Inflationary cosmology: Natural inflation

A Thesis

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Dedicated to my parents.

Certificate

This is to certify that this dissertation entitled "**Inflationary cosmology: Natural inflation**" towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by Shilbhushan Shambharkar at Physical Research Laboratory Ahmedabad under the supervision of Dr. Subhendra Mohanty, Professor at Theoretical Physics Division PRL Ahmedabad during the academic year 2018-2019.

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Declaration

I hereby declare that the matter embodied in the report entitled "**Inflationary cosmology: Natural inflation**" are the results of the work carried out by me at the Theoretical Physics Division, PRL Ahmedabad under the supervision of Dr. Subhendra Mohanty, Professor at Theoretical Physics Division PRL Ahmedabad and the same has not been submitted elsewhere for any other degree.

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Abstract

Standard model of cosmology describes the origin of universe successfully, however it does not explain the reason for flatness of universe and horizon problem. The most popular approach to solve these problems is to introduce an era of exponential expansion in early universe, known as inflation. In this thesis, first we discussed the problems that standard model of cosmology suffers then how inflation solve them. There are several models of single field inflationary cosmology, e.g., chaotic, natural and hybrid. These models must satisfy some criteria that include sufficient inflation, CMB anisotropy data. Single field model requires nearly flat potential (satisfy slow roll conditions) that involve very weakly coupled field. Thus it has naturalness problem. Natural inflation solve this problem because it include flat potential naturally. We analyze the natural inflation model and studied their phenomenology in present observation data from Planck.

Chapter 1

Inflation

Big bang theory successfully explains the origin of the universe. It assumes the universe to be isotropic and homogeneous. At large scale observation of cosmic microwave background (CMB) radiation agrees with it up to a large extent. However, precise measurements show anisotropy in temperature fluctuations. Apart from it, there are other problems in the standard model of cosmology that can not be resolved, e.g., flatness problem, the horizon problem, and density fluctuation in the CMB. Inflation was proposed [1] as a solution to these cosmological problems. Cosmological inflation is an epoch of exponential expansion assumed to happen in the very early universe. Inflation provides correct initial conditions for the standard big bang cosmology. It also solves the problems at the macroscopic scale, i.e., density fluctuation. There are many models of inflation that have been proposed [2], which are in good agreement with the observational data. However, the exact initial conditions are still unknown. This question is very important because inflation was initially proposed to solve similar initial condition problems of the big bang model. In this thesis, we will describe these problems, mentioned above, of the standard model of the big bang and discuss inflation as a possible solution. Additionally, we will analyze the popular model of inflation i.e., natural inflation.

In this chapter, we discuss the standard model of cosmology after that, the theory of inflation paradigm will be introduced. For a detailed review on cosmology and inflation see [3, 4, 5, 6, 7] and references therein.

1.1 Standard Model of Cosmology

The big bang model assumes that the universe is homogeneous and isotropic on large scale. The observation of the cosmic microwave background radiation (CMBR) is the evidence for this assumption. Therefore, Universe can be described by the Friedmann-Robertson-Walker (FRW) metric which is the most general metric of a homogeneous and isotropic Universe,

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \right], \tag{1.1}$$

where r, θ and ϕ are comoving coordinates–independent of time. t is the physical time, can be written as $a^{-1}(t)dt = d\tau$, where τ is the conformal time and $a(t)$ is the scale-factor. $k = 1/0/-1$, is releted with curveture of the spatial-part of FRW metric, reprsents the positively curved closed universe/flat universe/negatively curved open universe respectiviely, for a homogeneous and isotropic universe. From Einstein general theory of relativity we can obtain Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \tag{1.2}$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0, \tag{1.3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P), \tag{1.4}$$

where P is the total pressure, ρ is the total energy density, G is the gravitational constant. First equation (1.2) represents the total energy density of the universe, second equation (1.3) represents the evolution of the energy density with time and third equation (1.4) is derived from first and second equation and it quantifies acceleration of the universe.

Equation 1.3 is satisfied for all the individual components that include matter (baryonic as well as dark), radiation, and the cosmological constant. Conservation of total stress energy tensor of the universe,

$$\nabla_\mu T^{\mu\nu} = 0, \tag{1.5}$$

where $T^0_0 = \rho$, $T^i_j = -P\delta^i_j$ and $T^0_i = T^i_0 = 0$ for a homogenous and isotropic universe. Equation (1.2-1.4) can also be obtained from the equation (1.5). This is due to the Bianchi identity, which is an alternative way to derive Einstein equations. Physical distance $x_{ph}(t)$ can be written as $a(t)\ x_{co}$, where x_{co} is the comoving distance. For a homogeneous and isotropic universe $\dot{x}_{ph}(t) = \dot{a}(t)\ x_{co}$. Therefore, expansion of the universe can be described in the terms of the Hubble parameter:

$$H(t) = \frac{\dot{x}_{ph}(t)}{x_{ph}(t)} = \frac{\dot{a}(t)}{a(t)}. \tag{1.6}$$

The equation of state for ideal fluid can be written as $P = \omega\rho$, where ω is a constant. For the different components, ω is given as,

- Radiation/hot matter: $\omega = \frac{1}{3}$.
- Cosmological constant: $\omega = -1$.
- Dust/cold non relativistic matter: $\omega = 0$.

Using above equation of state, we find,

$$\rho = \rho_0 \left(\frac{a_0}{a}\right)^{3(1+\omega)}, \tag{1.7}$$

where a_0 is the scale factor for energy density ρ_0. Scale factor is normalized such that, today $a_0 = 1$. After substituting values of ω in the equation 1.7 we get,

- For radiation/hot matter: $\rho(t) \propto a^{-4}(t)$. Energy density of the radiation changes not only because of the change in number density, but also due to the change in the energy.
- Vacuum energy: $\omega = -1$, $\rho =$ constant.
- For dust/cold non relativistic matter $\rho(t) \propto a^{-3}(t)$. Energy density dilutes due to the change in number density,

Let us describe the parameters that determine the characteristics of the universe,

Hubble parameter H: The expansion rate of the universe.

Dimensionless density parameter: $\Omega(t)$ and its value today Ω_0.

Therefore, first Friedmann equation can be written as,

$$\Omega(t) - 1 = \frac{k}{a^2H^2}, \quad \Omega(t) \equiv \frac{\rho(t)}{\rho_c(t)} = \frac{8\pi G\rho}{3H^2}, \quad \rho_c(t) = \frac{3H^2}{8\pi G}. \tag{1.8}$$

From the equation (1.8), we can see that $\Omega(t)$ is the measure of the spatial curvature of the universe. There are three possibile scenarios,

- $\Omega(t) = 1 \rightarrow k = 0$ and $\rho = \rho_c \longrightarrow$ flat universe.
- $\Omega(t) > 0 \rightarrow sign(k) > 0$ and $\rho > \rho_c \longrightarrow$ closed universe.
- $\Omega(t) < 0 \rightarrow sign(k) < 0$ and $\rho < \rho_c \longrightarrow$ open universe.

We can see from the first expression in equation 1.8, that cosmological evolution does not alter the sign of right hand side, therefore, if the universe started with $\rho > \rho_c$ then $\rho(t)$ will always be greater than $\rho_c(t)$.

The most accepted model for stanadard cosmological evolution, i.e. in agreement with CMB [11], large scale structures[12], Hubble constant measurement and the expansion of the universe [13, 14], is ΛCDM model. Observational data suggests that our universe is spatially flat (k = 0) to a great accuracy, $\Omega_0 = 1.0005 \pm 0.00333$ [11], with $\Omega_m \simeq 0.3$ and $\Omega_\Lambda \simeq 0.7$. Value of H is around 70 Km s^{-1} Mpc^{-1}. See recent Planck results for values of cosmological parameters [15].

Using the Friedmann equation (1.2) and equation (1.7) assuming spatially flat universe we can rewrite the evolution of scale factor,

$$(\dot{a})^2 \propto a^{-(1+3\omega)}. \tag{1.9}$$

Subsequently, if $a(t)$ has power law behaviour, $a(t) \propto t^q$, then it can be shown that,

$$a(t) \propto t^{2/(3+3\omega)}, \quad \text{when} \quad \omega \neq 1, \tag{1.10}$$

hence, in the radiation dominated era: $a(t) \propto t^{1/2}$, and for matter dominated era: $a(t) \propto t^{2/3}$.

Comoving horizon is defined as the distance travelled by the light since $t = 0$. This is given by,

$$\eta = \int_0^t \frac{dt'}{a(t')}. \tag{1.11}$$

The light emitted from a distant source is redshifted because of expansion of the universe. The red shift z is given by,

$$1 + z \equiv \frac{\lambda_{observed}}{\lambda_{emitted}} = \frac{a_0}{a}, \tag{1.12}$$

Since $a(t)$ is a function of time, we can also write the redshift in terms of time t for matter/radiation dominated era.

1.2 Short Comings of Big Bang Cosmology

Standard big bang model is successful, but it has some of the shortcomings

- The flatness problem,

- The horizon problem,
- Temperature anisotropy in CMB.

1.2.1 The Flatness Problem

Friedmann equation:

$$H^2 = \frac{1}{3M_{pl}^2}[\frac{\rho_r}{a^4} + \frac{\rho_m}{a^3} + ...] - \frac{k}{a^2} \tag{1.13}$$

Where, ρ_r and ρ_m are the radiation and matter density respectively. We choose to work in the unit of M_{pl} and the conversion is given by $\frac{1}{3M_{pl}^2} = 8\pi G$.

Observation suggests that the universe is spatially flat, but the reason for flatness can not be explained by the standard model of cosmology; it has to be taken to set the initial conditions in ΛCDM. In Friedmann equation 1.13, on the left-hand side, we can see Hubble parameter square, which is the total energy of the universe on the right-hand side, is the various components of the universe that contribute to the total energy density of the universe. Friedmann equation also tells the evolution of different denisties with the scale factor, e.g., ρ_r evolved as $1/a^4$ whereas ρ_m as $1/a^3$. Curvature constant k decreases as a^{-2}.

$$1 = \Omega_{\text{total}} - \frac{k}{a^2H^2} \tag{1.14}$$

Ω_{total} is dimensionless parameter corresponds to total energy density of the universe. We can define the parameter to measure the flatness as:

$$|1 - \Omega_{\text{total}}| = \frac{1}{a^2H^2} = \frac{1}{\dot{a}^2} \tag{1.15}$$

We can see that deviation of Ω_{total} with 1 is the ratio of curvature and energy density. We can go backward in time and as we know this deviation $|1 - \Omega_{\text{total}}|$today we further can track $|1-\Omega_{\text{total}}|$ at the epoch of BBN,

$$\frac{|1-\Omega_{\text{total}}|_{T=T_N}}{|1-\Omega_{\text{total}}|_{T=T_0}} = \left(\frac{\dot{a}_N^2}{\dot{a}_0^2}\right) \simeq \left(\frac{T_0^2}{T_N^2}\right) \sim \mathscr{O}(10^{-16}), \tag{1.16}$$

where we have used, $T \sim \frac{1}{a}$, $T_0 \sim 10^{-13}$ GeV is the present temperature of CMB radiation and nucleosynthesis happened at a temperature $T_N \sim 1$ MeV [5].

In order to get the correct value of $|1-\Omega_{\text{total}}|$ today, its value must be fine-tuned to extremely close to zero at early times. However, there is no reason for such fine-tuned value of $|1-\Omega_{\text{total}}|$ in the early universe.

1.2.2 The Horizon Problem

We observed universe to be isotropic and homogeneous at large scale. The radius of observable universe today can be given as $R_{obs}(t_0) \sim \frac{1}{H_0} \sim t_0$. The evolution of R can be given as:

$$R_{obs}(t_i) = R_{obs}(t_0)\left(\frac{a_i}{a_0}\right) \approx t_0\left(\frac{a_i}{a_0}\right). \tag{1.17}$$

We can compare the horizon size with the Hubble size. Horizon measures the distance light travel within an amount of time. It tells how far light travel since the begining. Let us assume scale factor $a(t) \sim \frac{1}{t^{\varepsilon}}$ then size becomes:

$$R_{causal}(t_i) = a(t_i)\int_0^{t_i} \frac{dt}{a(t)} \approx \frac{\varepsilon}{\varepsilon-1} t_i. \tag{1.18}$$

This is horizon radius. Let us calculate the ratio of radius of Hubble and horizon,

$$\frac{R_{obs}(t_i)}{R_{causal}(t_i)} \sim \frac{\varepsilon-1}{\varepsilon}\left(\frac{\dot{a}_i}{\dot{a}_0}\right) >> 1 \tag{1.19}$$

This imply that observed region today is much much bigger than the light can travel (horizon). This leads to a problem how causally disconnected patches will communicate to make the isotropic and homogeneous universe as we see today.

1.2.3 Temperature anisotropy in CMB

Universe is homogeneous and isotropic on large scale i.e. $\gg 1Mpc$. As we come on smaller scale $\ll 1Mpc$ universe is dominated by non-linearities (galaxy, cluster, super-cluster etc.). These fluctuatuions in the energy densities $\frac{\delta\rho}{\rho}$ also seen in Cosmic background radiation (CMB). Scale factor is inversely proportional to the temperature of the universe ($a(t) \sim \frac{1}{T}$) thus ratio of size of the observed universe to causal horizon today can be calculated as:

$$\sim \left(\frac{10^{15}\text{GeV}}{10^{-13}\text{GeV}}\right)^{\varepsilon-1} \sim e^{60(\varepsilon-1)}. \tag{1.20}$$

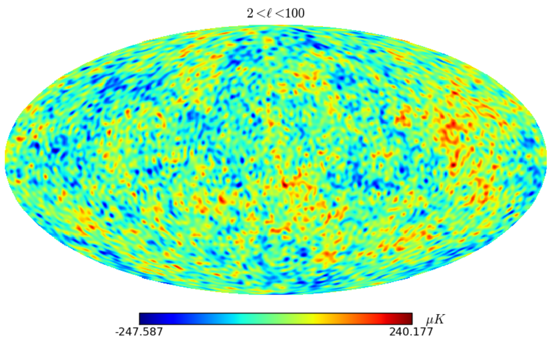

FIGURE 1.1: CMB measure of sky reported by WMAP

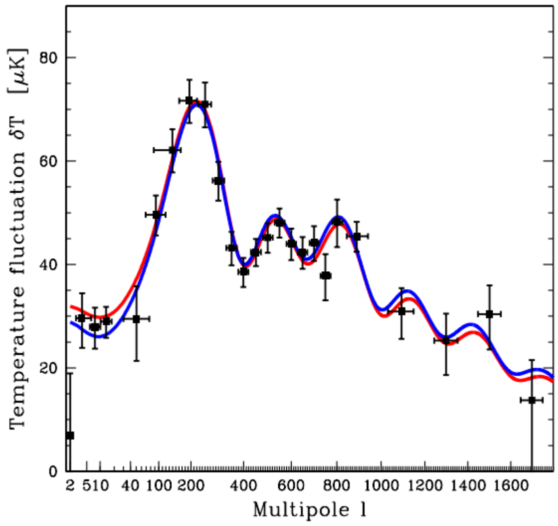

FIGURE 1.2: Variation of temperature fluctuations (δT) with l.

We represent the CMB temperature map measured by WMAP in figure 1.1. The average temperature measurement of CMB is around 2.7K. The color difference in figure 1.1 represents anisotropy in temperature of CMB where blue color represents over density, and red color describes under density regions that cause anisotropy in the temperature of the last scattering surface i.e., CMB. These temperature fluctuations are on the average is the order of $\delta T/T \sim 10^{-5}$. Figure 1.2, shows the variation in these temperature fluctuations as a function of multipole l.

1.3 Inflationary cosmology

These problems can be addressed by introducing smoothing fluid component density ρ_s which evolves as $a^{-2\varepsilon}$. Therefore Friedman equation becomes:

$$H^2 = \frac{1}{3M_{pl}^2}[\frac{\rho_r}{a^4} + \frac{\rho_m}{a^3} + ...] - \frac{k}{a^2} + \frac{\rho_s}{a^{2\varepsilon}}. \tag{1.21}$$

When $\varepsilon < 1$ then ρ_s will dominate the energy density of the universe. Let us assume the equation of state parameter of this new fluid as:

$$\varepsilon = \frac{3}{2}(1+w). \tag{1.22}$$

Here $w = \frac{p}{\rho}$ is the equation of state of the fluid. Inflation is exponential expansion due to ρ_s. The small patch expands exponentially and becomes so big to match the present universe horizon. The amount of inflation can be defined as:

$$N = \ln\frac{a_{end}}{a_{in}} = \int_{t_{in}}^{t_{end}} H dt. \tag{1.23}$$

Here a_{in} and a_{end} are the scale factor at the time when inflation starts and when it ends. N is the number of e-fold before end of inflation. The number of e-fold can be approximated as:

$$\frac{a_{end}}{a_{in}} \leq \frac{H_{inf}^{-1}}{H_0^{-1}} \tag{1.24}$$

Here H_{inf} is the size at the time of inflation and H_0 is size at today. Therefore this tells the amount of inflation needed to match the isotropy and homogeneity of the universe today. Now, $H^{-1} \sim 1/T^2$ and $a \sim 1/T$ so the number of e-fold can be written as:

$$N_{total} \geq \ln\frac{T_{end}}{T_0} \sim 60 \tag{1.25}$$

From the CMB observation we need $N_{total} \sim 60$ which is similar what we need in Eq. 1.20. Therefore inflation solves horizon as well flatness problem together.

1.4 Inflationary Models

There are many possible models for inflation. The simplest one could be cosmological constant; however, this is ruled out because it will never end the inflation. Therefore it should include a field with its dynamics so that after sufficient inflation, exponential expansion ceases.

There are many particle physics models are proposed that include single or multi fields, e.g. Power Law model [21], Hilltop model [22], Natural inflation [23, 24], D-brane inflation [25, 26, 27], Exponential potential in supergravity inspired models [28, 29, 30, 31, 32], Hybrid model [33, 34], α attactors [35], Higgs inflation [36, 37, 38, 39] etc. In this thesis we will discuss natural inflation and its impact on observations.

1.4.1 Single Field Slow-Roll Theories of Inflation

This model is based on a single field that has nearly flat potential so that its energy density changes very slowly during inflation. Most of the contributions are from the field itself. Contributions from others are negligible. The field that drives inflation is known as inflaton. When potential decreases sufficiently then inflation

When inflation ends, the field starts oscillating in the vacuum, and the reheating phase started. During reheating, inflaton radiates into other particles.

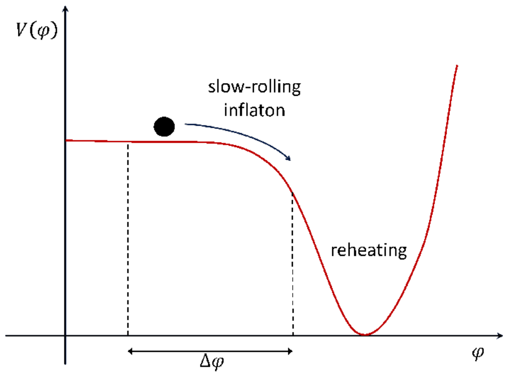

FIGURE 1.3: Schematic diagram of potential for slow roll inflaton. Inflation era is represented by verticle dashed line. [48].

Let us consider an inflaton field ϕ, and its potential is represented by $V(\phi)$. The energy density and pressure of ϕ can be written as,

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{1.26}$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{1.27}$$

The equation of motion of the inflaton field is,

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0, \tag{1.28}$$

where $\dot{\phi}$ is the derivative of homogeneous inflaton field with respect to t and V' is the derivative of the potential with respect to the field ϕ.

To check a potential to be slow roll, it is useful to construct parameters known as slow-roll parameters [6],

$$\varepsilon_V \equiv \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2, \qquad |\eta_V| \equiv M_{pl}^2 \frac{|V''|}{V}, \tag{1.29}$$

where $M_{pl}^2 = \frac{1}{8\pi G}$.

For slow roll, $\varepsilon_V << 1$ and $|\eta_V| << 1$. We further discuss it in the next chapter. First condition corresponds to $\dot{\phi}^2 << V(\phi)$ which is responsible to start inflation. The second parameter tells about the amount of inflation. Inflation stops when $\varepsilon_V \sim \mathscr{O}(1)$ and $|\eta_V| \sim \mathscr{O}(1)$. Amount of inflation can be quantify by number of e-fold which is given by,

$$N = \frac{1}{M_{pl}^2}\int_{\phi_{end}}^{\phi} \frac{V}{V'} d\phi. \tag{1.30}$$

The lower bound for sufficient inflation is in the range $N = 50 - 60$ [49].

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Chapter 2

Natural inflation

Inflation was proposed to solve the problems of Standard model of cosmology [2], i.e., the horizon problem, flatness of the observed universe, and existence of monopole. Inflationary cosmology assumes there is an epoch when the universe expands exponentially (e^{Ht}). We have discussed how inflation solves these problems in detail in the previous chapter.

There are various models proposed to provide the inflation including single or more field inflation: chaotic [3], natural [4], hybid [5] etc. These models must satisfy constraints: provide sufficient inflation and constraints from CMB anisotropy [1, 6] and structure formation. Inflationary model drive from single field suggest to satisfy the observations field potential need to be flat. More specifically single field potential should satisfy [7] $\Delta V/(\Delta\phi)^4 \leq \mathscr{O}(10^{-6}-10^{-8})$.

2.1 Slow roll field

Let us study the slow roll conditions in details. The energy density and pressure density of scalar field ϕ that act as inflaton are given by:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi),$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{2.1}$$

Using (2.1) in Einstein equations we get:

$$H^2 = \frac{8\pi}{3M_{pl}^2}\Big[\frac{1}{2}\dot{\phi}^2 + V(\phi)\Big],$$
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{2.2}$$

During inflaton era accelerated expansion of universe takes place so

$\ddot{a} > 0 \implies \dot{\phi}^2 < V(\phi)$. To have sufficient inflation to explain the present observations single field must satisfy the following conditions:

$$\dot{\phi}^2 << V(\phi) \text{ and } \ddot{\phi} << 3H\dot{\phi} \tag{2.3}$$

Another way to write these conditions is to define slow roll parameters which are given as [8]

$$\varepsilon = \frac{M_{pl}^2}{16\pi}\Big(\frac{V'(\phi)}{V(\phi)}\Big)^2, \quad \eta = \frac{M_{pl}^2}{8\pi}\frac{V''(\phi)}{V(\phi)} \tag{2.4}$$

For slow roll $\varepsilon << 1$ and $|\eta| << 1$. Inflation will end when this conditions start violating thus the end of the inflation can be parametrized from number of e-folding. It is defined as:

$$N = \log\frac{a_f}{a_i} = \int_{t_i}^{t_f} H dt$$

For potential $V(\phi)$, if start of inflation is at ϕ_1 and end at ϕ_2 the number of e-folding can be written as:

$$N = \frac{-8\pi}{M_{pl}^2}\int_{\phi_1}^{\phi_2}\frac{V(\phi)}{V'(\phi)}d\phi. \tag{2.5}$$

The simplest one field inflation model of particle physics require quartic self couplings to be very small, i.e., $\lambda < \mathscr{O}(\frac{\Delta V}{(\Delta\phi)^4}) \lesssim \mathscr{O}(10^{-12})$ [9, 10] which is unnatural model.

2.2 Natural inflation

One of the plausible ways to resolve this problem is if some symmetry restricts couplings to be very small. The most popular way to achieve a weakly coupled field is known as natural inflation [4]. In this scenario, if there is global symmetry breaking, then pseudo Nambu Goldstone boson arises with flat potential. However, if an additional symmetry breaks explicitly, then the potential is almost flat. The couplings are inversely proportional to the scale of breaking; thus, for a large breaking scale field will be very weakly coupled as required for inflation.

In this chapter, we discuss one of the model of natural inflation. The potential is of the form [4, 11]

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{n\phi}{f}\right) \right] \tag{2.6}$$

We assume n=1 to have single minima. The potential is plotted in Fig 2.1. Mininum is at πf. The inflation is in the interval ϕ_1 and ϕ_2 both are in $[0, \pi f]$. Slow roll parameters can

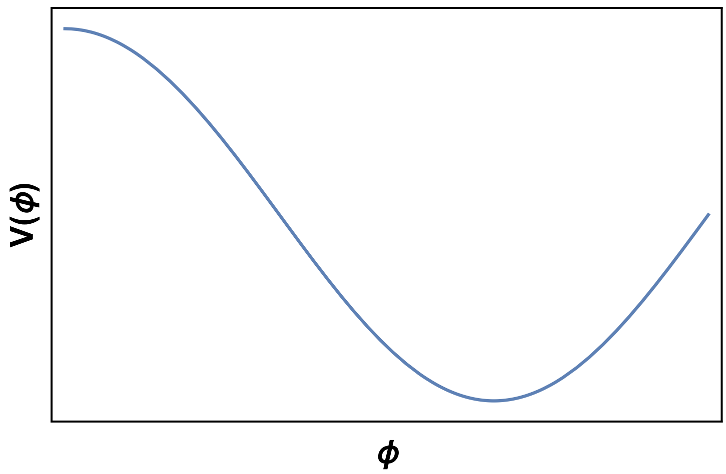

FIGURE 2.1: Inflaton potential.

be computed as:

$$\varepsilon = \frac{M_{pl}^2 \sin^2[\phi/f]}{16 f^2 \pi (1+\cos\frac{\phi}{f})^2}, \tag{2.7}$$

$$\eta = \frac{M_{pl}^2 \cos\frac{\phi}{f}}{8 f^2 \pi (1+\cos\frac{\phi}{f})}. \tag{2.8}$$

Therefore the conditions can be written as:

$$\frac{\sin\frac{\phi}{f}}{(1+\cos\frac{\phi}{f})} < \frac{\sqrt{48\pi} f}{M_{pl}} \tag{2.9}$$

$$\sqrt{\frac{2|\cos\frac{\phi}{f}|}{(1+\cos\frac{\phi}{f})}} \quad < \quad \frac{\sqrt{48\pi}f}{M_{pl}} \tag{2.10}$$

This requires $f \geq \frac{M_{pl}}{\sqrt{48\pi}}$. The inflation ends when these conditions are violated. Deepending on f the end of inflation ϕ_2 will be set. As f increases ϕ_2 approaches to πf.

As we discussed earlier, any model must provide sufficient inflation. This is quantized in terms of e-folding. In this framework the number of e-folding is calculated to be:

$$N = \frac{8\pi f}{M_{pl}^2}\int_{\phi_1}^{\phi_2}\frac{(1+\cos\frac{\phi}{f})}{\sin\frac{\phi}{f}} \tag{2.11}$$

$$= \frac{8\pi f}{M_{pl}^2}\int_{\phi_1}^{\phi_2}\cot\frac{\phi}{2f} \tag{2.12}$$

$$= \frac{8\pi f}{M_{pl}^2}\log\left[\frac{\sin\frac{\phi_2}{2f}}{\sin\frac{\phi_1}{2f}}\right] \tag{2.13}$$

To explain the observations N must be ≥ 60.

Another constraint one needs to consider in the model is due to the precise measurement of CMB anisotropy, i.e., $P_\xi^{1/2}(k) \sim 10^{-5}$. Therefore it sets a limit on Λ.

Power is defined as: $P_\xi^{1/2}(k) = \frac{15}{2}\frac{\delta\rho}{\rho}$.

Where $\frac{\delta\rho}{\rho}$ is perturbation amplitude. Let us consider after 60 e folding the inflation ends and largest amplitude produced by $\phi^{max} << \pi f$ as the value of ϕ. In this model, power is calculated to be:

$$P_\xi^{1/2}(k) = \frac{\pi^2 f}{M_{pl}^3}\frac{9}{2\pi}(\frac{8\pi}{3})^{3/2}\frac{(1+\cos\frac{\phi^{max}}{f})^{3/2}}{\sin\frac{\phi^{max}}{f}} \tag{2.14}$$

This can be approximated to

$$P_\xi^{1/2}(k) \approx \frac{1.4\pi^2 f}{M_{pl}^3}(\frac{16\pi}{3})^{3/2}\frac{f}{\phi^{max}} \tag{2.15}$$

$\frac{\phi^{max}}{f}$ can be obtained from the limit on the number of e-folding. If $\Lambda \sim 10^{15}$ GeV (Grand unified theory scale) mass of field comes out to be $m_\phi \sim 10^{11} - 10^{13}$ GeV. The comoving length scale of the fluctuation k^{-1} crosses the radius $(Ha)^{-1}$ at the time of inflation. The field value is ϕ_k. The number of e-folds at the time can be computed from the Eq. (2.13). Todays horizon size is $3000h^{-1}Mpc$ therefore the comoving length scale can be written as:

$$k^{-1} \sim (3000h^{-1}Mpc)e^{N(k)-60}, \tag{2.16}$$

here $N(k)$ is the number of e-fold from ϕ_k to end of inflation. Perturbation amplitude at the Hubble radius crossing the length scale k^{-1} is given by:

$$(\frac{\delta\rho}{\rho})_k \approx P_\xi^{1/2}(k) \approx k^{-\frac{M_{pl}^2}{16\pi f^2}} \tag{2.17}$$

2.3 Results and Conclusions

To constraint a inflationary model from CMB anisotropy usually power spectrum is defined as power law in k as $|\delta k|^2 \sim k^{n_s}$, and the parameter n_s is given by:

$$n_s = 1 - \frac{M_{pl}^2}{8\pi f^2} \tag{2.18}$$

for $f \sim 1$ spectrum is scale invariant. However for $f = M_{pl}/\sqrt{8\pi}$, $ns = 0$. Another bounds due to density fluctuation can be given on amount of tensor fluctuation. To constraint usually defined a parameter r which account for tensor to scalar ratio and defined as:

$$r = \frac{P_T^{1/2}}{P_\xi^{1/2}} = 16\varepsilon \tag{2.19}$$

ε is slow roll paramter when $\phi = \phi^{max}$:

$$\varepsilon \approx \frac{1}{32\pi^2}\left(\frac{M_{pl}}{f}\right)^2\left(\frac{\phi^{max}}{f}\right)^2 \tag{2.20}$$

To test any inflatiory model one can plot the constraints on $[r,n]$ plane. The current value on r and n_s is given by [1]:

$$n_s = 0.9649 \pm 0.0042(\text{ TT, TE,EE+lowE+lensing}), \tag{2.21}$$

$$r < 0.068(\text{ TT, TE,EE}) \tag{2.22}$$

For the model, let us choose some benchmark for $f = 0.6, 0.7, 1, 2, 5$ of M_{pl}

TABLE 2.1: Spectral index in natural inflation model.

f	n_s
$0.4\ M_{pl}$	0.75132
$0.5\ M_{pl}$	0.840845
$0.6\ M_{pl}$	0.889476
$0.7\ M_{pl}$	0.918798
M_{pl}	0.960211

We have provided the spectral index n_s for various value of f in the Table 2.1. Precise measurement from Planck put constraint, i.e., $f < 0.7 M_{pl}$.

The value of tensor to scalar ratio r lowered to 0.068 that can constrain the interval number of e-fold as it sets the ϕ^{max}. Planck provides the constrained plots in $[r, n]$ plane. Here we have presented the resulting plot in 2.2 from Planck 2018 [1]. The plot from

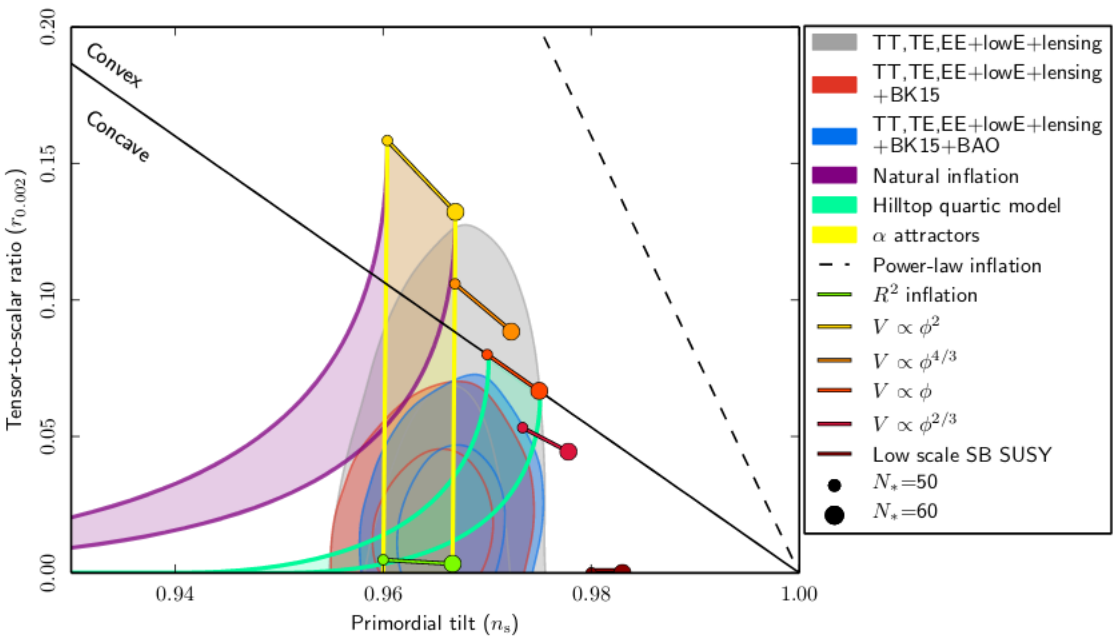

FIGURE 2.2: Observed limit on $(n_s - r)$ plane from Planck 2018 [1]. Here r is at $k = 0.0002\ Mpc^{-1}$

Planck (see Figure 2.2), show the region of allowed parameter space in n and r at at $k = 0.0002\ Mpc^{-1}$ plane. Colored regions are for different models, e.g., several polynomials of fields, Low scale SUSY, R^2 inflation, natural inflation etc. Legends are self explanatory.

Small circle corresponds to number of e-fold 50 and $n = 60$ represented by larger circle. Blue, red and gray region are corresponding to observation data from Planck. Violet colored region show the allowed parameter space for natural inflation. We can see still some of the region is allowed from current Planck data. However, future measurement seems to push it further and it might not remain allowed in its simplest form.

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