deterministic finite state automaton is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ [2]

Q, Σ are finite sets, $q_0 \in Q$,$F \subseteq Q$ and δ is a function from $Q \times \Sigma$ to Q.
Q is the set of states,Σ is the Alphabet, q_0 is the initial state, F is the set of accept states and δ is the transition function.

Consider a game G=(A,B,r), and an automaton $M = (Q, q_1, f, g)$. f is an action map from Q to A. g is a map from $Q \times B$ to Q.

The automaton plays a repeated game as follows. The aumaton chooses a strategy in A, given by the function f. State changes based on the values of the functions f and g.
$a_n = f(q_n), q_{n+1} = g(q_n, b_n)$,where b_n is the action of the other players in the n^{th} round.

Every Automaton induces a pure strategy $s \in S$.
Conversely every strategy s can be simulated by an automata(possibly of infinite size).

If a pure strategy s is equivalent to that induced by an automaton, it is said to be implementable by that automaton.

2.2.7 Completely Uncoupled Dynamics

An interated play of a game is said to be Completely uncoupled dynamics if at every time, every player knows just his action set and history of own past actions and the payoffs; He doesn't pocess information about the actions and payoffs of others.

Babichenko's paper's proves the following:
(i) There exists no completely uncoupled dynamics which leads play to almost sure convergence, to pure Nash equilibria, in almost all games possessing pure Nash equilibria.
(ii)Result (i) doesn't hold for Nash ϵ-equilibrium. We will show that completely uncoupled dynamics which lead to almost sure convergence of play to Nash ϵ-equilibrium

However, there is no strategy mapping, in games of incomplete information.

Chapter 3

Known Results

3.1 Results

The following results have been proved in Babichenko's paper *Completely uncoupled dynamics and Nash equilibria*.
Theorem 2, Theorem 6, Theorem 13 are negative results, they specify conditions under which we don't have a strategy mapping which can lead play to convergence to a nash equilibrium.

Theorem 5, Theorem 7, Theorem 9, Theorem 10 specify the conditions under which we have such a strategy mapping.

Theorem 2. *Let $A = A^1 \times A^2 \times A^3 \dots A^n$ be an action profile set such that $A^1 = A^2$. Then there is no completely uncoupled strategy mappin g that leads to a.s. convergence of Let $A = A^1 \times A^2 \times A^3 \times ...A^n$ be an action profile set such that $A^1 = A^2$. Then there is no completely uncoupled strategy mapping play to pure Nash equilibria in every generic n-person game with action profile set A that possesses a pure Nash equilibrium, and also in every generic $(n-1)$ person game with action profile set $A^2 \times A^3 \times ...A^n$ that possesses a pure Nash equilibrium.[3]*

Corollary 3. *There is no completely uncoupled strategy mapping that leads to a.s. convergence of play to pure Nash equilibria in every finite generic game.[3]*

Since no strategy mapping exists when we have two action sets being equal, we don't have such a strategy mapping in a general case.

Corollary 4. *There is no completely uncoupled strategy that leads to a.s. convergence of play to mixed Nash equilibrium in every finite generic game.[3]*

In the proof of Theorem2, we show a set of games, in which such a strategy mapping doesn't exist. From that set, we derive another set in which we have no strategy mapping which can guaranteed convergence to a pure or mixed Nash equilibrium. This is used to prove the corollary.

Theorem 5. *There exists a completely uncoupled strategy mapping that leads a.s. convergence of play to a pure Nash equilibrium in every finite game with distinct action sets that has a pure Nash equilibrium.[3]*

Theorem 6. *For every $\epsilon < 1/2$ there is no completely uncoupled strategy mapping that leads to play of pure Nash equilibria with frequency $1-\epsilon$, in every game with more than two players, where such an equilibrium exists. [3]*

Theorem 7. *If either,*
(1) The additional information of every player $i \in N$ is his own index; i.e., $\alpha^i = i$
or
(2) The additional information of every player $i \in N$ is the total number of players, i.e. $\alpha^i = n$
then there exists a completely uncoupled strategy mapping with additional information α that leads to a.s. convergence of play to pure Nash equilibrium in every finite generic game where such an equilibrium exists.[3]

By considering proof of theorem7, we conclude that additional information doesn't effect the proof. Hence we conclude that no strategy mapping exists, even with additional information. This argument is used to prove the corolllary.

Corollary 8. *For every $\epsilon < 1/2$ there is no completely uncoupled strategy mapping with uncoupled additional information which leads to pay of pure Nash equilibria with frequency $1-\epsilon$, in every finite game with more than two players, where such an equilibrium exists.[3]*

Theorem 9. *For, every $\epsilon > 0$, there exists a completely uncoupled strategy mapping that leads to a.s. convergence of play to Nash ϵ-equilibrium in every finite generic game.[3]*

Theorem 10. *For every $\epsilon > 0$, there exists a completely uncoupled strategy mapping that leads to play of Nash ϵ-equilibrium with frequency $1-\epsilon$ in every finite game.[3]*

Proof of Theorem13 is quite complex and is included in Babichenko's discussion paper. I will not be stating the proof of Theorem13, However, the following remarks which follow from Theorem 13 are quite interesting and important.

Remark 11. Corollaries 8 and 9, the negative results, hold in the finite memory dynamics case even if convergence to pure/mixed Nash equilibria is required with frequency 1 (rather than demanding a stricter almost sure convergence)

Remark 12. All the positive results hold also for the case of finite memory, if we assume that unique encoding in finite memory is possible for every payoff in the game. This is particularly important as it helps us in handling the finite memory case by assuming that unique finite encoding of payoffs is possible

Theorem 13. *For $\epsilon \leq 1/8$, there is no completely uncoupled strategy mapping into finite-memory strategies that leads to play of Nash ϵ-equilibria with frequency 1 in every game. [3]*

3.2 Proofs

Theorem 2: Let $A = A^1 \times A^2 \times A^3 \dots A^n$ be an action profile set such that $A^1 = A^2$. Then there is no completely uncoupled strategy mapping that leads to a.s. convergence of Let $A = A^1 \times A^2 \times A^3 \times ...A^n$ be an action profile set such that $A^1 = A^2$. Then there is no completely uncoupled strategy mappinplay to pure Nash equilibria in every generic n-person game with action profile set A that possesses a pure Nash equilibrium, and also in every generic $(n-1)$ person game with action profile set $A^2 \times A^3 \times ...A^n$ that possesses a pure Nash equilibrium. [3]

Proof: Proof by Contradiction. Let the action profile set be $A = A^1 \times A^2 \times A^3 ...A^n$. We are interested in the case when not all the action sets are distinct, so atleast two sets must be equal. Without loss of generality let us assume that $A^1 = A^2$. The other action sets $A^3, A^4, A^5, \dots A^n$ may or may not be distinct.

Now we need to prove that there is no completely uncoupled strategy mapping which almost surely converges the game to pure Nash equilibria, in every generic n-person game with action profile set A which possesses a pure Nash equilibrium, and also in every generic $(n-1)$ person game with action profile set $A^2 x A^3 ...A^n$ which possesses a pure Nash equilibrium.

Let us suppose such a strategy mapping ϕ does exist. The strategy mapping ϕ should lead play to convergence to a pure Nash equilibria in generic games on both the action profile sets- A, A^{-1} and A^{-2}. Since the strategy mapping ϕ leads play to pure Nash equilibria in generic games with action profile sets A^{-1} and A^{-2}. We will try to produce a set of payoff functions Q with a positive measure such that every game in it has a unique pure nash equilibrium and that the strategy mapping doesn't converge the game to the unique pure Nash equilibrium. Now, we need to prove the existence of a Q, the set of such payoff functions.

We will construct Q with the help of sets $SPN_{A^{-1}}$ and Z_h. Let $SPN_{A^{-1}}$ be the set of games over action set A^{-1} such that
(i) all the payoffs are bound by a fixed $L > 0$
(ii) Each game has a unique pure Nash equilibrium.

Claim: $\lambda(SPN_{A^{-1}}) > 0$.
Proof: We can construct such a positive measure set by providing every player with a strictly dominant action, and then taking a permutation of the payoffs in order to get a positive measure set. For every $v \in SPN_{A^{-1}}$, let $b(v)$ be the corresponding unique pure Nash equilibrium. The strategies $(f_{m^i})_{i=2}^n$ of players $2, 3 \dots n$ lead to $b(v)$ for almost every game v. Let $S \subset SPN_{A^{-1}}$ be the set of games such

that the strategy profiles leads the game to a unique pure nash equilibrium in it. Let b(v) be the corresponding unique pure Nash equilibrium. Since the strategies $(f_{A^i})_{i=2}^n$ leads to $b(v)$, we have $\lambda(S) = \lambda(SPN_{A^{-1}}) > 0$.

We will now construct a set Z_h from a history h(length t). Z_h is the subset of all games such that there is a chance of history h being realized and that the h is an absorbing state-once players play h, they will continue to play a fixed action continuously, with probability 1. Formally, for every $h \in H_{t,A^{-1}}$, let Z_h be the subset of all games v over action set A^{-1} such that:

1. h is realizable by $(f_m^i)_{i=2}^n$
2. If h is played, then from the time t+1, and on, the players play some action $b \in A^{-1}$ with probability 1.

Then

$$\cup_{h \in H^*_{A^{-1}}} Z_h \supset S$$

This is true because the game converges to $b(v)$ in every member of S, after a finite amount of time. Thus every member of S is a member of Z_{h_1} for some history h_1.

f_{A^i} is Borel measurable $\forall i \in \mathbb{N}$. So, Z_h is a measurable set. There is a countable number of histories $h \in H^*_{A^{-1}}$ and S has a positive measure; So, we have a history $\bar{h}(\bar{t}) \in H_{A^{-1}}$(a history of length $\bar{t}$) such that $Z_{\bar{h}}$ has a positive measure. We will denote $R = Z_{\bar{h}}$.

We have $A^1 = A^2$, and so their strategies will identical as the strategy mapping is a function of action set alone, i.e. $f_{A^1} = f_{A^2}$; So, the history $\bar{h}(\bar{t}) \in H_{\bar{t},A^{-2}}$ and the subset of games $R = Z_{\bar{h}}$ satisfy the following conditions:

1. $\bar{h}$ is realizable by $(f_{m^i})_{i=1,i\neq 2}^n$
2. if $\bar{h}$ is played, then from time $\bar{t}$+1 on the players play the action $(\bar{a^1}, \bar{a^3}, \bar{a^4} \ldots, \bar{a^n}) \in A^{-2}$ with probability 1.

We will now define Q. We will define it such that it's games have payoff functions agreeing with the previous game along the diagonal. We will construct the payoff matrix in such a way that equilibrium lies outside the diagonal.

Q is the set of all the games with payoff functions $u = (u^1, u^2, \ldots u^n)$ such that on the diagonal $a^1 = a^2$ the payoffs $u^{-1}|_{a\in A|a^1=a^2}$ and $u^{-2}|_{a\in A|a^1=a^2}$ are payoffs that belong to the subset of P. More importantly, we would like a^1_2 to be player-ONE's dominant action, and a^i_1 to be player-$i \neq 1$'s dominant action.

We will construct an n-1 player game $\tilde{u}^{-1}$ from n-player game u by making Player TWO choose the actions for both Player ONE and TWO.
(i.e. for every $i \neq 1$, $\tilde{u}^{-1}(a^2, a^3, \dots a^n) = u^i(a^2, a^3, a^3 \dots a^n)$).

Consider Q the set of all the payoff functions $u = (u^1, u^2, \dots u^n)$ s.t.:

1. $\tilde{u}^{-1}(a^2, a^3, \dots a^n), \tilde{u}^{-2}(a^1, a^3, a^4 \dots a^n) \in P$ for every action $a = (a^1, a^2, \dots a^n) \in A$ such that $a^1 = a^2$.
2. $M + 1 < u^i(a) \leq M + 2$ for all actions a s.t. $a^1 \neq a^2, i \neq 1$, and $a^i \neq a^i_1$.
3. $M + 1 < u^i(a) \leq M + 2$ for all actions a s.t. $a^1 \neq a^2, i = 1$, and $a^i \neq a^i_2$.
4. $M + 3 < u^i(a) \leq M + 4$ for all actions a s.t. $a^1 \neq a^2, i \neq 1$, and $a^i = a^i_1$.
5. $M + 3 < u^i(a) \leq M + 4$ for all actions a s.t. $a^1 \neq a^2, i = 1$, and $a^i = a^i_2$.

A small example will help in understanding the construction:

Let $M = 12$ and $A^1 = A^2 = \{e_1, e_2\}, A^3 = \{e^3_1, e^3_2\}$.
Consider the following set of games P

Table 3.1: P

	e^3_1	e^3_2
e_1	$[2, 3], [2, 3]$	$[1, 2], [3, 4]$
e_2	$[2, 3], [1, 2]$	$[6, 7], [5, 6]$

We will now construct Q from P. When players ONE and TWO play the same action we have payoffs agreeing with P. When they don't play the same action, we get the interval of the payoffs by substituting $M = 12$ in rules (2)-(5).

Table 3.2: Q

	e^2_1	e^2_2
e^1_1	$[2,3],[2,3],[2,3],$	$[13,14],[13,14],[15,16]^*$
e^1_2	$[15,16],[15,16],[15,16]^*,$	$[2,3],[2,3],[1,2]$

	e^2_1	e^2_2
e^1_1	$[1,2],[1,2],[3,4],$	$[13,14],[13,14],[13,14]^*$
e^1_2	$[15,16],[15,16],[13,14]^*,$	$[6,7],[6,7],[5,6]$

We will now show that

(1) Q has positive measure,
(2) every game in Q has a unique pure Nash equilibrium
(3) In every game of Q, there is a positive probability that the strategy mapping doesn't lead the play to the unique pure Nash equilibrium(as assumed in (2).

Once we prove these three statements, we would have a subset of games Q with unique pure Nash equilibrium, with positive measure, s.t. with positive probability, the strategy mapping doesn't lead the game to the unique pure Nash equilibrium. This will prove that the strategy mapping doesn't lead to almost sure convergence to unique pure Nash equilibrium in every generic game having a unique pure Nash Equilibrium. This will complete the proof.

Part (1):
For every payoff function u that satisfies(1)-(5), conditions (2)-(5) restrict the payoffs out of the diagonal to be in some interval with length 1. Consider the Lebesgue measure μ' on the space $\mathbb{R}^{n|A^{-1}|}$. So,

$\mu(Q) = \mu'(Q|\{a \in A|a^1 = a^2\}).1^{n(|A|-|A^{-1}|)} = \mu'(Q|\{\{a \in A|a^1 = a^2\}\})$ (1^*)

Let $D = \{d = (u^1, u^2, u^{-\{1,2\}})|(u^1, u^{-\{1,2\}}) \in P$ and $(u^2, u^{-\{1,2\}}) \in P\}$.
Since elements of P have $u^1 = u^2$, we have $D = \{a \in A|a^1 = a^2\}$.

Lemma 14. *For every $i, j \in \mathbb{N}$ and for every set $V \subset \mathbb{R}^{i+j}$ with a positive measure $\lambda(V) > 0$, the set $\mathbb{C} = \{c = (p,q,r) \in \mathbb{R}^i \times \mathbb{R}^i \times \mathbb{R}^j|(p,r) \in V$ and $(q,r) \in V \subset \mathbb{R}^{2i+j}\}$ has a positive measure.[3]*

In the above Lemma, we substitute $i = |A^{-1}|$, $j = (n-2)|A^{-1}|$, $V = P$ and we get $\mu(Q|\{a \in A|a^1 = a^2\}) > 0$.

And by $(1^*), \mu(Q) > 0$.

Part (2):

Consider the action $a = (a^1_2, a^2_1, a^3_1, \ldots a^n_1)$. The payoff received by all the players are in the interval $[M+3, M+4]$. If any player switches his strategy, then his payoff reduces to less than $M+2$, by construction. Hence $a = (a^1_2, a^2_1, a^3_1, \ldots a^n_1)$ is the unique pure Nash equilibrium in every game of Q.

Part (3):

$\bar{h} \in H_{\bar{t},A^{-1}}$, the history of player ONE is the same as the history of player TWO $\bar{h} \in H_{\bar{t},A^{-2}}$. Let us denote the by by $(c(t'), c(t'), a^3(t'), \ldots a^n(t'))^t_{t'=1}$ where $c(t')$ is the action of player1/player2,if $\bar{h}$ is an element of $H_{\bar{t},A^{-2}} // H_{\bar{t},A^{-1}}$ respectively.

Define $\tilde{h} \in H_{\bar{t},A}$ by $\tilde{h} = (c(t), c(t), a^3(t), a^4(t), \ldots a^n(t))^{\bar{t}}_{t=1}$.

There is a positive probability that at first period $(c(1), c(1), a^3(1), a^4(1), \ldots, a^n(1))$ will be played. This is because in the the first period in $\bar{h}$ will occur with a positive probability.

If at time $(t-1) = 1, 2, \ldots, \bar{t}-1$ be the history of play is the first $(t-1)$ periods in $\tilde{(}h)$, then all the played actions are on the diagonal $(\bar{a^1}, \bar{a^2})$ where their payoffs are from the set of games P.

Therefore the observations sequence of every player at period $t-1$ is exactly the same as if $\bar{h}$ occured.

Therefore $(c(t), c(t), a^3(t), a^4(t), \ldots a^n(t))$ will be played with a positive probability till time t. And by by induction $\tilde{h}$ is realizable by the strategies $(f_{A^i})^n_{i=1}$. Thus the players continue playing actions within the diagonal and the equilibrium is clearly outside the diagonal.

So, we have proved that the history $\tilde{h}$ will occur with a positive probability. We can observe that this ensures that the played is confined within the diagonal $a^1 = a^2$. This is because the action $(\bar{a^1}, \bar{a^2}, \bar{a^3}, \ldots, \bar{a^n})$ will be played with probability 1, after time $\bar{t}$. But $(\bar{a^1}, \bar{a^2}, \bar{a^3}, \ldots, \bar{a^n})$ is not a Nash equilibrium in the game u since player 1 can increases his payoff above $M+1$ by switching. Hence the strategy mapping doesn't lead the game to the unqiue pure Nash equilibrium which it has.

Theorem 5: There exists a completely uncoupled strategy mapping that leads a.s. convergence of play to a pure Nash equilibrium in every finite game with distinct action sets that has a pure Nash equilibrium [3]

Proof: We will prove the existence of a 'completely uncoupled strategy mapping' which leads play to convergence to equilibrium in every game with different payoffs. This will prove that it converges play to equilibrium in almost every game, as the set of all games with atleast payoffs being equal is a zero measure set

We assumed that the collection of all actions of players is countable. So, the set $\mathfrak{B}$ of all finite subsets of the collection is also countable. So, we have an injective function $\gamma : \mathfrak{B} \to \mathbb{N}$. So for every unique game Γ (unique action set) has a unique index through this map, i.e. the numbers $\gamma(A^1), \gamma(A^2), \ldots, \gamma(A^n)$ are distinct.

To construct the strategy mapping, let ϕ be the function which assigns a strategy $f(\gamma(A^i))$ to every action set $A^i \in \mathfrak{B}$.

The strategy $f_{A^i}(l)$ will consist of five main steps. In every step we specify the action of a player and the resultant behaviour of all players combined. In each step, the behaviour of a player only dependent on his payoff, as required in a completely uncoupled game.

The dynamic tries to facilitate communication between the players. At time $t = 1$, every player plays his first action a_1^i and records the payoff $u^i(a_1^1, a_1^2, \ldots, a_1^n)$, we will call it $u^i(1)$. The players use the action as a binary signal, they judge the signal from other players, based on whether the payoff was $u^i(1)$ or different from $u^i(1)$.

Step1 is called the identification of index:
The players aim at generating a unique index from $1, 2, \ldots, n$.
They review natural numbers $k = 1, 2, \ldots$. While reviewing his number $\gamma(A^i) = k$, each player signals to the others as follows: The player i having $\gamma(A^i) = k$ plays a_2^i , while the others play a_1^j . This lets them know if a player with $\gamma(A) = k$ exists;

We have an another problem. The players don't when to stop reviewing as the total number of players is unknown. We solve this problem by adding another step between reviewing of k and $k + 1$. The use the signalling mechanism to find-out if there is a player with payoff greater than k. If there is no such player, they can stop reviewing. Each player signals by playing a_2^i if his number $\gamma(A^i)$ is above k, otherwise he plays a_1^i.

Once each player finds all the $\gamma(A^j)$, each player evaluates his own index by $|\{j|\gamma(A^j) \leq \gamma(A^i)\}|$.

Step2 is called the identification of the action profile set:

In this step each player tries to identify the number of actions with the other players. Firstly, the index1 player plays his actions in the following order: $a^i_2, a^i_3, \ldots, a^i_{m^i}, a^i_1$, and the other players constantly play a^i_1 . The other players know that player1 has finished, when the players receive the payoff $u^i(1)$ and they record the number of actions of player1. The procedure then continues for players with indexes $2, 3, \ldots, n$.

If the players find a player with one action, they will assume that such a player doesn't exist. Such a player doesnt affect the outcome of the game in anyway.

Step3 is called the identification of the *own payoff function*:

From Step2, all players know the structure of the action set A.
The players go through all the actions $a \in A$ in lexicographic order, and record it.

Step4 is called *finding a pure Nash equilibrium*:

From *Step 3*, players know their own payoff function.
A player reviews all the actions $a = (a^i, a^i)$ in lexicographic order, and then plays a^i_1 if a^i is a best reply to a^i ; If not, he plays a^i_2 . If his payoff is $u^j(1)$, then he knows that the other players are best replying and so everyone is currently playing a pure Nash equilibrium. If not, they will continue reviewing.

Step5 is called *playing the pure Nash equilibrium*. In this step, the players will repeatedly play the pure Nash equilibrium.

Each of the steps(1-4) take finite amount of time and from then on, a pure Nash equilibrium will be played. 2 □

Theorem 6: For every $\epsilon < 1/2$ there is no completely uncoupled strategy mapping that leads to play of pure Nash equilibria with frequency $1 - \epsilon$, in every game with more than two players, where such an equilibrium exists.[3]

Proof. The requirement of convergence in 'every' game is very strong. We will produce an example of two games such that the probability of player3 playing the equilibrium action is not more than half. That will complete the proof.

Table 3.3: Γ_1

	a_1^1	a_2^2		a_1^1	a_2^2
a_1^1	$7,7,7^*$	$7,7,7^*$	a_1^1	$7,2,7$	$2,7,7$
a_2^1	$7,7,7^*$	$7,7,7^*$	a_2^1	$2,7,7$	$7,2,7$

Table 3.4: Γ_2

	a_1^1	a_2^2		a_1^1	a_2^2
a_1^1	$7,2,7$	$2,7,7$	a_1^1	$7,7,7^*$	$7,7,7^*$
a_2^1	$2,7,7$	$7,2,7$	a_2^1	$7,7,7^*$	$7,7,7^*$

In these games player THREE has payoff '7' irrespective of the actions of the players and the game being played, but his choice determines the actions of other players so as to produce an equilibrium.
Game Γ_1's pure Nash equilibria are $\{(i, j, 1)\}_{i,j=1}^2$. Game Γ_2's pure Nash equilibria are $\{(i, j, 2)\}_{i,j=1}^2$.

Player THREE employs the same strategy in both the games and his history doesnt depend on actions of other players. Player THREE doesn't play one of the actions a_1^3 or a_2^3 with limit frequency greater than 0.5 with probability 1. We will call this action $a_i^3 (i = 1, 2)$. So, in game Γ_i the strategies can't lead play to a pure Nash equilibria.

Such an example can be constructed for a different number of players and different number of actions. We need to construct the payoff and ensure that a player has play two different actions with limit frequency greater than 0.5, so as to lead play to converge to Nash equilibrium. □

Theorem 7: If either,
(1) The additional information of every player $i \in N$ is his own index; i.e., $\alpha^i = i$
or
(2) The additional information of every player $i \in N$ is the total number of players, i.e. $\alpha^i = n$

Then, there exists a completely uncoupled strategy mapping with additional information α that leads to a.s. convergence of play to pure Nash equilibrium in every finite generic game where such an equilibrium exists. [3]

Proof. We will prove that one of the following conditions is sufficient for the existence of a strategy mapping.

Condition 1: If the players have a unique index from $1, 2, \ldots, n$, then we can use the Theorem5, since the items of the collection $A_i \times j$, where j is the index, can be used in place of action sets. Action sets will now be unique and so we will have a strategy mapping which leads play to convergence to a pure Nash equilibrium, if it exists.

Condition 2: If the players know the total number of players, then we can use n in the description of the strategy mapping. We will define the strategy-mapping, by defining the strategy of a player i as follows:

Firstly, all the players play a step called random identification of action profile set of n players:

In this step, each player uniformly randomizes a natural number $1 \leq c^i \leq n$.

He will then use this random number as his index and play the step *identification of action profile set*. He will record the action set and finish this step, if he finds that there are n-players. If not, all the players will randomize again.

After this step, the player continues the step chains as prescribed in Theorem5 - *identification of payoff function*→*finding of pure Nash equilibrium*→playing the Nash equilibrium.

If $(c^1, c^2 \ldots, c^n)$ is a permutation of $(1, 2, \ldots, n)$, then players will use it as a unique index and can proceed as suggested in the proof of Condition(i) If more than one player has guessed the same number, let us call the smallest number which is not in $\{c^1, c^2, \ldots, c^n\}$ as j. That will help all the players in concluding that there are $j - 1$ players and so they try to pick a permutation once again.

Players guess a permutation with a probability $n!/n^n$ in every randomization. So, they will eventually pick a permutation in some iteration and then the dynamics will reach a pure Nash equilibrium. □

Theorem 9: For, every $\epsilon > 0$, there exists a completely uncoupled strategy mapping that leads to a.s. convergence of play to Nash ϵ-equilibrium in every finite generic game.[3]

Proof: We would like to discretize $\Delta(A^i)$ for all i, by having only the actions which are integeral multiples of ν. Consider a constant $\nu = \nu(\epsilon)$, a number small enough, s.t. every game has a Nash ϵ-equilibrium, with integer multiplications of ν as the actions. $\nu = \epsilon/4K$, where K is the bound on all the payoffs, is one such number small enough, to be assigned as ν. Such a ν exists, because every game has a Nash equilibrium, and we can approximate it by integer multiplications of ν We will denote this discretization by $\tilde{\Delta}(A^i)$. Note that it is a finite set. So $\tilde{\Delta}(A) = \tilde{\Delta}(A^1) \times \tilde{\Delta}(A^2) \times \; \times \tilde{\Delta}(A^n)$ is also a finite set, and we can define a lexicographic order over $\tilde{\Delta}(A)$.

The new step will called "searching Nash $\epsilon/2$-equilibrium".
The players will search all the mixed actions in $\tilde{\Delta}(A)$ in a lexicographic order. For every action $x = (x^i, x^{-i}) \in \tilde{\Delta}(A)$, player i will play a^i_1 if x^i is an $\epsilon/2$-best response to x^i ; if not, he will plays a^i_2. If his payoff was $u^i(1)$, then he records x^i as the Nash $\epsilon/2$-equilibrium mixed action. If not, he moves on to the action following it, in the lexicographic order.

We will define the state of the strategy, of a player:
State k.1: The player has the information that the number of players is at least k and that within k players, he couldn't find an $\epsilon/2$-Nash equilibrium.
State k.2: The player has the information that the number of players is at least k players, and that within k players, he has found an $\epsilon/2$-Nash equilibrium. If in this state,the player will record his k-players payoff function $\tilde{u}^i_k$ and the equilibrium action x^i. The initial state for a player is state 1.1.
At state k.1, Each player i will follow these steps: *random identification of action profile set of k players→identification of payoff function→ finding an $\epsilon/2$-Nash equilibrium.*

The Nash $\epsilon/2$-equilibrium action x^i, and payoff function $\tilde{u}^i_k$, containing equilibrium, are recorded by the player. After this step, player i will move to state k.2.

Consider a number ξ small enough s.t. for every Nash $\epsilon/2$-equilibrium $x = (x^i)^n_{i=1}$, the mixed actions profile $y = (y^i)^n_{i=1}$ defined by

$$y^i = (1 - \xi)x^i + \xi(\tfrac{1}{|A^i|}, \tfrac{1}{|A^i|}, \ldots, \tfrac{1}{|A^i|})$$

is a Nash ϵ-equilibrium with full support over A.

At *state k.2*, player will choose the mixed action y^i. If the player i gets a payoff not in $\tilde{u}^i_k$, he will then change to state $k + 1.1$. If not, he will continue in state

$k.2$.

Let the number of players be n. There is a positive probability of randomize a k-permutation if all the players are at state $k.1$ for every $k \leq n$ and then all players move simultaneously to state $k.2$. Consider any for every $k < n$, the played action y has full support if every player is in state $k.2$.

Therefore, there is a positive probability of playing an action that is not in $\tilde{u}_k$; in this case the received payoff of every player i does not exist in $\tilde{u}_k^i$, so all the players move simultaneously to state $k + 1.1$

Finally, the players will get to state $n.2$. Note that state $n.2$ is an absorbing state. In state $n.2$, the players play $(y^i)_{i \in \mathbb{N}}$, which is a Nash ϵ-equilibrium. The player always records an actual payoff value and so he will never get a payoff which doesnt exist in his function. So he will stay in state $n.2$ all the time. □

Theorem 13: For $\epsilon \leq 1/8$, there is no completely uncoupled strategy mapping into finite-memory strategies that leads to play of Nash ϵ-equilibria with frequency 1 in every game. [3]

Proof:. Proof by contradiction. Suppose such a strategy mapping does exist. We will examine the matching pennies game, in which a player having actions set $\{a_1, a_2\}$ plays against another player who has an action set $\{a_1^3, a_2^3\}$:

Table 3.5: Γ

	a_1^3	a_2^3
a_1	11, 7	7, 11
a_2	7, 11	11, 7

There exists a history, realizable by the strategy, $h = (a(t'), a^3(t'))_{t'=1}^{t}$ s.t. if h occurs, then the played actions ϵ close to $(1/2, 1/2)$ provided that their payoffs remain 7 or 11.

Now let us consider a three-player game where players ONE and TWO have the actions set $\{a_1, a_2\}$, same as in previous game and player3 has actions set $\{a_1^3, a_2^3\}$:

Table 3.6: Γ'

	a_1^2	a_2^2		a_1^2	a_2^2
a_1^1	$11, 11, 7$	$11, 7, 11^*$	a_1^1	$7, 7, 11$	$2, 11, 7, 11^*$
a_2^1	$7, 11, 11^*$	$7, 7, 11$	a_2^1	$7, 11, 11^*$	$11, 11, 7$

Players ONE and TWO will be playing the matching pennies game against THREE, on the diagonal $a^1 = a^2$. Players ONE/TWO get 11 if a_1^i is played by him ,and 7 if a_2^i is played by him, Out of the diagonal $a^1 = a^2$. THREE gets 7, in any case, out of the diagonal $a^1 = a^2$. $((1/2, 1/2), (1/2, 1/2), (1/2, 1/2))$ or any ϵ -perturbation of it is not a Nash ϵ -equilibrium for $\epsilon \leq 1/8$ (because player can increase his payoff to a value larger than $1/8$, by deviating).

$\tilde{h} := (a(t'), a(t'), a^3(t'))$ is a realizable history in Γ', as we have a positive probability of all the players playing the history h. If h occurs then players will remain on the diagnol and will continue to play $(1/2, 1/2)$, because all the payoffs in the game *Gamma* are either 7 or 11. So, if h does occur, then the players will continue playing actions ϵ close to $(1/2, 1/2)$.

Chapter 4

Model

4.1 Motivation

The assumption that a player can see his actions, overall outcome and payoff alone, is a very strong condition. This is equivalent to the game being a black-box.

The assumption that players cant see the number of players is legit in a large game(large number of players).

We can develop a model in which players can see a probability distribution over the action set. They may not know the action sets of other players, but they know the probability distribution with which nature allots them an action set.

4.2 Model

We have an n-player game with each player receiving an action-set from a finite collection A. The players payoffs are given by functions $p_i : A^n \longrightarrow \mathbb{R}$
After the start of the game, each player knows

1. His index i
2. His action set A_i
3. His payoff function p_i

Assumptions

1. Generic game
2. Each player receives a distinct action set

Problem: Existence of a strategy mapping which guarantees almost sure convergence of the game to a Nash equilibrium.

We will see a mechanism to ensure faster convergence to pure Nash equilibrium(if one exists) in the next chapter-Dynamics.

4.3 Negative Results

The negative results proved by Babichenko, which proves that no-strategy mapping exists which can lead play to Nash-equilibrium(if it exists) come up when one of these conditions are not satisfied-'distinct action sets' and 'generic game'.

If the 'distinct action set' condition is not satisfied, the strategy mapping has no mechanism to differentiate between two players pocessing the same action set. They will thus play the strategy. So, there is a positive probability of both the players staying on the diagnol(both playing same action). Thus, we wont have a stratey mapping in the games having equilibrium outside the diagnol. This is important since we insist on almost sure convergence, so even if there is a small positive probability of the game not converging to equilibrium(as a result of getting absorbed within the diagnol), then we have a negative result.
This negative result continues to hold in our model.

If the 'generic game' condition is not satisfied, then we can come with up games where a player has to play two actions with limit frequency, greater than 0.5, as seen in Theorem 6.

Secondly, the Dynamics also fail when there are multiple equilibria. We thus operate under the assumption that there is a unique pure Nash equilibrium.

Although these results couldn't be fixed, there are limitations in the Dynamics suggested by Babichenko. The Dynamics suggested in Theorem 5 require the players to go through all the actions and then explore an equilibrium. However, players may be able to see patterns and discover the equilibrium without actually going through the entire matrix. This motivates the development of a model where the information possessed by the players is intermediate between that in a coupled game(where entire payoff matrix is available and players can compute nash equilibrium using the matrix, if it is unique) and a completely uncoupled game.

Chapter 5

Dynamics

Each set $A_0 \in A$ is mapped to f_{A_0} by strategy mapping ϕ.
The strategy f_{A_0} is composed of two main steps

1. Search

2. Explore

Step1: $L_{i,f_{A_{0_i}}} = \{a \in \{A_0 \times A^{n-1}\} \setminus \beta \quad | \; p_i(a) \geq p_i(b, a_{-i}) \forall b \in A\}$
Players go through actions of $L_{i,f_{A_{0_i}}}$ in lexicographic order and record it. Reviewing all the actions $a^j = (a_i^j, a_{-i}^j)$ in lexicographic order, the player plays a_i^1 if a_i^j is the best response to a_{-i}^j
Otherwise, he plays a_i^2

If his payoff is $p_i(a^1)$ then all players are best replying.
Hence a Nash equilibrium is found.

Players will continue to play a_i^j.

Step2:

1. L_i is an order of actions of A(with $|A|^{(n-1)}$ elements) such that $X_{i=1}^n \; L_i$ is bijective with $X_{i=1}^n A_i$.
   Player i plays first action from L_i.

2. If $\exists A_1 \in A$ s.t. $p_i(a)! = p_i(b) \forall b \in A$ s.t. $b_j = a_j$ then send positive signal
   If β, updated after recording the payoff, it has cardinality $|A|^{n-1} - 1$ then send positive signal.
   Player sends positive signal and an element of L_i untill all players send

positive signal.
Once a positive signal is received from all players, they play *Search* step.

5.1 Example

Suppose there are two players.
$\mu = (a, b, c, d, e, f)$
$\lambda = ((a, b), (c, d), (e, f))$
Suppose the payoffs are identical for both the players Suppose Player1 receives

Table 5.1: Payoffs

X	a	b	c	d	e	f
a	6	0	51	50	51	70
b	2	9	2	29	35	36
c	1	2	3	4	1	6
d	7	8	23	1	41	21
e	3	5	4	1	31	19
f	5	14	23	15	17	18

(a,b) and Player2 receives (c,d)

Step1:
Player1's list-$\{(a, a), (b, b), (a, c), (a, d), (a, e), (a, f)\}$
Player2's list-$\{(a, d), (b, d), (c, d), (d, c), (e, d), (f, d)\}$

Player1's order is a,b
Player2's order is c,d

They play a,c
Player1 signals negatively against a,c

They play a,d
Both players signal positively

An equilibrium is found and the protocol terminates.

Note: An equilibrium was found within the amount of information known to the players and hence the protocol terminated without proceeding to step2.

If there was no equilibrium within the known information, we would witness an iteration of both the steps.

5.2 Proof of Correctness

Proposition 15. *If* $\cap_{i=1}^{n} L_{i,\phi(A_{0_i})} \neq \phi$ *then* ϕ *converges play to equilibrium.*

Proof: If $a \in \cap_{i=1}^{n} L_{i,\phi(A_{0_i})}$ then

1. a is a nash equilibrium
2. ϕ converges to a nash equilibrium

Proof of (i): If $a \in \cap_{i=1}^{n} L_{i,\phi(A_{0_i})}$, the a_i is a best response to a_{-i}.
Each player is best replying, in a.
Hence a is a Nash Equilibrium.

Proof of (ii) Proof by Contradiction, Suppose the strategy mapping doesn't converge play to 'a'. $a \in \cap_{i=1}^{n} L_{i,\phi(A_{0_i})} \Rightarrow$ a occurs in the lexicographic ordering of all actions tried by players, unless the game converged to a nash equilibrium.

Suppose the game didn't converge already, then each player send positive signal after 'a' is played in the order. The play then converges to 'a'. Hence proved □

Proposition 16. *Either the strategy mapping leads to Nash Equilibrium or to a root node.*

Proof: Since n, A are finite, total number of combinations of $|A|^{n-1}$ is finite.

Each step in Explore reduces potential number of combinations by atleast 1.
Thus, each player reaches root node by $|A|^{n-1}$ steps.

Proposition 17. *The strategy mapping checks for existence of a Nash Equilibrium.*

Proof: By proposition2, either ϕ converges play to Nash Equilibrium or to root node. By Proposition1 play converges to nash equilibria in root node, if it exists.

Hence the strategy mapping leads play to nash equilibrium,if one exists. □

Chapter 6

Conclusion

6.1 Further Questions

There is a tradeoff between exploring more information about the game and finding an equilibrium within the known information?

Can further optimization of time be possible within the model?
Can we have a model in which a strategy mapping can lead play to a nash equilibrium even when action sets needn't be distinct?

Can we have a model in which a strategy mapping can lead play to a nash equilibrium in every game(not just generic games)?

Can we optimize the dynamics using the probability distribution(belief structure over the collection of action sets)?

What is the nature of the Dynamics if we consider an equilibrium concept other than pure Nash, mixed Nash and ϵ-Nash?

6.2 Conclusion

Dynamics in Games of Complete Information is well understood.

Yakov Babichenko, in his paper *Games of incomplete information and nash equilibria* presents some impossibility theorems in the incomplete setting.

We are exploring models in between Complete and Incomplete information. It turns out that it is not possible to fix those impossibilities, without proceeding to an equivalent of games with complete information.

However, Yakov Babichenko concludes in his paper, that the dynamics presented in the paper are ugly in the sense that they assume complete coordination. We are exploring better dynamics, in a model which fits into most situations.

This model comes in, in a situation where the players have access to their complete payoff matrix and the Universal Action Set structure, but do not know the actual Action set with the players. There is a tradeoff between locating equilibrium within the knowledge system; and exploring more information about the game. Players make Bayesian updates about the game, through the recorded payoffs.

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