NOISE ATTENUATION TECHNIQUES FOR GAS EXHAUST FROM HEAT ENGINES

A thesis submitted towards partial fulfilment of
BS-MS Dual Degree Programme

by

Ankur Sharma

under the guidance of

Mr.Ashish Arora

Escorts Agri Machinery, Faridabad

Indian Institute of Science Education and Research
Pune

Certificate

This is to certify that this thesis entitled ”NOISE ATTENUATION TECHNIQUES FOR GAS EXHAUST FROM HEAT ENGINES” submitted towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research Pune represents original research carried out by ” Ankur Sharma” at ”Escorts Agri Machinery, Faridabad”, under the supervision of ”Mr.Ashish Arora” during the academic year 2012-2013.

Student
ANKUR SHARMA

Supervisor
MR.ASHISH
ARORA

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Abstract

The work involved in this thesis was to develop a new muffler/silencer for tractors. As we all know that designing is an iterative process and one design remains good only till a better one comes up. Thus I started developing the methodology for designing of muffler. The new muffler designs were analysed using the principles of Computational Fluid Dynamics (CFD) so as to generate precision results. There are many ways by which we can attenuate the sound waves. I studied several attenuation techniques which can be used in a component like a muffler. Mathematical calculation for the attenuation by different configurations of mufflers was done to develop the understanding of different elements of muffler.

A design procedure was formed and based on that, few new muffler models were developed. These model were first mathematically checked and the models that were giving approximately desired results were taken for CFD analysis. A procedure for the aero-acoustic analysis using CFD was developed and validated by comparing the results with the experimental results. The new mufflers were taken for CFD analysis to compare noise level and engine back pressure with the existing mufflers. The work was fully aimed at producing better mufflers for tractor industry which can be commercially used on new range of tractors that organization intends to roll-out.

Contents

Chapter 1

Introduction

Mufflers or silencers are devices used to reduce the noise produced by engine in a vehicle. There are many sources of noise in the vehicle including engine, fan, transmission etc. But the major amount of noise is generated by engine. Engine noise consists of air borne noise and structural noise. Mufflers are attached to the gas exhaust outlet of engine to reduce the air borne noise which is almost 100-150dB. Government has fixed the standards for the maximum allowable noise level that can be produced by a vehicle in different environments. The maximum allowed noise by a tractor on road according to Indian Standards is 88dB. So, it is important to reduce the noise levels. There are two important factors for a muffler, noise reduction and back pressure. Noise reduction is the basic function of a muffler and the main factor that has to be considered while designing. Back pressure defines the amount of negative pressure generated by the muffler that reduces the flow of exhaust gas. Back pressure is produced due to the obstacles in the path of exhaust gas because of the complicated designs of muffler. Back pressure reduces the efficiency of engine, so lesser back pressure is desired. The best muffler design should be the one which reduce the desired amount of noise without producing much back pressure.

Figure 1.1: A silencer reducing the amplitude of sound wave entering the silencer

1.1 The history

Milton O. Reeves and Marshal T. Reeves[18] have invented many improvements in Exhaust mufflers for engine. Their aim was to use the high tension explosion engine in bikes so that they can attain maximum power with a minimum weight of engine. But there is a major problem with that, a huge amount of noise has been produced due to the sudden release of exhaust gas from engine. This noise is very disturbing for the driver and so noise reduction became an important matter of concern. They produced exhaust mufflers with expansion chamber and placed it between the engine exhaust and the actual exhaust of vehicle. These muffler attenuated the desired amount of noise and muffler became an important element of the vehicle. Mufflers are designed in many ways to reduce the noise depending on requirements.

1.2 What are the current requirements and the objectives of project?

Most of the mufflers are effective in reducing the noise of higher frequencies only. But the maximum amount of noise produced by engine is at the first few orders of firing frequency of the engine. Thus the first objective of this project is to design a muffler that can reduce the noise of lower frequencies. Second objective is to reduce the back pressure. There is an analogy in the industry that if we reduce back pressure, noise will increase and vice versa. So the mufflers in this project are designed such that they reduce the noise as well as back pressure. As the muffler is designed for tractors, the design requirements of tractor industry are being considered. These requirement are cost effectiveness, low on maintenance and easily manufacturable.

Chapter 2

Theoretical background: Sound, Silencer and CFD

The study for this project has been done in various steps. At the beginning, project starts with the understanding of objectives of the project. Here the basic quantities have been introduced that are used to define muffler properties and experiment values. This step also includes the study of sound generated by engine. The next step is the study of sound and acoustics. Then the various methods have been introduced for the attenuation of sound. The next step includes the mathematical analysis for the attenuation of sound for basic muffler shapes using plane wave theory. Here all the elements of muffler and their behaviour are studied. Next step is the development of muffler design procedure and formation of new designs following that procedure. The next step is the study of computational fluid dynamics (CFD) and a procedure is developed for the aero acoustic analysis using the available software Star-CCM+. Then the analysis of all new mufflers is done using the developed procedure using CFD software. The following are the quantities used to define various parameters of a muffler:

1. Transmission loss (TL): It is the inherent property of a muffler, defined as the difference of sound pressure level (SPL) between the inlet and the outlet of the muffler. TL completely depends on the structure of the muffler.

$$TL = SPL_{inlet} - SPL_{outlet}$$

2. Insertion loss (IL): Insertion loss is the quantity that is calculated in the experiment. To calculate the IL, sound pressure level is calculated at a particular distance from the outlet of silencer. Then the silencer is removed and a straight pipe is substituted at its position and SPL is calculated at the same distance from the gas outlet keeping same physical conditions. Difference between the sound pressure level of straight pipe and the silencer is called as the Insertion loss of silencer.

$$IL = SPL_{straightpipe} - SPL_{silencer}$$

3. Noise reduction (NR): Noise reduction is a quantity that defines how much noise has been reduced by the muffler. It is the difference between the SPL at the exhaust with and without silencer.

4. Back pressure: Back pressure is the pressure drop because of insertion of the silencer. It is measured as the difference of pressure at inlet and outlet of the silencer. The more the back pressure, the more it is difficult for the exhaust gas to come out. So, lesser back pressure is desirable to increase the engine efficiency.

There are mainly two types of Mufflers available in the market, reactive/reflective mufflers and dissipative/absorptive mufflers. Brief description of these mufflers is as follows:

1. Reactive Mufflers: Reactive mufflers(fig 2.1) are those mufflers which reduce the sound using the phenomenon of destructive interference, expansion and resonance. A reactive muffler may consist of multiple number of expansion or resonating chambers to reduce the desired noise. These mufflers produce large back pressure as they consist of obstacles in the path of exhaust gas.

2. Dissipative mufflers: Dissipative mufflers (fig 2.2) use absorption to reduce the noise. As sound travels through a sound absorbing material like glass wool, sound waves vibrate the fibre of the material and convert the energy into heat. There are many advance materials are being invented for sound absorption. These mufflers reduce noise mostly for high frequencies only. Absorptive material has small life span and so the muffler requires maintenance/replacement in 2-3 years. These mufflers do not produce much back pressure but they are not good in noise reduction too. Absorptive mufflers are not used in tractor industry because of their high maintenance and

Figure 2.1: Reactive muffler

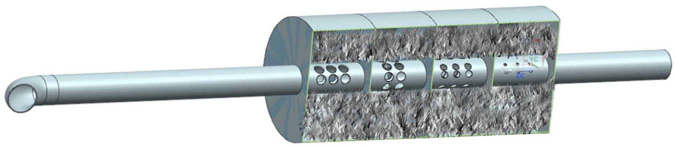


Figure 2.2: Absorptive muffler

small life. So they are not being considered in the project.

2.1 Sound and acoustics

2.1.1 Sound and noise

Sound wave is any mechanical disturbance that is propagated in an elastic medium, which may be gas, liquid or solid. Sound travels by producing a oscillatory pressure fluctuation in medium due to compression and rarefaction of particles, also called as pressure wave.

In physics point of view, noise is the broadband spectrum in Fourier domain. Noise can be defined as any sound that is unpleasant for human being in general.

2.1.2 Sound pressure, intensity and power

Sound is generated as the difference in pressure because of to and fro motion of the particles of medium. This variation of pressure above and below atmospheric pressure is called sound pressure. A normal human being can hear sound in the frequency range of roughly 20Hz to 20,000Hz. This is called as the audible frequency range. The pure tone of a sound wave is represented by the equation.

$$p(t) = p_0 \sin(\omega t)$$

where $p(t)$ is the sound pressure ω is the angular frequency and p_0 is the atmospheric pressure. As this is sine wave, the average value of a full cycle will be equals to zero. So root mean square (rms) value of sound pressure is calculated, also called as effective value.

$$p_{rms}^2 = p_0^2$$

$$p_{rms} = 0.707 p_0$$

Sound intensity is defined as the power carried by a sound wave per unit area. For a freely travelling wave, sound pressure is related to the maximum sound intensity as:

$$p_{rms}^2 = I_{max}(\rho/c)Pa^2$$

Where, p_{rms} = rms sound pressure
I_{max} = maximum sound intensity
ρ = density of medium
c = speed of sound in the medium

Sound power is the total amount of power that is emitted by a source. If we consider an imaginary sphere of radius r with the sound source at the centre, total sound power emitted by that source is given by the product of sound intensity at a distance r and the area of the sphere.

$$W_s = (4\pi r^2)I_s(r)W$$

where, $I_s(r)$ = maximum sound intensity at a distance r from the sound source
W_s = total sound power emitted by the source

2.1.3 Decibel, sound power level and sound pressure levels

As the human ear has wide range of hearing, sound pressure is not a very convenient way of calculating sound. So, sound is measured in a logarithmic relative quantity, Decibels.

$$Decibel(dB) = 10 \log(\frac{A}{A_{ref}})$$

Where, A is a physical quantity of measurement and A_{ref} is the reference value for that physical quantity.

The quantities that are measured in decibels are defined as levels. Sound pressure level and sound power level are mostly used quantities.

Sound pressure level(SPL): Sound pressure level is a relative quantity used to measure the sound pressure. SPL id defined as:

$$SPL = 10 \log \frac{(P)^2}{(P_{ref})^2} \tag{2.1}$$

Where, P is the sound pressure at the point of measurement and P_{ref} is the reference pressure which is taken to be $20\mu Pa$ for air.

For sound power level, the squared sound pressure is replaced by the sound power.

2.1.4 Near field and far field

For a sound source, near field is described as the region where the sound pressure changes very rapidly with distance. The near field region is considered up to the distance closer than the wavelength. The region which is at the distance more than the wavelength is considered as the far field. There is a transition region from near to far field, called as mid field.

2.2 Attenuation of sound

The reduction of sound power because of any reason is called as attenuation of sound. Sound can be attenuated by four ways: Reflection, expansion, absorption and diffraction. A silencer is designed to attenuate the noise using one or more of these techniques. Descriptions of these techniques are given in the subsections.

2.2.1 Reflection/Interference

When the sound wave crosses a region of changing cross section, scattering of wave takes place. Some part of the wave gets transmitted and rest reflects back. Then interference2.3 takes place between incident and reflected waves. Destructive interference results into the attenuation of sound.

Figure 2.3: Interference of waves in water (http://www.scienceclarified.com/everyday/Real-Life-Physics-Vol-2/Interference-Real-life-applications.html)

2.2.2 Expansion

As described above, when sound wave crosses a region of changing cross sectional area, scattering of wave takes place. This way sound wave gets expanded when the cross sectional area increases. The scattering of the wave leads to the dissipation of sound power. Thus, expansion is a method for the attenuation of sound.

2.2.3 Absorption

There are few materials that have properties of sound absorption when sound passes through them. These are mostly fibrous materials or foams. When sound passes through a sound absorbing material, the fibrous part starts vibrating and converts some part of the total energy into heat energy. A sound absorbing panel is shown in fig2.4 that is used for sound insulation inside a room.

2.2.4 Diffraction

Diffraction is defined as the bending of a wave around any obstacle and its spreading through small holes. Sound, being transverse wave also gets diffracted by any obstacle and spread out of holes. The amount of diffraction depends on the size of obstacle or hole. Sound coming through a small hole in the door is an example of diffraction of sound. Diffraction is also

Figure 2.4: Sound absorption panel for wall mounting to create sound insulation inside a room (http://www.archiexpo.com/prod/lomakka/felt-sound-absorption-panels-52089-109406.html)

being used for attenuating sound in many ways. One example is anechoic chambers. In anechoic chambers, panels of different shapes are placed on the walls. These panel bend the sound waves in many directions and reduce the amount of reflected sound from the wall. Anechoic chambers are the quietest place on earth where sound pressure level can be reduced up to -9 dB. Another example of attenuation of sound using diffraction is the roof of airports. Above the roof, a grid of rods is made so that they can diffract the sound of lower frequencies coming from the blowers of air-planes that are flying above the airport.

Diffraction can only be obtained if the diffraction element is of equivalent dimension of the wavelength. The maximum noise generated by the engine is of the low frequencies or long wavelengths (up to few meters). It is not possible to create such large diffraction elements in a silencer, so diffraction can not be used for attenuation of sound in silencers.

2.3 Classification of mufflers based on the attenuation techniques

Based on the attenuation techniques described above, mufflers can be classified in to two categories: Expansion chamber mufflers and Resonator mufflers. These both mufflers use expansion and reflection/ interference to attenuate the sound. The details of these mufflers are given below:

Expansion chamber mufflers: An expansion chamber is a cylindrical

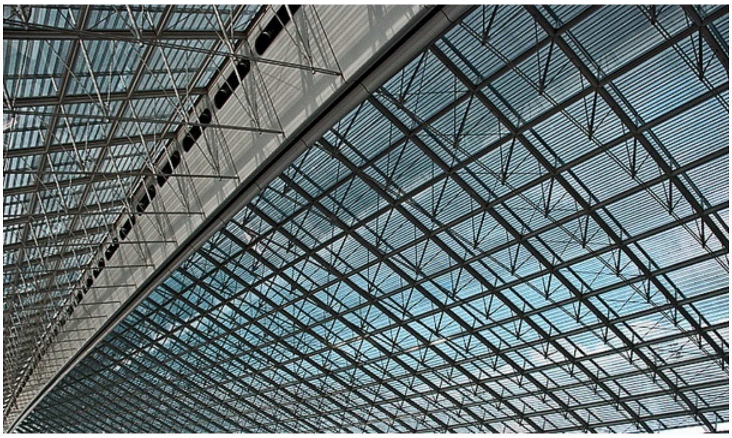


Figure 2.5: Roof of an airport consisting grid of pipes to capture the sound coming from the airplanes flying above the airport (http://www.flickr.com/photos/ilkeruzuner/5105444860/)

object which is connected to the exhaust gas pipe of the engine. These mufflers consist of one or more expansion chambers connected in series to reduce the noise of the engine. The sound coming from the engine reflects back when crossing the region of changing cross sectional area. So the expansion chamber consists of incident, reflected and transmitted waves. These different sound waves of different amplitudes and moving in different directions interfere to kill the sound. The attenuation depends on the size of chamber and number of chambers. The formulations for the attenuation of expansion chambers has been done and presented in Appendix A.

Resonator Mufflers: These mufflers work on the concept of Helmholtz resonator. In these muffler, resonating chamber/s is/are attached to the gas exhaust pipe by a connecting tube. When sound wave enters the resonator, sound corresponding to a particular frequency is captured by the resonator. This frequency is equal to the intrinsic resonating frequency of Helmholtz resonator. Most commonly used resonator mufflers are designed as an expansion chamber concentric to the exhaust pipe and the pipe is having different amount of holes in it. Expansion chamber works as the resonating chamber and the holes act as connecting tube. The formulations for the attenuation by a resonator muffler has been given in the Appendix A.

2.4 Mathematical analysis of the attenuation properties of basic muffler shapes

The attenuation for expansion chamber mufflers and resonator mufflers has been calculated. Various parameters of muffler are changed and the attenuation has been plotted against frequency to understand the functionality of each parameter. All the cases are explained below along with the graphs.

2.4.1 Expansion chamber

The attenuation of a muffler which consists of a single expansion chamber can be given by (Appendix A):

$$Attenuation = 10\log[1+\frac{1}{4}(m-\frac{1}{m})^2\sin^2(kl_e)] \tag{2.2}$$

where, m = expansion ratio, equals to the ratio of cross sectional area of the expansion chamber to the cross sectional area of exhaust pipe.
k = wavelength constant, $\frac{2\pi f}{c}$
l_e = length of expansion chamber

Effect of m: The effect of expansion ratio on attenuation is shown in the fig 2.6. It is clear from the figure that the attenuation increases with the increase of m. But as we can see that there is not much difference in the attenuation for m = 36 and m = 64, but the size of the muffler will increase by significant amount. So, very large expansion ratio is not a smart choice for increasing the attenuation. There are other ways used to increase the attenuation that are discussed in further cases.

Effect of length: The effect of length on attenuation is shown in fig 2.6. Here we can see that for short length muffler good attenuation is not obtained at low frequency values. But for a long muffler, as the case 4, high amplitudes are attained even at very low frequency because of the increase of nodes in the frequency range.

Effect of number of expansion chamber- Internally and externally connected: Dual expansion chamber mufflers are shown in fig 2.7. As we can see the the maximum attenuation has been increased with the increase in the number of expansion chambers. But there are few regions of low attenuation has formed that are called as the frequency band pass. Also

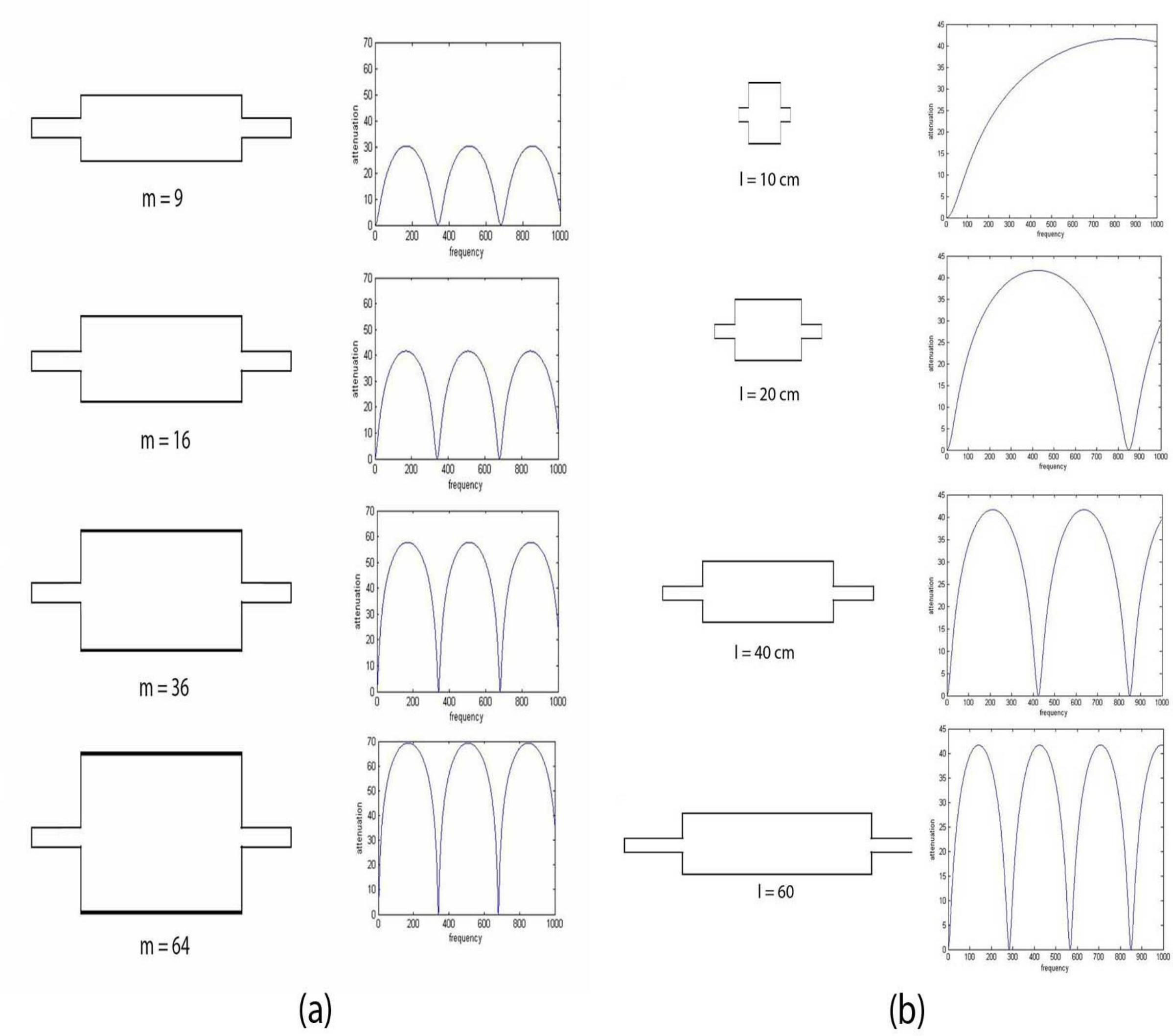


Figure 2.6: (a) Effect of expansion ratio; (b) Effect of length

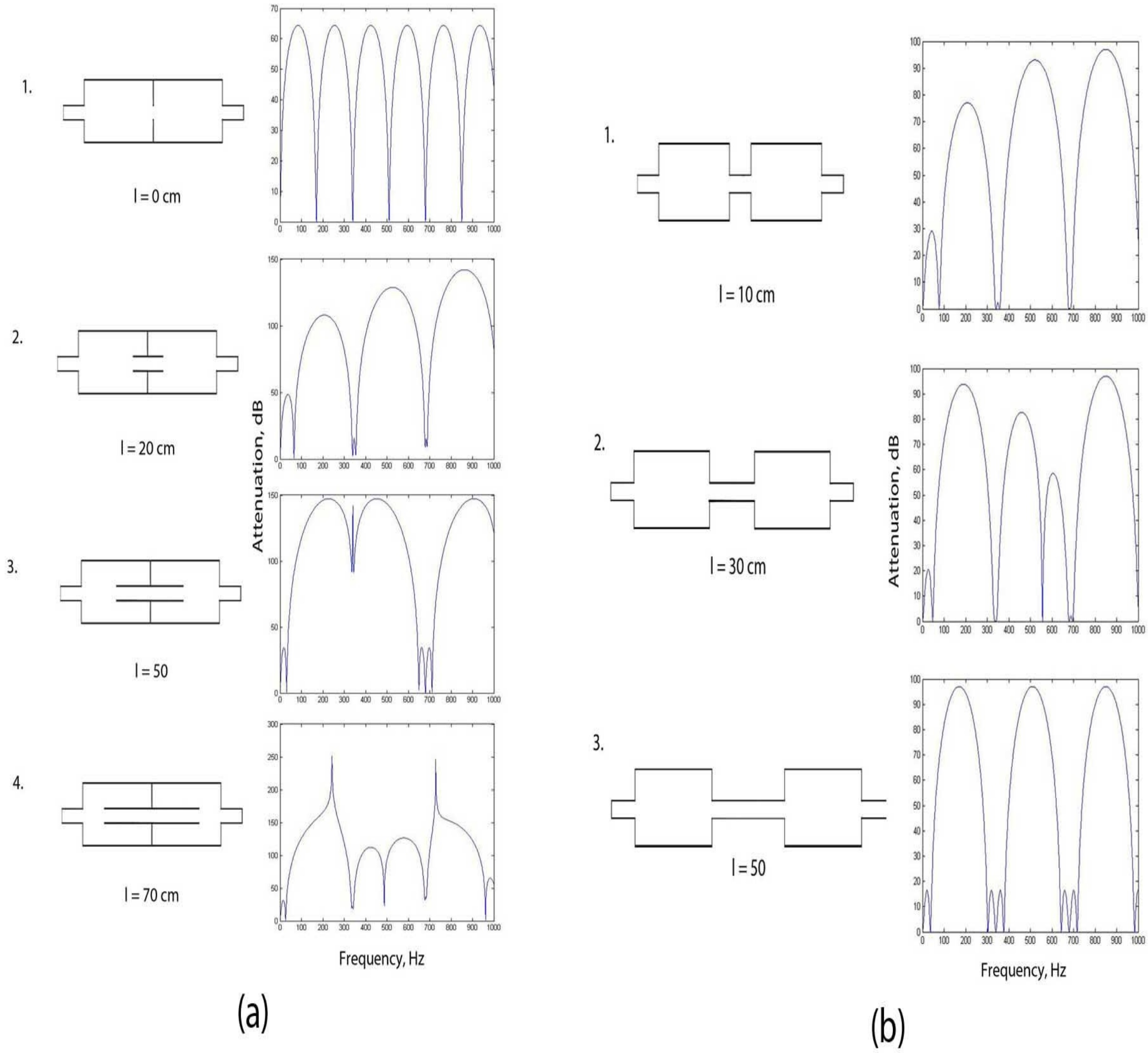


Figure 2.7: (a) Multiple chambers with internal connecting tube;(b) Multiple chambers with externally connecting tube

a cut-off frequency limit is formed in the case of multiple expansion chamber. Here a very interesting case is found, which is the muffler 3 of part (a) of the figure. In this case, the first band pass has been eliminated and a good attenuation has been attained for a long range. The size of the connecting pipe in this case is set to be exactly equal to the size of single expansion chamber. This geometry leads to the resonance in the chamber that gives the result as shown in the figure.

Dual expansion chamber: Special case: As discussed in the previous case, if the size of internal connecting tube is equal to the length of expansion chamber then the first band pass vanishes because of the resonance and hence good attenuation is obtained. Mufflers of different sizes have been plotted and the results are shown in fig.2.8.

2.4.2 Resonator

A Resonator[1] is an instrument that can capture particular frequencies of noise at its resonance frequencies. The attenuation properties of a resonator completely depends on two elements, volume of the resonating chamber and conductivity of the connecting tube. To define the attenuation properties, two parameters are defined, Resonance parameter(RP) and Attenuation parameter(AP).

$RP = \sqrt{\frac{c_0}{v}}$

$AP = \frac{\sqrt{c_0 v}}{S}$

where, v = volume of the resonating chamber; c_0 = conductivity of the connecting tube; S = cross sectional area of exhaust pipe;

Here, conductivity is a function of cross sectional area and length of the tube connecting exhaust pipe and resonating chamber.

Effect of volume: Figure2.9(a) is showing the effect of volume on the attenuation by keeping the conductivity constant.Here, we can see that as the volume is decreasing, the attenuation is also decreasing with a shift towards higher frequency.

Effect of conductivity and volume, keeping resonance parameter constant: As the resonance parameter is kept constant, the location of peak is same in all the three cases. But as we have seen in the previous section, the

[1] Helmholtz resonator is called as Resonator in the complete report

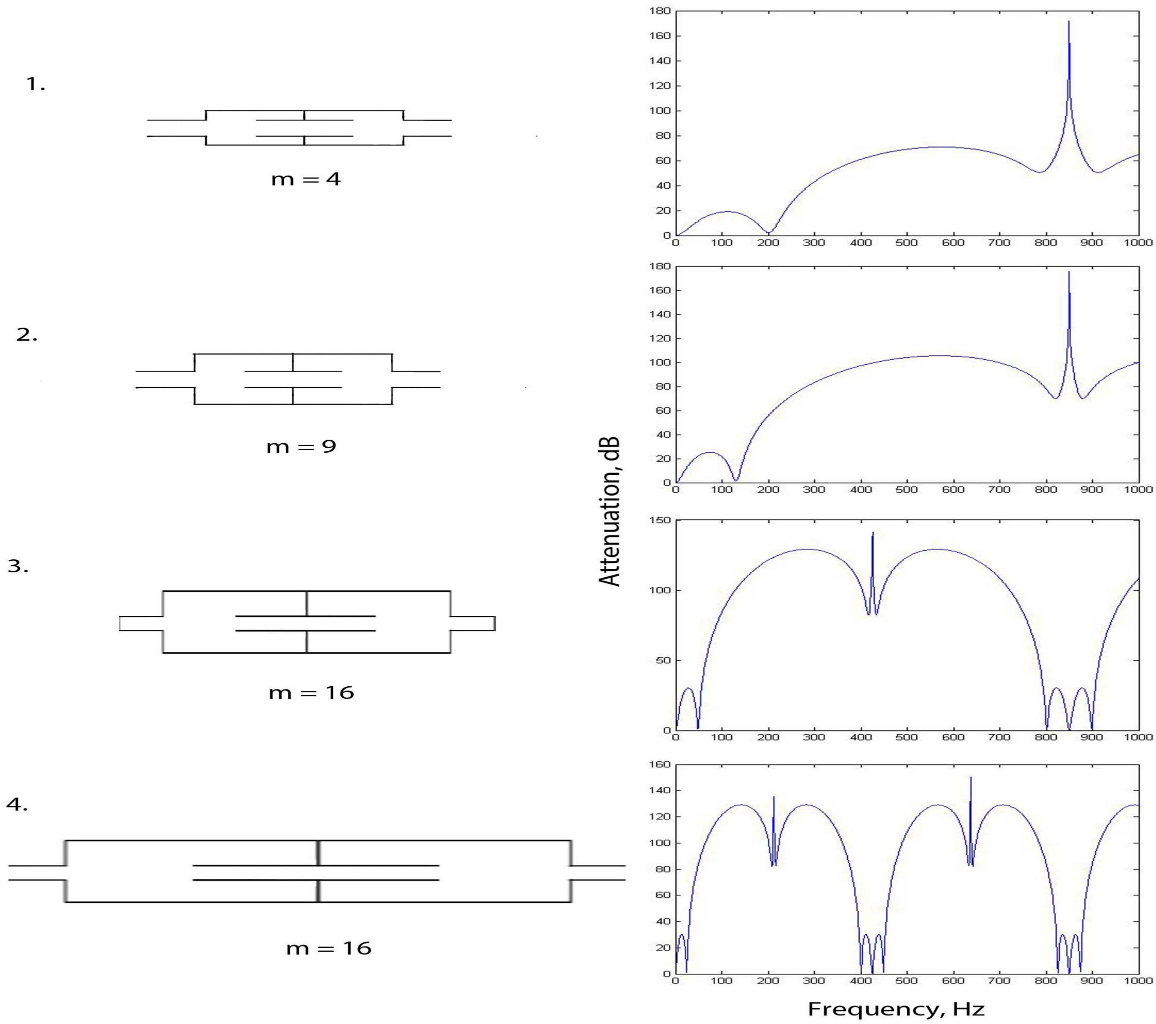


Figure 2.8: Dual chamber: Special case

resonator with larger volume is having more attenuation. So, the attenuation is maximum in the top case and least in the bottom case.

2.5 Computational fluid dynamics

2.5.1 What is CFD?

Computational fluid dynamics or CFD is the analysis of fluid flow, heat transfer, acoustics, chemical reactions and related phenomena by means of computer based simulation. This technique is widely used for industrial and non-industrial purposes presently. CFD codes use numerical methods to solve the conservation equations of fluid dynamics with user defined initial conditions. There are many CFD codes available today like FLUENT,Star-CCM, Phoenix etc. Star-CCM+ has been used in this project to perform all the CFD analysis. Simulation is a very good means for research because it saves huge amount of time and money as compared to experimental analysis.

2.5.2 How CFD works?

All the CFD codes contain three main elements: pre-processing, sover and post processing.

Pre-Processing: Pre-processing is the stage where the geometry and initial conditions of the problems has been defined so that solver can use it. The following activities are performed in pre-processing:

1. Definition of geometry for the region of computational domain.

2. Mesh formation: This is the discretization of geometry into a number of small elements that are not overlapping to each other. These element can be uniform or non-uniform. The solver solve equations at the nodes of these elements. So, finer mesh(more elements) gives better results but increase the computational time. So optimisation required in mesh formation. Mesh can be 2-dimensional(surface) or 3-dimensional(volume).

3. Selection of physical models as per the problem.

4. Definition of fluid properties.

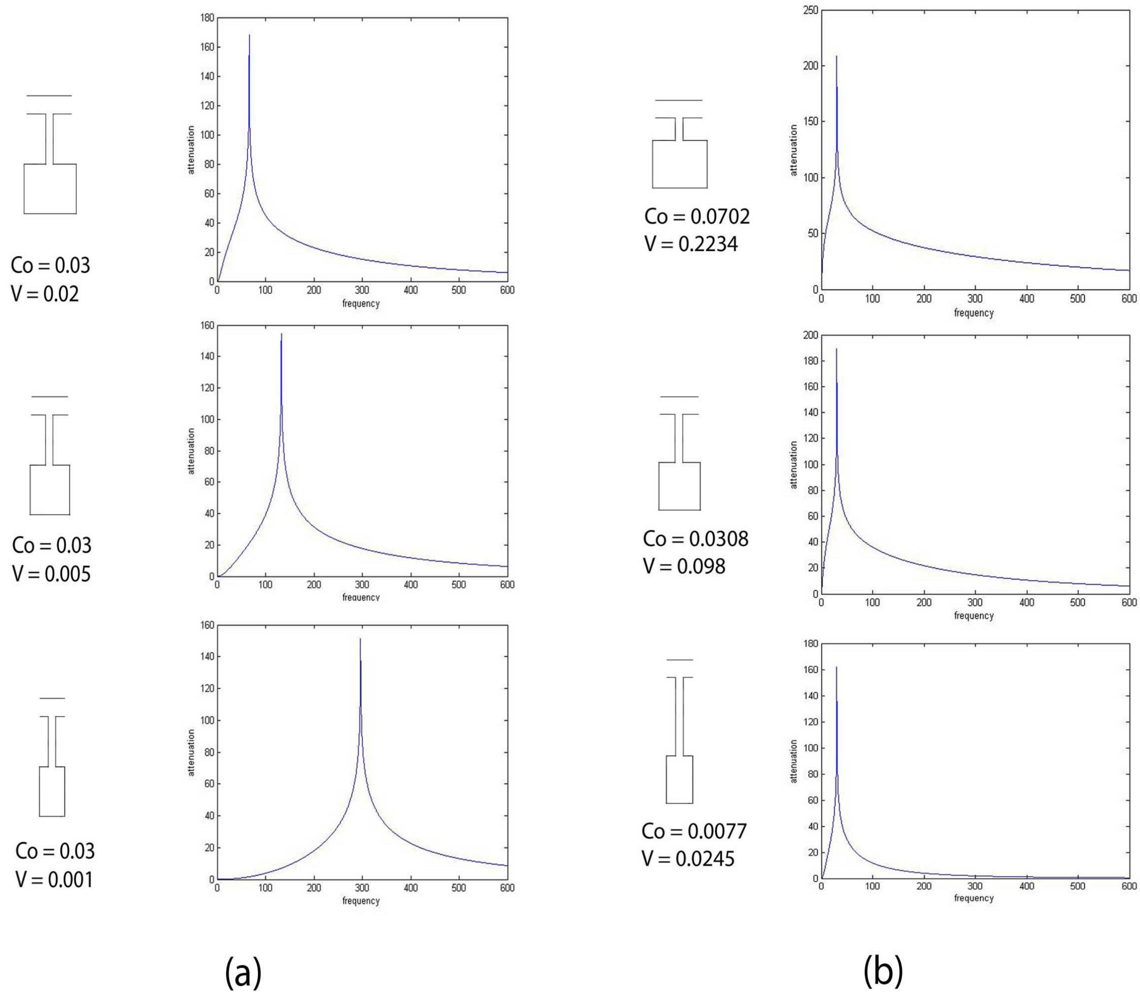


Figure 2.9: (a) Effect of volume of resonator on attenuation (b) Effect of conductivity and volume, keeping resonance parameter constant

5. Specify initial boundary conditions based on the problem.

Solver: There are three numerical solution techniques used: finite difference, finite element and finite volume method. The solver involved following step:

1. Integration of the governing equations on all the elements of the domain.
2. Conversion of integral equations to algebraic equations.
3. Solution of algebraic equations using iterative method.

Finite volume method(FVM) is majorly used for the problems involving CFD.

Post-Processing: Post-Processing includes mostly the visualization of results. It can include the visualization of:

1. Generation of Contour plots.
2. Generation of Scalar plots for required physical quantities.
3. Vector plots.
4. Processing of data such as Fourier transform.

2.5.3 Governing equations

The governing equations are basically the mathematical representation of conservation laws of fluid dynamics. It consists of the following laws:

1. Mass conservation: The mass of a fluid is conserved.
2. Momentum conservation: The rate of change of the momentum is equal to the sum of forces acting on the body.
3. Energy conservation: Change in the total energy of a system is equal to the sum of net energy flow through the boundaries and net change in energy due the pressure on boundaries.

Mass conservation:

Net rate of change of mass = Net incoming or outgoing mass

This statement can be mathematically represented as:

$$\frac{\partial \rho}{\partial t} + div(\rho u) = 0 \tag{2.3}$$

where, ρ = density of fluid;

u = velocity vector of fluid particles.

Momentum conservation:

Change in momentum = Total force acting on fluid particles

It can be mathematically written as:

$$\frac{\partial(\rho u)}{\partial t} + div(\rho u^2 + p) = 0 \tag{2.4}$$

Where, p = pressure.

Energy conservation:

change in total energy = net energy flow through boundaries + net energy change due to pressure on the walls

It can be mathematical written as:

$$\frac{\partial(\rho e_T)}{t} + div(\rho u e_T + pu) = \frac{\partial(\rho e_T)}{t} + div(\rho u h_T) = 0 \tag{2.5}$$

Where, e_T = Total internal energy of fluid particles;
h_T = Total enthalpy of fluid particles.

These equations are the core of CFD. All these are solved using numerical methods and iterative technique is used to obtain the accurate result. Most of the commercial software use finite volume method to solve the equation which is explained in the next section.

2.5.4 Star CCM+

Star-CCM+ is the commercial code developed by the company CD-adapco in 2004. This software is specifically created for CFD and completely based on the finite volume method. In Star CCM, CCM stands for computational continuum mechanics. This software delivers a powerful CFD tool in an easy to use environment.

Chapter 3

Computational and experimental procedure for aero-acoustic analysis of silencer

3.1 Muffler design procedure

The procedure for the designing of muffler has been established which is as follows:

1. Sound level spectrum of engine has been calculated experimentally at different conditions using the procedure described in subsection 3.3. This spectrum is called as the "Engine noise spectrum"

2. An allowable spectrum has been estimated based on the government standards for allowable noise. According to Indian government standards for the maximum allowable noise by a tractor on road is fixed to 88dB.

3. The difference of the engine noise spectrum and the allowable spectrum is calculated and defined as "Required attenuation Spectrum". All the designs are made by taking into consideration of the noise at different frequencies in this spectrum.

4. Compare the required attenuation spectrum with design curves (attenuation verses frequency) developed in the subsection14. Create combinations of expansion chambers and resonators to attain an attenuation which is more than the required attenuation throughout the frequency range.

5. From the required cut-off frequency, calculate the minimum tailpipe

length. This is an important step to define the cut-off frequency for finite tailpipe muffler. Using that tailpipe length, compute the locations of frequency band pass and add components to reduce them.

6. Several muffler designs should be developed to get the required attenuation and then smaller muffler should be sorted out. Detailed calculations are huge and not necessary until the final configuration has been closely approached.

7. Finalized designs are then taken for developing the CAD models and analysed using CFD.

8. Cross check of CFD results has been done using LMS. Then the silencers that are giving good results are manufactured and tested experimentally.

3.2 Analysis using CFD

CFD analysis of silencers is done on software Star-CCM+ for this project. First, the procedure for the aero-acoustic analysis has been established. Using this procedure, IL for the existing silencer of PT-439 is calculated. For the measurement of IL, the SPL vs Frequency plot of silencer and straight pipe is measured and the difference is IL. Then the IL for same configuration has been measured experimentally as explained in section3.3. The IL obtained by CFD is compared with the experimental result to validate the CFD procedure.

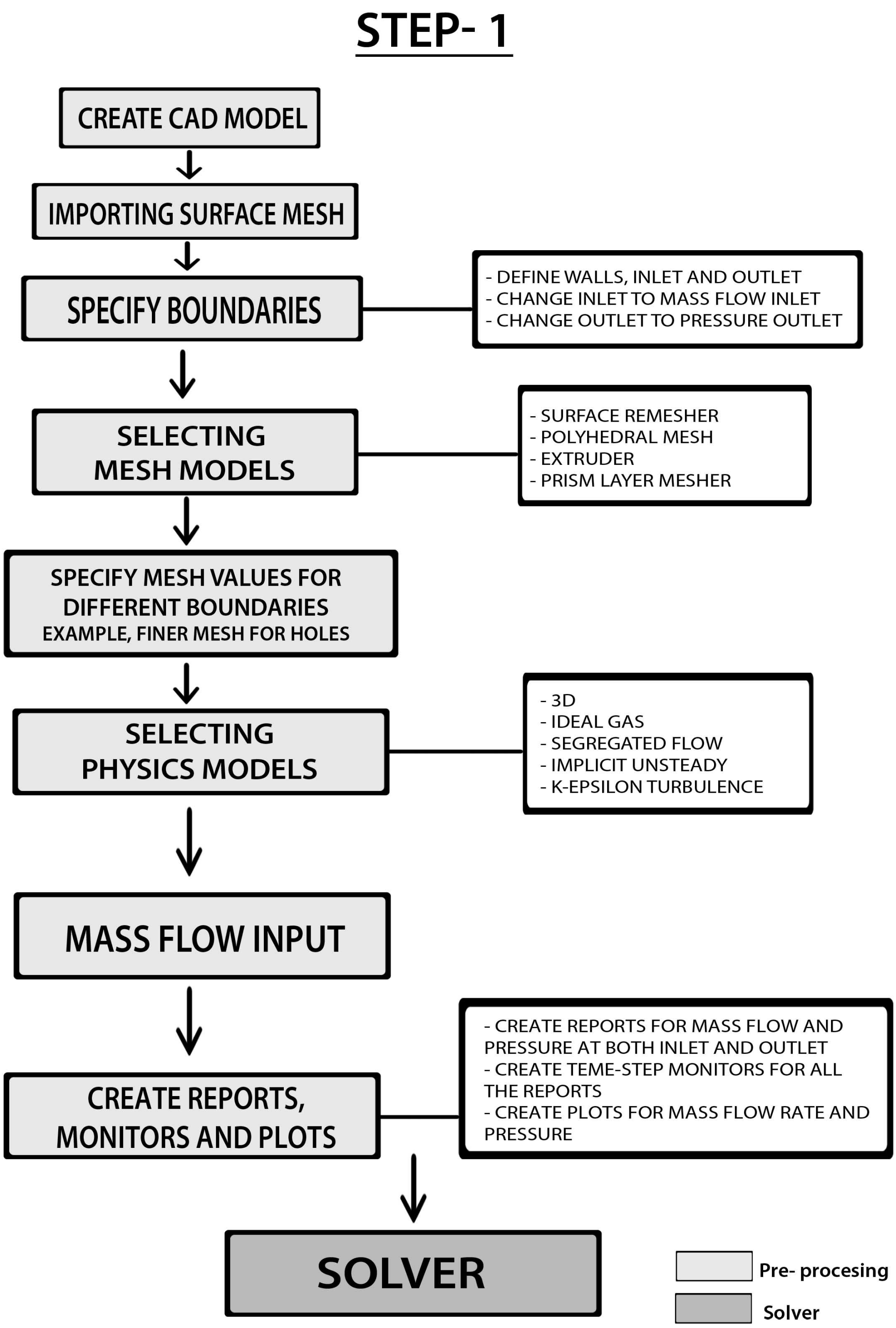
The procedure for aero-acoustic analysis using CFD is divided in two steps.

STEP-1:

1. The CAD model of silencer is created by the design software Unigraphics-NX.

2. Parasolid file has been created by UG-NX is imported to Star-CCM+.

3. Different boundaries are created for separating the regions of different physical conditions and mesh requirements. Inlet, outlet and different wall regions are created separately. Inlet is set to mass flow inlet and outlet is set to pressure outlet.


STEP- 1
CREATE CAD MODEL
IMPORTING SURFACE MESH
SPECIFY BOUNDARIES
- DEFINE WALLS, INLET AND OUTLET
- CHANGE INLET TO MASS FLOW INLET
- CHANGE OUTLET TO PRESSURE OUTLET
SELECTING MESH MODELS
- SURFACE REMESHER
- POLYHEDRAL MESH
- EXTRUDER
- PRISM LAYER MESHER
SPECIFY MESH VALUES FOR DIFFERENT BOUNDARIES
EXAMPLE, FINER MESH FOR HOLES
SELECTING PHYSICS MODELS
- 3D
- IDEAL GAS
- SEGREGATED FLOW
- IMPLICIT UNSTEADY
- K-EPSILON TURBULENCE
MASS FLOW INPUT
CREATE REPORTS, MONITORS AND PLOTS
- CREATE REPORTS FOR MASS FLOW AND PRESSURE AT BOTH INLET AND OUTLET
- CREATE TEME-STEP MONITORS FOR ALL THE REPORTS
- CREATE PLOTS FOR MASS FLOW RATE AND PRESSURE
SOLVER
Pre- procesing
Solver

4. Mesh models are selected: Surface remesher is selected for the generation of 2D surface mesh. Polyhedral mesh is selected for the generation of 3D volume mesh. Extruder is selected to create a region of FW-H surface in step-2. And prism layer mesher is selected to create a finer mesh at the boundaries.

5. Mesh values are specified for different regions depending on the size of the region and minimum required mesh size. Extruder values are given for the outlet to create small outward extruded region from the outlet. Then the mesh is generated.

6. Physics Models are selected: 3D is selected for space, Implicit unsteady is selected for time, Ideal gas and segregated flow are selected to define the medium properties, k-epsilon model is selected for turbulence specification.

7. Mass flow input is created for PT-439 which has a 3-cylinder, 4-stroke engine. Mass flow rate is generated for each time step. The number of time steps are calculated using the method given in A.

8. Reports, monitors and plots are created for mass flow and pressure at inlet and outlet.

9. Solver has given the time step which is 7.58E-5s and the maximum run time is set for 2 cycles which is 0.109091s and the simulation is started. Each time step includes 10 inner iterations.

The result must be converged before proceeding to the next step. Here, simulation is given for 1440 time step which is sufficient for the convergence.

STEP-2:

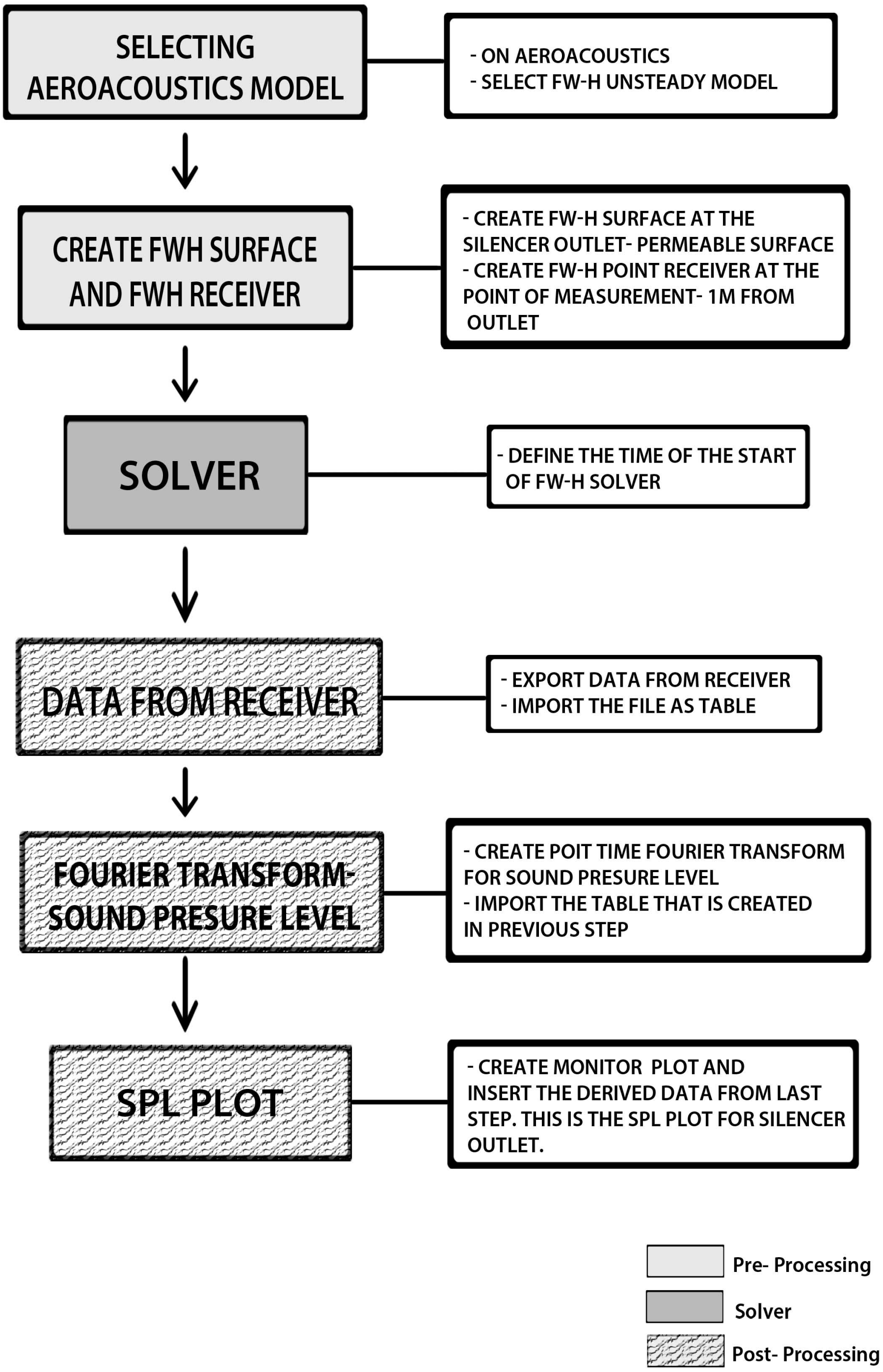
1. Aero-acoustic model is selected from the physics model. Ffowcs Williams-Hawkings (FW-H) unsteady model is selected for acoustics.

2. Create FW-H surface at the interface of the outlet and the extruded region. This is the region where the sound is measured. FW-H receiver is formed at a distance of 1m from the outlet and at angle of 45 degrees (see section3.3 for details).

3. The start time of the FW-H solver and the time for run is given to the solver. Now the simulation is given run for 4000 time steps. These much time steps are required to produce the data up to 7000Hz which is our sampling frequency. This sampling frequency produce accurate results upto 350Hz.

4. After the finish of simulation, data produced at the point receiver is exported to a file. This file then imported to the software in form of a table.

STEP- 2



SELECTING AEROACOUSTICS MODEL
- ON AEROACOUSTICS
- SELECT FW-H UNSTEADY MODEL
CREATE FWH SURFACE AND FWH RECEIVER
- CREATE FW-H SURFACE AT THE SILENCER OUTLET- PERMEABLE SURFACE
- CREATE FW-H POINT RECEIVER AT THE POINT OF MEASUREMENT- 1M FROM OUTLET
SOLVER
- DEFINE THE TIME OF THE START OF FW-H SOLVER
DATA FROM RECEIVER
- EXPORT DATA FROM RECEIVER
- IMPORT THE FILE AS TABLE
FOURIER TRANSFORM- SOUND PRESURE LEVEL
- CREATE POIT TIME FOURIER TRANSFORM FOR SOUND PRESURE LEVEL
- IMPORT THE TABLE THAT IS CREATED IN PREVIOUS STEP
SPL PLOT
- CREATE MONITOR PLOT AND INSERT THE DERIVED DATA FROM LAST STEP. THIS IS THE SPL PLOT FOR SILENCER OUTLET.
Pre- Processing
Solver
Post- Processing

5. Point time Fourier transform is created and the output function is set to sound pressure level. The imported table is inserted to the above created function to get the SPL results. The window function of the point time Fourier function is set to Hanning.

6. New monitor plot is created and the derived data from previous step is inserted.

The obtained plot is SPL vs Frequency plot for the noise produced at outlet of silencer and measured at a distance of 1m and 45 degrees angle.

3.3 Experimental procedure

As per the Indian Standards for the allowable noise that can be generated by tractors:

1. Maximum allowed noise at the operator ear level measured at full throttle is 98dBA with load and 92dBA without load.
2. Maximum allowed noise at the pass-by level measured at full throttle is 88dBA.

Experiment for the measurement of noise spectrum has been performed to compare the insertion loss results with CFD results. This way, the CFD procedure has been validated.

3.3.1 Experimental Setup

For the calculation of IL, a standard experimental procedure is followed in which the noise levels are measured at a distance of 0.5m. But as the CFD results are valid only for far fields, all the CFD calculations are done at a distance of 1m, which is the far field limit of the first order, 55Hz. So the experiment is done to measure the Noise Spectrum at 1m distance from exhaust outlet of existing muffler and straight pipe. In this way, the measurement of overall noise levels in dBA and the octave spectrum is done in the frequency range of 20Hz to 8000Hz.The IL is calculated as the difference in sound levels of straight pipe and silencer.

In experiment, microphone was located at a distance of 1m and at an angle of 45 degrees from the exhaust outlet. Noise was measured by keeping the engine speed at rated rpm and stationary condition.

3.3.2 Instrumentation

LMS SCADAS recorder is used to record the input data and free field microphone is used to capture the desired noise. The maximum limit of the microphone is upto 8kHz. Tachometer/ Magnetic pick-up is used to measure the rpm of the engine.

The incoming signal from the microphone is recorded by LMS SCADAS recorder and then the analysis is done using the software LMS Testlab.

Chapter 4

Simulation results for Back Pressure and Noise

4.1 Validation of CFD analysis

The existing silencer is shown in fig4.1 and fig4.2. This is a core plug silencer[5] which is a combination of expansion chamber and resonator. The inner pipe is sealed at the centre of the silencer and exhaust gas is forced to move through holes. This seal is called as Core Plug. This core plug results into high back pressure. So the new silencers are designed in a fashion that they produce low back pressure and reduce noise equal or more than the existing silencer.

The acoustic analysis of silencers are performed using CFD. So, initially we need to validate the CFD procedure by comparing CFD results with experimental results.

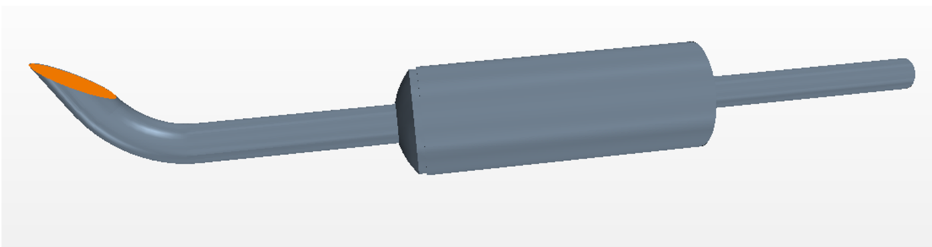


Figure 4.1: Existing silencer

Figure 4.2: Existing silencer from inside

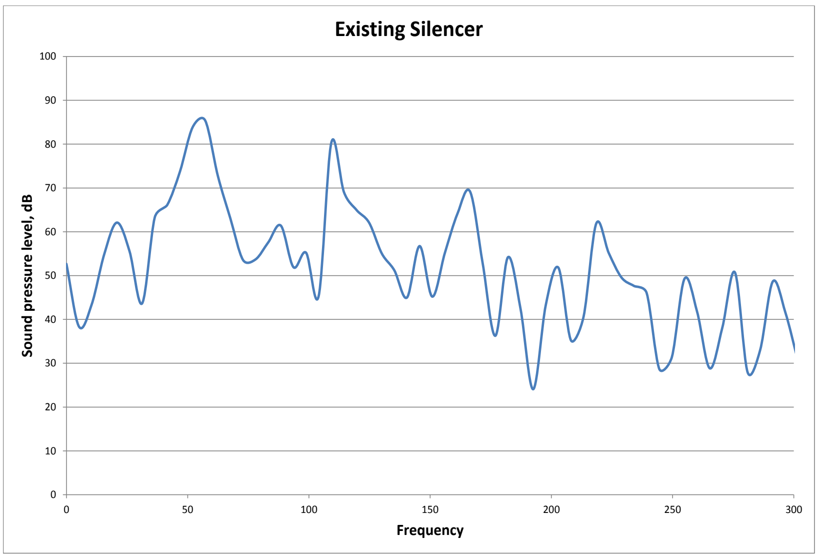


Figure 4.3: Noise spectrum of existing silencer produced by CFD

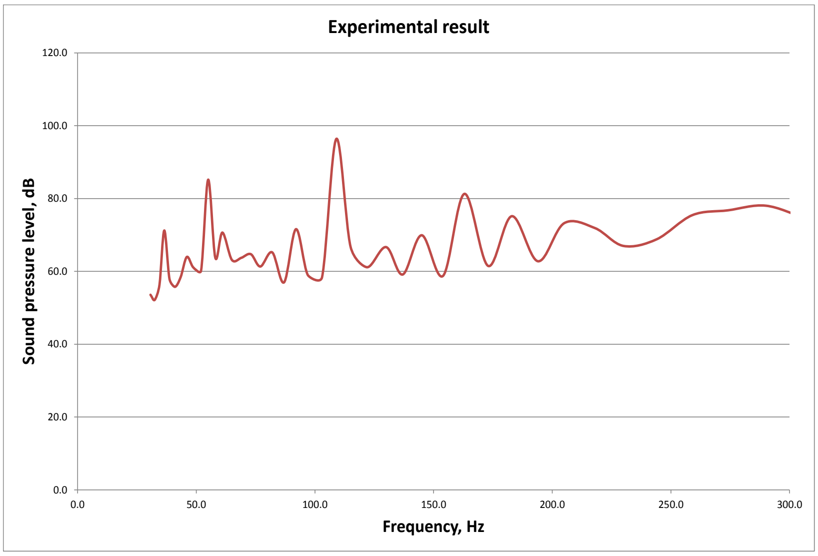


Figure 4.4: Experimental result: Noise spectrum of existing silencer

4.1.1 CFD results for existing silencer

Fig4.3 is showing the noise spectrum of existing silencer. It is the sound pressure level versus frequency plot obtained from Star-CCM+. Here peaks are obtained at the orders of firing frequency. Here, the calculations are done at 2200rpm of a 3-cylinder, 4-stroke engine. $rpm = 2200$ $rps = \frac{2200}{60}$

For 4 stroke engine, we obtain 2 power strokes in one complete cycle. So, the rps value is multiplied by 2 and because it is 3-cylinder engine, first order is obtained at: $\frac{rps}{2} \times 3$ which is 55Hz.

Thus, we get peaks at: 55Hz, 110Hz, 165Hz, 220Hz, 275Hz... In the noise spectrum, we can observe the peaks at these frequencies.

4.1.2 Experimental results

Figure 4.4 is showing the noise spectrum of existing silencer that is obtained experimentally. Here also we can see the peaks at the orders of firing frequency.

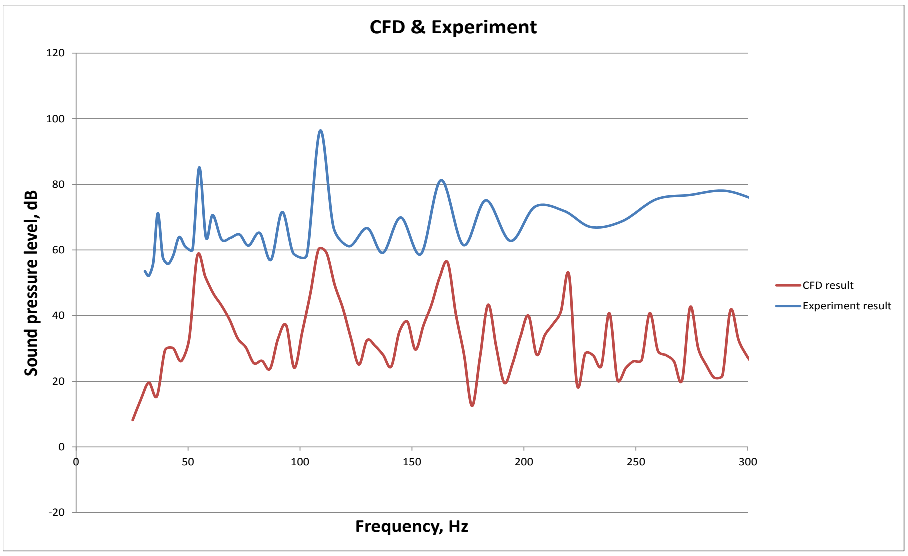


Figure 4.5: Comparison between CFD analysis and experimental analysis.

4.1.3 Comparison

Figure4.5 shows the comparison between the CFD and experimental results. We can see that the peaks are matching in both the plots. There is a gap in the absolute values. Following could be the reasons for gap:

1. Experimental result includes the structural noise of the engine that is not present in CFD results.

2. Experimental has been performed in open air condition which results in the addition of background noises.

3. Experimental results are plotted in the software LMS Testlab and CFD results are plotted in Star-CCM+. The reference values for SPL calculation could be different in these software that leads to the difference in absolute values.

The CFD method is giving the similar noise level spectrum but not giving the exact dB values. So, this method can be used to compare the noise reduction properties of different silencers. CFD comparisons can provide the exact difference in decibel values of noise, though absolute value may not be correct.

Figure 4.6: Silencer A

4.2 New Designs

Three new designs are designed and analysed using CFD. These all designs are made such that they are easy in construction, cost effective and maintenance free. All first and second silencers are simple expansion chamber mufflers and the third one is resonator muffler.

4.2.1 Silencer A

Design brief

Silencer A is an elliptical muffler with two expansion chambers and two exhaust pipes. The design is shown in fig4.6. This muffler is based on the mufflers analysed in the fig2.8. It consists of two expansion chambers which care connected internally with two connecting pipes. Both the internal connecting pipes are located at focus of the ellipse. Outlet pipes are located exactly at the extended positions of the connecting pipes. Because of this, back pressure will be reduced by significant amount. The expansion ratio of this muffler is set to 20 and depending on this, the major and minor axes of the ellipse are calculated. End corrections are introduced in the inlet and outlet pipes based on (reference). The inlet and outlet pipes are expended by 14.6mm inside the chamber for end corrections. As the designs are based on

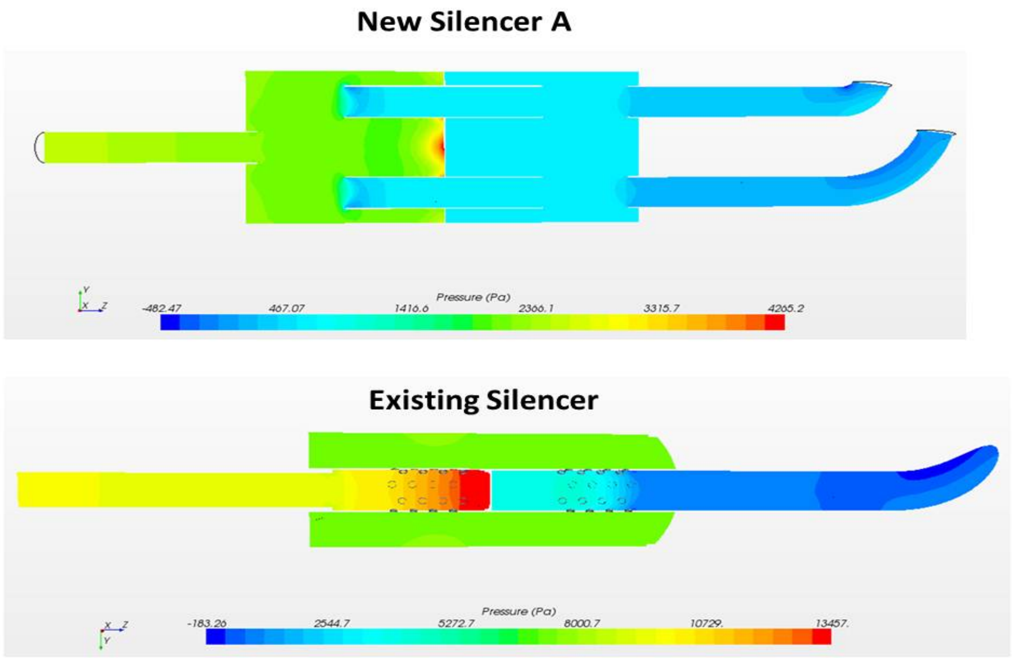


Figure 4.7: Back pressure comparison of new silencer A and existing silencer.

special case of dual expansion chamber(fig2.8), the size of internal connecting pipes are equal to the length of expansion chamber.

Back pressure analysis

Back pressure is an important factor for developing a muffler because back pressure effects the efficiency of engine and so the life. Fig4.7 shows the pressure plots of silencer A as well as existing silencer.

The back pressure produced by:

Existing silencer = 98.48 mbar.

Silencer A = 25.41 mbar

The difference in back pressure is 73.07 mbar which is almost 74 percent of the back pressure value of existing silencer. Thus, the silencer is very good in terms of back pressure.

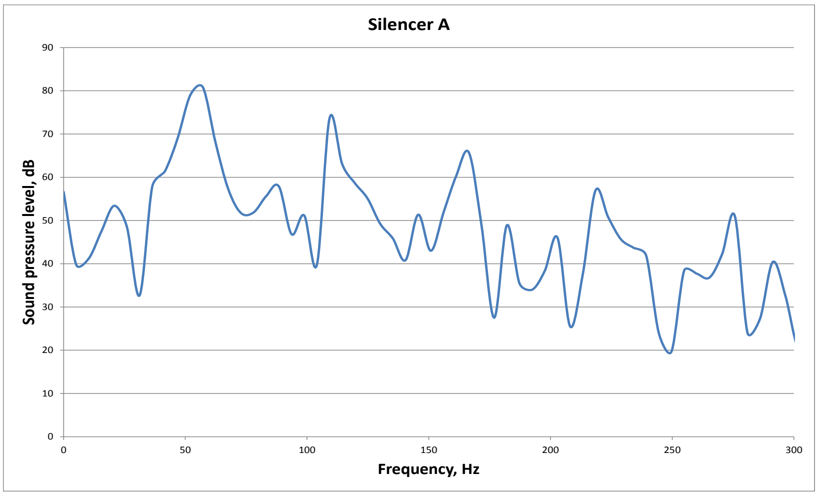


Figure 4.8: Noise spectrum of silencer A

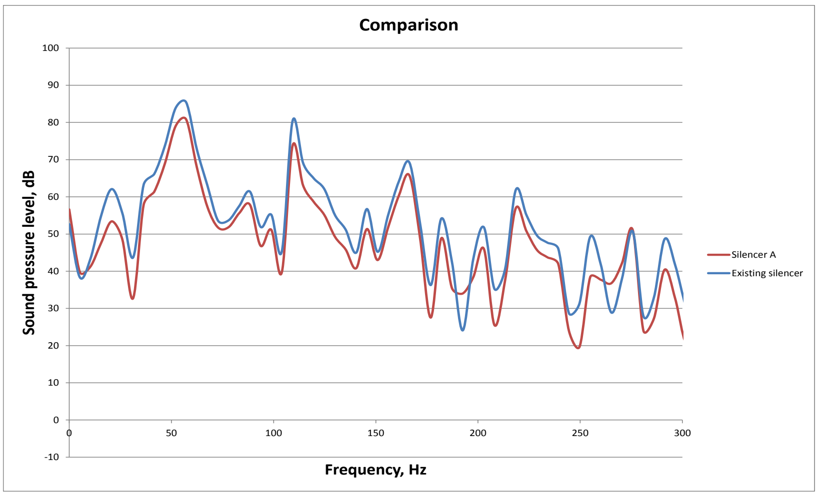


Figure 4.9: Comparison between silencer A and existing silencer on the basis of noise spectrum.

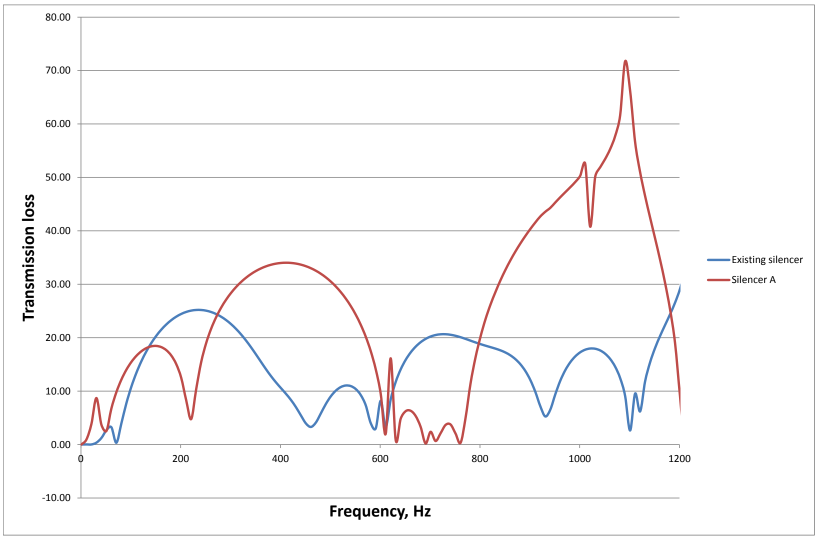


Figure 4.10: Transmission loss comparison between Existing silencer and silencer A

Noise analysis

The noise spectrum of existing silencer and silencer A are shown in the figure4.3 and figure4.8 respectively.

A comparative plot is shown in figure4.9. Here, we can see that the spectrum of silencer A is lying below the spectrum of existing silencer for almost all the frequency values. A difference of almost 5dB can be seen at the peaks.

Transmission loss has been measured by the software LMS labview for both the silencers and a comparative plot is shown in the fig4.10. These results are plotted up-to 1200Hz. From the transmission loss vs frequency plot, we can see a frequency band pass in the TL of silencer A in the range of 600Hz to 800Hz. Rest of the region is showing good attenuation by silencer A.

So, by CFD as well as TL results, we can say that Silencer A is attenuating more sound than the existing silencer.

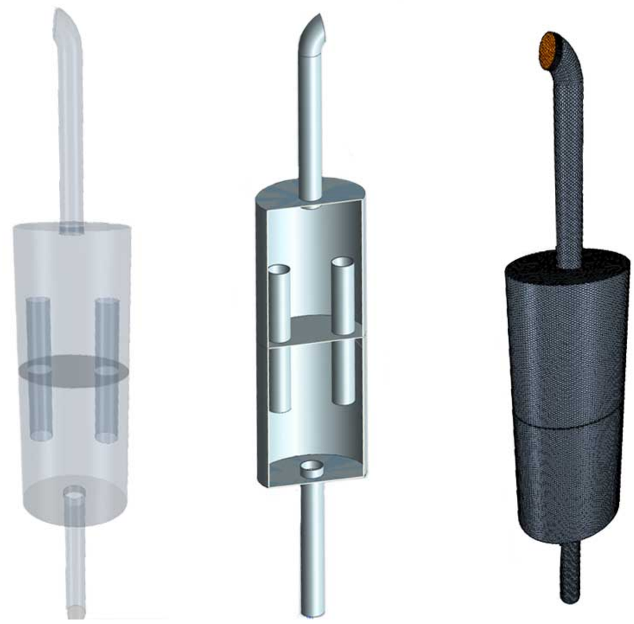


Figure 4.11: Silencer B

4.2.2 Silencer B

Design brief

Silencer B, shown in figure4.11, is of the same configuration as silencer A except the silencer B is having only one outlet pipe at the centre. As the new aesthetic look is not accepted very easily by the common public, Silencer A has given the look related to existing silencer and results into Silencer B.

Back pressure analysis

As the outlet pipes are reduced to one as compared to silencer A, there must be much difference in the back pressure. Also the location of the outlet pipe is changed as compared to the silencer B. Figure4.12 shows the pressure plots for existing silencer and silencer B.

The back pressure produced by:
Existing silencer = 98.48 mbar
Silencer B = 76.97 mbar

The difference in back pressure is 21.51 mbar which is almost 22 percent of the back pressure value of existing silencer. So, Silencer B is better than

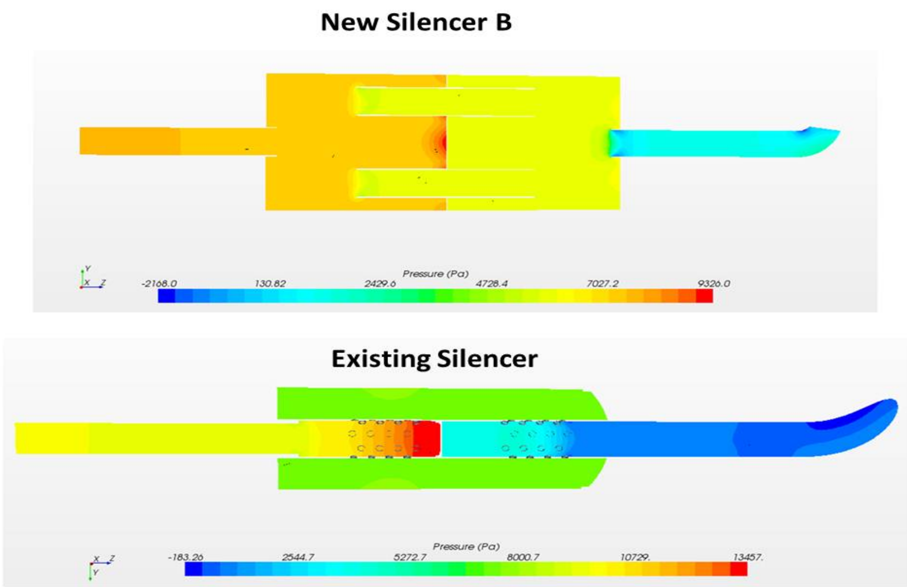


Figure 4.12: Back pressure comparison of new silencer B and existing silencer.

the existing silencer in terms of back pressure.

Noise analysis

The noise spectrum of existing silencer and silencer B are shown in the figure4.3 and figure4.13 respectively. A comparative plot is made in figure4.14. Here we can see reduction in sound pressure level at the peaks. Almost 5dB of difference is obtained at the first few orders. TL plots of silencer B and existing silencer are shown in the fig4.15. The TL of silencer B is showing a similar frequency band pass as silencer A. For the rest of the region, this silencer is giving good results. So we can conclude that silencer B is better than the existing silencer in terms of noise levels.

4.2.3 Silencer C

Design brief

Silencer C is shown in the figure4.16. This silencer is completely a resonator type muffler with an expansion chamber concentric to the exhaust pipe. The expansion chamber is divided into 4 parts of 10cm length and each part works as a resonance chamber. The internal pipe is having holes of different size and configuration for each part. This holes are designed in a pattern that they form a separate resonator for each closed volume to attenuation the

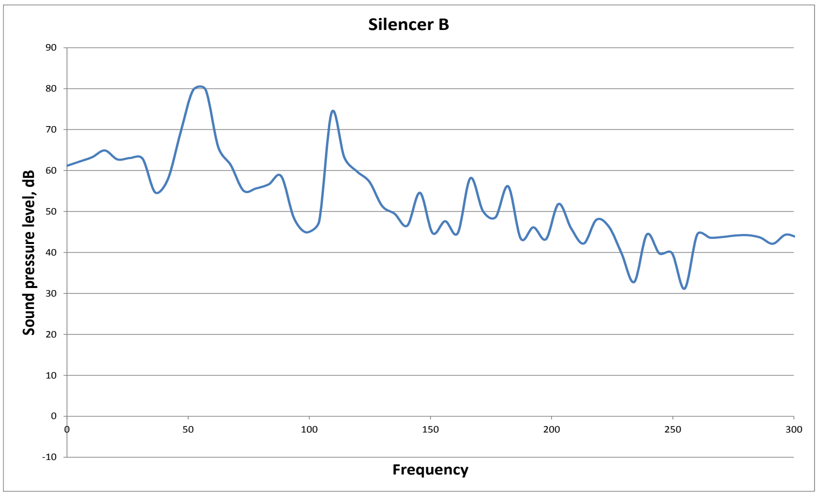


Figure 4.13: Noise spectrum of Silencer B

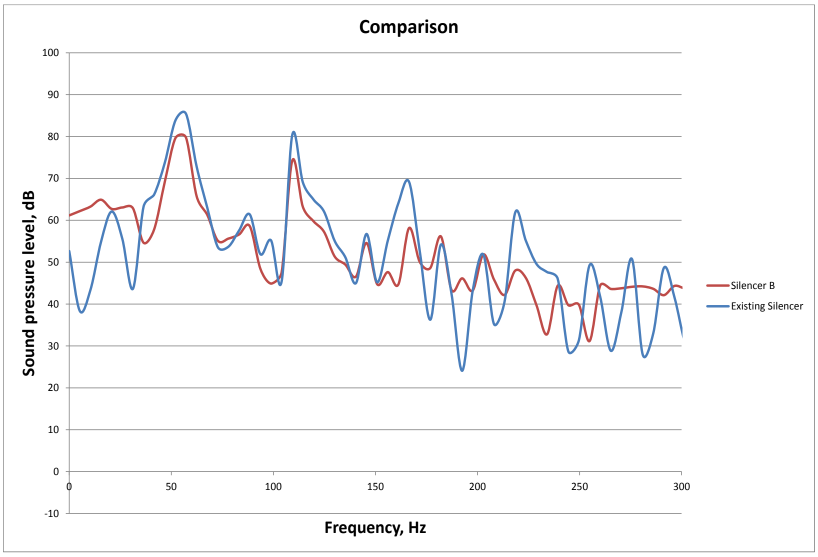


Figure 4.14: Comparison between Silencer B and Existing silencer on the basis of noise reduction.

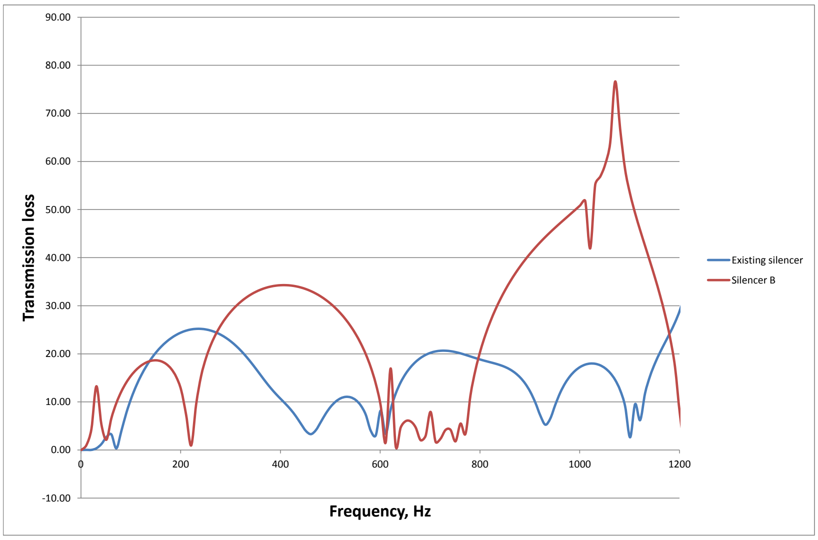


Figure 4.15: Transmission loss comparison between Existing silencer and silencer B

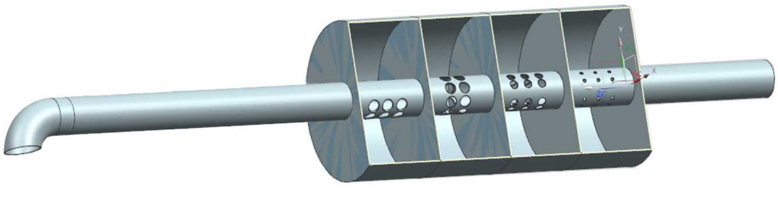


Figure 4.16: Silencer C

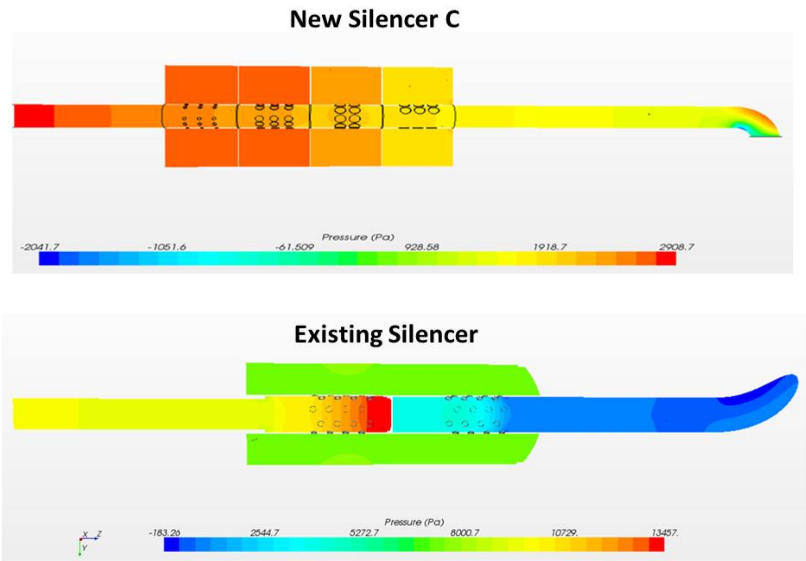


Figure 4.17: Back pressure comparison of new silencer C and existing silencer.

first 4 orders of firing frequency. The attenuating properties of a resonator depends completely on two quantities, volume and conductivity. In this design, volumes of all the four sections are fixed equal so as to make it easy to manufacture. Now the attenuation depends only on the conductivity or, we can say, holes. The holes are structured in a fashion that the four sections attenuate 55Hz, 110Hz, 165Hz and 220Hz separately.

Back pressure analysis

As there is no restriction in the flow of exhaust as, there must be very less back pressure generated by this silencer. Figure4.17 shows the pressure plots of existing silencer and Silencer C.

The back pressure produced by:
Existing silencer = 98.48 mbar
Silencer C = 28.69 mbar

The difference in the back pressure is 69.8 mbar which is almost 71 percent of back pressure produced by existing silencer. Thus, this silencer is very good as compared to the existing silencer in terms of back pressure.

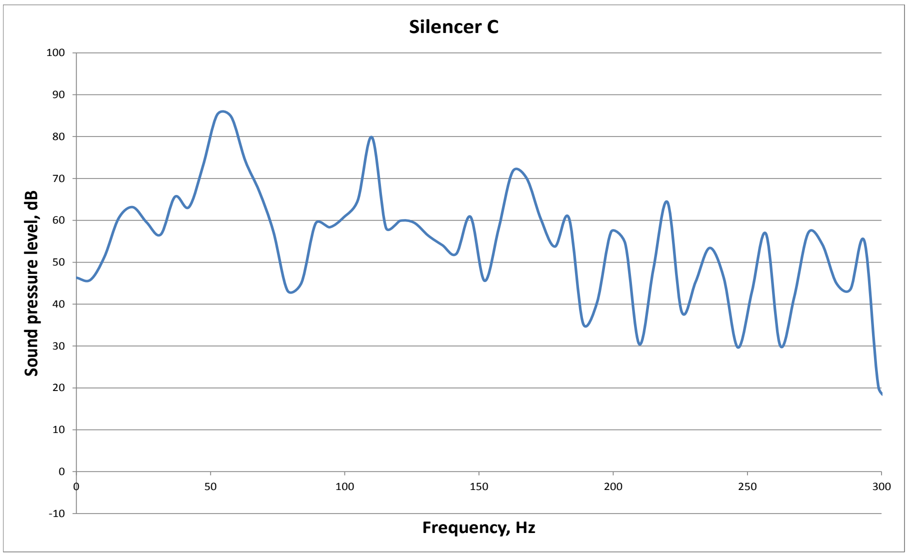


Figure 4.18: Noise spectrum of Silencer C

Noise analysis

The noise spectrum of existing silencer and silencer C is shown in figure4.3 and figure4.18 respectively. A comparative plot is shown in figure4.19.

Here, the noise produced by both the silencers at same conditions are almost equal. The peak values are almost matching for both the plots. TL for existing silencer and silencer C is plot in the fig4.20. Here, we can see that the silencer C is showing equivalent performance as the existing silencer 700Hz. The CFD results are showing the same behaviour of this silencer as existing silencer in the range of 20 Hz to 300 Hz. But the TL plot is showing a huge increment in the attenuation by silencer C from 800Hz to 1500Hz. So we can conclude that this silencer is much better than the existing silencer in this range of frequency.

4.3 Conclusion

The analysis of back pressure and noise of new silencers has been performed in the previous section. The analysis results of all the new silencers are compared with the existing silencer. As per the comparison, all the new silencers are better than the existing silencer in terms of back pressure. Silencer A

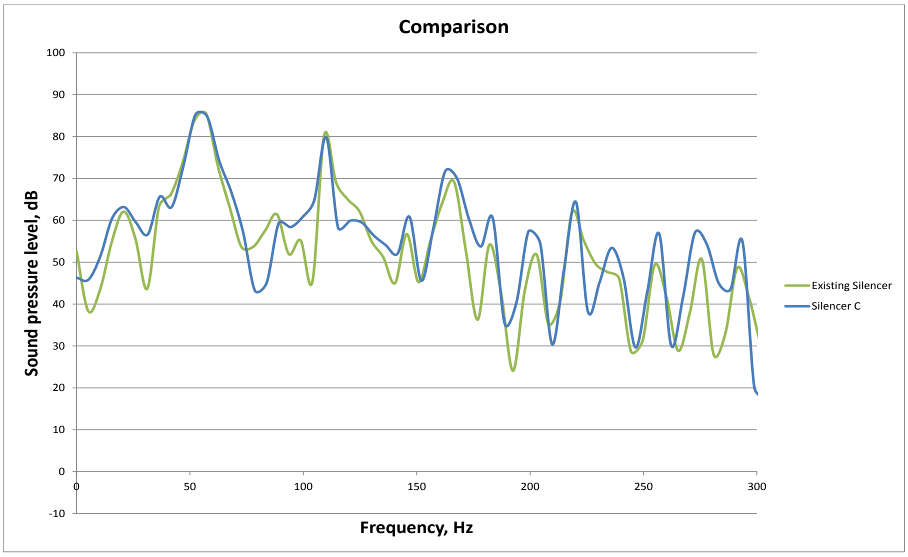


Figure 4.19: Comparison between Silencer C and existing silencer on the basis of noise reduction.

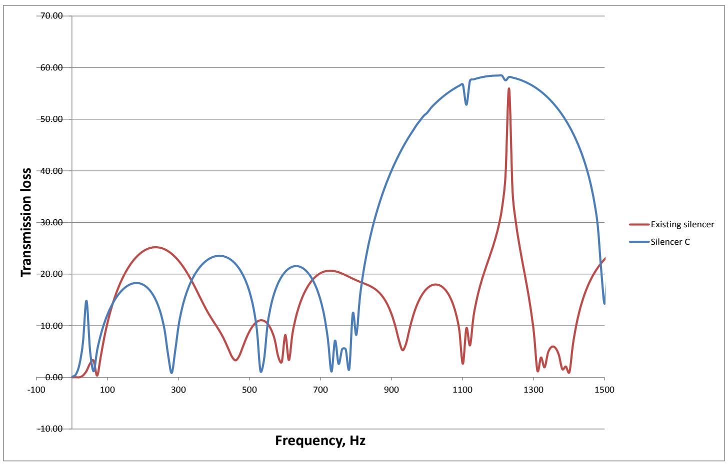


Figure 4.20: Transmission loss comparison between Existing silencer and silencer C

and silencer C are giving very significant reduction in back pressure.

For the noise reduction, silencer A and silencer B are better than the existing silencer. Existing silencer has an insertion loss of almost 14dB as per the experimental results. Silencer A and silencer B are producing lesser noise of 2dB-5dB. So, the IL of new silencers should be increased by 2dB-5dB. The silencer C is producing almost same noise as the existing silencer up-to 700Hz. From 800Hz to 1500Hz silencer C is giving much better result than the existing silencer.

Combining the back pressure and noise attenuation results, all the three silencers are considered better than the existing silencer and can be used as per the requirement.

Chapter 5

Discussion

Designing is an iterative process but each iteration makes some increment to the knowledge. I have developed many concepts that may lead to solve the problem but most of them were rejected by me during the process. I want to discuss about few of these concepts in this section.

First I have started designing a model that can reduce the noise by restricting the exhaust gas in chamber. As we can see in the existing model, the core plug is restricting the gas and without that plug sound reduction would be very less. Using the same method, I developed a muffler, as shown in the fig.(5.1), which consisted of an expansion chamber divided into 4 sections using three rotating discs. A shaft had been placed concentric to the cylindrical expansion chamber that was used to rotate the discs. Each section then divided into three isolated volumes by creating walls from central axis to the wall of expansion chamber. A hole was made in each disc at different locations corresponding to different isolated volumes. The working of the silencer goes in the following way- *When the exhaust gas enters the chamber, it is captured in one volume and stays there till the hole of next disc reaches that volume. This way, the gas moves from one section to other after staying there for some amount of time.* But this method definitely leads to the increment in back pressure. To reduce the back pressure, a fan was assembled at the outlet that took power from the shaft. I developed the design and performed back pressure analysis at the first step because that was the point of doubt and resulted in a certain increase of back pressure. So, the design was rejected at that level.

When the problem of band pass came up, I have started thinking of different ways by which these band pass can be compensated. As we can see in the TL plot of the new designs, there is a visible band pass in a small frequency range. The most appealing method of attenuation that came to my mind to reduce the noise at those frequencies is diffraction. I have introduced

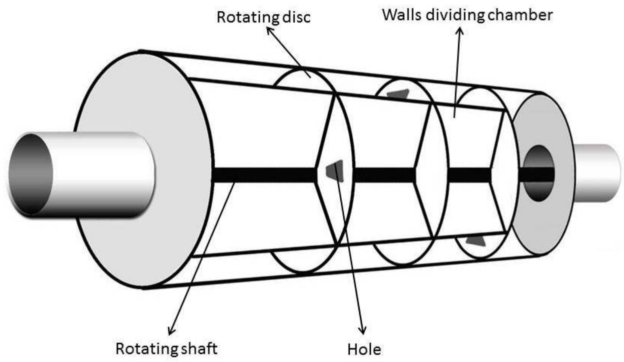


Figure 5.1: Muffler with rotating discs.

fins inside the **Silencer A** on the wall which is separating the two chambers. These fins should be the size of quarter wavelength of interest. The band pass in this silencer is at 600Hz to 800Hz. At 600Hz the quarter wavelength is almost equals to 14cm. So the depth of the fins should have minimum size of 14cm which is almost half the size of a chamber. The band pass can be shifted to higher frequency range if we reduce the length of chamber and then we can introduce shorter fins to compensate this problem. But this will lead to the reduction of amplitude of attenuation at lower frequencies that are the matter of concern. Thus, fins are rejected for these designs. Diffraction can also be obtained at these frequencies by creating a grid of quarter wavelength size inside the chamber. But introducing a grid inside the chamber is not easy to manufacture at present.

The problem of band pass can only be sorted out by shifting it to particular range of frequencies where no higher peaks are dominating. Thus all the silencers are designed such that the band pass lies in the range of lower noise.

In this project, we developed a procedure for designing automotive muffler for specific requirements. Starting from the plane wave theory, we established different techniques to attenuate sound. Applying these ideas, we can produce mufflers to attenuate sound of particular frequencies. This method can be very useful to design mufflers for automotive applications as the noise produced by engine is dominating at specific frequencies.

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Appendix A

Physical quantities and Formulation

Physical quantities used in the report

There are many physical quantities used in the report, the list of all the quantities is as follows:

dB = decibel
SPL = sound pressure level
TL = Transmission loss
IL = Insertion loss
NR = Noise reduction
BP = Back pressure
c = speed of sound
t = time
p = sound pressure
ρ = density of medium
k = wavelength constant, $\frac{2\pi f}{c}$
m = expansion ratio, equals to the ratio of cross sectional area of the expansion chamber to the cross sectional area of exhaust pipe
l = l_e = length of expansion chamber
l_c = length of connecting tube
S = cross sectional area of exhaust pipe
v = volume of resonator
c_0 = conductivity of connecting pipe of resonator
RP = resonance parameter
AP = attenuating parameter

dBA = A-weighted sound pressure level
rpm = revolutions per minute
rps = revolutions per second

Formulae used

Attenuation[8]:

$$Attenuation = 10\log(\frac{average\ incident\ sound\ power}{average\ transmitted\ sound\ power}) \tag{A.1}$$

Single expansion chamber:

$$Attenuation = 10\log[1 + \frac{1}{4}(m - \frac{1}{m})^2 \sin^2(kl_e)] \tag{A.2}$$

Double expansion chamber with externally connecting pipe:

$$a = \frac{1}{16m^2}[4m(m+1)^2 \cos 2k(l_e + l_c) - 4m(m-1)^2 \cos 2k(l_e - l_c)] \tag{A.3}$$

$$b = \frac{1}{16m^2}[2(m^2+1)(m+1)^2 \sin 2k(l_e+l_c) - 2(m^2+1)(m-1)^2 \sin 2k(l_e-l_c) - 4(m^2-1)^2 \sin 2kl_c] \tag{A.4}$$

$$Attenuation = 10\log(a^2 + b^2) \tag{A.5}$$

Double expansion chamber with internally connecting pipe:

$$a = \cos 2kl_e - (m-1)\sin 2kl_e \tan kl_c \tag{A.6}$$

$$b = (m + \frac{1}{m})\sin 2kl_e + (m-1)\tan kl_c[(m + \frac{1}{m})\cos 2kl_c - (m - \frac{1}{m})] \tag{A.7}$$

$$Attenuation = 10\log a^2 + b^2 \tag{A.8}$$

Single resonator with conductivity c_0 and volume V

$$Attenuation = 10\log[1 + (\frac{\frac{\sqrt{c_0 V}}{2S}}{\frac{f}{f_r} - \frac{f_r}{f}})^2] \tag{A.9}$$

$$f_r = \frac{c}{2\pi}\sqrt{\frac{c_0}{V}} \tag{A.10}$$

Relation for time step size and frequency resolution[17]

$$Sampling\ frequency = 20 \times Interest\ frequency \tag{A.11}$$

$$Number\ of\ time\ steps = \frac{2 \times sampling\ frequency}{Frequency\ resolution} \tag{A.12}$$

$$Time\ step\ size = \frac{1}{2 \times sampling\ frequency} \tag{A.13}$$