A STUDY OF THE HIGGS MECHANISM IN YANG-MILLS AND CHERN-SIMONS FIELD THEORY

A thesis submitted towards partial fulfilment of
BS-MS Dual Degree Programme

by

K.SriHariTej

under the guidance of

Prof.Sunil Mukhi

Indian Institute of Science Education and Research Pune

IISER PUNE

Certificate

This is to certify that this thesis entitled 'A Study of the Higgs Mechanism in Yang-Mills and Chern-Simons Field Theory' submitted towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research Pune represents original research carried out by K.SriHariTej at IISER Pune, under the supervision of Prof.Sunil Mukhi during the academic year 2013-2014.

Student
K.SriHariTej

Supervisor
Prof.Sunil Mukhi

Acknowledgements

I wish to express my sincere gratitude to Prof. Sunil Mukhi, my thesis supervisor. Without his expert guidance and constant encouragement, this project wouldn't have been possible. Despite his busy schedule, he always devoted his precious time to me and patiently cleared all my (sometimes silly) doubts and nudged me to push to the best of my capacity.

I am also grateful to Dr. Sudarshan Ananth, my mentor for recommending me to Prof. Sunil Mukhi. It was his constant motivation and guidance that encouraged me to pursue a career in Physics research. I would also like to extend my gratitude to Dr. Mukul Kabir and Dr. Arjun Bagchi, for their support and encouragement throughout my fifth year. I would also like to thank all the Physics faculty for all courses and suggestions, which has laid a strong background. I would also like to thank Mr. Prabhakar for providing administrative help.

I would like to express my special gratitude to my friends Aashay, Arun, Avani, Bhavesh, Priyatham, Punya, Satish, Siddhartha Das and Tarun. I have learnt a lot from all of them. Without them, my IISER journey would have been quite boring.

Finally, I deeply thank my parents and brother for their support and encouragement throughout my studies.

The work done in this thesis was supported by the INSPIRE grant from the Department of Science and Technology, Government of India.

Abstract

In this work, we focus on the Higgs mechanism.First, we study the symmetry breaking patterns in the fundamental and adjoint Higgs field. Next,we study the symmetry breaking patterns in the case of direct product symmetry groups with bi-fundamental matter using Singular Value Decomposition. We present here two important applications of Singular Value Decomposition to study the bi-fundamental Higgs mechanism:
1) We derive the symmetry breaking patterns for bi-fundamental $SU(2)\times SU(2)$ and $SU(3)\times SU(3)$ theories,
2) We investigate the spectrum of the difference Chern-simons theory in the presence of a generic bi-fundamental VEV, generalizing the novel Higgs mechanism in the literature.

Contents

Chapter 1

Introduction

1.1 Quantum field theory

Quantum field theory(QFT) is a mathematical tool that is essential for the study of Elementary Particle Physics. In order to deal with particles moving with a speed close to the speed of light, we require a relativistic framework. On the other hand, in order to study phenomena at nanoscopic length scales, one requires the framework of quantum mechanics. However, many problems arise when we have to deal with phenomena at high speeds and small length scales. QFT tries to address this problem by creating a unified framework. In QFT, we describe particles using fields that can interfere constructively or destructively resulting in the birth and death of particles. The classical fields are represented as operators acting in a Hilbert space.

Based on the general principles of QFT, many consistent theories have been constructed. One of the most interesting among them is the Yang-Mills theory. Lie groups, along with matter particles assigned to their respective group representations, are used to characterize Yang-Mills theory. There are large number of choices for Lie groups and additionally, for each group, there are various possibilities of group representations. For example, in the case of SU(2) Yang-Mills theory there are two group representations, viz., doublet representation and adjoint representation. We discuss in detail about these groups and their corresponding matter particles in the following chapters.

1.2 Yang-Mills theory

Yang-Mills theory is a gauge theory characterized by the Lie group. It is named after Chen Ning Yang and Robert Mills. The famous Standard model of particle physics is constructed in the framework of Yang-Mills theory. Unification of electromagnetic force and weak force, known as the Glashow-Salam-Weinberg theory and the strong force are explained using Yang-Mills theory. Further details of Yang-Mills theory are discussed in chapter 2.

1.3 Chern-Simons theory

Chern-Simons theory is a quantum field theory in 2+1 dimensions. One of the special features of Chern-Simons theory is that its kinetic term is first order in the derivatives[1],[2],[3]. Since its action is proportional to the Chern-Simons 3-form, it is named as Chern-Simons theory. Unlike the Yang-Mills theory, the Chern-Simons action is gauge invariant only if boundary terms are handled properly. It can be made gauge independent when the gauge field strength vanishes at all the boundaries.As Yang-Mills theory, Chern-Simons theory is also characterized by the Lie groups. This theory has interesting practical applications in the planar Condensed Matter Physics and (2+1)d gravity. In Condensed Matter Physics, it explains the topological order in Quantum Hall effect.

1.4 Higgs Mechanism

The process by which elementary gauge particles acquire mass is known as the Higgs mechanism. It was initially proposed by Philip Anderson but the relativistic model was later developed by Peter Higgs and Richard Brout with Francois Englert. It predicts the existence of a particle called Higgs boson,which is generated along with the other massive gauge bosons. The Higgs boson doesn't have spin, charge and color charge. The discovery of Higgs boson by ATLAS and CMS groups at the Large Hadron Collider on 4 July 2012 confirmed that the fundamental particles get their mass through Higgs mechanism. With the discovery of the Higgs boson, we can say that the Standard Model is in some sense complete. Francois Englert and Peter Higgs have been awarded the 2013 Physics Nobel Prize for their contribution towards the understanding of the origin of subatomic particles. In the following chapters, we will discuss the Higgs mechanism in detail.

1.5 Novel Higgs Mechanism(NHM)

Chern-Simons gauge theories in 2+1 dimensions with multiple gauge fields exhibit novel properties. One of the striking feature is the possibility of a non-propagating Chern-Simons field acquiring a massless propagating mode via the Higgs mechanism. NHM was originally discovered in the context of M-theory and sufficient conditions are found for it to occur. NHM helps in understanding the world-volume theory on multiple membranes in M-theory.In the third chapter, we study NHM in the case of difference Chern-Simons action without any reference to M-theory or Supersymmetry. We find that only under certain conditions, non-propagating gauge fields turn into a massless propagating field (Yang-Mills field).

Chapter 2

Higgs mechanism

2.1 Introduction

Higgs mechanism is the answer to the most important question in the particle physics:“how do elementary particles acquire mass?”. Initially it was proposed by Anderson, but the relativistic model was developed by Higgs and Englert and Brout that answers this question. According to Higgs mechanism,particles acquire mass,when the local symmetry is broken. Spontaneous breaking of different symmetry groups results in giving mass to various elementary particles. In this chapter,we will explore Higgs mechanism in great detail.

2.2 Spontaneous Breaking of Symmetry

Consider a complex scalar field $\phi(x)$.Lagrangian in terms of complex scalar field is given as :

$$L = \frac{1}{2}\partial_\mu \phi^*(x)\partial^\mu \phi(x) - \frac{1}{2}m^2\phi(x)^2 - \frac{\lambda}{4!}\left(\phi^*(x)\phi(x)\right)^2 \tag{2.1}$$

Take global phase transformation of the complex scalar field:

$$\phi(x) \to \phi(x)e^{i\alpha} \tag{2.2}$$

Under the above transformation, the kinetic term in the Lagrangian changes as follows:

$$\partial_\mu \phi^*(x)\partial^\mu \phi(x) \to \partial_\mu (e^{-i\alpha}\phi^*(x))\partial^\mu (\phi(x)e^{i\alpha)}) \tag{2.3}$$

It is seen from above that the scalar field theory is invariant under global phase transformation.Now, let us choose a potential such that ϕ gets a minimum at a finite value.

$$L = \frac{1}{2}\partial_\mu \phi^*(x)\partial^\mu \phi(x) + \frac{1}{2}m^2\phi(x)^2 - \frac{\lambda}{4!}\left(\phi^*(x)\phi(x)\right)^2 \tag{2.4}$$

The potential term is now:

$$V(\phi) = -\frac{1}{2}m^2\phi(x)^2 + \frac{\lambda}{4!}\left(\phi^*(x)\phi(x)\right)^2 \tag{2.5}$$

The above potential has a local maximum at $|\phi| = 0$ and local minimum at $|\phi| = \sqrt{\frac{6m^2}{\lambda}}$. We make a change of field variables and write complex scalar field as:

$$\phi(x) = R(x)e^{i\theta(x)} \tag{2.6}$$

In the above variables the local minimum is :

$$R = \sqrt{\frac{6m^2}{\lambda}} \tag{2.7}$$

Inserting the above form of $\phi(x)$ in the Lagrangian, we find that

$$L = \frac{1}{2}\partial_\mu R\partial^\mu R - \frac{1}{2}m^2R^2 - \frac{\lambda}{4!}R^4 + \frac{R^2}{2}\partial_\mu\theta\partial^\mu\theta \tag{2.8}$$

In these new variables of $\phi(x)$, the global phase transformation is just a constant shift :

$$\theta \to \theta + \alpha \tag{2.9}$$

After the transformation, the Lagrangian transforms as:

$$L = \frac{1}{2}\partial_\mu R\partial^\mu R - \frac{1}{2}m^2R^2 - \frac{\lambda}{4!}R^4 + \frac{1}{2}R^2\partial_\mu(\theta+\alpha)\partial^\mu(\theta+\alpha) \tag{2.10}$$

Since α is a constant, the above Lagrangian is invariant under global transformation.

In order to study the theory around minimum, we define a new field as:

$$\widetilde{R} = R - \sqrt{\frac{6m^2}{\lambda}} \tag{2.11}$$

Inserting $\widetilde{R}$ in the above Lagrangian(eq.(2.8)), we find that:

$$\begin{aligned} L =& \frac{1}{2}(\partial_\mu\widetilde{R}\partial^\mu\widetilde{R}) - m^2\widetilde{R}^2 - \frac{\lambda}{4!}\widetilde{R}^4 - \frac{m}{2}\sqrt{\frac{3\lambda}{2}}\widetilde{R}^3 \\ &+ \frac{3m^2}{\lambda}(\partial_\mu\theta)^2 + \sqrt{\frac{6m^2}{\lambda}}\widetilde{R}(\partial_\mu\theta)^2 + \frac{1}{2}\widetilde{R}(\partial_\mu\theta)^2 \end{aligned} \tag{2.12}$$

In the above Lagrangian, $\tilde{R}$ has a sensible mass term $\sqrt{2}m$ and θ is massless.

Therefore, a continuous symmetry is spontaneously broken, because the potential was minimized by nonzero ϕ. We gave the field a definite nonzero value in the vacuum ($R = \sqrt{\frac{6m^2}{\lambda}}$,$\theta = 0$). In the radial variables, θ=constant is a vacuum configuration for any constant. Therefore,the theory has one real massive scalar and one axionic scalar(Goldstone boson) with vanishing potential.Now we are in a position to understand the statement of the Goldstone theorem, which states that for every spontaneously broken continuous symmetry,the theory must contain a massless particle and the massless fields that arise through spontaneous symmetry breaking are called Goldstone bosons. In this case,the massless field is $\theta(x)$ and the scalar associated with $\theta(x)$ is called Goldstone boson.

2.3 The Higgs mechanism:

In this section we begin by applying the Higgs mechanism to an Abelian *U(1)* gauge theory to demonstrate how the mass of the corresponding gauge boson comes about. The Abelian example will then be generalized in a straightforward way to the system with the non-Abelian gauge symmetry in the following section.

Let us take local phase transformation of the complex scalar field.For this,we need to promote α to a local parameter $\alpha(x)$.Under local phase transformation:

$$\phi(x) \to \phi(x)e^{i\alpha(x)} \tag{2.13}$$

The kinetic term in the Lagrangian changes as follows:

$$\partial_\mu \phi^*(x)\partial^\mu \phi(x) \to \partial_\mu(e^{-i\alpha(x)}\phi^*(x))\partial^\mu(\phi(x)e^{i\alpha(x)}) \tag{2.14}$$

$$\begin{aligned}\partial_\mu(e^{-i\alpha(x)}\phi^*(x))\partial^\mu(\phi(x)e^{i\alpha(x)}) =& \partial_\mu \phi^*(x)\partial^\mu \phi(x) - i\partial_\mu \alpha \phi^*(x)\partial^\mu \phi(x) \\ &+ i\partial_\mu \alpha \partial_\mu \phi^*(x)\partial^\mu \phi(x) + \partial_\mu \alpha \partial_\mu \alpha \phi^*(x)\partial^\mu \phi(x)\end{aligned} \tag{2.15}$$

The above scalar field theory is not invariant under local phase transformation.we couple a gauge field A_μ to the complex scalar field, such that the combined system has local gauge invariance.

Consider the following replacement in the action:

$$\partial_\mu \phi \to (\partial_\mu - iA_\mu)\phi \tag{2.16}$$

Under the gauge transformation,we know that :

$$\begin{aligned}&\phi(x) \to \phi(x)e^{i\alpha(x)} \\ &\phi^*(x) \to e^{-i\alpha(x)}\phi^*(x) \\ &A_\mu \to A_\mu + \partial_\mu \alpha\end{aligned} \tag{2.17}$$

Using the above equations,we may check that $(\partial_\mu - iA_\mu)$ transforms as:

$$(\partial_\mu - iA_\mu)\phi \to e^{i\alpha(x)}(\partial_\mu - iA_\mu)\phi \tag{2.18}$$

This above formalism helps to write the generalized kinetic term as:

$$(\partial_\mu + iA_\mu)\phi^*(\partial_\mu - iA_\mu)\phi \tag{2.19}$$

The above kinetic term is gauge invariant, as are the potential terms that depend only on the scalar field. We have seen that,if a field has the local transformation law,then it's covariant derivative has the same transformation law. The same conclusion can also be applied to the commutator of covariant derivatives:

$$[D_\mu, D_\nu]\phi(x) \to e^{i\alpha(x)}[D_\mu, D_\nu]\phi(x) \tag{2.20}$$

Expanding the above term:

$$\begin{aligned} &[D_\mu, D_\nu]\phi(x) = i(\partial_\mu A_\nu - \partial_\nu A_\mu)\phi(x) \\ &\Rightarrow [D_\mu, D_\nu]\phi(x) = iF_{\mu\nu}\phi(x) \end{aligned} \tag{2.21}$$

Where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.22}$$

The factor of $\phi(x)$ on the right-hand side of the equation accounts for the entire transformation law, so the multiplicative factor $F_{\mu\nu}$ must be gauge invariant.

The *U(1)* gauge invariant kinetic term is given by:

$$L_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.23}$$

One can see that L_A is invariant under the transformation: $A_\mu \to A_\mu + \partial_\mu\alpha$. When we add the above term to the Lagrangian, we end up with a coupled theory that is gauge invariant.

From the previous section,we know that the potential has a local maximum at $|\phi| = 0$ and local minimum at $|\phi| = \sqrt{\frac{6m^2}{\lambda}}$. As in the case of global phase transformation, we will write the field($\phi(x)$) in terms of R and θ.

$$\begin{aligned} L = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu R\partial^\mu R - \frac{1}{2}m^2R^2 \\ &-\frac{\lambda}{4!}R^4 + \frac{R^2}{2}\partial_\mu\theta\partial^\mu\theta \end{aligned} \tag{2.24}$$

Under the local phase transformation :

$$R(x)e^{i\theta(x)} \to R(x)e^{i\theta(x)}e^{i\alpha(x)} \tag{2.25}$$

$\theta(x)$ independent terms are all invariant, but the last term of the Lagrangian changes as:

$$\frac{1}{2}R^2\partial_\mu(\theta+\alpha)\partial^\mu(\theta+\alpha) \tag{2.26}$$

From above, phase invariant Lagrangian may be achieved from the following transformation:

$$\partial_\mu\theta \to \partial_\mu\theta - A_\mu \tag{2.27}$$

Under the above transformation, the Lagrangian changes as:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu R\partial^\mu R) - \frac{1}{2}m^2R^2 - \frac{\lambda}{4!}R^4 + \frac{R^2}{2}(\partial_\mu\theta - A_\mu)^2 \tag{2.28}$$

The above Lagrangian is invariant under the transformation:

$$\begin{aligned} &\theta(x) \to \theta(x) + \alpha(x) \\ &A_\mu \to A_\mu + \partial_\mu\alpha \end{aligned} \tag{2.29}$$

As we are interested in studying the theory around the minimum, we shall define a new field:

$$\widetilde{R} = R - \sqrt{\frac{6m^2}{\lambda}} \tag{2.30}$$

Substituting $\widetilde{R}$ in the above Lagrangian(2.28), which then changes to:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\widetilde{R}\partial^\mu\widetilde{R}) - m^2\widetilde{R}^2 - \frac{\lambda}{4!}\widetilde{R}^4 + \frac{1}{2}(\widetilde{R}+\sqrt{\frac{6m^2}{\lambda}})^2(\partial_\mu\theta - A_\mu)^2 \tag{2.31}$$

consider the quadratic term in the Lagrangian:

$$\frac{3m^2}{\lambda}(\partial_\mu\theta - A_\mu)^2 \tag{2.32}$$

This is not a good quadratic term, as it has a cross term between A_μ and $\partial_\mu\theta$.This motivates us to perform the change of variables:

$$A_\mu \to A_\mu + \partial_\mu\theta \tag{2.33}$$

Under the above transformation, quadratic term changes to

$$\frac{3m^2}{\lambda}A_\mu A^\mu \tag{2.34}$$

After all the transformations, the Lagrangian is:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\widetilde{R}\partial^\mu\widetilde{R}) - m^2\widetilde{R}^2 - \frac{\lambda}{4!}\widetilde{R}^4 + \frac{1}{2}(\widetilde{R}+\sqrt{\frac{6m^2}{\lambda}})^2 A_\mu A^\mu \tag{2.35}$$

The Lagrangian consists of a mass term for the gauge field A_μ. This process of generating mass to the gauge bosons is called the Higgs mechanism and $\widetilde{R}$

is called the Higgs field. This above theory has a real massive scalar $\widetilde{R}$ of mass $\sqrt{2}m$ and a massive gauge field of mass $\mu = \sqrt{\frac{3m^2}{\lambda}}$ .By comparing this with the global symmetry case, we can see that the Goldstone term(θ) has been completely disappeared from the theory and one can say that the Goldstone boson has been eaten away to give the photon mass. Let us count degrees of freedom before and after spontaneous symmetry breaking has occurred. We started with a massless photon,which has two degrees of freedom and a complex scalar field,which has two degrees of freedom. After spontaneous symmetry breaking ,we have one massive photon,which has three degrees of freedom and a real scalar field ,which has one degree of freedom. So,the total number of degrees of freedom of the theory are conserved.

2.3.1 Why does a photon have only two degrees of freedom?

The massless vector field A_μ has four components. Hence it may seem that A_μ has four degrees of freedom,but quantizing A_μ gives only two degrees of freedom rather than four.In this section, we will study the reason behind this interesting fact.

The equation of motion for the electromagnetic field is :

$$\partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu = 0 \tag{2.36}$$

Gauge invariance allows us to choose a gauge. In this case, let us choose Lorentz gauge.

$$\partial_\mu A^\mu = 0 \tag{2.37}$$

Under Lorentz gauge the equation of motion reduces to:

$$\partial_\mu \partial^\mu A^\nu = 0 \tag{2.38}$$

The above equation has solutions of the form:

$$A_\mu = \xi_\mu(p) e^{-ip.x} \tag{2.39}$$

where ξ_μ is the four component polarization vector and p is the photon four momentum. The above equation has to satisfy the Lorentz condition.

$$\xi_\mu p^\mu = 0 \tag{2.40}$$

Inserting this constraint into eq.(2.39), we find that there are only three independent components.

Next we will consider an additional gauge transformation, which is given as:

$$\begin{aligned} A'_\mu &= A_\mu + \partial_\mu \lambda \\ \nabla^2 \lambda &= 0 \end{aligned} \tag{2.41}$$

Let us choose $\lambda = ibe^{-ip.x}$.For this choice, we find the following:

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \lambda = (\xi_\mu + bp_\mu)e^{-ip.x} \quad (2.42)$$

From the above equation, we find that the electromagnetic field is invariant under the transformation $\xi'_\mu \to \xi_\mu + bp_\mu$. we can choose b such that $\xi_0 = 0$(Coulomb gauge).Therefore, from eq.(2.40), we find that:

$$\overrightarrow{\xi}.\overrightarrow{p} = 0 \quad (2.43)$$

From the eq.(2.43), we infer that there exists only two independent components, both perpendicular to the photon's momentum. Thus, there are only two degrees of freedom for the photon and they are transverse.

In the case of massive vector field, the Lagrangian is given as:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu \quad (2.44)$$

Equation of motion for the above Lagrangian is :

$$\partial_\mu F^{\mu\nu} = -2m^2 A^\nu \quad (2.45)$$

We can see that the above Lagrangian is not invariant under the gauge transformation due to the second term in the equation(2.44). Taking divergence of the equation(2.44), we find that:

$$\partial_\nu \partial_\mu F^{\mu\nu} = -2m^2 \partial_\nu A^\nu \quad (2.46)$$

Since $F^{\mu\nu}$ is anti-symmetric, the above equation reduces to:

$$\partial_\nu A^\nu = 0 \quad (2.47)$$

For a massive vector boson,we can see that there exists only one constraint equation reducing degrees of freedom from four to three. It is not possible to have any further reduction of degrees of freedom. Therefore,a massive vector field boson has three degrees of freedom.

We have seen that a complex scalar field and a massless vector field, both with two degrees of freedom, as a result of the Higgs mechanism were transformed into one real scalar field with one degree of freedom and a massive vector boson field with 3 degrees of freedom respectively. A massless spin 1 particle has two transverse polarized states, while a massive spin 1 particle has an additional longitudinal polarized state.

2.4 Basic facts about Non-Abelian gauge theory:

In order to work with the non-Abelian gauge theories in the following sections,it is worthwhile to glance through the general transformation properties of the non-Abelian gauge theory.

Consider a set of complex scalar fields $\phi_I(x)$, where $I = 1, 2, ...N$. The natural generalization of a phase transformation on a single field would be a unitary transformation on this set of fields:

$$\phi_I(x) \to U_{IJ}\phi_J(x) \tag{2.48}$$

where U_{IJ} satisfies $U^\dagger U$ =I

Let us take the unitary symmetry to be a local one, which means that U is an arbitrary function $U(x)$. The Lagrangian above fails to be invariant because:

$$\partial_\mu\phi^\dagger\partial^\mu\phi \to \partial_\mu(\phi^\dagger U^{-1})\partial^\mu(U\phi) \tag{2.49}$$

Expanding right hand side of the above equation we get:

$$(\partial_\mu\phi^\dagger U^{-1} + \phi^\dagger\partial_\mu U^{-1})(\partial^\mu U\phi + U\partial^\mu\phi) = \partial_\mu\phi^\dagger\partial^\mu\phi + \partial_\mu\phi^\dagger(U^{-1}\partial^\mu U)\phi + \phi^\dagger(\partial_\mu U^{-1}U)\partial^\mu\phi + \phi^\dagger(\partial_\mu U^{-1}\partial^\mu U)\phi \tag{2.50}$$

The terms that are made out of the U_{IJ} and sandwiched between $\phi_I^\dagger$ and ϕ_J can be canceled by introducing a matrix valued vector field $A_{\mu IJ}$ and generalizing the derivative to:

$$\partial_\mu\phi_I \to (D_\mu\phi)_I = (\partial_\mu\delta_{IJ} - ieA_{\mu IJ})\phi_J \tag{2.51}$$

It can be written more implicitly as:

$$\partial_\mu\phi \to D_\mu\phi = (\partial_\mu - ieA_\mu)\phi \tag{2.52}$$

We need to arrange in such a way that under the transformation $\phi(x) \to U(x)\phi(x)$,

$$D_\mu\phi \to U(x)D_\mu\phi \tag{2.53}$$

such that kinetic term $D_\mu\phi^\dagger D^\mu\phi$ remains invariant.This will be possible if A_μ transforms to $A_\mu^{'}$ in such a way that:

$$(\partial_\mu - ieA_\mu^{'})\phi \to U(\partial_\mu - ieA_\mu)\phi \tag{2.54}$$

$$\implies \partial_\mu U\phi + U\partial_\mu\phi - ieA_\mu^{'}U\phi = U\partial_\mu\phi - ieUA_\mu\phi \tag{2.55}$$

Since the above equation is true for every ϕ, we have:

$$A_\mu^{'} = UA_\mu U^{-1} - \frac{i}{e}\partial_\mu UU^{-1} \tag{2.56}$$

Therefore, the commutator of the vector field and the gauge parameter enters into the transformation law .Now, to find the kinetic term that will make the matrix A_μ into a propagating field, let us consider the commutator of covariant derivatives($[D_\mu, D_\nu]$).Under the local transformation of the field, covariant derivative also has the same transformation law Thus, the second covariant derivative of ϕ also transforms as eq.(2.52). Therefore, we can conclude that the above transformation law applies for commutator of derivatives.

$$[D_\mu, D_\nu]\phi(x) \to U(x)[D_\mu, D_\nu]\phi(x) \tag{2.57}$$

Inserting D_μ in the above equation and expanding the commutator, we find that :

$$[D_\mu, D_\nu] = -ieF_{\mu\nu} \tag{2.58}$$

where,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu] \tag{2.59}$$

we can find the transformation law for $F_{\mu\nu}$ by inserting eq.(2.56) in eq.(2.59), which gives:

$$F'_{\mu\nu} = U^{-1}F_{\mu\nu}U \tag{2.60}$$

For a general case , any unitary matrix can be written as: $U = e^{i\Lambda}$, where Λ is a Hermitian matrix. The results for an infinitesimal gauge transformation by assuming the matrix has small entries and dropping terms of second and higher orders are:

$$\delta A_\mu = \frac{1}{e}\partial_\mu\Lambda + i[\Lambda, A_\mu] \tag{2.61}$$

$$\delta F_{\mu\nu} = -i[\Lambda, F_{\mu\nu}] \tag{2.62}$$

The above equation tells us that the field strength is no longer a gauge-invariant quantity.However, we can find gauge invariant combinations of the field strengths. One such gauge-invariant kinetic energy term for A^i_μ is:

$$-\frac{1}{4}tr(F_{\mu\nu}F^{\mu\nu}) \tag{2.63}$$

The term " $-\frac{1}{4}tr(F_{\mu\nu}F^{\mu\nu})$" is called Yang-Mills Lagrangian. Pure Yang-Mills theory is an interacting theory, which we can see this by expanding Yang-Mills Lagrangian:

$$\begin{aligned}-\frac{1}{4}tr(F_{\mu\nu}F^{\mu\nu}) = &-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{i}{e}(\partial_\mu A_\nu - \partial_\nu A_\mu)[A_\mu, A_\nu] \\ &+ \frac{e^2}{4}[A_\mu, A_\nu][A_\mu, A_\nu]\end{aligned} \tag{2.64}$$

This term $-\frac{1}{4}tr(F_{\mu\nu}F^{\mu\nu})$ contains both free terms and interaction terms that are precisely dictated: a 3-point and a 4-point interaction with related coefficients.

2.4.1 Systematics of the Higgs mechanism:

The Higgs mechanism explained in the section 2.3 extends straightforwardly to systems with non-Abelian gauge symmetry. Let us first derive the general relation through which a set of scalar field VEV values lead to the appearance of gauge bosons[4] and in the following sections we will apply this to *SU(2)* gauge symmetry.

Consider a system of scalar fields ϕ_i in a Lagrangian,which is invariant under a symmetry group G that is represented by the transformation:

$$\phi_i \rightarrow (1 + i\alpha^a t^a)_{ij}\phi_j \tag{2.65}$$

In general, t^a are Hermitian. To write the ϕ_i as real-valued fields, t^a must be pure imaginary and antisymmetric.

$$t^a_{ij} = iT^a_{ij} \tag{2.66}$$

where T^a are real and antisymmetric.

We promote symmetry group G to a local gauge symmetry, then the co-variant derivative on the ϕ_i is:

$$D_\mu\phi = (\partial_\mu - iA^a_\mu t^a)\phi = (\partial_\mu + A^a_\mu T^a)\phi \tag{2.67}$$

Then the kinetic energy term reads as:

$$\frac{1}{2}(D_\mu\phi)^2 = \frac{1}{2}(\partial_\mu\phi)^2 + A^a_\mu(\partial_\mu\phi_i T^a_{ij}\phi_j) + \frac{1}{2}A^a_\mu A^{b\mu}(T^a\phi)_i(T^b\phi)_i \tag{2.68}$$

Let the VEV of ϕ be

$$\langle\phi_i\rangle = (\phi_0)_i \tag{2.69}$$

When we expand the ϕ_i about these VEV, the last term of eq(2.68) contains a term with the structure of a gauge boson mass,

$$\frac{1}{2}(D_\mu\phi)^2 = \frac{1}{2}m^2_{ab}A^a_\mu A^{b\mu} + \text{.........................} \tag{2.70}$$

where $m^2_{ab} = (T^a\phi)_i(T^b\phi)_i$ evaluated at VEV. Therefore, all the gauge bosons will receive positive masses and when a particular generator T^a of G leaves the vacuum invariant, the corresponding gauge boson remains massless.

2.4.2 *SU(2)*-Higgs Mechanism

In this section,we will apply the above mentioned general formalism to the *SU(2)* representation[5],[6].*SU(2)* group has two important representations:
1)Doublet representation
2)Triplet representation
We will discuss the Higgs mechanism for each of the above representations. In the case of *SU(2)*, the unitary matrix is given by:

$$U(x)=exp(i\alpha^i(x)\frac{\sigma^i}{2}) \tag{2.71}$$

Doublet Representation

The covariant derivative associated with local *SU(2)* symmetry:

$$D_\mu = \partial_\mu - iA^a_\mu \frac{\sigma^a}{2} \tag{2.72}$$

The covariant derivative acting on ϕ is:

$$D_\mu\phi = (\partial_\mu - iA^a_\mu \frac{\sigma^a}{2})\phi \tag{2.73}$$

The Lagrangian for the complex scalar field is :

$$\mathcal{L} = Tr(D_\mu\phi^\dagger D^\mu\phi) + \frac{g}{4}(\phi^\dagger\phi - \frac{v^2}{2})^2 \tag{2.74}$$

From the above Lagrangian,we find that the V(ϕ) has local minimum at $|\phi| = \frac{v}{\sqrt{2}}$.Using the freedom of rotations of *SU(2)*, we can write the expectation value in the form:

$$\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \tag{2.75}$$

In order to study the theory around the minimum, we will define a new field:

$$\tilde{\phi} = \phi - \phi_0 \tag{2.76}$$

This new field $\tilde{\phi}$ has its minimum value at 0 and therefore we can expect this field to have decent properties. Inserting $\tilde{\phi}$ in the above eq.(2.73), which then becomes:

$$D_\mu\phi = D_\mu\tilde{\phi} + D_\mu\phi_0 \tag{2.77}$$

The gauge boson masses arise from:

$$|D_\mu\phi|^2 = \frac{1}{2}\begin{pmatrix} 0 & v \end{pmatrix}\frac{\sigma^a}{2}\frac{\sigma^b}{2}\begin{pmatrix} o \\ v \end{pmatrix} A^a_\mu A^{b\mu} + \ldots\ldots\ldots\ldots\ldots\ldots \tag{2.78}$$

Symmetrizing the matrix product under the interchange of a and b, we find that :

$$|D_\mu\phi|^2 = \frac{v^2}{8}A^a_\mu A^{a\mu} + ... \tag{2.79}$$

where $\left\{\frac{\sigma^a}{2}, \frac{\sigma^b}{2}\right\} = \frac{1}{2}\delta^{ab}$.Therefore, all three gauge bosons become massive with mass :

$$m_A = \frac{v}{2} \tag{2.80}$$

This tells us that all three generators of *SU(2)* are broken by the VEV (ϕ_0). For complex scalar doublet there are four degrees of freedom and there are six degrees of freedom for the vector bosons. Through the Higgs mechanism, the Lagrangian is transformed into one real scalar, three massive vector bosons. The single remaining scalar is identified with the Higgs boson. After Higgs mechanism,again counting degrees of freedom, we have one from the Higgs and nine from the massive vector bosons, adding up to ten.

Adjoint Representation:

In this section,we will discuss the above derived formalism in the case of *SU(2)* gauge field coupled to Higgs field in the adjoint representation. The covariant derivative of ϕ is written as:

$$D_\mu\phi = \partial_\mu\phi - i[A_\mu, \phi] \tag{2.81}$$

where $\phi = \phi^a\tau^a$

$$\begin{aligned} \phi &= \phi^a\tau^a \\ A_\mu &= A^a_\mu\frac{\sigma^a}{2} \end{aligned} \tag{2.82}$$

In terms of matrices,A_μ is:

$$A_\mu = \frac{1}{2}\begin{bmatrix} A^3_\mu & A^1_\mu - iA^2_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu \end{bmatrix} \tag{2.83}$$

This is represented as:

$$A_\mu = \frac{1}{2}\begin{bmatrix} a_\mu & W_\mu \\ W^*_\mu & -a_\mu \end{bmatrix} \tag{2.84}$$

where

$$\begin{aligned} a_\mu &= A^3_\mu \\ W_\mu &= A^1_\mu - iA^2_\mu \end{aligned} \tag{2.85}$$

The Lagrangian is :

$$\mathcal{L} = Tr(D_\mu\phi^\dagger D^\mu\phi) + \frac{g}{4}(tr(\phi^2) - \frac{v}{2}^2)^2 \tag{2.86}$$

In the adjoint representation ,ϕ is a 2×2 traceless Hermitian matrix,whose effects can be analyzed by diagonalizing it(any 2×2 traceless Hermitian matrix can be generated by taking a particular linear combination of generators of *SU(2)*).

From eq.(2.86), we see that V(ϕ) has local minimum at :

$$Tr(\phi^2) = \frac{v^2}{2} \tag{2.87}$$

$$\Longrightarrow a^2 + b^2 = \frac{v^2}{2} \tag{2.88}$$

Using the degree of freedom of *SU(2)* group, the VEV is reduced to :

$$\phi_0 = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \tag{2.89}$$

where $a^2 + b^2 = \frac{v^2}{2}$ and since ϕ is a traceless Hermitian matrix,we have $a^2 = \frac{v^2}{4}$. In order to study the theory around the minimum, we will define a new field:

$$\tilde{\phi} = \phi - \phi_0 \tag{2.90}$$

This new field $\tilde{\phi}$ has its minimum value at 0. The Lagrangian in terms of the new field is:

$$D_\mu \phi = D_\mu \tilde{\phi} + D_\mu \phi_0 \tag{2.91}$$

$$\Rightarrow D_\mu \phi = D_\mu \tilde{\phi} - i[A_\mu, \phi_0] \tag{2.92}$$

The mass terms come from the square of the kinetic term.

$$(D_\mu \phi)^2 = (D_\mu \tilde{\phi})^2 - [A_\mu, \phi_0]^2 - iD_\mu \tilde{\phi}[A^\mu, \phi_0] - i[A_\mu, \phi_0]D^\mu \phi \tag{2.93}$$

Consider $[A_\mu, \phi_0]^2$, which may be expanded as:

$$[A_\mu, \phi_0]^2 = \frac{1}{4} \begin{bmatrix} W_\mu W^{\mu *}(2ba - \frac{v^2}{2}) & 0 \\ 0 & W_\mu W^{\mu *}(2ba - \frac{v^2}{2}) \end{bmatrix} \tag{2.94}$$

Using $(a+b)^2 = 0$ and $a^2 + b^2 = \frac{v^2}{2}$, we find that:

$$a^2 + b^2 = -2ab \tag{2.95}$$

$$\Rightarrow (2ba - \frac{v^2}{2}) = -v^2 \tag{2.96}$$

$$tr([A_\mu, \phi_0]^2) = -\frac{1}{2} v^2 W_\mu W^{\mu *} \tag{2.97}$$

$$tr([A_\mu, \phi_0]^2) = -\frac{1}{2}v^2(A_\mu^1)^2 - \frac{1}{2}v^2(A_\mu^2)^2 \tag{2.98}$$

Therefore,one can see that the $tr(D_\mu\phi)^2$ picks up mass terms:

$$tr(D_\mu\phi)^2 = \frac{1}{2}m^2((A_\mu^1)^2 + (A_\mu^2)^2) + \ldots\ldots\ldots\ldots\ldots\ldots \tag{2.99}$$

Where $m = v$.The gauge bosons corresponding to the generators 1 and 2 acquire masses, while the boson corresponding to the generator 3 remain massless. This means that the VEV destroys the symmetry of rotation about two axis of rotation,but the symmetry of rotation about the third axis of rotation that corresponds to the massless gauge boson is preserved. The resulting theory has unbroken $U(1)$ symmetry. For a scalar triplet there are three degrees of freedom and there are six degrees of freedom for the vector bosons. Through the Higgs mechanism, the Lagrangian is transformed into one real scalar, two massive vector bosons. After the Higgs mechanism, again counting degrees of freedom, we have one from the Higgs, six from the massive vector bosons and two from the massless vector boson, adding to nine

Chapter 3

Bifundamental Higgs mechanism

3.1 Introduction

A bifundamental representation is a tensor product of two fundamental or anti fundamental representations. An example is the PQ-dimensional representation $(P, \bar{Q})$ of the group *SU(P)*×*SU(Q)*. In this chapter, we will study the Higgs mechanism in the context of different gauge groups in the bifundamental representation. Our considerations apply to Yang-Mills in (3+1)d as well as both Yang-Mills and Chern-simons in (2+1)d.

3.2 Spontaneous Breaking of *SU(2)* × *U(1)*

In this section, we will study the spontaneous breaking of *SU(2)* × *U(1)* , which gives the correct description of the weak interaction. This theory is called "The Glashow-Weinberg-Salam Theory of Electroweak Interactions." The gauge transformation for this group is given as:

$$\phi \rightarrow e^{i\alpha^a \tau^a} e^{i\beta/2} \phi \tag{3.1}$$

Now if the field acquires a VEV, then general form of the VEV is given as:

$$\langle \phi \rangle = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \tag{3.2}$$

where v_1 and v_2 are complex. General element of *SU(2)* is given as :

$$\begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \tag{3.3}$$

Acting *SU(2)* on $< \phi >$, we get:

$$\begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} av_1 + bv_2 \\ -b^* v_1 + a^* v_2 \end{pmatrix} \tag{3.4}$$

we can choose a and b, such that $av_1 + bv_2 = 0$, which gives $\frac{a}{b} = \frac{-v_2}{v_1}$. This reduces VEV to:

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ \frac{a^*}{v_2^*}(|v_1|^2 + |v_2|^2) \end{pmatrix} \tag{3.5}$$

which can be further reduced to:

$$\begin{pmatrix} 0 \\ \upsilon e^{i\alpha} \end{pmatrix} \tag{3.6}$$

where υ and α are real. After acting *U(1)* on $< \phi >$ in (eq.(3.6)),VEV is :

$$\begin{pmatrix} 0 \\ \upsilon e^{i\alpha + i\chi} \end{pmatrix} \tag{3.7}$$

Again using degree of freedom of *SU(2)* group, we find that:

$$\begin{pmatrix} e^{i\alpha+i\chi} & 0 \\ 0 & e^{-i\alpha-i\chi} \end{pmatrix} \begin{pmatrix} 0 \\ \upsilon e^{i\alpha+i\chi} \end{pmatrix} = \begin{pmatrix} 0 \\ \upsilon \end{pmatrix} \tag{3.8}$$

where υ is real. Therefore, most general form of VEV in the case of *SU(2)* $\times$ *U(1)* gauge theory is given by:

$$\langle\phi\rangle = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ \upsilon \end{pmatrix} \tag{3.9}$$

where υ is real.

3.2.1 Mass spectrum of *SU(2)* $\times$ *U(1)*

In this section, we will work out the details of the mass spectrum generated by *SU(2)* $\times$ *U(1)* gauge theory by using the methods discussed in the previous sections.
The covariant derivative of ϕ is:

$$D_\mu\phi = (\partial_\mu - igA_\mu^a\tau^a - i\frac{g'}{2}B_\mu)\phi \tag{3.10}$$

where A_μ^a and B_μ are *SU(2)* and *U(1)* gauge bosons respectively and g,g' are coupling constants of *SU(2)* and *U(1)* gauge groups respectively. The gauge boson mass terms come from the square of the kinetic terms,evaluated at the VEV. The relevant terms are:

$$\Delta\mathcal{L} = \frac{1}{2}(0 \quad v)(gA_\mu^a\tau^a + \frac{g'}{2}B_\mu)(gA^{b\mu}\tau^b + \frac{g'}{2}B^\mu)\begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{3.11}$$

After evaluating the matrix product explicitly, we find that:

$$\Delta\mathcal{L} = \frac{1}{2}\frac{v^2}{4}[g^2(A_\mu^1)^2 + g^2(A_\mu^2)^2 + (-gA_\mu^3 + g'B_\mu)^2]. \tag{3.12}$$

This gives rise to three massive vector bosons,which we will notate as follows:

$$\begin{aligned} W_\mu^{\pm} &= \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2) \ \text{ with mass } \ m_W = g\frac{v}{2}; \\ Z_\mu^0 &= \frac{1}{\sqrt{g^2+g'^2}}(gA_\mu^3 - g'B_\mu) \ \text{ with mass } \ m_Z = \sqrt{g^2+g'^2}\frac{v}{2}. \end{aligned} \tag{3.13}$$

Since this theory contains four gauge bosons, out of which three are massive,there exists a fourth vector field,which is massless and orthogonal to Z_μ^0:

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gA_\mu^3 + g'B_\mu) \ \text{ with mass } \ m_A = 0. \tag{3.14}$$

This theory contains one massless gauge boson and three gauge bosons that acquire mass from the Higgs mechanism. These three gauge bosons were detected in proton-antiproton collisions at the converted Super Proton Synchrotron at mass values $m_{W^{\pm}}$=80.38 GeV/c^2 and m_Z=91.19 GeV/c^2. Thus, *SU(2)* $\times$ *U(1)* symmetry is broken to *U(1)* .

3.3 Spontaneous breaking of bifundamental *SU(2)* $\times$ *SU(2)*

In this section,we shall discuss the breaking of bifundamental *SU(2)* $\times$ *SU(2)* group. The gauge transformation for this group is given as:

$$\phi \to e^{i\alpha^a\tau^a}e^{-i\alpha'^b\tau^b/2}\phi. \tag{3.15}$$

where a=1,2,3 and b=1,2,3.

If the field acquires VEV,it is given as :

$$\langle\phi\rangle = \begin{pmatrix} v_1' & v_2' \\ v_3' & v_4' \end{pmatrix} \tag{3.16}$$

where v_1', v_2', v_3', v_4' are complex entries.Now we shall focus on the problem of diagonalizing the above VEV matrix using *SU(2)* $\times$ *SU(2)* symmetry group. To diagonalize the above matrix,we need to discuss an important theorem in the linear algebra called Singular Value Decomposition (SVD).

Singular Value Decomposition(SVD):

Singular Value Decomposition (SVD) is a factorization of a real or complex matrix. This has many useful applications in signal processing and statistics.

Statement of the theorem(SVD):

Suppose M is an m×n matrix whose entries come from the field K, which is either the field of real numbers or the field of complex numbers. Then there exists a factorization of the form

$$M = U\Sigma V^{\dagger} \tag{3.17}$$

where U is an m×m unitary matrix over K, the matrix Σ is an m×n diagonal matrix with positive real numbers on the diagonal, and the n×n unitary matrix $V^{\dagger}$ is the conjugate transpose of the n×n unitary matrix V. Such a factorization is called a singular value decomposition of M[7].

The diagonal entries σ_i of Σ are known as the singular values of M. The diagonal matrix Σ is uniquely determined by M (though the matrices U and V are not).

Consider the following transformation:

$$U = U'P \quad V = V'Q \tag{3.18}$$

where P,Q are diagonal matrices and U',V' are special unitary matrices. Diagonalizing the matrix M in the eq.(3.17), using U' and V':

$$U'P\Sigma Q^{\dagger}V'^{\dagger} \tag{3.19}$$

which reduces to :

$$U'\tilde{\mathbf{\Sigma}}V'^{\dagger} \tag{3.20}$$

From the eq.(3.20), we arrive at an interesting result that when the unitary matrices are restricted to only special unitary matrices, this leads to a diagonal matrix $\tilde{\mathbf{\Sigma}}$ with complex entries on the diagonal. So we may write the above theorem(SVD), when restricted to special unitary matrices.

Singular Value Decomposition(SVD)
(restricted to special unitary matrices):

Suppose M is an $m \times n$ matrix whose entries come from the field K, which is either the field of real numbers or the field of complex numbers. Then there exists a factorization of the form

$$M = U'\tilde{\mathbf{\Sigma}}V'^{\dagger} \tag{3.21}$$

where M is an $m \times m$ special unitary matrix over K, the matrix $\tilde{\mathbf{\Sigma}}$ is an $m \times n$ diagonal matrix with complex entries on the diagonal. This implies that given any matrix and bifundamental symmetry group, we can diagonalize it using SVD. Going back to our discussion on Spontaneous Breaking of *SU(2)* $\times$ *SU(2)* , using SVD the VEV in eq.(3.16) can be reduced to :

$$\langle\phi\rangle = \begin{pmatrix} \upsilon_1 & 0 \\ 0 & \upsilon_2 \end{pmatrix} \tag{3.22}$$

where υ_1 and υ_2 are complex numbers.

3.3.1 Mass spectrum of *SU(2)* $\times$ *SU(2)*

In this section, we will explore the mass spectrum generated by *SU(2)* $\times$ *SU(2)* gauge theory by using the methods discussed in the previous sections. The covariant derivative of ϕ is:

$$D_\mu\phi = \partial_\mu\phi - igA^a_\mu\tau^a\phi - ig'\phi\tilde{A}^b_\mu\tau^b \tag{3.23}$$

where A^a_μ and $\tilde{A}^b_\mu$ are *SU(2)* $\times$ *SU(2)* gauge bosons and g,g' are coupling constants of the *SU(2)* gauge groups. The gauge boson mass terms come from the kinetic term of the Lagrangian evaluated at the VEV. The relevant terms are:

$$\Delta\mathcal{L} = tr[gA^a_\mu\tau^a\phi + g'\phi\tilde{A}^a_\mu\tau^a][g\phi^*A^{\mu b}\tau^b + g'\tilde{A}^{\mu b}\tau^b\phi^*] \tag{3.24}$$

$$\begin{aligned}\Rightarrow \Delta\mathcal{L} =& tr[g^2A^a_\mu\tau^a\phi\phi^*A^{\mu b}\tau^b + gg'A^a_\mu\tau^a\phi\tilde{A}^{\mu b}\tau^b\phi^* \\ &+ g'g\phi\tilde{A}^a_\mu\tau^a\phi^*A^{\mu b}\tau^b + g'^2\phi\tilde{A}^a_\mu\tau^a\tilde{A}^{\mu b}\tau^b\phi^*]\end{aligned} \tag{3.25}$$

where

$$\begin{aligned} A_\mu = A^a_\mu\tau^a &= \frac{1}{2}\begin{bmatrix} A^3_\mu & A^1_\mu - iA^2_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu \end{bmatrix}; \\ \tilde{A}_\mu = \tilde{A}^a_\mu\tau^a &= \frac{1}{2}\begin{bmatrix} \tilde{A}^3_\mu & \tilde{A}^1_\mu - i\tilde{A}^2_\mu \\ \tilde{A}^1_\mu + i\tilde{A}^2_\mu & -\tilde{A}^3_\mu \end{bmatrix} \end{aligned} \tag{3.26}$$

Using VEV of eq.(3.22), we will explicitly write down each of the four terms in the above eq.(3.25).

$$\begin{aligned} tr(g^2A^a_\mu\tau^a\phi\phi^*A^{b^\mu}\tau^b) =& \frac{g^2}{4}[|\upsilon_1|^2A^3_\mu A^{\mu 3} + |\upsilon_2|^2(A^1_\mu A^{\mu 1} + A^2_\mu A^{\mu 2}) \\ &+|\upsilon_1|^2(A^1_\mu A^{\mu 1} + A^2_\mu A^{\mu 2}) + |\upsilon_2|^2A^3_\mu A^{\mu 3}] \end{aligned} \tag{3.27}$$

$$\begin{aligned} tr[gg'A^a_\mu\tau^a\phi\tilde{A}^{\mu b}\tau^b\phi^*] =& \frac{gg'}{4}[|\upsilon_1|^2A^3_\mu\tilde{A}^{\mu 3} + \upsilon_1^*\upsilon_2(A^1_\mu - iA^2_\mu)(\tilde{A}^{\mu 1} + i\tilde{A}^{\mu 2}) \\ &+ \upsilon_2^*\upsilon_1(A^1_\mu + iA^2_\mu)(\tilde{A}^{\mu 1} - i\tilde{A}^{\mu 2}) + |\upsilon_2|^2A^3_\mu\tilde{A}^{\mu 3}] \end{aligned} \tag{3.28}$$

$$
\begin{aligned}
tr[g'g\phi\tilde{A}^a_\mu\tau^a\phi^* A^{\mu^b}\tau^b] =&\frac{g'g}{4}[|\upsilon_1|^2\tilde{A}^3_\mu A^{\mu3} + \upsilon_1\upsilon_2^*(\tilde{A}^1_\mu - i\tilde{A}^2_\mu)(A^{\mu1} + iA^{\mu2}) \\
&+ \upsilon_1^*\upsilon_2(\tilde{A}^1_\mu + i\tilde{A}^2_\mu)(A^{\mu1} - iA^{\mu2}) + |\upsilon_2|^2\tilde{A}^3_\mu A^{\mu3}]
\end{aligned}
\tag{3.29}
$$

$$
\begin{aligned}
tr[g'^2\phi\tilde{A}^a_\mu\tau^a\tilde{A}^{\mu b}\tau^b\phi^*] =&\frac{g'^2}{4}[|\upsilon_1|^2\tilde{A}^3_\mu\tilde{A}^{\mu3} + |\upsilon_2|^2(\tilde{A}^1_\mu\tilde{A}^{\mu1} + \tilde{A}^2_\mu\tilde{A}^{\mu2}) \\
&+|\upsilon_1|^2(\tilde{A}^1_\mu\tilde{A}^{\mu1} + \tilde{A}^2_\mu\tilde{A}^{\mu2}) + |\upsilon_2|^2\tilde{A}^3_\mu\tilde{A}^{\mu3}]
\end{aligned}
\tag{3.30}
$$

Inserting the above equations (eq.(2.27)- eq.(3.30)) in eq.(3.25), we find that:

$$
\begin{aligned}
\Delta\mathcal{L} =&\frac{1}{4}[g^2(|\upsilon_1|^2 + |\upsilon_2|^2)(A^1_\mu)^2 + g^2(|\upsilon_1|^2 + |\upsilon_2|^2)(A^2_\mu)^2 + g^2(|\upsilon_1|^2 + |\upsilon_2|^2)(A^3_\mu)^2 \\
&+ 2gg'(|\upsilon_1|^2 + |\upsilon_2|^2)A^\mu_3\tilde{A}^{\mu3} + 2gg'\upsilon_1^*\upsilon_2(A^1_\mu - iA^2_\mu)(\tilde{A}^{\mu1} + i\tilde{A}^{\mu2}) \\
&+ 2gg'\upsilon_2^*\upsilon_1(A^{\mu1} + iA^{\mu2})(\tilde{A}^1_\mu - i\tilde{A}^2_\mu) + g'^2(|\upsilon_1|^2 + |\upsilon_2|^2)(\tilde{A}^1_\mu)^2 \\
&+ g'^2(|\upsilon_1|^2 + |\upsilon_2|^2)(\tilde{A}^2_\mu)^2 + g'^2(|\upsilon_1|^2 + |\upsilon_2|^2)(\tilde{A}^3_\mu)^2]
\end{aligned}
\tag{3.31}
$$

Based on the different choices of VEV, we have three cases:

Case 1 : $\upsilon_1 = \upsilon_2 = 0$

$$
\Rightarrow \Delta\mathcal{L} = 0 \tag{3.32}
$$

This choice of VEV doesn't break the symmetry of *SU(2)* $\times$ *SU(2)* group. Hence, there are no massive gauge bosons.

Case 2 : $\upsilon_1 \neq \upsilon_2 \neq 0$

$$
\begin{aligned}
\Rightarrow \Delta\mathcal{L} =&\frac{1}{4}[2|g\upsilon_1 W^+_\mu + g'\upsilon_2\tilde{W}^+_\mu|^2 + 2|g\upsilon_2 W^-_\mu + g'\upsilon_1\tilde{W}^-_\mu|^2 \\
&+ (|\upsilon_1|^2 + |\upsilon_2|^2)(gA^3_\mu + g'\tilde{A}^3_\mu)^2]
\end{aligned}
\tag{3.33}
$$

where

$$
\begin{aligned}
W^\pm_\mu &= \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu) \\
\tilde{W}^\pm_\mu &= \frac{1}{\sqrt{2}}(\tilde{A}^1_\mu \mp i\tilde{A}^2_\mu)
\end{aligned}
\tag{3.34}
$$

For this ($\upsilon_1 \neq \upsilon_2 \neq 0$) choice of VEV, *SU(2)* $\times$ *SU(2)* group is broken to *U(1)* , therefore the theory consists of five massive gauge bosons and one massless vector field. For a bifundamental complex scalar field there are eight degrees of freedom and there are twelve degrees of freedom for the vector bosons. Through the Higgs mechanism, the Lagrangian is transformed into three singlet fields, five massive vector bosons and one massless boson. After Higgs mechanism, again counting degrees of freedom, we have two from the massless boson, three from the singlet fields and fifteen from the massive vector bosons, adding up to twenty.

Case 3 : $\upsilon_1 = \upsilon_2 = \upsilon$

$$\begin{aligned} &\Rightarrow \Delta\mathcal{L} = \frac{1}{2}\upsilon^2[(gA^1_\mu + g'\tilde{A}^1_\mu)^2 + (gA^2_\mu + g'\tilde{A}^2_\mu)^2 + (gA^3_\mu + g'\tilde{A}^3_\mu)^2] \\ &\Rightarrow \Delta\mathcal{L} = \frac{1}{2}\upsilon^2[2|gW^-_\mu + g'\tilde{W}^-_\mu|^2 + (gA^3_\mu + g'\tilde{A}^3_\mu)^2] \end{aligned} \tag{3.35}$$

From eq.(3.35), we find that when $\upsilon_1 = \upsilon_2 = \upsilon$, breaking of *SU(2)* $\times$ *SU(2)* symmetry group gives rise to three massive gauge bosons and three massless gauge bosons. The massless gauge bosons orthogonal to the massive gauge bosons are :

$$(g'A^1_\mu - g\tilde{A}^1_\mu), (g'A^2_\mu - g\tilde{A}^2_\mu), (g'A^3_\mu - g\tilde{A}^3_\mu) \tag{3.36}$$

Therefore,for this($\upsilon_1 = \upsilon_2 = \upsilon$) choice of VEV, spontaneous breaking of *SU(2)* $\times$ *SU(2)* symmetry group leads to diagonal *SU(2)* group. For a bifundamental complex scalar field there are eight degrees of freedom and there are twelve degrees of freedom for the vector bosons. Through the Higgs mechanism, the Lagrangian is transformed into scalar triplet, three massive vector bosons, two scalar singlets and three massless bosons. After Higgs mechanism,again counting degrees of freedom, we have six from the massless bosons,two from the scalar singlets, three from the scalar triplet and nine from the massive vector bosons, adding up to twenty. One can see that the case 2 is a result of sequential Higgs mechanism of case 3.

3.4 Spontaneous breaking of bifundamental *SU(3)*$\times$*SU(3)*

As in the previous section, in this section we will study the spontaneous breaking of *SU(3)*$\times$*SU(3)* symmetry group. The gauge transformation for this group is given as:

$$\phi \to e^{i\alpha^a\tau^a}e^{-i\alpha'^b\tau^b/2}\phi. \tag{3.37}$$

Where a=1,2,3,...8 and b=1,2,3,....8. If the field acquires VEV,it is given as :

$$\langle\phi\rangle = \begin{pmatrix} \upsilon'_1 & \upsilon'_2 & \upsilon'_3 \\ \upsilon'_4 & \upsilon'_5 & \upsilon'_6 \\ \upsilon'_7 & \upsilon'_8 & \upsilon'_9 \end{pmatrix} \tag{3.38}$$

Using Singular Value Decomposition, the above form of VEV can be reduced to:

$$\langle\phi\rangle = \begin{pmatrix} \upsilon_1 & 0 & 0 \\ 0 & \upsilon_2 & 0 \\ 0 & 0 & \upsilon_3 \end{pmatrix} \tag{3.39}$$

where υ_1,υ_2 and υ_3 are complex entries.

3.4.1 Mass spectrum of *SU(3)*×*SU(3)*

In this section, we will study the mass spectrum of *SU(3)* × *SU(3)* group. The covariant derivative of ϕ is:

$$D_\mu\phi = \partial_\mu\phi - igA^a_\mu\tau^a\phi - ig'\phi\tilde{A}^b_\mu\tau^b \tag{3.40}$$

where A^a_μ and $\tilde{A}^b_\mu$ are *SU(3)*× *SU(3)* gauge bosons and g,g' are coupling constants of the *SU(3)* gauge groups. The gauge boson mass terms come from the kinetic term of the Lagrangian evaluated at the VEV. The relevant terms are:

$$\Delta\mathcal{L} = tr[gA^a_\mu\tau^a\phi + g'\phi\tilde{A}^a_\mu\tau^a][g\phi^*A^{\mu b}\tau^b + g'\tilde{A}^{\mu b}\tau^b\phi^*] \tag{3.41}$$

$$\begin{aligned}\Rightarrow \Delta\mathcal{L} =& tr[g^2A^a_\mu\tau^a\phi\phi^*A^{\mu b}\tau^b + gg'A^a_\mu\tau^a\phi\tilde{A}^{\mu b}\tau^b\phi^* \\ &+ g'g\phi\tilde{A}^a_\mu\tau^a\phi^*A^{\mu b}\tau^b + g'^2\phi\tilde{A}^a_\mu\tau^a\tilde{A}^{\mu b}\tau^b\phi^*]\end{aligned} \tag{3.42}$$

After expanding each term and finding the trace, we find that:

$$\begin{aligned}\Delta\mathcal{L} =& 2|g\upsilon_1W^-_\mu + g'\upsilon_2\tilde{W}^-_\mu|^2 + 2|g\upsilon_2W^+_\mu + g'\upsilon_1\tilde{W}^+_\mu|^2 \\ &+ 2|g\upsilon_1K^-_\mu + g'\upsilon_3\tilde{K}^-_\mu|^2 + 2|g\upsilon_3K^+_\mu + g'\upsilon_1\tilde{K}^+_\mu|^2 \\ &+ 2|g\upsilon_2M^-_\mu + g'\upsilon_3\tilde{M}^-_\mu|^2 + 2|g\upsilon_3M^+_\mu + g'\upsilon_2\tilde{M}^+_\mu|^2 \\ &+ |\upsilon_1|^2(X_\mu + Z_\mu)^2 + |\upsilon_2|^2(X_\mu - Z_\mu)^2 + 4|\upsilon_3|^2X^2_\mu\end{aligned} \tag{3.43}$$

where

$$\begin{aligned}&W^\pm_\mu = \frac{A^1_\mu \mp iA^2_\mu}{\sqrt{2}}, K^\pm = \frac{A^4_\mu \mp iA^5_\mu}{\sqrt{2}}, \\ &M^\pm_\mu = \frac{A^6_\mu \mp iA^7_\mu}{\sqrt{2}}, \\ &\tilde{W}^\pm_\mu = \frac{\tilde{A}^1_\mu \mp i\tilde{A}^2_\mu}{\sqrt{2}}, \tilde{K}^\pm = \frac{\tilde{A}^4_\mu \mp i\tilde{A}^5_\mu}{\sqrt{2}}, \\ &\tilde{M}^\pm_\mu = \frac{\tilde{A}^6_\mu \mp i\tilde{A}^7_\mu}{\sqrt{2}}, \\ &Z_\mu = gA^3_\mu + g'\tilde{A}^3_\mu, X_\mu = \frac{gA^8_\mu + g'\tilde{A}^8_u}{\sqrt{3}}\end{aligned} \tag{3.44}$$

We will discuss three cases based on the choice of the VEV:

Case 1 : $\upsilon_1 = \upsilon_2 = \upsilon_3 = 0$

$$\Rightarrow \Delta\mathcal{L} = 0 \tag{3.45}$$

This choice of VEV doesn't break the symmetry of *SU(3)×SU(3)* group. Therefore, there are no massive gauge bosons.

Case 2 : $\upsilon_1 \neq \upsilon_2 \neq \upsilon_3 \neq 0$

$$\begin{aligned}\Rightarrow \Delta\mathcal{L} =& \frac{1}{4}[2|g\upsilon_1 W_\mu^- + g'\upsilon_2\tilde{W}_\mu^-|^2 + 2|g\upsilon_2 W_\mu^+ + g'\upsilon_1\tilde{W}_\mu^+|^2 \\ &+ 2|g\upsilon_1 K_\mu^- + g'\upsilon_3\tilde{K}_\mu^-|^2 + 2|g\upsilon_3 K_\mu^+ + g'\upsilon_1\tilde{K}_\mu^+|^2 \\ &+ 2|g\upsilon_2 M_\mu^- + g'\upsilon_3\tilde{M}_\mu^-|^2 + 2|g\upsilon_3 M_\mu^+ + g'\upsilon_2\tilde{M}_\mu^+|^2 \\ &+ |\upsilon_1|^2(X_\mu + Z_\mu)^2 + |\upsilon_2|^2(X_\mu - Z_\mu)^2 + 4|\upsilon_3|^2 X_\mu^2\end{aligned} \tag{3.46}$$

From eq.(3.46),we find that the theory gives 14 massive gauge bosons and two massless gauge bosons. Therefore, for this choice($\upsilon_1 \neq \upsilon_2 \neq \upsilon_3 \neq 0$) of VEV, *SU(3)×SU(3)* symmetry group is broken to *U(1)* × *U(1)* group. For a bifundamental complex scalar field there are eighteen degrees of freedom and there are 32 degrees of freedom for the vector bosons. Through the Higgs mechanism, the Lagrangian is transformed into four scalar singlets,fourteen massive vector bosons and two massless bosons. After Higgs mechanism, again counting degrees of freedom, we have four from the massless bosons, four from the scalar singlets and forty two from the massive vector bosons, adding up to fifty.

Case 3 : $\upsilon_1 = \upsilon_2 = \upsilon_3 = \upsilon$

$$\begin{aligned}\Rightarrow \Delta\mathcal{L} =& \frac{\upsilon^2}{4}[4|gW_\mu^- + g'\tilde{W}_\mu^-|^2 + 4|gK_\mu^- + g'\tilde{K}_\mu^-|^2 \\ &+ 4|gM_\mu^- + g'\tilde{M}_\mu^-|^2 + 2Z_\mu^2 + 6X_\mu^2]\end{aligned} \tag{3.47}$$

$$\begin{aligned}\Rightarrow \Delta\mathcal{L} =& \frac{1}{2}\upsilon^2[(gA_\mu^1 + g'\tilde{A}_\mu^1)^2 + (gA_\mu^2 + g'\tilde{A}_\mu^2)^2 + (gA_\mu^4 + g'\tilde{A}_\mu^4)^2 \\ &+ (gA_\mu^5 + g'\tilde{A}_\mu^5)^2 + (gA_\mu^6 + g'\tilde{A}_\mu^6)^2 + (gA_\mu^7 + g'\tilde{A}_\mu^7)^2 \\ &+ (gA_\mu^3 + g'\tilde{A}_\mu^3)^2 + (gA_\mu^8 + g'\tilde{A}_\mu^8)^2]\end{aligned} \tag{3.48}$$

From eq.(3.48),we find that there are 8 massive gauge bosons and 8 massless gauge bosons. Therefore, for this($\upsilon_1 = \upsilon_2 = \upsilon_3 = \upsilon$) choice of VEV, *SU(3)×SU(3)* symmetry group is broken to diagonal *SU(3)* group. For a bifundamental complex scalar field there are eighteen degrees of freedom and there are 32 degrees of freedom for the vector bosons. Through the Higgs mechanism, the Lagrangian is transformed into two scalar singlets, eight massive vector bosons, one scalar triplet and eight massless bosons. After Higgs mechanism,again counting degrees of freedom, we have sixteen from the massless bosons, two from the singlet fields, eight from the scalar triplet and twenty four from the massive vector bosons, adding up to fifty. One can notice that the case 2 is a result of sequential Higgs mechanism of case 3.

3.5 Discussion

In this chapter, we derived the mass spectrum of the bifundamental symmetry groups. We found the following results:
1)When entries of the VEV are equal, bifundamental $SU(3)\times SU(3)$ and $SU(2) \times SU(2)$ groups are broken to diagonal $SU(3)$ and $SU(2)$ respectively.
2) When the entries of the VEV are not equal, bifundamental $SU(3)\times SU(3)$ and $SU(2) \times SU(2)$ are broken to $U(1) \times U(1)$ and $U(1)$ respectively.The mass spectrum of the bifundamental theories depend on the choice of the VEV. We may generalize the above symmetry breaking patterns to bifundamental $SU(N)\times SU(N)$ symmetry group. We would like to study the symmetry breaking patterns in any bifundamental $SU(N)\times SU(M)$ group and generalize our results.

Chapter 4

Novel Higgs Mechanism(NHM)

4.1 Introduction

Chern-Simons gauge theories in 2+1 dimensions with multiple gauge fields exhibit novel properties. One of its striking features is the possibility of a non-propagating Cherm-Simons field acquiring a massless propagating mode via the Higgs mechanism[8]. This phenomenon is called Novel Higgs Mechanism. NHM was discovered and applied to study the Moduli space of extended super-conformal field theories describing multiple membranes in M-theory. In this chapter, we will study NHM in the case of difference Chern-Simons theory[9].

4.2 Difference Chern-Simons theory

Non-Abelian difference Chern-Simons action is:

$$L_{CS} = \frac{k}{4\pi} tr(A \wedge dA + \frac{2}{3} A \wedge A \wedge A - \tilde{A} \wedge d\tilde{A} - \frac{2}{3} \tilde{A} \wedge \tilde{A} \wedge \tilde{A}) \tag{4.1}$$

where $A_\mu = A^a_\mu T^a$ and $trT^aT^b = \frac{-1}{2}\delta^{ab}$.

A_μ and $\tilde{A}_\mu$ in matrix form are:

$$A_\mu = \frac{i}{2} \left[\begin{array}{cc} a_\mu & A^+_\mu \\ A^-_\mu & -a_\mu \end{array} \right] \tag{4.2}$$

where

$$\begin{aligned} A^\pm_\mu &= A^1_\mu \mp A^2_\mu \\ a_\mu &= A^3_\mu \end{aligned} \tag{4.3}$$

$$\tilde{A}_\mu = \frac{i}{2}\begin{bmatrix} \tilde{a}_\mu & \tilde{A}^+_\mu \\ \tilde{A}^-_\mu & -\tilde{a}_\mu \end{bmatrix} \tag{4.4}$$

where

$$\begin{aligned} \tilde{A}^\pm_\mu &= \tilde{A}^1_\mu \mp \tilde{A}^2_\mu \\ \tilde{a}_\mu &= \tilde{A}^3_\mu \end{aligned} \tag{4.5}$$

After substituting the above matrices(eq.(4.2) and eq.(4.3)) in the Lagrangian(eq.(4.1)), we find that:

$$\begin{aligned} L_{CS} = \frac{k}{4\pi}(-\frac{1}{2})(a \wedge da + A^+ \wedge dA^- + ia \wedge A^+ \wedge A^- - \tilde{a} \wedge d\tilde{a} \\ - \tilde{A}^+ \wedge d\tilde{A}^- - i\tilde{a} \wedge \tilde{A}^+ \wedge \tilde{A}^-) \end{aligned} \tag{4.6}$$

It is preferred to work in the basis, which is obtained by taking the following linear combinations:

$$\begin{aligned} &B^\pm = (\frac{A^\pm - \tilde{A}^\pm}{2}), b = (\frac{a - \tilde{a}}{2}) \\ &C^\pm = (\frac{A^\pm + \tilde{A}^\pm}{2}), c = (\frac{a + \tilde{a}}{2}) \\ &F^c = -dc - \frac{i}{2}(C^+ \wedge C^-), \\ &F^{C^+} = -dC^+ + i(C^+ \wedge C), F^{C^-} = -dC^- - i(C^- \wedge C). \end{aligned} \tag{4.7}$$

In the above basis, the Lagrangian is:

$$\begin{aligned} L_{CS} = \frac{k}{4\pi}(-2b \wedge dc - B^+ \wedge dC^- - B^- \wedge dC^+ - i(c \wedge B^+ \wedge C^- \\ +c \wedge C^+ \wedge B^- + b \wedge B^+ \wedge B^- + b \wedge C^+ \wedge C^-)) \end{aligned} \tag{4.8}$$

From chapter 2, we know that the mass can be given to the gauge field through bifundamental Higgs mechanism. Consider the complex scalar field (ϕ) coupled to the difference Chern-Simons action.
In this case the covariant derivative is:

$$D_\mu\phi = \partial_\mu\phi + A_\mu\phi - \phi\tilde{A}_\mu \tag{4.9}$$

The scalar kinetic term is normalized as :

$$\frac{k}{4\pi} tr(D_\mu\phi^\dagger D^\mu\phi) \tag{4.10}$$

The above kinetic term gives rise to the interaction term:

$$\frac{k}{4\pi} tr\left|A_\mu\phi - \phi\tilde{A}_\mu\right|^2 \tag{4.11}$$

The Higgs VEV is :

$$\begin{pmatrix} v_1' & v_2' \\ v_3' & v_4' \end{pmatrix} \tag{4.12}$$

Using Singular Value Decomposition, the above form is reduced to:

$$< \phi >= \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} \tag{4.13}$$

The VEV in the eq.(4.12) is written as:

$$< \phi >= vI + w\sigma^3 \tag{4.14}$$

where

$$\begin{aligned} v &= \frac{v_1 + v_2}{2} \\ w &= \frac{v_1 - v_2}{2} \end{aligned} \tag{4.15}$$

Expanding the interaction term:

$$\begin{aligned} tr\left|A_\mu\phi - \phi\tilde{A}_\mu\right|^2 =& \frac{1}{4}[|v_1 A_\mu^- - v_2\tilde{A}_\mu^-|^2 + |v_2 A_\mu^+ - v_1\tilde{A}_\mu^+|^2 \\ &+ |v_1|^2 + |v_2|^2(a_\mu - \tilde{a}_\mu)] \end{aligned} \tag{4.16}$$

In terms of the basis in eq.(4.7) and eq.(4.15), the above eq.(4.16) is:

$$\begin{aligned} tr|A_\mu\phi - \phi\tilde{A}_\mu|^2 =& -\frac{w^2}{2}(C^+ \wedge {}^*C^-) - \frac{v^2}{2}(B^+ \wedge {}^*B^-) \\ &- \frac{(v^2 + w^2)}{2}(b \wedge {}^*b) \end{aligned} \tag{4.17}$$

The Lagrangian after adding the mass term:

$$\begin{aligned} L_{CS} =& [2b \wedge F^c + B^+ \wedge F^{C^-} + B^- \wedge F^{C^+} - ib \wedge B^+ \wedge B^- \\ &- \frac{w^2}{2}(C^+ \wedge {}^*C^-) - \frac{v^2}{2}(B^+ \wedge {}^*B^-) - \frac{(v^2 + w^2)}{2}(b \wedge {}^*b)] \end{aligned} \tag{4.18}$$

From the above Lagrangian, Equation of motion for b is:

$$\begin{aligned} &2F^c - iB^+ \wedge B^- - (v^2 + w^2)\,{}^*b = 0 \\ &\Rightarrow b = -\frac{1}{(v^2 + w^2)}[2\,{}^*F^c - i\,{}^*(B^+ \wedge B^-)] \end{aligned} \tag{4.19}$$

Equation of motion for B^+ is:

$$\begin{aligned} &F^{C^-} + ib \wedge B^- - \frac{v^2}{2}\,{}^*B^- = 0 \\ &\Rightarrow B^- = -\frac{2}{v^2}[\,{}^*F^{C^-} + i\,{}^*(b \wedge B^-)] \end{aligned} \tag{4.20}$$

Equation of motion for B^- is:

$$
\begin{aligned}
&F^{C+} - ib\wedge B^+ - \frac{v^2}{2}\,{}^*B^+ = 0 \\
&\Rightarrow B^+ = -\frac{2}{v^2}[\,{}^*F^{C+} - i\,{}^*(b\wedge B^+)]
\end{aligned}
\tag{4.21}
$$

From eq.(4.20) and eq.(4.21), eq.(4.19) is:

$$
b = -\frac{1}{(v^2+w^2)}[2\,{}^*F^c - i\frac{4}{v^4}\,{}^*(\,{}^*F^{C+}\wedge\,{}^*F^{C-}) + ...] \tag{4.22}
$$

From eq.(4.19), eq.(4.20) is:

$$
B^- = -\frac{2}{v^2}[\,{}^*F^{C-} + i\frac{4}{v^2(v^2+w^2)}\,{}^*(\,{}^*F^c\wedge{}^*F^{C-}) + ...] \tag{4.23}
$$

From eq.(4.19), eq.(4.21) is :

$$
B^+ = -\frac{2}{v^2}[\,{}^*F^{C+} - i\frac{4}{v^2(v^2+w^2)}\,{}^*(\,{}^*F^c\wedge{}^*F^{C+}) + ...] \tag{4.24}
$$

The terms in .... contain powers of F^{C^+},F^{C^-} and F^c appearing in combinations.
From the above equations,the term $b\wedge B^+\wedge B^-$ in the Lagrangian is:

$$
b\wedge B^+\wedge B^- = -\frac{8}{v^4(v^2+w^2)}(\,{}^*F^c\wedge\,{}^*F^{C+}\wedge\,{}^*F^{C-}) + ... \tag{4.25}
$$

Inserting the above equations in the Lagrangian, we find that:

$$
\begin{aligned}
\mathcal{L} =&\frac{k}{4\pi}(-\frac{2}{v^2+w^2}\,{}^*F^c\wedge F^c - \frac{2}{v^2}\,{}^*F^{C-}\wedge F^{C+} \\
&+ i\frac{20}{v^4(v^2+w^2)}(\,{}^*F^c\wedge\,{}^*F^{C+}\wedge\,{}^*F^{C-}) - \frac{w^2}{2}C^+\wedge\,{}^*C^- + ...)
\end{aligned}
\tag{4.26}
$$

For the above Lagrangian, we need to consider different cases:

Case 1 : $v\to\infty$ and $w\to\infty$:

In this limit, we can neglect higher order terms in v^{-1} and w^{-1}. Thus, we will consider terms only upto second order in both v^{-1} and w^{-1}.We can ignore the higher order terms in F^{C^+},F^{C^-} and F^c. In this limit equations of motion are:

$$
\begin{aligned}
b &= -\frac{2}{(v^2+w^2)}\,{}^*F^c \\
B^- &= -\frac{2}{v^2}\,{}^*F^{C-} \\
B^+ &= -\frac{2}{v^2}\,{}^*F^{C+}
\end{aligned}
\tag{4.27}
$$

In this limit, the Lagrangian(eq.(4.26)) is:

$$\mathcal{L} = \frac{k}{4\pi}(-\frac{2}{v^2+w^2}\,{}^*F^c \wedge F^c - \frac{2}{v^2}\,{}^*F^{C^-} \wedge F^{C^+} - \frac{w^2}{2}C^+ \wedge {}^*C^- + ...) \tag{4.28}$$

One of the interesting points in the above Lagrangian is that the mass term and the Yang-Mills coupling constant are dependent on each other.

Case 2 : $v \to \infty$ and $\frac{w}{v} \to 0$:

In this limit, we can neglect higher order terms in v^{-1}. In this case, the Lagrangian is:

$$\mathcal{L} = \frac{k}{4\pi}(-\frac{2}{v^2}\,{}^*F^c \wedge F^c - \frac{2}{v^2}\,{}^*F^{C^-} \wedge F^{C^+} - \frac{w^2}{2}C^+ \wedge {}^*C^- + ...) \tag{4.29}$$

In this limit,Chern-Simons Lagrangian reduces to massive Yang-Mills Lagrangian. The coupling constant of Yang-Mills term is $\frac{v}{\sqrt{k}}$ and the mass of the gauge boson is "w".

Case 3 : $v \to \infty$ and $w = 0$:

In this limit,the Lagrangian is:

$$\mathcal{L} = \frac{k}{4\pi}(-\frac{2}{v^2}\,{}^*F^c \wedge F^c - \frac{2}{v^2}\,{}^*F^{C^-} \wedge F^{C^+} + ...) \tag{4.30}$$

In this case, the Lagrangian reduces to the massless Yang-Mills Lagrangian. In the above Lagrangian, v^2 plays the role of the Yang-Mills coupling constant [8]. In the same limit that decouples the higher-order terms, the Yang-Mills term becomes strongly coupled.This can be avoided by simultaneously scaling $k \to \infty$,$v \to \infty$ keeping $\frac{k}{v^2}$ fixed[10]. In this limit ($k \to \infty$,$v \to \infty$) the higher-order terms drop out,but the Yang-Mills coupling $\frac{v}{\sqrt{k}}$ remains finite and can be chosen arbitrarily.

4.3 Discussion

It is already known that a non-propagating gauge field absorbs the degree of a Higgs field and turns into a massless propagating gauge field. In this chapter, we showed how this can be generalized. Some of the interesting results which we derived are:
1)In a certain limit,non-propagating Chern-Simons field reduces to massless Yang-Mills field and
2)In certain other limit, the non-propagating Chern-Simons field reduces to massive Yang-Mills term with mass of the gauge boson dependent on the coupling constant of the Yang-Mills term.

We wish to further study the above mentioned results and their implications in Condensed matter systems and (2+1)d gravity.

Chapter 5

The Higgs mechanism for more general potentials

5.1 Introduction

In the previous chapters, we studied some interesting physics when a complex scalar field is coupled to a gauge field. Mass of a gauge field depends on the VEV. So, VEV plays an important role to study the symmetry breaking patterns. Number of massive bosons and massless bosons depends on the choice of VEV. For example, when VEV is equal to zero, we don't see any massive gauge bosons. In this chapter, we will study the dependency of mass of the gauge bosons resulting from the Higgs mechanism on the coefficients of general potential of the complex scalar field.

5.2 Bifundamental Scalar field in $SU(2)\times SU(2)$

In this section, we will discuss the dependency of mass of the gauge bosons of bi fundamental $SU(2)\times SU(2)$ group on the coefficients of a general potential of the complex scalar field.

For a complex scalar field ϕ, Let us consider the following form of the potential

$$V = \alpha tr(\phi\phi^\dagger) + \beta(tr(\phi\phi^\dagger))^2 + \gamma tr(\phi\phi^\dagger\phi\phi^\dagger) \tag{5.1}$$

$$\begin{aligned} &\frac{\delta V}{\delta \phi^\dagger} = 0 \\ &\Rightarrow \alpha\phi + 2\beta tr(\phi\phi^\dagger)\phi + 2\gamma\phi\phi^\dagger\phi = 0 \end{aligned} \tag{5.2}$$

From chapter 3, VEV is :

$$< \phi >= \begin{pmatrix} v_1 & v_2 \\ v_3 & v_4 \end{pmatrix} \tag{5.3}$$

Using singular value decomposition, the above form can be reduced to :

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \tag{5.4}$$

The VEV in eq.(5.4) should satisfy eq.(5.2),

$$\alpha \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} + 2\beta(|a|^2 + |b|^2) \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} + 2\gamma \begin{pmatrix} a|a|^2 & 0 \\ 0 & b|b|^2 \end{pmatrix} = 0 \tag{5.5}$$

From above eq.(5.5), we find that:

$$a\alpha + 2a\beta(|a|^2 + |b|^2) + 2\gamma a|a|^2 = 0 \tag{5.6}$$

and

$$b\alpha + 2b\beta(|a|^2 + |b|^2) + 2\gamma b|b|^2 = 0 \tag{5.7}$$

From chapter 3, we know that for *SU(2)*$\times$*SU(2)* symmetry group, gauge boson mass terms come from the following lagrangian:

$$\begin{aligned}\Delta\mathcal{L} =& \frac{1}{4}[g^2(|a|^2 + |b|^2)(A^1_\mu)^2 + g^2(|a|^2 + |b|^2)(A^2_\mu)^2 + g^2(|a|^2 + |b|^2)(A^3_\mu)^2 \\ &+ 2gg'(|a|^2 + |b|^2)A^\mu_3 \tilde{A}^{\mu 3} + 2gg'a^*b(A^1_\mu - iA^2_\mu)(\tilde{A}^{\mu 1} + i\tilde{A}^{\mu 2}) \\ &+ 2gg'b^*a(A^{\mu 1} + iA^{\mu 2})(\tilde{A}^1_\mu - i\tilde{A}^2_\mu) + g'^2(|a|^2 + |b|^2)(\tilde{A}^1_\mu)^2 \\ &+ g'^2(|a|^2 + |b|^2)(\tilde{A}^2_\mu)^2 + g'^2(|a|^2 + |b|^2)(\tilde{A}^3_\mu)^2]\end{aligned} \tag{5.8}$$

We like to study the mass spectrum of the above Lagrangian for various choices of a and b:

Case 1: $a = 0$, $b = 0$

$$\Delta\mathcal{L} = 0 \tag{5.9}$$

For this coice of a and b,the mass spectrum doesn't contain massive gauge bosons. Therefore for this choice of VEV, *SU(2)*$\times$*SU(2)* symmetry in not broken.

Case 2: $a \neq 0$; $b \neq 0$

From eq.(5.6) and eq.(5.7),we find that:

$$\alpha + 2\beta(|a|^2 + |b|^2) + 2\gamma|a|^2 = 0 \tag{5.10}$$

$$\alpha + 2\beta(|a|^2 + |b|^2) + 2\gamma|b|^2 = 0 \tag{5.11}$$

subtracting the above two equations:

$$(|a|^2 - |b|^2)\gamma = 0 \tag{5.12}$$

For the above equation, we need to consider two possibilities:

a) when $\gamma \neq 0$,eq.(5.12) gives:

$$|a|^2 = |b|^2 \tag{5.13}$$

Using the above eq.(5.13) in eq.(5.9) and eq.(5.10),we find that:

$$\alpha + (4\beta + 2\gamma)|b|^2 = 0 \ \Rightarrow |a|^2 = |b|^2 = \frac{-\alpha}{4\beta + 2\gamma} \tag{5.14}$$

For the above choice of VEV,the mass spectrum is:

$$\begin{aligned} \Delta\mathcal{L} &= \frac{1}{2}|a|^2[(gA^1_\mu + g'\tilde{A}^1_\mu)^2 + (gA^2_\mu + g'\tilde{A}^2_\mu)^2 + (gA^3_\mu + g'\tilde{A}^3_\mu)^2] \\ \Rightarrow \Delta\mathcal{L} &= \frac{1}{2}|a|^2[2|gW^-_\mu + g'\tilde{W}^-_\mu|^2 + (gA^3_\mu + g'\tilde{A}^3_\mu)^2] \end{aligned} \tag{5.15}$$

where

$$\begin{aligned} W^\pm_\mu &= \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu), \\ \tilde{W}^\pm_\mu &= \frac{1}{\sqrt{2}}(\tilde{A}^1_\mu \mp i\tilde{A}^2_\mu), \\ |a|^2 &= \frac{-\alpha}{4\beta + 2\gamma}. \end{aligned} \tag{5.16}$$

When γ in the potential term (eq.(5.1)) is not equal to zero, the mass spectrum consists of three massive gauge bosons with mass proportional to $|a|^2$.

b)when $(|a|^2 - |b|^2 \neq 0)$, eq.(5.12) gives:

$$\gamma = 0 \tag{5.17}$$

From the above equations, eq.(5.10) and eq.(5.12),we find that:

$$|a|^2 + |b|^2 = \frac{-\alpha}{2\beta} \tag{5.18}$$

For this choice of VEV,the mass spectrum is:

$$\Rightarrow \Delta\mathcal{L} = \frac{1}{4}[2|gaW_\mu^+ + g'b\tilde{W}_\mu^+|^2 + 2|gbW_\mu^- + g'a\tilde{W}_\mu^-|^2 + (|a|^2+|b|^2)(gA_\mu^3 + g'\tilde{A}_\mu^3)^2] \tag{5.19}$$

where a and b satisfy: $|a|^2 + |b|^2 = \frac{-\alpha}{2\beta}$. When $\gamma = 0$ in the potential term,the mass spectrum consists of five massive gauge bosons whose mass is proportional to $|a|^2 + |b|^2$.

5.3 Bifundamental Scalar field in *SU(N)*×*SU(N)*

In the this section, we will extend the above mentioned formalism for a general bifundamental *SU(N)*×*SU(N)* group. We will obtain a relationship between VEV and coefficients of the scalar potential.

$$<\phi>= \begin{pmatrix} \upsilon_{11} & \upsilon_{12} & \cdots & \upsilon_{1n} \\ \upsilon_{21} & \upsilon_{22} & \cdots & \upsilon_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \upsilon_{n1} & \upsilon_{n2} & \cdots & \upsilon_{n,n} \end{pmatrix} \tag{5.20}$$

Using singular value decomposition, we can reduce the above form of VEV as follows:

$$<\phi>= \begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & a_{nn} \end{pmatrix} \tag{5.21}$$

where $a_{11}, a_{22}, a_{33}, ..., a_{nn}$ are complex.

Based on the various choices of $a_{11}, a_{22}, a_{33}, ..., a_{nn}$, we will discuss different cases:

Case 1: $a_{11} = a_{22} = ... = a_{nn} = 0$.

$$\Rightarrow <\phi>= 0 \tag{5.22}$$

Case 2: $a_{11} \neq a_{22} \neq .... \neq a_{nn} \neq 0$.

For this case, we get n equations by substituting $<\phi>$ in eq.(5.2),

$$
\begin{aligned}
&a_{11}\alpha + 2\beta(|a_{11}|^2 + |a_{22}|^2 + ... + |a_{nn}|^2)a_{11} + 2\gamma a_{11}|a_{11}|^2 = 0 \\
&a_{22}\alpha + 2\beta(|a_{11}|^2 + |a_{22}|^2 + ... + |a_{nn}|^2)a_{22} + 2\gamma a_{22}|a_{22}|^2 = 0 \\
&a_{33}\alpha + 2\beta(|a_{11}|^2 + |a_{22}|^2 + ... + |a_{nn}|^2)a_{33} + 2\gamma a_{33}|a_{33}|^2 = 0 \\
&... \\
&... \\
&... \\
&... \\
&... \\
&a_{nn}\alpha + 2\beta(|a_{11}|^2 + |a_{22}|^2 + ... + |a_{nn}|^2)a_{nn} + 2\gamma a_{nn}|a_{nn}|^2 = 0
\end{aligned} \tag{5.23}
$$

Subtracting any two equations in the above eq.(5.23) gives:

$$\gamma(|a_{ii}|^2 - |a_{ij}|^2) = 0 \tag{5.24}$$

where, i $\neq$ j=1,2,.....,n.

For the eq.(5.24), we have two possibilities:

a) when $\gamma \neq 0$, we find that:

$$a_{11} = a_{22} = a_{33} = ... = a_{nn} \tag{5.25}$$

Substituting the eq.(5.25) in eq.(5.23), we find that:

$$
\begin{aligned}
&a_{22} = a_{33} = ... = a_{nn} = \sqrt{\frac{-\alpha}{2\beta n + 2\gamma}}, \\
&|a_{11}|^2 = \frac{-\alpha}{2\beta n + 2\gamma}
\end{aligned} \tag{5.26}
$$

In the above eq.(5.26), we made use of the fact that all the diagonal entries of the $< \phi >$ can be made real except one entry by using the freedom of rotations of *SU(N)* group.In this case a_{11} is complex and all other entries are real.

b) When $(|a_{ii}|^2 - |a_{ij}|^2) \neq 0$, we find that:

$$\gamma = 0 \tag{5.27}$$

From eq.(5.23), we find that:

$$|a_{11}|^2 + |a_{22}|^2 + ... + |a_{nn}|^2 = \frac{-\alpha}{2\beta} \tag{5.28}$$

Case 3: $a_{11} = 0; a_{22}, a_{33}......a_{nn} \neq 0.$

In this case, we get n-1 equations by substituting $< \phi >$ in eq.(5.2):

$$\begin{aligned}
&\alpha + 2\beta(|a_{22}|^2 + ... + |a_{nn}|^2) + 2\gamma|a_{22}|^2 = 0 \\
&\alpha + 2\beta(|a_{33}|^2 + ... + |a_{nn}|^2) + 2\gamma|a_{33}|^2 = 0 \\
&... \\
&... \\
&... \\
&... \\
&... \\
&\alpha + 2\beta(|a_{22}|^2 + ... + |a_{nn}|^2) + 2\gamma|a_{nn}|^2 = 0
\end{aligned} \tag{5.29}$$

Subtracting any two equations in eq.(5.29), we find that:

$$\gamma(|a_{ii}|^2 - |a_{jj}|^2) = 0 \tag{5.30}$$

where, i $\neq$ j=2,.....,n

For eq.(5.30), we have two possibilities:

a) when $\gamma \neq 0$, we find that:

$$a_{22} = a_{33} = ... = a_{nn} \tag{5.31}$$

Substituting the above eq.(5.31) in eq.(5.29), we find that:

$$a_{22} = a_{33}............ = a_{nn} = \sqrt{\frac{-\alpha}{2\beta(n-1) + 2\gamma}} \tag{5.32}$$

In the above eq.(5.32), $a_{22}, a_{33}, ..., a_{nn}$ are real.

b) When $(|a_{ii}|^2 - |a_{ij}|^2) \neq 0$, we find that:

$$\gamma = 0 \tag{5.33}$$

From eq.(5.23), we find that:

$$|a_{22}|^2 + ... + |a_{nn}|^2 = \frac{-\alpha}{2\beta} \tag{5.34}$$

Case 4: $a_{11} = 0; a_{22} = 0, a_{33}......a_{nn} \neq 0.$ For this case, we get n-2 equations by substituting $<\phi>$ in eq.(5.2):

$$\begin{aligned}
&\alpha + 2\beta(|a_{33}|^2 + ... + |a_{nn}|^2) + 2\gamma|a_{33}|^2 = 0 \\
&... \\
&... \\
&... \\
&... \\
&... \\
&\alpha + 2\beta(|a_{33}|^2 + ... + |a_{nn}|^2) + 2\gamma|a_{nn}|^2 = 0
\end{aligned} \tag{5.35}$$

Subtracting any two equations in eq.(5.35) gives:

$$\gamma(|a_{ii}|^2 - |a_{jj}|^2) = 0 \tag{5.36}$$

where, i $\neq$ j=3,.....,n

For the eq.(5.36), we have two possibilities:

a) When $\gamma \neq 0$, we find that:

$$a_{33} = a_{44} = ... = a_{nn} \tag{5.37}$$

Substituting the above eq.(5.37) in eq.(5.35), we find that:

$$a_{33} = a_{44}............ = a_{nn} = \sqrt{\frac{-\alpha}{2\beta(n-2) + 2\gamma}} \tag{5.38}$$

where $a_{33}, ..., a_{nn}$ are real.

b) When $(|a_{ii}|^2 - |a_{ij}|^2) \neq 0$, we find that:

$$\gamma = 0 \tag{5.39}$$

From eq.(5.35), we find that:

$$|a_{33}|^2 + ... + |a_{nn}|^2 = \frac{-\alpha}{2\beta} \tag{5.40}$$

Case 5: $a_{11} = 0; a_{22} = 0, a_{33} = 0, ...a_{n-1n-1} = 0, a_{nn} \neq 0$.
For this case, we have only one equation by substituting $<\phi>$ in eq.(5.2):

.

$$\alpha + 2\beta|a_{nn}|^2 + 2\gamma|a_{nn}|^2 = 0 \tag{5.41}$$

From the above eq.(5.41), we have the following equations:

a)When $\gamma \neq 0$, we find that:

$$a_{nn} = \sqrt{\frac{-\alpha}{2\beta + 2\gamma}} \tag{5.42}$$

where a_{nn} is real.

b) When $\gamma = 0$, we find that:

$$|a_{nn}|^2 = \frac{-\alpha}{2\beta} \tag{5.43}$$

When $\gamma \neq 0$, we notice that the value of the entries of the VEV depend on the number of non-zero entries of VEV and when $\gamma = 0$ we get a sphere, whose radius depends on the coefficients of the potential. The above derived relations can be used directly to study the mass spectrum of bifundamental *SU(N)*×*SU(N)* symmetry group for a given scalar potential.

5.4 Discussion

In section 5.2, we found that how mass spectrum of a bifundamental symmetry group depends on the potential of a complex scalar field. In addition to this, we derived mass of the gauge bosons in terms of coefficients of the scalar potential. Therefore, once we know the mass spectrum of bifundamental *SU(2)*$\times$*SU(2)* group, we can get the mass of the gauge bosons for any arbitrarily given scalar potential.

In section 5.3, we extended the formalism in section 5.2 to a general bifundamental *SU(N)*$\times$*SU(N)* symmetry group. We found that the values of the VEV entries follow an interesting pattern based on the number of nonzero entries. This formalism is very useful when we want to study the mass spectrum of a bifundamental symmetry groups for different potentials. In addition to this, using the above derived relations we can know the mass of the gauge bosons by inserting the coefficients of the potential.In this chapter, we only focused on the mass spectrum of bifundamental *SU(2)*$\times$*SU(2)* group,but we can also easily derive these relations for the mass spectrum of bifundamental *SU(3)*$\times$*SU(3)* group, which we derived in chapter 3.

Appendix A

Convention

We work with the (2+1)d metric:

$$\eta_{\mu\upsilon} = diag(-,+,+) \tag{A.1}$$

The differential form notation can be translated into conventional index notation using the following identities:

$$\begin{aligned} A \wedge {}^*A &= -A_\mu A^\mu \\ A \wedge dA &= \epsilon^{\mu\nu\lambda} A_\mu \partial_\upsilon A_\lambda \\ dA \wedge {}^*dA &= -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \end{aligned} \tag{A.2}$$

In the non-Abelian context with compact groups, we have:

$$A = A^a T^a, F = F^a T^a \tag{A.3}$$

where $T^a = \frac{i\sigma^a}{2}$ are anti-Hermitian,and:

$$tr(T^a T^b) = -\frac{1}{2}\delta^{ab} \tag{A.4}$$

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{A.5}$$

References

[1] S. Deser, R. Jackiw, S. Templeton, Topologically Massive Gauge Theories, Annals Phys. 140 (1982) 372–411. doi:10.1016/0003-4916(82)90164-6.

[2] R. Jackiw, S. Templeton, How super-renormalizable interactions cure their infrared divergences, Phys. Rev. D 23 (1981) 2291–2304. doi:10.1103/PhysRevD.23.2291.
URL http://link.aps.org/doi/10.1103/PhysRevD.23.2291

[3] J. F. Schonfeld, A Mass Term for Three-Dimensional Gauge Fields, Nucl.Phys. B185 (1981) 157. doi:10.1016/0550-3213(81)90369-2.

[4] M. Peskin, D. Schroeder, An Introduction to Quantum Field Theory, Advanced book classics, Addison-Wesley Publishing Company, 1995.
URL http://books.google.co.in/books?id=i35LALN0GosC

[5] P. W. Higgs, Broken symmetries and the masses of gauge bosons, Phys. Rev. Lett. 13 (1964) 508–509. doi:10.1103/PhysRevLett.13.508.
URL http://link.aps.org/doi/10.1103/PhysRevLett.13.508

[6] F. Englert, R. Brout, Broken symmetry and the mass of gauge vector mesons, Phys. Rev. Lett. 13 (1964) 321–323. doi:10.1103/PhysRevLett.13.321.
URL http://link.aps.org/doi/10.1103/PhysRevLett.13.321

[7] L. Trefethen, D. Bau, Numerical Linear Algebra, Society for Industrial and Applied Mathematics, 1997.
URL http://books.google.co.in/books?id=bj-Lu6zjWbEC

[8] S. Mukhi, C. Papageorgakis, M2 to D2, JHEP 0805 (2008) 085. arXiv:0803.3218, doi:10.1088/1126-6708/2008/05/085.

[9] S. Mukhi, Unravelling the novel Higgs mechanism in (2+1)d Chern-Simons theories, JHEP 1112 (2011) 083. arXiv:1110.3048, doi:10.1007/JHEP12(2011)083.

[10] J. Distler, S. Mukhi, C. Papageorgakis, M. Van Raamsdonk, M2-branes on M-folds, JHEP 0805 (2008) 038. arXiv:0804.1256, doi:10.1088/1126-6708/2008/05/038.