Quantum Fields in de Sitter Space

A thesis submitted towards partial fulfilment of
BS-MS Dual Degree Programme

by

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under the guidance of

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Declaration

I hereby declare that the matter embodied in the report entitled "Quantum Fields in de Sitter Space" are the results of the investigations carried out by me at the Department of Physics, Chennai Mathematical Institute, under the supervision of Dr. Alok Laddha, and the same has not been submitted elsewhere for any other degree.

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Certificate

This is to certify that this thesis entitled QUANTUM FIELDS IN DE SITTER SPACE submitted towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research Pune represents original research carried out by HIMANSHU BADHANI at CHENNAI MATHEMATICAL INSTITUTE, under the supervision of DR. ALOK LADDHA during the academic year 2015-2016.

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Abstract

In this thesis we analyze interacting massive scalar fields in de-Sitter space, specifically the issue of infrared divergences which cause the conventional in-out perturbation theory inapplicable. We then review the use of Schwinger-Keldysh formalism which is the most reliable tool to study non-equilibrium systems like QFT in de-Sitter spacetime. We review the previous work on correlation functions (specifically the propagators) the re-summation of propagators and use the technique to show that the propagators calculated have well defined flat-space limit. Next, we discuss the work done on the construction of an S-matrix on the global de sitter space and provide our own construction of an S-matrix in the expanding Poincaré patch. We show that our S-matrix shows good behavior under CPT operation, is unitary and has the expected flat space limit.

Contents

Chapter 1

Introduction

Free quantum field theories in de Sitter have been understood very well for quite some time now. Bruce Allen's work in 1986 [4] showed that unlike the case for Minkowski space, the vacuum state invariant under the de Sitter symmetry group $O(1,4)$ is not unique. Instead it is a (complex) one parameter family of vacuum state called α vacuua. The $\alpha = 0$ state is special in the sense that it coincides with the vacuum state of the Euclidean de-Sitter space, namely S^4. This state is commonly known as Bunch-Davies vacuum.
There have been studies ([8]) in the past which show that as the two point function of a free scalar field in any vacuum apart from the Euclidean vacuum has additional singularities, as a result of which interacting field theories in such states are ill defined. In light of this, we will restrict our attention simply to the Euclidean vacuum.
The interacting QFTs on de-Sitter space have been under intense scrutiny during last two decades due to their relevance to early universe cosmology. Conceptually these theories are far more intricate then their counterparts in Minkowski space due to the inflating geometry of de-Sitter space. As we will see, this causes the loop expansion which is an indispensable tool for QFTs in Minkowski spacetime to breakdown and one has to resort to non-perturbative techniques like Re-summation to make sense of quantum theory.
Another aspect of this breakdown in perturbation theory was nicely illustrated by Higuchi [9], who showed that due to the time dependence of the spacetime metric, there is a continuous particle creation during any quantum process as a result of which the in/out perturbation theory breaks down. Higuchi argued that if on the other hand one uses in-in perturbation theory as that used in non-equilibrium statistical mechanics [see [12]] then the quantum theory produces sensible results.
This failure of these perturbative methods is mainly because of the fact that the de Sitter space is not a conservative system. The global de Sitter metric is given as:

$$ds^2 = -dt^2 + \frac{Cosh^2(Ht)}{H} d\Omega^2_{D-1} \tag{1.1}$$

With $g_{\mu\nu}$ being time dependent, t is not a killing vector and hence Hamiltonian is time dependent. This shows that the de Sitter space is essentially a system in non-equilibrium state. As we will see, conventional Feynman contour that runs from $t = -\infty$ to $t = \infty$ cannot work in a non equilibrium system.

Various alternative techniques have been proposed to calculate propagators in de Sitter. They are broadly divided into two categories: (1) the closed time path (CTP) method as used in Schwinger Keldysch technique and (2) Euclidean continuation from S^D.

Euclidean continuation: The d-dimensional de Sitter space is a wick rotated d dimensional hypersphere. Euclidean continuation makes use of this fact and calculates the propagators in de Sitter space by analytical continuation of the propagators calculated in the hypersphere with respect to the Euclidean vacuum (see for example [15] and [16]). Since the hypersphere is compact, the propagators will naturally be free from any kind of IR divergence. Also, the propagators have good flat space limits which corresponds to the radius of the hypersphere going to infinity

Closed Time Path: This technique has been in use, especially in condensed matter physics as tool to study non-equilibrium systems. This method will be discussed in some detail in the next chapters.

Although there are arguments that these two approaches are equivalent in the sense that analytically continued propogators in Euclidean approach match (order by order in perturbation theory) with corresponding propagators obtained via in-in formalism (see for example [10]). However several puzzles remain. The Euclidean approach leads one to conclude that at any order in perturbation theory corelator computed via in-in formalism should be free of infrared divergences, which however is not the case. The approach we have taken in the thesis is to always work with the in-in formalism as we believe this is the most unambiguous method to study non-equillibrium systems.

In addition to the subtleties in the infra-red behavior of quantum fields, de Sitter spacetime QFT is challenging for another key reason. There is till date no satisfactory definition of S Matrix in de Sitter space. Lack of timelike killing field, spacelike nature of past and future infinity as well as the aforementioned IR divergences, make construction of S matrix technically as well as conceptually challenging. In a seminal work, Marolf et al proposed a definition of S Matrix for interacting QFT on global de Sitter space. Their S matrix satisfies a number of desirable properties like unitarity and right behaviour under CPT. As the in-in formalism that we have studied in this thesis is appropriate for studying QFT in EPP (or CPP) , as a natural application of the in-in formalism and Marolf Morrisson ideas, we construct an S matrix for interacting quantum field theory in the (expanding) Poincare Patch of de-Sitter space. This is our main result.

The outline of the thesis is as follows.

In Chapter 1 we study the geometry of de Sitter spacetime, the different nature of Klien Gordon fields in global and Poincaré patches and see how one

can define a de Sitter invariant vacuua for massive scalar fields.
In Chapter 2 the issue of IR divergences arriving from the in-out perturbation theory is discussed. These problems can be resolved by the use of Schwinger Keldysh formalism and as we show, the re-summed propagators do indeed go to flat space values in large mass limit. We use this observation in chapter 4 to formalize LSZ equivalent rules in EPP for a perturbative construction of S-matrix.

Chapter 2

Vacuum states in de Sitter

de Sitter spacetime is one of the maximally symmetric solution of the Einstein's equation with positive cosmological constant. A maximally symmetric spacetime is the one which has the maximum number of killing vectors, which, for a spacetime of dimension D is $D(D+1)/2$. In this chapter, we will discuss the geomtry of de Sitter spave and study the free scalar fields in de Sitter backgrond.

2.1 de Sitter Geometry

We will discuss only the global and static coordinates system. Other systems can be found in [14]. The de Sitter space S^D is defined by the following hyper-surface on the $D+1$ dimensional manifold:

$$-{X_o}^2 + {X_1}^2 + {X_2}^2 + ... + {X_D}^2 = H^{-2} \tag{2.1}$$

It looks like a hyperboloid, the circles representing the S^d sphere. [1]

The $d+1$ points on this sphere are parametrized as:

$$\begin{aligned} \omega^i &= Sin\theta_1 Sin\theta_2 .... Sin\theta_{i-1} Cos\theta_i \quad with \quad 1 \leq \theta_i \leq \pi \; and \; 1 \leq i \leq d \\ \omega^D &= Sin\theta_1 Sin\theta_2 .... Sin\theta_{d-1} Sin\theta_d \quad with \quad 1 \leq \theta_d \leq 2\pi \end{aligned} \tag{2.2}$$

The *global coordinates*(τ, θ_i) are given by:

$$\begin{aligned} X^0 &= \frac{Sinh(H\tau)}{H} \\ X^i &= \frac{\omega^i Cosh(H\tau)}{H} \end{aligned} \tag{2.3}$$

Then, the infinitesimal distance is given by:

$$\begin{aligned} ds^2 &= -dX_0^2 + \Sigma \; dX_i^2 \\ &= -d\tau^2 + \frac{Cosh^2(H\tau)}{H} d\Omega_d^2 \end{aligned} \tag{2.4}$$

[1] Throughout the thesis, unless otherwise stated, we will work on the de sitter space of dimension D and use the convention: $d = D - 1$.

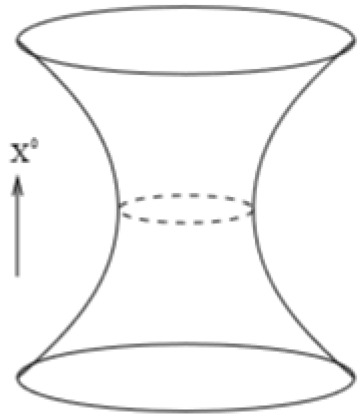

Figure 2.1: The de Sitter hyperboloid [image source [14]]

where $d\Omega_d^2$ is the metric on S^d with

$$d\Omega_d^2 = \sum (d\omega^i)^2 = \sum_{j=1}^{d} \left(\prod_{i=1}^{j-1} Sin^2\theta_i \right) d\theta_j^2 \tag{2.5}$$

The *conformal coordinates* (T, θ_i) are related to the global coordinates by:

$$Cosh(H\tau) = \frac{1}{CosT} \tag{2.6}$$

with $\pi/2 \leq T \leq \pi/2$. The metric takes the following form:

$$ds^2 = \frac{1}{H^2 Cos^2 T}(-dT^2 + d\Omega_d^2) \tag{2.7}$$

We define a new metric, $d\bar{s}^2 = (Cos^2 T) ds^2$, hence

$$d\bar{s}^2 = -dT^2 + d\Omega_d^2 \tag{2.8}$$

From this form of the metric it is apparent that the de Sitter dS_D space is conformal to a cylinder (product of R and d sphere).

The light rays are parallel to the diagonal, and hence for an observer in, say, north pole, only upper half of the diagonal is visible. A light message cannot be sent to all of the space, and light message from all of the universe cannot be received.

Poincaré Patches

The de Sitter hyperboloid in (2.1) can also be solved using the following constraints:

$$\begin{aligned} -X_o^2 + X_D^2 &= 1/H - x_i^2 e^{2H\tau} \\ X_1^2 + ... + X_{D-1}^2 &= e^{2H\tau} \end{aligned} \tag{2.9}$$

Then we parametrize the de Sitter space with (x_i, τ) and the resulting induced metric on the region $-X_o + X_D = -e^{H\tau}/H < 0$ is given as

$$ds_+^2 = -d\tau^2 + e^{2H\tau} d\mathbf{x}^2 \tag{2.10}$$

This region of space is referred to as the Expanding Poincare Patch or EPP. A similar metric can be defined in the region $-X_o + X_D = -e^{H\tau}/H > 0$ called the Contracting Poincaré Patch or CPP.

$$ds_-^2 = -d\tau^2 + e^{-2H\tau} d\mathbf{x}^2 \tag{2.11}$$

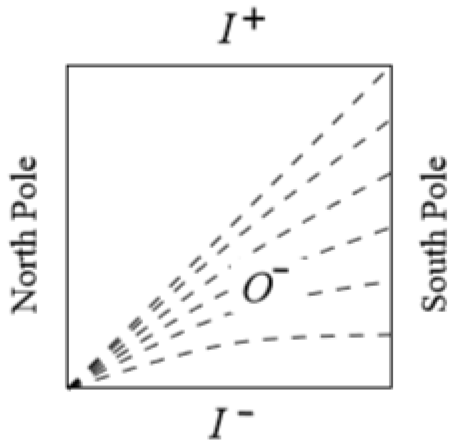

Figure 2.2: Conformal diagram of de Stter . The dashed lines show the constant τ surfaces [image source [14]]

One should note that the spatial part of the global de Sitter is the metric on S_{D-1}. While the Poincare Patches have the $D-1$ dimensional euclidean planes as the spatial metric. This have some interesting consequences for the quantum fields when studied in the global de Sitter and when studied in the Poincaré patches. As an example, the Klein Gordon equation for the scalar fields of mass m in global coordinates is given as:

$$\left[-\partial_t^2 + (D-2)tanh(t)\partial_t + \frac{\bigtriangledown_{D-1}(\Omega)}{cosh^2}(t) - m^2 \right] \phi(x) = 0 \tag{2.12}$$

Where $\bigtriangledown_{D-1}(\Omega)$ is the Laplacian in spherical coordinates, hence naturally the modes of the field ϕ are proportional to the $D-1$ dimensional spherical harmonics Y_l^m. Hence the fields in the global de Sitter have associated angular momentum index and are represented by $\phi_{l,m}$.

In the expanding Poincaré patch the Klein Gordon equation take the form

$$\big[-\eta^2\partial_\eta^2+(D-2)\eta\partial_\eta+\bigtriangledown_{(D-1)}-m^2\big]\phi(x)=0 \tag{2.13}$$

where $\bigtriangledown_{(D-1)}$ is the flat space Laplacian due to which the modes of the fields in EPP are spinless. The solution $\phi(x)$ is in terms of Hankel functions of first kind $H^{(1)}_{i\mu}$ with $i\mu=i\sqrt{m^2-(d/2)^2}$. Fields corresponding to real values of μ belongs belong to the "principle series" representation of the de Sitter group O(1,4). The fields with imaginary μ belong to the "complementary series".
The different nature of these modes in global and the Poincaré patch has important implications. One of these is the application of time reversal operation and behavior under CPT which we will analyze in the penultimate chapter on S-matrix in EPP.
We define a quantity $Z(x,y)$, called the hyperbolic distance, as:

$$Z(x,y)=H^2\eta_{ab}X^a(x)Y^b(y) \tag{2.14}$$

where $X^a(x)$ is the vector at at point x. Hence $Z(x,y)$ is the analog of the angular distance on a sphere. The distance between x and y is given by the following formula:

$$d(x,y)=H^{-1}cos^{-1}Z(x,y) \tag{2.15}$$

In the de Sitter space described by (2.1), the timelike geodesics will be described by the negative value of the first fundamental form (metric), $ds^2<0$. By the above given form of the metric (2.8), it is clear that the timelike distance should have imaginary length, and matching with (2.15) one concludes that $Z>1$ for timelike separated points.

2.2 Quantum Fields in global dS

A general de Sitter invariant state

Let ϕ_n be the modes in the Fourier expansion of the Euclidean vacuum state $|0>$, then such that

$$\phi_n(\overline{x})=\phi_n^*(x) \tag{2.16}$$

That such modes are always possible has been proved in the appendix A of [3]. One performs a trivial Bogoliubov transformation on the conventionally defined modes

$$\psi_{klm}(x)=y_k(t)Y_{klm}(\Omega) \tag{2.17}$$

the transformation being:

$$\phi_{klm}(x)=e^{i\pi/2k}[e^{i\pi/4}\psi_{klm}(x)+e^{-i\pi/4}\psi_{kl-m}(x)]/\sqrt{2} \tag{2.18}$$

A Bogoliubov transformation is a unitary transformation that preserves the canonical commutation/anti-commutation relation. Its general form is

$$\phi_n=\Sigma_m(\alpha_{nm}\psi_m+\beta_{nm}\psi_m^*) \tag{2.19}$$

If all β_{nm} are zero, the transformation is called "trivial" Bogoliubov transformation; such transformation represents equivalent vacuum state. (A proof is given in Birrell and Davis, Quantum Fields in Curved Space [5] page 46). Given these modes of the vacuum states, one does a Bogoliubov transformation to define a new set of modes by

$$\overline{\phi}_n = A\phi_n(x) + B\phi_n^*(x) \tag{2.20}$$

Now one defines the following inner product of two scalar functions in de Sitter:

$$(\phi_n, \phi_m) = i\int_\Sigma (\phi_m^* \nabla_\mu \phi_n - \phi_m^* \nabla_\mu \phi_n) d\Sigma^\mu = \delta_{mn} \tag{2.21}$$

Given this inner product, we see that

$$(\overline{\phi}_m, \overline{\phi}_n) = (|A|^2 - |B|^2)\delta_{mn} \tag{2.22}$$

This implies $|A|^2 - |B|^2 = 1$, and the transformation, up to an overall phase can be written as:

$$\overline{\phi}_n(x) = \phi(x)Cosh\alpha + e^{i\beta}\phi_n^*(x)sinh\alpha \tag{2.23}$$

This newly defined 2-parameter (α, β) family of vacuum state can be shown to be de Sitter invariant. The two symmetric and anti-symmetric 2-point functions in (α, β) are given by:

$$G^{(1)}_{\alpha,\beta} = <\alpha, \beta|\Phi(x)\Phi(y) + \Phi(y)\Phi(x)|\alpha, \beta> \tag{2.24}$$

$$iD_{\alpha,\beta} = <\alpha, \beta|\Phi(x)\Phi(y) - \Phi(y)\Phi(x)|\alpha, \beta> \tag{2.25}$$

It is easily shown that the 2-point function can be written as

$$G^{(1)}_{\alpha,\beta}(x, y) = G^{(1)}_0(Z)cosh\alpha + sinh2\alpha[G^{(1)}_0(-Z)cos\beta - D_0(\overline{x}, y)sin\beta] \tag{2.26}$$

Where $G^{(1)}_0$ is the $G^{(1)}_{\alpha,\beta}$ with $\alpha = 0$. As was shown in [3], $D_0(x, y)$ is not a function of Z, hence unless $\beta = 0$, $G^{(1)}_{\alpha,\beta}$ is not de-Sitter invariant. This provides us with a one parameter $(\alpha, 0)$ family of states, which are invariant under de Sitter symmetry group.

The Euclidean Vacuum

The nature of the measure of distance in dS, especially the fact that $d(\overline{x}, y) = d(x, y)$, allows us to leave the α unconstrained. But in minkowski spacetime, the symmetry group is the poincare group, and the invariant distance is translation invariant, and hence $G^{(1)}_0(\overline{x}, y)$ is not a function of the distance measure. To make the two point function invariant in flat spacetime, where the distances are translation invariant, one must impose $\alpha = 0$. Hence in the flat spacetime, the vacuum state is unique.

The $\alpha = 0$ vacuum is often called the *Euclidean Vacuum* (the terminology we will use henceforth) as it is also obtained from $SO(4)$ invariant vacuum state for a free Euclidean scalar field theory defined on S^4 (see the work of Marolf et al[15]). As the only singularity in this state is at $Z = 1$ which is when points are light-like separated, it has the same short distance singularity structure as Minkowski vacuua , a property we expect vacuua in any curved spacetime to satisfy (More commonly this property is called Hadamard property of a state.)

Green's functions

The hyper-geometric equation [see [19]] is given by

$$z(1-z)y'' + [c - (a+b+1)z]y' - aby = 0 \tag{2.27}$$

Solving it with Frobenious method, using the ansatz $y = \Sigma A_n z_n$ gives

$$f(Z) = y = A_0[1 + \frac{ab}{1!c}z + \frac{a(a+1)b(b+1)}{2!c(c+1)}z^2 + ...] = A_0\ {}_2F_1(a,b;c;z) \tag{2.28}$$

${}_2F_1(a,b;c;z)$ is called the hyper-geometric function. For $m^2 > 0$ it has a singularity at $Z = 1$. It is defined within the disk $|z| < 1$. By making a cut on $(1, \infty)$ the function can be analytically continued to the whole complex plane.

A symmetric two-point function is invariant under the complete de Sitter group if it depends only two points x and y via the geodesic distance $d(x, y)$. Hence

$$\begin{aligned} G_\lambda^{(1)} &= <\lambda|\{\phi(x), \phi(y)\}|\lambda> \\ &= F(d(x,y)) \\ &= F(Z) \end{aligned} \tag{2.29}$$

Hence $G_\lambda^{(1)}$ is a function of $Z(x, y)$. With the free theory action given as:

$$S[\phi] = \int \sqrt{|g|}[\partial_\alpha\phi\partial^\alpha\phi + m^2\phi^2]d^Dx \tag{2.30}$$

This two point function must satisfy the Klien Gordon equation, and for massive scalar fields

$$(\Box_x - m^2)G_\lambda^{(1)} = 0 \tag{2.31}$$

which in terms of Z can be written as follows [7]:

$$(Z^2 - 1)\frac{d^2}{dZ^2}F(Z) + 4Z\frac{d}{dZ}F(Z) + m^2H^{-2}F(Z) = 0 \tag{2.32}$$

This has the form of an hyper-geometric equation and whose solutions are hyper-geometric function ${}_2F_1(c, 3-c; 2; (1+Z)/2$ and ${}_2F_1(c, 3-c; 2; (1-Z)/2$ since the equation (2.32) is symmetric under $Z \to -Z$. Here c is such that $c(3-c) = m^2H^{-2}$.

Also since the hyper-geometric equation is invariant under $Z \to -Z$, both $f(Z)$ and $f(-Z)$ are the solutions. Hence a general solution is of the form:

$$F(Z) = af(Z) + bf(-Z) \tag{2.33}$$

This solution will have poles at $Z = 1$ (when x and y are connected by the null geodesics) and $Z = -1$ (when x and $\overline{y}$ are connected by the null geodesics). We define signed distance as

$$\overline{Z}(x,y) = \begin{cases} Z(x,y) + i\epsilon \text{ if } x \text{ is to the future of } y \\ Z(x,y) - i\epsilon \text{ if } y \text{ is to the future of } x \end{cases} \tag{2.34}$$

Motivating the $i\epsilon$ prescription from the prescription used in Minkowski space, the time ordered functions given by $G(Z + i\epsilon)$

Chapter 3

The IR divergences

Field theories in de Sitter are IR divergent, by this one means that the naive calculation two point functions diverge at low frequencies. As discussed in the previous section, the correlators are functions of $Z(x,y)$. In particular the feynman propagator for a conformally-coupled scalar field of mass m in $D=4$ is given by:

$$G_F(x,y) = \frac{\Gamma(a_+)\Gamma(a_-)}{16\pi^2} {}_2F_1(a_+, a_-; 2; Z(x,y) - i\epsilon) \tag{3.1}$$

Where

$$a_\pm = \frac{1}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 - m^2} \tag{3.2}$$

In the massless case with $d=3$, the propagator takes the following form:

$$G_F^o(x,y) = \frac{1}{(16\pi^2)(1 - Z_{xy} + i\epsilon)} \tag{3.3}$$

Now if we introduce a ϕ^2 perturbation with mass m[1]:

$$H_I(\lambda) = \frac{m^2}{2}\int \frac{d^3x}{(H\tau)^3} : \phi^2(x) : \tag{3.4}$$

Ideally, we should recover the propagator for mass m. However, if we use the usual perturbation theory, we find [Higuch and Lee [9]] that the correction to the lowest order diverges:

(3.5)

$$\begin{aligned} G_F^1(x,y) &= \int d^4x' \sqrt{-g(x')} G_F^o(x,x')(-im^2) G_F^o(x',y) \\ &= -\frac{im^2}{12\pi^2} \int \frac{d\tau_{x'} d^3x'}{\tau^4{}_{x'}} \frac{1}{(1 - Z_{xx'} + i\epsilon)(1 - Z_{x'y} + i\epsilon)} \end{aligned} \tag{3.6}$$

[1] In the remaining of the chapter we will use τ as the notation for the conformal time in the Expanding Poincare Patch. This is done in order to be able to refer the work of [11] more easily as we borrow their technique extensively

The propagator at the lowest order of correction diverges as λ^{-2}
Higuchi [9] shows that if you start with a massless field and perform a Bogolyubov transformation to the fields with mass m, the two point function diverges due to the divergence of $|\beta(\eta)|^2$ as $\eta \to 0$. For a Bogolyubov transformation from states u_1 to u_2, value of $|\beta|^2$ tells us the number of particles that the vacuum of u_2 mode contains in u_1 mode [see [5] for proof of this statement]. the divergence of $|\beta(\eta)|^2$ term suggest an uncontrolled particle creation due to a small perturbation.

Marolf et al. [15] explain this divergence as coming from the expansion of the de Sitter spacetime. The measure of the integral in the corrections contributes a factor of $\tau^{(D-1)}$. As we will see, the propagators have a fall of term of the type $\tau^{-(D-1)/2}$. Thus the integral diverges as $\int d\tau \sim \tau^{(D-1)/2}$

These divergences are generally seen as the failure of the conventional techniques when applied to de Sitter space. We will discuss the reasons these techniques fail and then outline the ramifications that have been proposed in the literature.

3.1 Reasons for the failure of in-out theory

Consider a two-point time ordered green's function in an interacting field theory with the interacting vacuum Ω

$$iG(x,t;x',t') =< \Omega|T[\phi(x,t)\phi(x',t')]|\Omega > \tag{3.7}$$

$S(t,t')$ be the evolution operator (or the S-matrix) with respect to the perturbed Hamiltonian $H = H_0 + H'(t)$, then the time ordered Green's function (with the assumption of adiabatic switching on of the interaction) is given by:

$$iG(x,t;x',t') = \frac{< 0|T[S(-\infty,\infty)\hat{\phi}(x,t)\hat{\phi}(x',t')]|0 >}{< 0|S(-\infty,\infty)|0 >} \tag{3.8}$$

Where $|0>$ is the free theory vacuum and $\hat{\phi}$ represents the field operator in interaction picture. The operators in the Heisenberg picture are related to their interaction picture counterparts by:

$$\phi(x,t) = S(0,t)\hat{\phi}(x,t)S(t,0) \tag{3.9}$$

The S-matrix $S(-\infty,\infty)$ is explicitly given as:

$$S(-\infty,\infty) = Texp[-i\int_{-\infty}^{\infty} dt_1 \hat{H}'(t_1)] \tag{3.10}$$

$|\Omega>$ is the interacting vacuum state evolved from free vacuum in the asymptotic past $|O(-\infty)>$ or from the free vacuum in the asymptotic future

$|O(\infty)>$. Feynman-Dyson expression in equation 3.8 holds when the free vacuum sates in the asymptotic past and future differ only by a phase factor. This is true when the interactions are turned on adiabatically. As per the adiabatic theorem, both the states will be the eigenstates of the Hamiltonian H.
Now consider a non-equilibrium Hamiltonian $H(t) = H + H'(t)$, with $H'(t)$ being the non-equilibrium perturbation. Out of equilibrium, the adiabatic theorem cannot be used because it involves non adiabatic evolution of Hamiltonian, and the asymptotic initial and final states do not belong to the same Hilbert state [see [13]]. This is the reason the usual perturbation theory fails when used in de Sitter field theory.

Schwinger Keldysh formalism

The idea of a Schwinger-Keyldysh contour is to avoid any reference to the initial and final state. Let $\hat{\phi}(\infty)$ be a field in the interacting picture at $t = \infty$. To avoid any reference to the final state in the asymptotic future, the trick is to simply rewind the time evolution back to the asymptotic past. [13]

$$\hat{\phi}(\infty) = S(\infty, -\infty)\hat{\phi}(-\infty) \tag{3.11}$$

The time ordered function is then given by

$$iG(x,t;x',t') =< \hat{\phi}(-\infty)|S(-\infty,\infty)T[S(\infty,-\infty)\hat{\psi}(x,t)\hat{\psi}^{\dagger}(x',t')]|\hat{\phi}(-\infty)> \tag{3.12}$$

with the Schwinger-Keldysh contour, enforced by a contour-ordering operator T_c, the time ordered two-point function has the form

$$iG(x,t;x',t') =< \hat{\phi}(-\infty)|T_c[S_c(-\infty,-\infty)\hat{\psi}(x,t)\hat{\psi}^{\dagger}(x',t')]|\hat{\phi}(-\infty)> \tag{3.13}$$

With the S-matrix, S_c is given by

$$S_c(-\infty,-\infty) = T_c \exp[-i\oint_C dt'\hat{H}(t')] \tag{3.14}$$

This is how S-matrix can be defined in a global de Sitter space, as is given in the work of Marolf et el. [17] and this is exactly how we will define our EPP S-matrix in chapter 4, but the in states will be defined at the boundary of the Poincaré Patch and the closed contour will traverse between $\tau = -\infty$ to $\tau = \infty$ in Poincaré coordinates of de Sitter.

3.2 Perturbative quantum field theory in Expanding Poincare Patch

In this chapter we continue with our analysis of interacting scalar field theories and use Schwinger Keldysch technique to derive perturbative corrections to the Propagator. Keeping in mind, our final goal of defining S-matrix in EPP, we will restrict our attention to EPP instead of global De-Sitter spacetime.

We first review the work of Jatkar et al [11] in which they derive 1-loop corrections to the propogator for cubic interactions. One consequence of this result is the explicit breakdown of perturbation theory at late times. Jatkar et al show that inspite of this breakdown, one can resum the perturbation series and derive the full quantum two point function for ϕ^3 theory. This is quite a remarkable result and among other thing it shows the decay of massive fields in de-Sitter space. A result which is consistent with the fact that in the absence of time-like killing symmetry and associated conservation of energy, massive particles in de-Sitter can decay to themselves.

Keyldysh Rotation

According to the Closed Time Path or CTP formalism [18], where an operator $\hat{Y}$ evolves as:

$$\hat{Y} =< 0, in|\hat{T}[exp(i\int_{t_{in}}^{t} H(t')dt')]\hat{Y}T[exp(-i\int_{t_{in}}^{t} H(t')dt')]|0, in > \qquad (3.15)$$

The field $\phi(x)$ is split into two parts, ϕ^+ and ϕ^- along the future and past contour. Past and future is with respect to an initial Cauchy surface, which in our case is τ=0 surface.
The Lagrangian density $L[\phi]$ breaks as $L[\phi^+]$-$L[\phi^-]$.We start with a Lagrangian density of a massive scalar field with cubic interaction:

$$\mathcal{L}[\phi_+, \phi_-] = \sqrt{-g}(-\partial_\mu\phi_+\partial^\mu\phi_+ + \partial_\mu\phi_-\partial^\mu\phi_- - m^2(\phi_+^2 + \phi_-^2) - \frac{\lambda}{3!}[\phi_+^3 - \phi_-^3] \qquad (3.16)$$

Now, the Keyldysh rotation involves the following change of basis to the fields $\phi^{(1)}$ and $\phi^{(2)}$:

$$\phi^{(1)} = \frac{\phi^+ + \phi^-}{2}, \quad \phi^{(2)} = \phi^+ - \phi^- \qquad (3.17)$$

This change in basis changes the Lagrangian density to

$$\mathcal{L}[\phi^{(1)}, \phi^{(2)}] = \sqrt{-g}(-\partial_\mu\phi^{(1)}\partial^\mu\phi^{(2)}) - m^2\phi^{(1)}\phi^{(2)} - \frac{\lambda}{3!}[3(\phi^{(1)})^2\phi^{(2)} + \frac{(\phi^{(2)})^3}{4}] \qquad (3.18)$$

This suggests the two kind of ϕ^3 vertices possible.

the retarded green's function

$$G^R(x, y) = i\theta(x^o - y^o)[< 0|\phi(x)\phi(y)|0 > - < 0|\phi(y)\phi(x)|0 >] \qquad (3.19)$$

In the $\phi^{(1)}$ and $\phi^{(2)}$ basis the Green's function is:

$$G^R(x, y) =< 0|T\phi^{(1)}(x)\phi^{(2)}(y)|0 > \qquad (3.20)$$

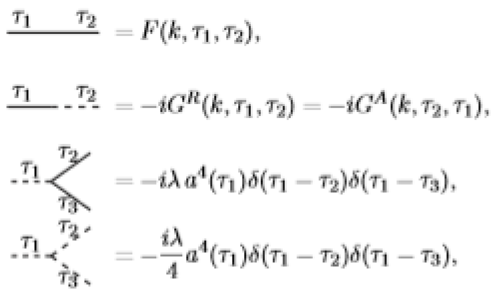

Figure 3.1: Pictorial representation of the propagators in new basis with the associated Feynman rules [source [18]]

represented by one bold-dashed line. Similarly the symmetric two point function

$$F(x,y) = \frac{i}{2} < 0|\phi(x)\phi(y)|0 > + < 0|\phi(y)\phi(x)|0 > \tag{3.21}$$

is given by

$$F(x,y) =< 0|T\phi^{(1)}(x)\phi^{(1)}(y)|0 > \tag{3.22}$$

represented by a bold line.

So what we see is that the Keldysh rotation has turned the retarded green's function and the Keldysh propagator into time-ordered two point functions. Effectively these are the new feynman propagators with respect to the contour-ordering operators T_c. One might expect that these "new Feynman functions" can be calculated perturbatively just like the usual Feynman propagators in flat space. As we will see in the coming sections, this is indeed the case. We will first review the work done on the use of this technique to calculate the decay rate of massive particles. Then we calculate the corrections to the mass of a field in ϕ^2 theory and show that the resulting re-summed propagator has well defined flat space limits.

Decay of massive fields: Loop corrections in ϕ^3 theory

Jatkar et al [11] re-sum loop corrections to the retarded green's function for massive fields in dS and find that there is an imaginary shift in the mass of the filed. This shift, for fields with , can be interpreted as the evidence that massive fields in dS can decay to themselves. While the particle interpretation in dS allows such kind of decay in the principal series, the ligh fields however are forbidden from this kind of decay kinematically. The Keldysh Propagator is given as:

$$\begin{aligned} F(k,\tau_1,\tau_2) &= \frac{1}{2}(\tau_1\tau_2)^{d/2} Re(h_\mu(-k\tau_1)h^*_\mu(-k\tau_2)) \\ &= (\tau_1\tau_2)^{d/2} f(k\tau_1, k\tau_2) \end{aligned} \tag{3.23}$$

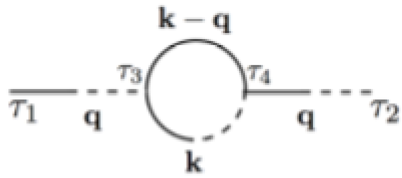

Figure 3.2: First order loop corrections to G^R. [image source [11]]

similarly the retarded Green's function is given as:

$$\begin{aligned} G^R(k,\tau_1,\tau_2) &= -\theta(\tau_1-\tau_2)(\tau_1\tau_2)^{d/2} Im(h_\mu(-k\tau_1)h_\mu^*(-k\tau_2)) \\ &= \theta(\tau_1-\tau_2)(\tau_1\tau_2)^{d/2} g^R(k\tau_1,k\tau_2) \end{aligned} \tag{3.24}$$

The first order loop correction[11] to the retarded Green's function

$$\begin{aligned} G^{R1}(q,\tau_1,\tau_2) = \lambda^2 \int \frac{d^3k}{(2\pi)^3} \int d\tau_3 d\tau_4 \frac{1}{(\tau_3\tau_4)^4} G^{R0}(q,\tau_1,\tau_3) \\ \times G^{R0}(k,\tau_3,\tau_4) F^0(k-q,\tau_3,\tau_4) G^{R0}(q,\tau_4,\tau_2) \end{aligned} \tag{3.25}$$

with $x_{3,4} = k\tau_{3,4}$ under the condition $k >> q$

$$\begin{aligned} g^{R1}(q,\tau_1,\tau_2) = \lambda^2 \int_q^\infty \frac{d^3k}{(2\pi)^3 k^d} \int_{k\tau_2}^{k\tau_1} dx_3 \int_{k\tau_2}^{x_3} dx_4 (x_3x_4)^{1/2} g^{R0}(q\tau_1, \frac{q}{k}x_3) \\ \times g^{R0}(x_3,x_4) f^0(x_3,x_4) g^{R0}(\frac{q}{k}x_4, q\tau_2) \end{aligned} \tag{3.26}$$

In the infrared limit, i.e. $-k\tau_{1,2}$, the integral gives the following result:

$$g^{R1}(q,\tau_1,\tau_2) = g^{R0}\sigma ln\frac{\tau_2}{\tau_1} \tag{3.27}$$

with

$$\sigma = \frac{i\lambda^2 S_{d-1}}{(4\pi)^d 2\mu} \int_{-\infty}^0 dx_3 \int_{-\infty}^0 dx_4 x_3^{-i\mu} x_4^{i\mu} (x_3x_4)^{1/2} g^{R0}(x_3,x_4) f^0(x_3,x_4) \tag{3.28}$$

The logarithmic term is the above equation (3.27) depends on the ratio τ_1/τ_2. In the late time limit, i.e. $\tau_1 >> \tau_2$, the first order term seems to diverge. Hence, even in this formalism we have encountered divergences. However as will be shown here, these terms can be summed up and the re-summed propagator is free of such a divergence.

For n^{th} order loop correction can similarly be solved to give:

$$g^{Rn}(q,\tau_1,\tau_2) = g^{R0}\sigma^n \frac{(ln\frac{\tau_2}{\tau_1})^n}{n!} \tag{3.29}$$

Hence the re-summed propagator becomes

$$\begin{aligned} g^R(q,\tau_1,\tau_2) &= \sum_n g^{Rn} \\ &= g^{R0} exp[-\sigma ln\frac{\tau_1}{\tau_2}] \\ &= \frac{i}{2\mu}\left(\left(\frac{\tau_1}{\tau_2}\right)^{i\mu-\sigma} - \left(\frac{\tau_1}{\tau_2}\right)^{-i\mu-\sigma}\right) \end{aligned} \tag{3.30}$$

Using equation 3.24, the re-summed propagator can be written terms of the Hankel function $H^{(1)}_{i\mu}(x)$ as

$$G^R(k,\lambda_1,\lambda_2) = \theta(\lambda_x - \lambda_y)\frac{\pi e^{-\pi\mu}}{2} Im\big(H^{(1)}_{i\mu-\sigma}(x) H^{(1)}_{(i\mu-\sigma)^*}(x)\big) \tag{3.31}$$

For $\mu \approx m \gg d/2$, this corresponds to an imaginary shift in mass which leads to a Breit-Wigner resonance with decay rate $\Gamma = -2\sigma$. A similar shift in mass has also been calculated by Marolf and Morrison[15] using Euclidean continuation methods.

3.3 Loop corrections to G^R in ϕ^2 theory

Higuchi [9] shows that the $m^2\phi^2$ correction to the Lagrangian density of a massive scalar field gives divergent results even at the lowest order of perturbation. This corresponds to the failure of the in-out perturbation theory.We calculate the retarded green's function using in-in perturbation theory and show that the results are indeed non-divergent and the propagators approach the flat space limit at $m >> d/2$.
the pertirbative Hamiltonian be given as:

$$H_I(\tau) = \frac{m^2}{1}\int \frac{d^3x}{(H\tau)^3} : \phi^2(x) : \tag{3.32}$$

In the limit $-k\tau_{1,2} << 1$, the retarded propagator is given (see [11]) by

$$g^R = \frac{i}{2\mu}\left(\left(\frac{\tau_1}{\tau_2}\right)^{i\mu} - \left(\frac{\tau_1}{\tau_2}\right)^{-i\mu}\right) \tag{3.33}$$

We can also represent g^R in terms of trigonometric function, which will prove useful later on,

$$g^R = -\frac{1}{\mu} Sin\left(\mu ln(\frac{\tau_1}{\tau_2})\right) \tag{3.34}$$

Similarly

$$f^0_{12} \equiv \left(\frac{\tau_1}{\tau_2}\right)^{i\mu} + \left(\frac{\tau_2}{\tau_1}\right)^{i\mu} = 2Cos\left(\mu ln(\frac{\tau_1}{\tau_2})\right) \tag{3.35}$$

With leading terms in $ln\left(\frac{\tau_2}{\tau_1}\right)$, the correction terms are given as [see appendix A.1 for the detailed calculations]

$$g_{12}^{R1} = \left(\frac{-1}{4\mu^2}\right) m^2 ln\left(\frac{\tau_1}{\tau_2}\right) f_{12}^0 \tag{3.36}$$

$$g_{12}^{R2} = \left(\frac{-1}{4\mu^2}\right) m^4 \frac{1}{2!}\left(ln\left(\frac{\tau_1}{\tau_2}\right)\right)^2 g_{12}^{R0} \tag{3.37}$$

$$g_{12}^{R3} = \left(\frac{-1}{4\mu^2}\right)^2 m^6 \frac{1}{3!}\left(ln\left(\frac{\tau_1}{\tau_2}\right)\right)^3 f_{12}^0 \tag{3.38}$$

$$g_{12}^{R4} = \left(\frac{-1}{4\mu^2}\right)^2 m^8 \frac{1}{4!}\left(ln\left(\frac{\tau_1}{\tau_2}\right)\right)^4 g_{12}^{R0} \tag{3.39}$$

the re-summed retarded propagator g^R is then given by

$$\begin{aligned} g^R &= g^{R0} + g^{R1} + g^{R3} + g^{R4} + ... \\ &= g^{R0} + g^{R2} + g^{R4} + ... + g^{R1} + g^{R3} + g^{R5} + ... \end{aligned} \tag{3.40}$$

We team the even and odd order of perturbations here

$$\begin{aligned} g^R = g^{R0}&\left[1 - \left(\frac{m^2}{2\mu}\right)^2 \frac{1}{2!}\left(ln\left(\frac{\tau_1}{\tau_2}\right)\right)^2 + \left(\frac{m^2}{2\mu}\right)^4 \frac{1}{4!}\left(ln\left(\frac{\tau_1}{\tau_2}\right)\right)^4 - ....\right] \\ - \frac{f^0}{2\mu}&\left[\frac{m^2}{2\mu} ln\left(\frac{\tau_1}{\tau_2}\right) - \left(\frac{m^2}{2\mu}\right)^3 \frac{1}{3!}\left(ln\left(\frac{\tau_1}{\tau_2}\right)\right)^3 + \left(\frac{m^2}{2\mu}\right)^5 \frac{1}{3!}\left(ln\left(\frac{\tau_1}{\tau_2}\right)\right)^5\right] \end{aligned} \tag{3.41}$$

With appropriately identifying the terms in the brackets, we arrive at the following

$$\begin{aligned} g_{12}^R(\mu) &= g^{R0} Cos\left(\frac{m^2}{2\mu} ln\left(\frac{\tau_1}{\tau_2}\right)\right) - \frac{f^0}{2\mu} Sin\left(\frac{m^2}{2\mu} ln\left(\frac{\tau_1}{\tau_2}\right)\right) \\ &= -\frac{1}{\mu} Sin\left(\mu ln(\frac{\tau_1}{\tau})\right) Cos\left(\frac{m^2}{2\mu} ln\left(\frac{\tau_1}{\tau_2}\right)\right) - \frac{1}{\mu} Cos\left(\mu ln(\frac{\tau_1}{\tau})\right) Sin\left(\frac{m^2}{2\mu} ln\left(\frac{\tau_1}{\tau_2}\right)\right) \\ &= -\frac{Sin\left((\mu + \frac{m^2}{2\mu}) ln(\frac{\tau_1}{\tau})\right)}{\mu} \\ &= \left(1 + \frac{m^2}{2\mu^2}\right) g_{12}^{R0}(\mu + \frac{m^2}{2\mu}) \end{aligned} \tag{3.42}$$

With the large mass and small perturbation assumption, we have the following relation

$$g_{12}^{R}(\mu) = g_{12}^{R0}(\mu + \frac{m^2}{2\mu}) \tag{3.43}$$

Now we try to determine the shift in mass M due to the perturbation

$$\begin{aligned} \mu \to \mu + \frac{m^2}{2\mu} &= \frac{2\mu^2 + m^2}{2\mu} \\ &= \frac{2M^2 + m^2 - d^2/2}{2\sqrt{M^2 - d^2/4}} \\ &\approx \sqrt{M^2 + m^2 - d^2/4} \end{aligned} \tag{3.44}$$

thus we see the following shift for the masses with $M^2 - (d/2)^2 >> m^2$

$$M^2 \to M^2 + m^2 \tag{3.45}$$

Hence this calculation is an important check that the re-summed propagator approaches the flat space limit for masses $M >> d/2$. Adding a $m^2\phi^2$ to massive field in flat space interacting qft gives the corrected mass as $M^2 + m^2$. Hence it can be deduced that heavy masses do not experience the curvature of the de Sitter space. Here we have taken the Hubble constant to be 1. With respect to the Hubble constant H, the condition for flat space limit is $M >> dH/2$. One can then infer that due to the small wavelength associated with the large masses,they do not experience the curvature of the background space.
Same would be true when we deal with the states at the boundaries of EPP and CPP in the next chapter. Due to the factor of $e^{2H\tau}$ in the spatial part of the metric, the states experience infinite blueshift in he past of the EPP and hence due to the small wavelength, not experience the effect of background curvature [similar arguments have been made in [2]].

Chapter 4

S matrix in the expanding Poincaré patch (EPP)

4.1 S-matrix on the global de Sitter

S-matrix is one of the most powerful tool in quantum field theory, both from the point of view of experimentatal physics and also as a theoretical tool because of its properties like invariance under field redefinition, covariance etc. Hence an important part of understanding quantum fields in de Sitter would be a construction of an S-matrix like quantity. In the global de Sitter space such an attempt has been made recently by Marolf et al. [17]. They have proposed a construction of an S-matrix with, using Schwinger-Keyldysh formalism. Marolf et al. define a procedure for the construction of LSZ equivalent in global de Sitter and they show that the resulting s-matrix satisfies the basic properties that one expects of an S-matrix.

4.2 Motivation for an S-matrix on EPP

In this section we define a S-matrix for quantum field theory on EPP. Our work is an extension of the work done by Marolf, Morrison and Srednicki on S-matrix in global De-Sitter spacetime. However as the (resummed) correlators in in-in perturbation theory which are IR finite are derived in EPP, it is a natural question to ask if there exists a S-matrix formulation of QFT in expanding Poincare Patch whose amplitudes (via LSZ type formulation) are related to the in-in correlators. Conceptually this would put such a QFT on a firmer footing exactly as in the case of Minkowski spacetime.

An argument against any S-matrix in the global de Sitter (articulated nicely in [17]) is that no observer in the global de Sitter has access to a complete set of ingoing and outgoing states as the past and future boundaries are spacelike and hence acausal. Thus no single observer has access to all the information contained in the S-matrix rendering it an uninteresting object to study phenomenologically. We have nothing to add to this interesting conceptual question in the thesis. However as emphasized above, we do believe that having a definition of S-matrix which is consistent with the in-in perturbation theory and has other essential properties like unitarity and correct flat space limit, in turn vindicates use of Schwinger Keldysch techniques for studying de-Sitter QFTs.

One of the key subtleties underlying definition of S-matrix in de-Sitter space (or one of it's subspaces like EPP or CPP) is that a definition of S-matrix relies on the notion of asymptotic states which are defined at past and future infinites as well as the definition of an S-matrix operator which maps the (asymptotic) in states to out states. Even though one can (as we review below) define asymptotic states for (global or otherwise) de-Sitter spacetime, the earlier calculation by Higuchi reveals that one can not appeal to Gell-Mann Low type construction to define Scattering amplitudes. Thus on one hand we want to construct scattering amplitudes between in and out states and on the other hand we do not have recourse to in/out perturbation theory. We will show below how this dichotomy between different structures is resolved by use of Schwinger Keyldish technique on one hand and appropriate construction of asymptotic states on the other. Thus in a nut-shell we want to define S-matrix using Schwinger-Keyldish formalism as discussed in section 3.1.

4.3 Initial and final states

Our initial state is situated at the conformal boundary ($\tau = \infty$) of the Expanding Poincaré Patch defined by the following metric

$$ds_+^2 = -d\tau^2 + e^{2H\tau}d\mathbf{x}^2 \tag{4.1}$$

With $-\infty < \tau < \infty$, the EPP is a space-time in itself. For this reason one expects that a well defined S-matrix can exist in EPP even though the in-states are not in the asymptotic past. In the above metric, the constant τ surfaces are$D-1$ dimensional Euclidean planes, and $\tau = -\infty$ where we define our initial state is a null surface.
As discussed in section 2.1, the Klien Gordon equation in EPP in terms of the conformal time η is given by:

$$\big[-\eta^2\partial^2 + (D-2)\eta\partial_\eta + \bigtriangledown_{(D-1)} - m^2\big]\phi(x) = 0 \tag{4.2}$$

hence $\bigtriangledown_{(D-1)}$ is the flat space Laplacian. the solution of this equation are the modes $u_{m,k}(x)$ given by

$$u_k(x) = \frac{\sqrt{\pi}}{2}\eta^{3/2}H^{(1)}_{i\mu}(k\eta)e^{ik.x} \tag{4.3}$$

with $H^{(1)}_\nu(k\lambda)$ being Hankel functions of order $i\mu$ where

$$\mu = \sqrt{m^2 - \left(\frac{d}{2}\right)^2} \tag{4.4}$$

A general state $|n>$ in the fock space, characterized by mass parameter μ and linear momentum k, is given by the action of a raising operator on the interacting vacuum:

$$|n>= a^{\dagger}{}_{n}(\eta)|\Omega> \tag{4.5}$$

Where with the $\eta = e^{-\tau}$ is the conformal time in the EPP coordinates. The raising operator given by:

$$a^{\dagger}{}_{m}(\eta) := -i \int d\Sigma^{\nu}(x)[\phi_m \overleftrightarrow{\nabla}_{\nu} u^{*}_{m}(x)] \tag{4.6}$$

Here the integration is on a constant time Cauchy surface. Since the Klein Gordon product in the EPP doesnot diverge (at least for the principal series), we can use the analogy from the free theory to define the particle state in this fashion. We take this analogy a bit further and hope that the initial/final states can be given as:

$$|n_1, n_2...., n_k >_{i/f} := \lim_{\eta \to +\infty/0} a^{\dagger}{}_{n_1}(\eta) a^{\dagger}{}_{n_2}(\eta).......a^{\dagger}{}_{n_k}(\eta)|\Omega> \tag{4.7}$$

$|n_1>$ represents particle with mass μ_1 and linear momentum k_1. This definition should hold exactly for the initial states, with $a^{\dagger}(\eta \to \infty)$ being the raising operator in the free field.

Covariance of these states

The states transform as metric tensor products under the action of the de Sitter group. Let U(g) be an element of SO(4,1), then a state transforms as:

$$U(g)|n_1, n_2...., n_k >_{i/f} = |gn_1, gn_2...., gn_k >_{i/f} \tag{4.8}$$

Where the action of $U(g)$ on $|n_i>$ is given as

$$g|n_i(k_i, \mu_i) >= |n_i(gk_i, \mu_i)> \tag{4.9}$$

This corresponds to the covariant transformation of states under de Sitter group. we have shown in the appendix 2 that CPT action also acts covariantly on these states.
Since the initial states do not experience the background curvature, we expect that the initial orthogonal states will have particle interpretation. Additionally, they can be expected to be tensor product of one particle states, just as the case is in the flat space. This would mean that the initial states $|n>_i$ are all mutually orthogonal. The distinct final states however do not have vanishing overlaps, and inner products like ${}_f< n'|n>_f$ contribute in the order by order calculation of S-matrix.

Next, we need to show the S-matrix behaves properly under CPT, is unitary, transforms covariantly and have a flat space limit.

CPT operation in the Poincaré Patch

In the global de Sitter QFT, let the modes be $|p, l_z, n>$. Let Θ represent a CPT operation on a particle $|p, l_z, n>$ where p is the momentum, l_z is the z-projection of the net angular momentum L and n is the quantum state of the particle and n^c denotes the antiparticle state.

$$\Theta|p, l_z, n >= (-1)^{L-l_z}|p, -l_z, n^c > \tag{4.10}$$

For the modes in global de Sitter, this action is the same as the antipodal map

$$u_{\mu L}(Ax) = (-1)^{L-l_z} u^*_{\mu L}(x) \tag{4.11}$$

Antipodal map maps modes across the patches, a map that is not allowed in our current domain of study.In EPP, the modes are spinless, given in terms of the Hankel functions $H^{(1)}_{i\mu}(k\lambda)$ as given in equation 4.3 we have shown in the appendix, theese modes are neither time reversal invariant nor parity invariant, but they are the eigenstates of the PT (or CPT) operator Θ with eigenvalue -1.

$$\Theta u_k(x) = -u_k(x) \tag{4.12}$$

The behavior of S-matrix under CPT is given as [see [6]] is readily satisfied:

$$S = \Theta^{-1} S^\dagger \Theta \tag{4.13}$$

4.4 LSZ formulation for EPP

The flatness at the cosmological horizon accounts for the fact that the initial states can be made orthogonal while retaining their particle interpretation, and hence they should not receive any $_i < n|n >_i$ corrections. This fact alone suggests that there should be no vertex connected directly to initial bra $< i|$. Hence the diagrammatic rules in EPP are not simple extension of the global dS.

Generating function

$J_+(x)$ and $J_-(x)$ be the sources on the forward and backward contours. The generating function with respect to the initial state on the cosmological horizon is given by

$$Z[J_+, J_-] = \sum_{out} < in|out >_{J_+} < out|in >_{J_-} \tag{4.14}$$

This can be rewritten as:

$$Z[J_+, J_-] = Z_0[J_+, J_-] \int [D\phi'_+][D\phi'_-] exp[iS_{int}(J(\phi'_+) - iS_{int}(J(\phi'_-)] \tag{4.15}$$

Where

$$Z_0[J_+,J_-] = exp\int_x\int_y[-\frac{1}{2}J_+(x)G(x,y)J_+(y)\theta_+(x)\theta_+(y)$$
$$-\frac{1}{2}J_-(x)G^*(x,y)J_-(y)\theta_-(x)\theta_-(y)$$
$$+J_+(x)W(x,y)J_-(y)\theta_+(x)\theta_-(y)] \quad (4.16)$$

A time ordered correlation function with respect to the Euclidean vacuum $|\Omega>$ can be calculated from this generating as follows:

$$<\Omega|T\phi(x_i)|\Omega>= \frac{\delta}{i\delta J_+(x_i)}Z[J_+,J_-]\Big|_{J_+=J_-=0,\phi_+=\phi_-=\phi} \quad (4.17)$$

Diagrammatic rules for scattering amplitudes

We hope that there exits an LSZ like formulation of S-matrix even for de Sitter space so that the scattering amplitudes of the form $_f<n_1|n_2>_f>$, $_i<n_1|n_2>_i$ or $_f<n_1|n_2>_i$ can be expressed in term of the two point correlators. We provide a set of diagrammatic rules to construct such a S-matrix in terms of two-point functions based on some educated guesses. To obtain the amplitudes, draw all possible Feynman diagrams of a particular order and then follow the these given diagrammatic rules:

1. A vertex connected directly to the final bra $<f|$ lies on the J_+ contour. Replace the Green's function connecting to the final bra with $u^*(x)$, x being the four vector integrated over the space-time.

2. A vertex connected directly to the final ket $|f>$ lies on the J_- contour. Replace the Green's function connecting to the final ket with $u(x)$, x being the four vector integrated over the space-time.

3. A vertex connected directly to the initial ket $|i>$ lies on the J_+ and J_- contours. Replace the Green's function connecting to the initial ket with $u(x)$, x being the four vector integrated over the space-time.

4. No vertex is directly connected to the initial bra $<i|$

5. A vertex not directly connected to the initial or final states can lie on any of the two contours.

6. Include the multiplicative factor ig for the vertex on the J_+ branch and $-ig$ for vertex on the J_- branch.

7. The use of Green's functions between any two vertices sitting on the two contours is as follows (see the generating function in equation (4.16)):

   (a) If both the vertices are situated on J_+ contour, use G(x,y).
   (b) If both the vertices are situated on J_+ contour, use G*(x,y).
   (c) If x vertex is on J_+ and y on J_- contour, use W(x,y)

4.5 $\mathcal{O}(g^2)$ corrections to $\phi_1 \to \phi_1$ amplitudes in EPP

We work with Lagrangian for a massive scalar field in ϕ^3 interaction:

$$\mathcal{L}[\phi_i] = \sum_i \partial_\mu \phi_i \partial^\mu \phi_i + m^2 \phi^2 + ig\phi_1\phi_2\phi_3 + \text{ counter terms} \tag{4.18}$$

We also assume that these fields ϕ_μ labeled by mass μ are in principle series. Using the above set of diagrammatic rules to derive the orthogonal states to calculate the $\mathcal{O}(g^2)$ corrections to single particle scattering. We first construct the orthogonal initial and final states.
Our diagrammatic rules tell us that the initial state does not require any corrections, as is expected from the flat nature of the cosmological horizon.

$$|n >_i^{ON} = |n >_i \tag{4.19}$$

The final state however will have corrections coming from amplitudes like ${}_f < n|n >_f^{(2)}$, since such amplitudes are non-zero in the asymptotic future.

$$|n >_f^{ON} = |n >_f [1 - \frac{1}{2} {}_f < n|n >_f^{(2)}] \tag{4.20}$$

Hence

$$\begin{aligned}
\mathcal{A}^{(2)}_{(\phi_1 \to \phi_1)} &= {}_f < n|n >_i^{ON(2)} \\
&= {}_f < n|n >_i^{(2)} - \frac{1}{2} {}_f < n|n >_f^{(2)} \\
&= (ig)^2 \int_x \int_y u_1(x) u_1^*(y) \big[G_2(x,y) G_3(x,y) - \frac{1}{2}\big(W_2(x,y) W_3(x,y)\big)\big] \\
&\qquad + \text{Counter terms}
\end{aligned} \tag{4.21}$$

Counter terms would exactly be the same as in global space, except that the region of integration will lie only on the EPP. However the nature of divergence will be completely different. In the global de Sitter case the second order correction is of the order $\eta^{-2\mu}$, where η is the conformal time, while in Poincare Patch, the second order correction in propagators have logarithmic divergences as shown in equation 3.27. To verify the optical theorem we need to compute the real part of the amplitude, which gives the following results:

$$2Re(\mathcal{A}^{(2)}) = (ig)^2 \int_x \int_y u_1(x) u_1^*(y) \mathcal{M}(x,y) \tag{4.22}$$

Where

$$\mathcal{M}(x,y) = \big[G_2(x,y) G_3(x,y) - \frac{1}{2} W_2(x,y) W_3(x,y)\big] + c.c.(x \leftrightarrow y) \tag{4.23}$$

With Feynman and Wightman correlators given respectively as:

$$\begin{aligned}G(x,y) &= \sum_k [\theta(x-y)u_k(x)u_k^*(y) + \theta(y-x)u_k(y)u_k^*(x)] \\ &and \\ W(x,y) &= \sum_k u_k(x)u_k^*(y)\end{aligned} \tag{4.24}$$

the following two relations hold generally

$$\begin{gathered}G_3(x,y)G_2(x,y) + G_3^*(y,x)G_2^*(y,x) = W_2(x,y)W_3(x,y) + W_2(y,x)W_3(y,x) \\ and \\ [W_2(x,y)W_3(x,y)]^* = W_2(y,x)W_3(y,x)\end{gathered} \tag{4.25}$$

This gives

$$\mathcal{M}(x,y) = W_2(y,x)W_3(y,x) \tag{4.26}$$

So finally

$$\begin{aligned}2Re(\mathcal{A}^{(2)}) &= (ig)^2 \int_x \int_y u_1(x)u_1^*(y)W_2(y,x)W_3(y,x)) \\ &= \left|(ig)\int_x u_1 u_2^* u_3^*(y)\right|^2 \\ &= \left|{}_f < n_2 n_3 | n_1 >_i \right|^2\end{aligned} \tag{4.27}$$

Which is the statement of the optical theorem

Its worth noting that the proof of the optical theorem follows solely from the LSZ/Feynman rules given above and do not depend on the form of the modes or their domain of validity.

Flat space limit

The flat space limit of the S-matrix corresponds to $\lim H \to 0$. We can argue, as we have, that the flatness at the cosmological horizon accounts for the fact that the initial states can be made orthogonal while retaining their particle interpretation, and hence they do not receive any ${}_i < n|n >_i$ corrections. This translates to the diagrammatic rules that we have given, that dictate that there are no vertex connected directly to initial bra $< i|$. Now suppose if we have the future boundary also flat, the final ket $|n >_f$ should also not receive any contribution from J_- branch on the account that an orthogonal state can be constructed without damaging its particle interpretation.
With this rule the second order correction to the amplitude in equation (4.21) will be given as:

$$\mathcal{A}^{(2)}_{(\phi_1 \to \phi_1)} = (ig)^2 \int_x \int_y u_1(x)u_1^*(y)G_2(x,y)G_3(x,y) \tag{4.28}$$

This clearly satisfies the flat space limit of our S-matrix because the Feynman functions $G_i(x, y)$ have well defined flat space limits as has been shown in the previous chapter with the help of a re-summed retarded propagator G^R.

Chapter 5

Conclusions and Discussions

In this thesis we have reviewed the interacting quantum field theories for massive scalar fields in de Sitter space, in particular the issues related to IR divergence was studied in detail. We saw that the closed time path techniques like the Schwinger-Keldysh formalism might again give IR divergences (for example the late time logarithmic divergences in loop corrections) but the re-summed results are free of such behavior and have good flat space limit [11].

We have calculated the mass corrections to the massive fields and saw that we get the expected flat space limit when $M >> dH/2$. This is an additional confirmations that large masses in de Sitter behave as if they are in flat space. This is because of the small de Broglie wavelength of the field due to which it does not experience the background curvature. This argument can be extended to the fields that are blue-shifted near the horizon of EPP (or CPP). The blue-shifted fields have very small associated de Broglie wavelength and hence can be considered to behave as if they are in flat space. This insight helps us to build up an LSZ formulation for the S-matrix in EPP whose in-states are at the horizon and hence can be treated as particles. Using these cues we construct an S-matrix which behaves properly under CPT, is unitary (and has been shown to satisfy optical theorem) and has good flat space limit.

For the future work, we notice that many questions remain open. We would like to continue our study of the analysis of S-matrix in de-Sitter Poincare Patch. In the expanding Poincare patch, initial states are close to the horizon and hence are highly blue-shifted resulting transplanckian energies. This is an issue which can cause complications in the UV physics of the theory which we remains to be investigated. We also note once again that in order to regulate IR divergences in the S-matrix computations, we had added some counter-terms whose only justification lies in the fact that the resulting IR finite S matrix is unitary. In Ordinary quantum field theory on flat spacetime, counter-terms are required to control UV divergences. Conceptually addition of such counter-terms to regulate the large volume (IR) divergences in de-Sitter QFTS need further investigation. Finally, there has been a resurgence in the study of S matrix in Ads spacetime in recent years. We would like to see if by doing the so-called double analytic continuation (in time as well as in cosmological constant) we can related the Scattering amplitudes in poincare patch with that in Poincare Patch of AdS spacetimes.

Appendix A

A.1 calculations for mass corrections

$$
\begin{aligned}
g_{12}^{R1}(q) &= \frac{m^2}{2}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}g_{13}^{R0}(q)g_{32}^{R0}(q) \\
&= -\frac{m^2}{8\mu^2}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}\left(\left(\frac{\tau_1}{\tau_3}\right)^{i\mu}-\left(\frac{\tau_3}{\tau_1}\right)^{i\mu}\right)\left(\left(\frac{\tau_3}{\tau_2}\right)^{i\mu}-\left(\frac{\tau_2}{\tau_3}\right)^{i\mu}\right) \\
&= -\frac{m^2}{8\mu^2}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}\left(\left(\frac{\tau_1}{\tau_2}\right)^{i\mu}+\left(\frac{\tau_2}{\tau_1}\right)^{i\mu}-\frac{(\tau_1\tau_2)^{2i\mu}}{\tau_3^{2i\mu}}-\frac{\tau_3^{2i\mu}}{(\tau_1\tau_2)^{2i\mu}}\right) \\
&= -\frac{m^2}{8\mu^2}\left[\left(\frac{\tau_1}{\tau_2}\right)^{i\mu}+\left(\frac{\tau_2}{\tau_1}\right)^{i\mu}\right]ln\left(\frac{\tau_1}{\tau_2}\right)-\frac{m^2}{8\mu^2}\frac{1}{2i\mu}\left[\frac{\tau_3^{2i\mu}}{(\tau_1\tau_2)^{i\mu}}-\frac{(\tau_1\tau_2)^{i\mu}}{\tau_3^{2i\mu}}\right]_{\tau_2}^{\tau_1} \\
&= -\frac{m^2}{8\mu^2}\left[\left(\frac{\tau_1}{\tau_2}\right)^{i\mu}+\left(\frac{\tau_2}{\tau_1}\right)^{i\mu}\right]ln\left(\frac{\tau_1}{\tau_2}\right)+\frac{m^2}{4\mu^2}\frac{i}{2\mu}\left[\left(\frac{\tau_1}{\tau_2}\right)^{i\mu}-\left(\frac{\tau_1}{\tau_2}\right)^{-i\mu}\right] \\
&= -\frac{m^2}{4\mu^2}\left[\frac{1}{2}ln\left(\frac{\tau_1}{\tau_2}\right)f_{12}^0-g_{12}^{R0}\right]
\end{aligned}
\tag{A.1}
$$

$$g_{12}^{R2}(q) = \frac{m^2}{2}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}g_{13}^{R0}(q)g_{32}^{R1}(q)$$

$$= -\frac{m^2}{2}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}g_{13}^{R0}(q)\frac{m^2}{4\mu^2}\left[\frac{1}{2}ln\left(\frac{\tau_3}{\tau_2}\right)f_{32}^0 - g_{32}^{R0}\right]$$

$$= -\frac{m^4}{16\mu^2}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}ln\left(\frac{\tau_3}{\tau_2}\right)g_{13}^{R0}(q)f_{32}^0 + \frac{m^4}{8\mu^2}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}g_{13}^{R0}g_{32}^{R0}$$

$$= -\frac{m^4}{16\mu^2}\left[\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}ln(\tau_3)g_{13}^{R0}f_{32}^0 - ln(\tau_2)\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}g_{13}^{R0}f_{32}^0\right] + \frac{m^2}{4\mu^2}g_{12}^{R1}$$

(A.2)

$$I_1 = \int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}[ln(\tau_3) - ln(\tau_2)]g_{13}^{R0}f_{32}^0$$

$$= \frac{i}{2\mu}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}[ln(\tau_3) - ln(\tau_2)]\left(\left(\frac{\tau_1}{\tau_3}\right)^{i\mu} - \left(\frac{\tau_3}{\tau_1}\right)^{i\mu}\right)\left(\left(\frac{\tau_3}{\tau_2}\right)^{i\mu} + \left(\frac{\tau_2}{\tau_3}\right)^{i\mu}\right)$$

$$= \frac{i}{2\mu}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}\left(\left(\frac{\tau_1}{\tau_2}\right)^{i\mu} - \left(\frac{\tau_1}{\tau_2}\right)^{-i\mu} + \frac{(\tau_1\tau_2)^{i\mu}}{\tau_3^{2i\mu}} - \frac{\tau_3^{2i\mu}}{(\tau_1\tau_2)^{i\mu}}\right)[ln(\tau_3) - ln(\tau_2)]$$

$$= g_{12}^{R0}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}[ln(\tau_3) - ln(\tau_2)] + \frac{i}{2\mu}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}\left(\frac{(\tau_1\tau_2)^{i\mu}}{\tau_3^{2i\mu}} - \frac{\tau_3^{2i\mu}}{(\tau_1\tau_2)^{i\mu}}\right)[ln(\tau_3) - ln(\tau_2)]$$

$$= \frac{g_{12}^{R0}}{2}\left(ln\left(\frac{\tau_1}{\tau_2}\right)\right)^2 + \frac{i}{2\mu}\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}ln(\tau_3)\left(\frac{(\tau_1\tau_2)^{i\mu}}{\tau_3^{2i\mu}} - \frac{\tau_3^{2i\mu}}{(\tau_1\tau_2)^{i\mu}}\right)$$

(A.3)

we find that

$$\int_{\tau_2}^{\tau_1}\frac{d\tau_3}{\tau_3}\left(\frac{(\tau_1\tau_2)^{i\mu}}{\tau_3^{2i\mu}} - \frac{\tau_3^{2i\mu}}{(\tau_1\tau_2)^{i\mu}}\right) = 0 \quad \text{(A.4)}$$

and using

$$\int x^k\frac{ln(x)}{x}dx = \frac{x^k ln(x)}{k} - \frac{x^k}{k^2} \quad \text{(A.5)}$$

so we have

$$\int_{\tau_2}^{\tau_1} \frac{d\tau_3}{\tau_3} ln(\tau_3) \left(\frac{(\tau_1\tau_2)^{i\mu}}{\tau_3^{2i\mu}} - \frac{\tau_3^{2i\mu}}{(\tau_1\tau_2)^{i\mu}} \right) = -\frac{1}{2i\mu} ln\left(\frac{\tau_1}{\tau_2}\right) f_{12}^0 - \frac{1}{i\mu} g_{12}^{R0} \tag{A.6}$$

$$g_{12}^{R2} = -\frac{m^4}{16\mu^2}\left[\frac{g_{12}^{R0}}{2}\left(ln\left(\frac{\tau_1}{\tau_2}\right)\right)^2 + \frac{i}{2\mu}[-\frac{1}{2i\mu} ln\left(\frac{\tau_1}{\tau_2}\right) f_{12}^0 - \frac{1}{i\mu} g_{12}^{R0}]\right] + \frac{m^4}{8\mu^2} g_{12}^{R1} \tag{A.7}$$

With leading terms in $ln\left(\frac{\tau_2}{\tau_1}\right)$, the correction terms are given as

$$g_{12}^{R1} = \frac{m^2}{8\mu^2} ln\left(\frac{\tau_2}{\tau_1}\right) f_{12}^0 \tag{A.8}$$

$$g_{12}^{R2} = -\frac{m^2}{8\mu^2}\frac{m^2}{4}\left(ln\left(\frac{\tau_1}{\tau_2}\right)\right)^2 g_{12}^{R0} \tag{A.9}$$

A.2 Behaviour of the states in EPP under CPT action

The field $\phi_k(x)$ of mass $m > d/2$, satisfying the KG equation in EPP is given by:

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3}[a_k u_k(x) + a_k^\dagger u_k^*(x)] \tag{A.10}$$

where the modes $u_k(x)$ with $x = k\lambda$, are given in terms of the Hankel functions $H_\nu^{(1)}(k\lambda)$ by

$$u_k(x) = \frac{\sqrt{\pi}}{2}\lambda^{3/2}H_{i\mu}^{(1)}(k\lambda)e^{ik.x} \tag{A.11}$$

with the order $i\mu$ given by

$$i\mu = i\sqrt{m^2 - \left(\frac{d}{2}\right)^2} \tag{A.12}$$

We wish to see the CPT invariance of the S-matrix. Let Θ denote CPT operation, then with respect to the Bessel functions of first kind $J_\nu(x)$ the Hankel function can be written as [see [1]]

$$H_{i\mu}^{(1)}(x) = csch(\pi\mu)\left[e^{\pi\mu}J_{i\mu}(x) - J_{-i\mu}(x)\right] \tag{A.13}$$

Action of time reversal operation $\hat{T}$ on a spin-less state is equivalent to a complex conjugate operation and hence we have

$$\hat{T}H_{i\mu}^{(1)}(x) = H_{i\mu}^{(1)^*}(x) \tag{A.14}$$

Now we use the fact that the complex conjugate of a holomorphic function $f(z)$ that maps $\mathbb{R}$ to $\mathbb{R}$ satisfies

$$f(z)^* = f(z^*) \tag{A.15}$$

Hankel function satisfies this property ad hence

$$H_{i\mu}^{(1)^*}(x) = H_{-i\mu}^{(1)}(x) = csch(\pi\mu)\left[e^{\pi\mu}J_{-i\mu}(x) - J_{i\mu}(x)\right] \tag{A.16}$$

Partity operation $\hat{P}$ on the spin less state has the effect of inversion of momentum, that is

$$\hat{P}H_{i\mu}^{(1)}(x) = H_{i\mu}^{(1)}(-x) \tag{A.17}$$

Since the Bessel function $J_{i\mu}(x)$ is given by

$$J_{i\mu}(x) = \sum_m \frac{(-1)^m}{m!\Gamma(i\mu + m + 1)}\left(\frac{x}{2}\right)^{2m+i\mu} \tag{A.18}$$

We have

$$J_{i\mu}(-x) = (-1)^{i\mu}J_{i\mu}(x) = e^{\pi\mu}J_{i\mu}(x) \tag{A.19}$$

the CPT operation Θ is acts on the

$$\begin{aligned}\Theta H^{(1)}_{i\mu}(x) &= csch(\pi\mu)\big[e^{\pi\mu}e^{-\pi\mu}J_{-i\mu}(x) - e^{\pi\mu}J_{i\mu}(x)\big] \\ &= csch(\pi\mu)\big[J_{-i\mu}(x) - e^{\pi\mu}J_{i\mu}(x)\big] \\ &= -H^{(1)}_{i\mu}(x)\end{aligned} \tag{A.20}$$

Hence the modes $u_k(x)$ are eigenstates of the CPT operator with eigenvalue -1, and the covariance of our initial and final states ensure that the S-matrix behaves properly under the action of CPT.

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