

Frequency Stabilization of 1064 nm Narrow Linewidth Laser

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by

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Certificate

This is to certify that this dissertation entitled Frequency Stabilization of 1064 nm Narrow Linewidth Lasertowards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by Thejas Chilkunda Seetharamu at Indian Institute of Science Education and Research under the supervision of Dr. Umakant Rapol, Associate Professor, Department of Physics, during the academic year 2020-2021.

A handwritten signature in black ink, appearing to read 'U. O. Rapol', enclosed within a circular scribble.

Dr. Umakant Rapol

Committee:

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Prof. Bhas Bapat

This thesis is dedicated to *Pilli*

Declaration

I hereby declare that the matter embodied in the report entitled Frequency Stabilization of 1064 nm Narrow Linewidth Laser are the results of the work carried out by me at the Department of Physics, Indian Institute of Science Education and Research, Pune, under the supervision of Dr. Umakant Rapol and the same has not been submitted elsewhere for any other degree.

A handwritten signature in black ink, consisting of stylized, overlapping loops and lines, positioned above the printed name.

Thejas Chilkunda Seetharamu

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Abstract

LIGO (Laser Interferometry Gravitational-Wave Observatory) detectors help detect gravitational waves (GW) that Albert Einstein first predicted in the early 20th century. These detectors on earth face an unprecedented challenge of detecting modulation in space-time that is one part in 10^{22} or trillionth the wavelength of the laser used by the detectors. These disturbances in the fabric of space-time originate from collisions of stellar mass objects like black hole mergers about a million light-years away. LIGO observatories adopt perpendicular Michelson Interferometry arms with arm lengths of about 4 kilometers and an ultra-stable high-power laser system to reach the sensitivity requirements necessary to enable these detections [1]. A high laser power is necessary to increase the signal to quantum noise ratio at high Fourier frequencies. Besides quantum noise, the technical noise in laser coupling into interferometer arms is a prominent noise source that requires attention [2]. Technical noise is alleviated by exploiting passive and active laser stabilization schemes. Designing optical cavities that exploiting symmetry is another avenue towards the reduction of laser noise [1].

During this Master's project, an attempt is made to address frequency noise of a 1064 nm laser (Make: Coherent, Model: Mephisto- Non-Planar Ring Oscillator (NPRO), Power: 1W). We attempt absolute frequency locking and relative frequency locking. Absolute frequency locking addresses frequency drift of a laser, while relative frequency locking addresses frequency noise at finite Fourier frequencies (or equivalently the laser linewidth). To reduce laser frequency drift, a ro-vibrational transition line in molecular cesium (Cs_2) is used as a frequency reference for the laser. Three cesium vapour cells are prepared. Characterization of the cell(s) is done using a 852 nm laser diode. The standard Saturation Absorption Spectroscopy (SAS) is done to probe sub-Doppler lines which can be later exploited to stabilize laser frequency. All the three vapour cells were lost to contamination of vacuum before any effort of stabilizing the laser could be attempted. On the other hand, a high finesse triangular optical resonator is developed (Courtesy: Prof. Rana Adhikari, Caltech, USA) to address laser linewidth reduction. This is a fixed length optical resonator and the mirrors are clamped on a stainless steel spacer that is locally fabricated. This resonator is also aimed at addressing spatial profile filtering and temporal filtering of the laser beam. The laser beam is phase modulated using an EOM (Electro-Optic Modulator) and PDH (Pound-Drever-Hall [3]) technique is adopted to extract an error signal from the resonator. The error signal is exploited to lock laser to the cavity and also cavity to the laser. An absolute frequency ref-

erence in conjunction with an optical resonator can be shown to be a promising combination for laser frequency stabilization.

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Chapter 1

Introduction

Since their invention in 1960 [4], lasers have found a myriad of applications. They are almost omnipresent in today's technology. Their unique properties, i.e., temporal and spatial coherence, high monochromaticity, and low divergence, have made them ideal candidates for accurate measurements within science and engineering. Spatial coherence of lasers enable high intensity at a tight focus that create thermal effects and perform laser cutting to treat certain ailments in medicine. Spatial coherence also enables collimation that can be exploited for everyday use such as laser pointers. Furthermore, they have found their use in precision metrology, that is, measurement of time and distance at quantum scales. The list of applications of lasers is unlimited. Though lasers that are available in market are quite stable as they are for many applications, they are noisy for certain applications requiring precise measurements. In such cases additional stabilization techniques are adopted to stabilize these devices further. Resulting systems called *stable laser systems* play a primal role in today's science.

In this chapter, we will introduce the terminologies involved in the topic of this thesis. We will then present motivation of the project. In section 1.1 we will introduce lasers and associated parameters. Section 1.2 will describe stable laser systems and explain the importance of laser stabilization. We will also describe motivation behind this project in section 1.3 and present two techniques that are adopted to address frequency noise of lasers.

1.1 What are Lasers?

Laser (Light amplification by stimulated emission of radiation) is a device that amplifies and generates coherent radiation at various optical frequencies. Schematic of a typical laser oscillator system along with its elements is found in figure 1.1. A laser consists of three essential components:

- Gain medium or active medium that does light amplification,
- Pump, that pumps energy into the the gain medium and excite certain energy levels to achieve population inversion,
- Optical resonator, that enables power build up and promotes single mode oscillation of the laser.

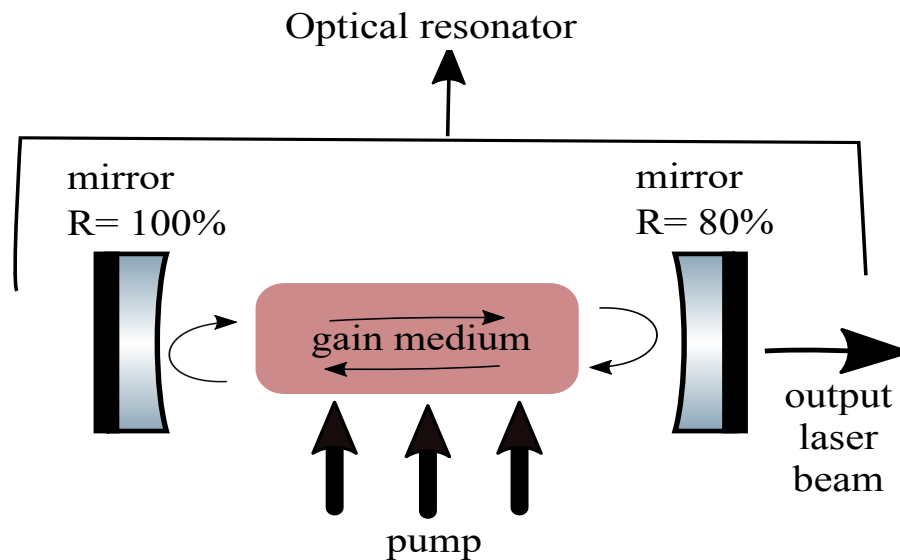


Figure 1.1: A typical laser oscillator and its elements [5]: the gain medium, the energy pump and optical resonator that is composed of two mirrors surrounding the aforementioned components.

There exist a plethora of laser types in today's world: Gas lasers, chemical lasers, dye lasers, solid-state lasers, semiconductor lasers to name a few. In this project we use a solid-state laser, specifically an Nd-YAG laser. Some important parameters of a laser beam are as follows:

- *Wavelength/frequency*: This is the spatial frequency of a laser beam. It is determined by the separation of energy levels in the gain medium that are involved in spontaneous emission process.
- *Power*: The output power of a laser is an important parameter as this influences a user's choice of a laser based on the application.
- *Coherence length*: This is the distance over which a laser beam maintains a fixed phase relationship at different spatial (spatial coherence) and different temporal coordinates (temporal coherence). Coherence of laser is a direct consequence of stimulated emission.
- *Output polarization*: This is the orientation of electric field of the output beam.
- *Beam quality factor* [6]: This parameter indicates how tightly a laser beam can be focused. It also alludes to higher order mode content in a laser beam.
- *Beam divergence*: This parameter indicates how fast a laser beam's diameter expands as it moves away from its waist.
- *Linewidth*: It is the half-width half-maximum (HWHM) of spectral lineshape of a laser (section 1.1.1).

1.1.1 Spectral lineshape

A laser is never monochromatic, and there is always profile about the laser's mean frequency which is referred to as its spectral lineshape. Consequently, there is a HWHM associated with this spectral profile which is called *laser linewidth* (figure 1.3). This lineshape is a consequence of noise associated with laser frequency. A laser's frequency noise spectrum consists of two noise components: White noise and Flicker noise ($1/f$). White noise originates from spontaneous emission thus is an intrinsic noise source, whereas the flicker noise occurs due to fluctuations in resonator length or electronics noise in the laser system (technical noise). White noise gives rise to a Lorentzian lineshape (natural linewidth) while flicker noise broadens this into a Gaussian lineshape [7] [5]. Convolution of the two forms of lineshapes gives rise to Voigt profile (figure 1.2) for a laser lineshape.

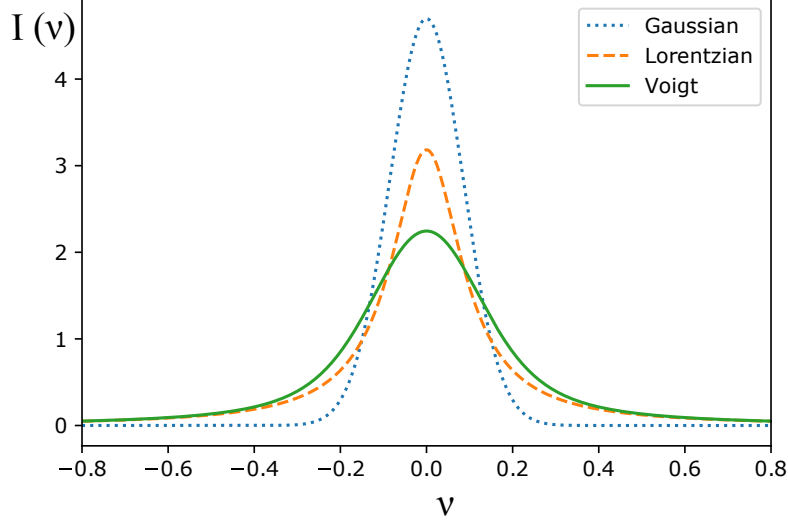


Figure 1.2: Voigt profile is a convolution of corresponding Gaussian profile and Lorentzian profile. Inhomogeneous broadening mechanisms like Doppler broadening give rise to Gaussian laser line profiles. While homogeneous broadening mechanisms like collisional broadening give rise to Lorentzian line profiles. In nature, both these broadening mechanisms together give rise to a convoluted Voigt line profile [7]

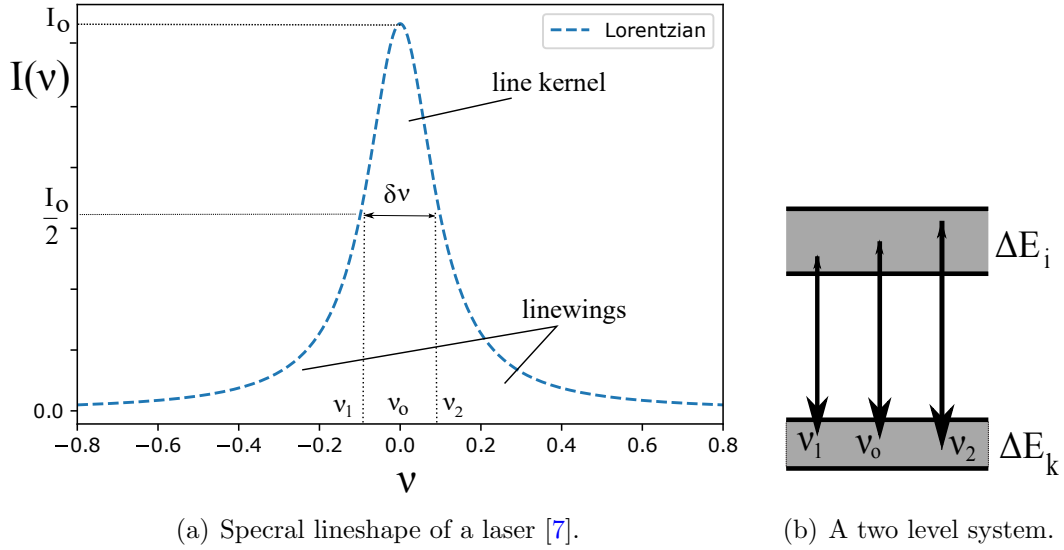


Figure 1.3: Figure (a) depicts natural linewidth of a laser, x-axis is frequency and y-axis is spectral intensity. The region inside the spectral width of this profile is called *line kernel*. The region outside the kernel on either side is called line wing(s). The origin of this spectral broadening is attributed to finite lifetimes for de-excitation from excited energy level (shown in figure (b)). From the uncertainty principle this can be realized as uncertainty in the energy eigenvalue for the two energy levels involved in the transition. As shown in figure (b), ΔE_i and ΔE_k indicate uncertainty in energy eigenvalue, thus the frequency of the transition [7].

Also, the mean frequency of a laser is not constant, it drifts over a long time range. This phenomenon is termed frequency drift of a laser.

1.1.2 Frequency noise

In the previous subsection we saw spectral lineshape and frequency drift of a laser. Together, these contribute to frequency noise of a laser. Let's understand at frequency noise in more detail.

Frequency noise refers to random fluctuations or deviation in the instantaneous frequency of the laser. The instantaneous frequency of laser is given by:

$$\nu = \frac{1}{2\pi} \frac{d\phi}{dt} \quad (1.1)$$

The Power Spectral Density (PSD) of frequency noise $S_\nu(f)$ (Hz^2/Hz) is given by:

$$S_\nu(f) = \int_{-\infty}^{+\infty} G_\nu(\tau) e^{i2\pi f\tau} d\tau \quad (1.2)$$

where

$$G_\nu(\tau) = \langle \nu(t) \nu(t + \tau) \rangle$$

is the auto-correlation function of $\nu(t)$.

Noise spectral density of a laser can tell everything about lineshape of that laser [8]. While frequency drift, which is the deviation of a laser's frequency from operating frequency in long run, can be measured with a wavemeter, measurement of noise PSD requires a spectrum analyzer since many traces of the variable of interest is necessary. Nevertheless, a rough estimate of PSD can be acquired with just one trace.

1.2 Stable laser systems

One of the fields where lasers have been exploited to perform measurements at unprecedented levels is the precision metrology. One of the techniques, optical heterodyne interferometry, has been a valuable tool in precision metrology since the 1960s. Researchers at MIT used this technique to perform an improved Michelson-Morley experiment [9], they also performed spectroscopy with unprecedented sensitivities and enormous resolution. Nevertheless, these measurements were fraught with spurious noises like frequency noise and frequency drifts. Some of the factors that influence laser stability are quantum noise (radiation pressure noise, shot noise) [10], frequency noise, power noise, beam quality and beam pointing jitter.

Thus, laser stabilization was and has always been at great demand. Stable laser systems enable applications like GW detection [1], precision measurement, quantum simulation, and so on. Since its inception in the mid-1960s [9], there have been several developments in the field of stabilization techniques. In the following subsection we will understand the need for stable laser systems in detail by picking one example.

1.2.1 Why is laser stabilization important?

Let us understand in detail the importance of stable laser systems for GW detection.

Ground based GW detectors detect space time modulation in the Fourier frequency range of 10 Hz-100 kHz. Frequency noise associated with the laser in this frequency range can be found in figure 1.4, and it should be noted that laser noise is above the requirements of the detector (indicated as black solid line in figure 1.4). Moreover, frequency noise of free running laser is always higher than this due to noise from external factors. This justifies necessity of laser stabilization in this frequency range. Advanced LIGO, for example, uses a three-stage frequency stabilization process [11]. In the first stage, laser beam is picked off from Pre-Mode Cleaner resonator [1] and fed into fixed length high finesse reference resonator [12], followed by a second stage stabilization to an Input Mode Cleaner resonator before the final stage stabilization of the laser to interferometers average arm length.

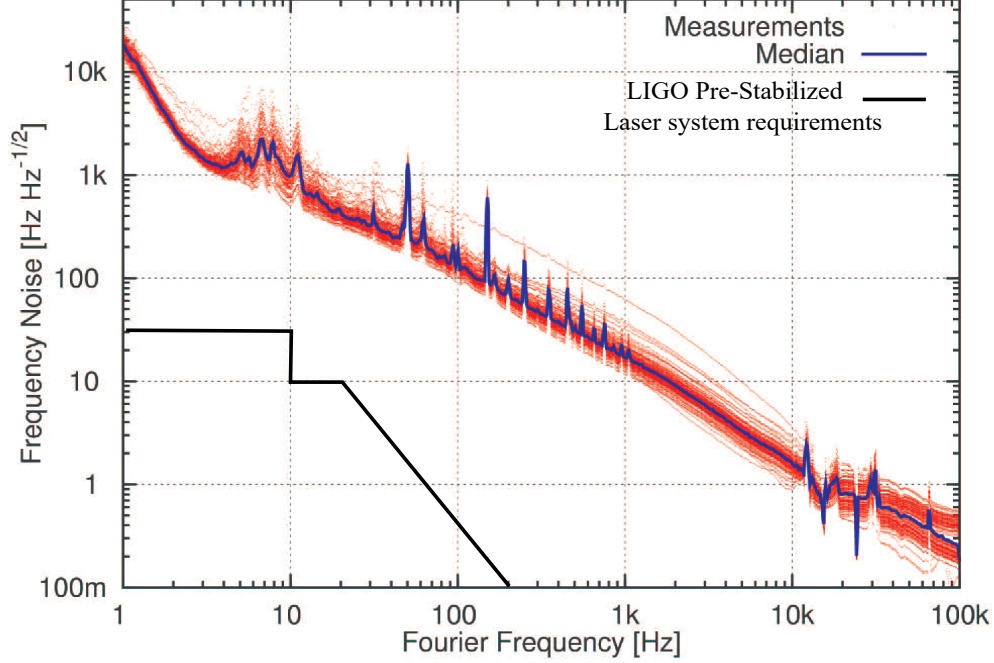


Figure 1.4: Frequency noise (amplitude noise spectral density) of a typical Mephisto NPRO laser (section 3.1). Blue solid line corresponds to frequency noise of a typical NPRO laser (adapted from Fig. 6 of [13]), it is the median of measurements indicated as red scatter plot; black solid line corresponds to requirements of LIGO [1]. Requirements are clearly well below the typical frequency noise profile thus demanding a frequency stabilized laser system.

Frequency drift over 10 minutes of a typical solid-state laser that is used for GW detection in LIGO is about 30 MHz. The laser in LIGO is locked to the interferometer's average arm length. This ensures that the laser stays in resonance with the interferometer arms, thus enabling GW detection. But, due to drifts associated with interferometer arm-lengths, the laser frequency drifts. This in-turn could lead to mode-hops of the laser. This is probably avoided with a separate control loop that locks the interferometer arms to the laser, thus restricting drift in the laser's frequency. However, these hassles could be avoided if an absolute frequency reference is used in the detector.

For several applications like phase-sensitive optical time domain reflectometer [14] a laser with lower drift is desirable. One more example for requirement of a laser with very low frequency drift is in research of materials with very thermo-refractive index noise index where length measurements need to be done with extreme accuracy.

Laser stabilization is crucial to probe atomic clock frequencies. These transitions have

spectral linewidths as low as mHz, so stabilization of laser frequency is necessary to enable probing of these transitions. Here, high finesse, high stable, optical resonators are used to frequency stabilize lasers. Optical resonators are a commonplace in such high precision experiments in physics and engineering.

Now that we seen how laser stabilization has been pursued by instrumentalists in one of the most sensitive detectors even built by humankind, let us understand in detail two methods used to address frequency stabilization of a laser in this project.

1.2.2 Frequency stabilization techniques

As we have seen in section 1.4, frequency noise of a laser can be treated in two stages, frequency drift and spectral linewidth. In this project we attempt to exploit *optical resonators* to address spectral linewidth, and exploit molecular transition lines to address frequency drift. The former is termed as *relative frequency stabilization*, while the latter is called *absolute frequency stabilization*.

A molecular species provides a stable long term frequency reference, but it could have phase noise associated at higher Fourier frequencies [12]. Also, a molecular reference can not be used to reduce laser linewidth. On the other hand, an optical resonator can be used to reduce linewidth to very low levels [15]. However, stabilizing a laser to high finesse cavities provides stability only for short time periods thus not guaranteeing a stable frequency reference in a long run. Thus, an absolute frequency reference in conjunction with a high finesse optical resonator promises to be one of the best combination for laser frequency stabilization.

Absolute frequency stabilization

Absolute frequency stabilization is achieved by locking a laser’s frequency to an atomic or molecular transition line. This reduces frequency drift of the laser, and provides a long term frequency stabilization.

Atoms at a finite temperature follow the Maxwell-Boltzmann velocity distribution of velocity [7]. This causes the resonance frequencies of atoms at different velocities to be differ-

ent or Doppler shifted which gives rise to Doppler broadening of resonance profiles. Locking a laser to such broad transition profiles renders a less stable frequency reference. Thus, a high resolution, sub-Doppler spectroscopy should be performed to remove the Doppler background. We adopt the *Saturation Absorption Spectroscopy (SAS)* technique to address Doppler broadening. This technique exploits saturation of light absorption upon shining the atoms with a high intensity laser beam. Upon saturation, a counter-propagating probe beam is used to detect sub-Doppler transition lines. An error signal derived from these narrow transition lines is used to lock the laser to a molecular transition. As a result, a reduction in frequency drift of the laser is achieved.

Relative frequency stabilization

Optical resonators are a set of mirrors arranged together such that laser light repeatedly reflects between them and allows single mode oscillation of laser beam (section 7.1). Relative frequency stabilization is achieved by locking a laser to an optical resonator [12]. A phase modulated laser beam is sent to an aligned optical resonator of round-trip length L . It is ensured that the input beam resonates with the cavity, and this is optimized further. This will be described in detail in this thesis. Light reflected off from the resonator is sent to a PDH (Pound-Drever-Hall [3]) locking setup to generate an error signal. This error signal is sent to the laser through a fast feedback servo (figure 7.9). The laser is now locked to a stable reference cavity that suppresses frequency noise at finite Fourier frequencies. This reduces spectral linewidth of the laser.

1.3 Outline and Overview

As we have seen in the previous sections, development of a stable laser system is essential for advancement of science and technology. In this thesis, we will describe our efforts to frequency stabilize a 1064 nm narrow linewidth laser (Make: Coherent, Model: Mephisto-Non-Planar Ring Oscillator (NPRO), Power:1W, refer section 3.1) and perform beam quality improvement.

In the first half of the thesis, that is, from chapters 2-6, we will present our efforts on absolute frequency stabilization of the laser. Chapter 2 will give a theoretical background

adequate to understand experimental details described in 3. In section 2.1 we briefly present ro-vibrational spectra of a diatomic molecules using the non-rigid rotor model. We also present details about the molecular spectra of Cs_2 molecule used to achieve frequency stabilization of the 1064 nm laser. In subsection 2.2.1 we briefly present spectral-line broadening mechanisms for a laser. The subsection 2.7 introduces theoretical details on SAS. In chapter 3, we will discuss about the laser in section 3.1, development of the vapour cell in section 3.2, characterization of the cell using an 852 nm diode laser in 3.4 and optics setup for spectroscopy in section 3.5. In chapter 4 we will present Cs_2 Doppler spectra, graphs that indicate formation of Cs_2 molecules around 140 °C and SAS signals. Now that we have seen experimental details and some observations, we will discuss challenges encountered in chapter 5. This chapter will give rise to more judicious future directions in chapter 6 that will open newer avenues.

The second half of the thesis consists chapters describing efforts to reduce spectral linewidth and improve laser beam quality. Chapter 7 consists of sections that briefly present necessary theoretical details on optical resonators and the PDH technique (subsection 7.3.1). In Chapter 8 we will discuss experimental details about cavity development, alignment and extraction of error signal. Later chapters 9 and 10 include preliminary observations from the cavity and challenges encountered. Chapter 11 will describe further steps that would be taken to improve the system and conclude the efforts.

Chapter 2

Cs₂ molecular spectroscopy

Long term frequency instability has the potential to hamper sensitive measurements that rely on a robust laser frequency. Reasons for frequency drift are mainly environmental factors such as temperature fluctuations, electronic noise and also air flow and lack of absolute reference to compare lasers frequency. Absolute frequency locking helps to reduce long term frequency drift of the laser.

Cs₂ is treated as a non-rigid rotor. This leads to ro-vibrational energy level structure as shown in figure 2.2. We will use one of these transitions occurring near 1064 nm to frequency stabilize a 1064 nm laser. We adopt the standard Saturation Absorption Spectroscopy technique to perform Doppler-free spectroscopy (section 2.7). Following this we will perform absolute reference locking with the corresponding ro-vibrational transition as a frequency reference.

This chapter presents a brief literature review on Cs₂ molecules and its energy levels, and it provides theoretical background required for performing absolute reference locking.

2.1 Energy levels of Cs₂

Cesium is a silvery golden alkali metal with 55 electrons and only one electron in its valence shell. It has a melting point of 28.5 °C. ¹³³Cs, the only stable isotope, is the species consid-

ered through out the thesis. Being the the least electronegative element, Cesium is highly pyrophoric thus reacts violently when exposed to air. It is transported in glass ampoules packed delicately in sealed metal containers.

Cesium molecules are formed upon heating cesium to temperatures of 140 °C and above [16]. Cs atoms have only one valence electron, so the bond strength between the alkali atoms in dimer form is weak. This is the case with all the alkali metal elements except Li due to its small size. Cs dimer can thus be treated as a non-rigid rotor [17](figure 2.1).

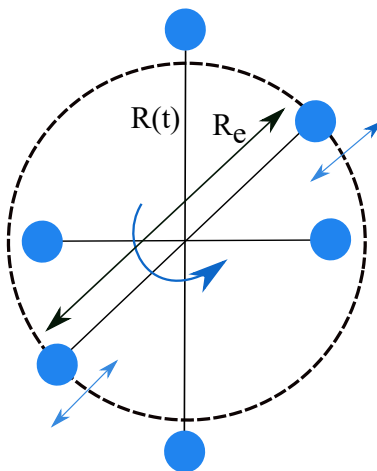


Figure 2.1: Cs₂ is considered as a non-rigid rotor [17]. Here the blue spheres depict Cs atoms. $R(t)$ corresponds to inter-atomic separation at time t . R_e corresponds to the equilibrium separation. Atoms here vibrate about their mean position such that inter-atomic distance varies periodically (indicated by double-headed blue arrows) as the system rotates about the centre of mass (indicated by the curved blue arrow).

In this model Cs atoms are held by a force dictated by *Morse potential*. This is given by the following equation:

$$E_{\text{pot}}(R) = E_B [1 - e^{-a(R-R_e)}]^2 \quad (2.1)$$

where R_e is the equilibrium internuclear distance, E_B is binding energy of the molecule and a is an experimentally determined parameter [17]. Total energy of a rotational-vibrational level is given by:

$$\begin{aligned}
E^{\text{Morse}}(v, J)/\hbar c = & E_e/\hbar c + \omega_e \left(v + \frac{1}{2} \right) \\
& - \omega_e x_e \left(v + \frac{1}{2} \right)^2 + B_v J(J+1) \\
& - D_v J^2(J+1)^2
\end{aligned} \tag{2.2}$$

where B_v, D_v are rotational constants:

$$B_v = \frac{\hbar}{4\pi c M} \int \psi_{\text{vib}}^*(v, R) \frac{1}{R^2} \psi_{\text{vib}}(v, R) dR \tag{2.3}$$

$$D_v = \frac{\hbar^3}{4\pi c k M^2} \int \psi_{\text{vib}}^* \frac{1}{R^6} \psi_{\text{vib}} dR \tag{2.4}$$

where ψ_{vib} is vibrational wavefunction. The absorption spectra of Cs_2 is determined by the transition matrix element:

$$\begin{aligned}
\mathbf{M}_{ik} &= \int \chi_i^* \left[\int \Phi_i^* \mathbf{p}_{\text{el}} \Phi_k d\tau_{\text{el}} \right] \chi_k d\tau_{\text{N}} \\
&= \int \chi_i^* \mathbf{M}_{ik}^{\text{el}}(R) \chi_k d\tau_{\text{N}}
\end{aligned} \tag{2.5}$$

where $\chi(\mathbf{R})$ is nuclear wavefunction and $\Phi(\mathbf{r}, \mathbf{R})$ is electronic wavefunction. Integral over $d\tau_{\text{N}}$ and $d\tau_{\text{el}}$ correspond to integral over nuclear and electronic coordinates respectively. Here $\mathbf{M}_{ik}^{\text{el}}(R)$ depends on internuclear distance R through the wavefunction $\Phi(\mathbf{r}, \mathbf{R})$, but in a large number of cases this dependence is weak [17] thus enabling us to Taylor approximate the element and consider the term $\mathbf{M}_{ik}^{\text{el}}(R_e)$ that is independent of R as a first approximation. When interaction between rotation and vibration is weak, the electronic wavefunction can be written as a product of rotational wavefunction and vibrational wavefunction such that the matrix element can be written as:

$$\begin{aligned}
\mathbf{M}_{ik} &= \mathbf{M}_{ik}^{\text{el}}(R_e) \int \psi_{\text{vib}}^*(v_i) \psi_{\text{vib}}(v_k) dR \\
&\cdot \int Y_{J_i}^{M_i} \hat{\mathbf{p}}_{\text{el}}(\vec{r}) Y_{J_k}^{M_k} \sin \vartheta d\vartheta d\varphi
\end{aligned} \tag{2.6}$$

where rotational level with angular momentum J and its projection $M\hbar$ onto the quantization axis.

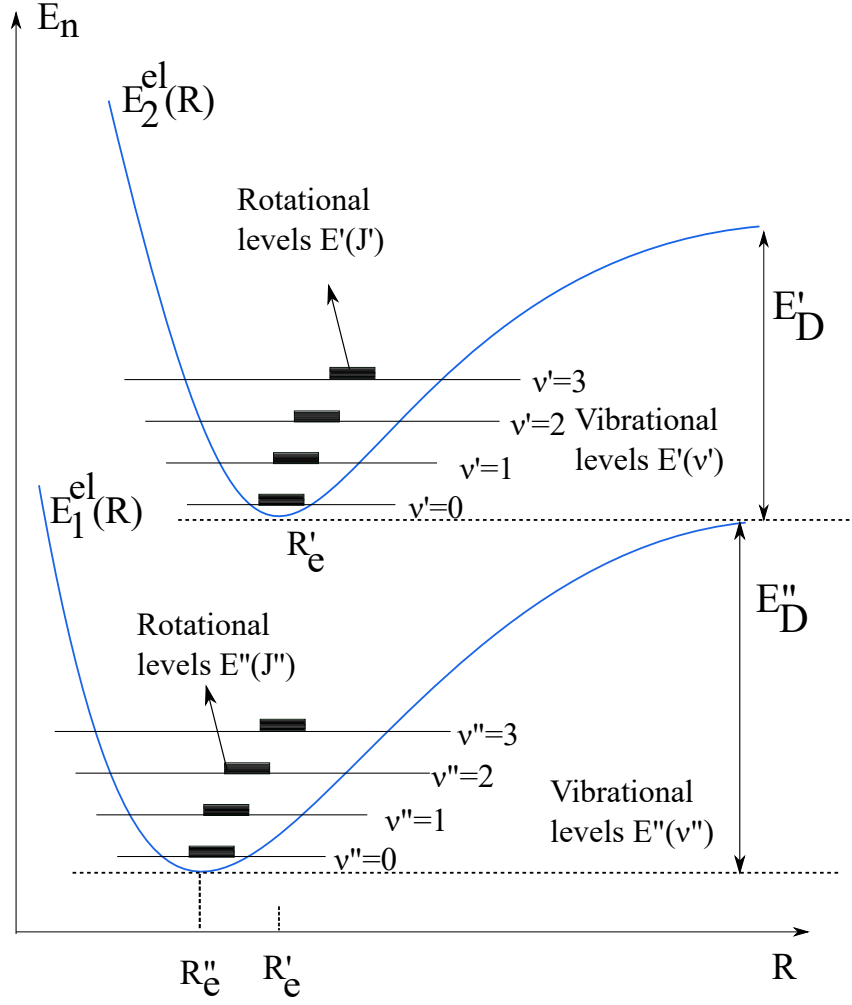


Figure 2.2: Ro-vibrational energy levels of two different electronic states $E_1^{el}(R)$ and $E_2^{el}(R)$ in a molecule are indicated by the solid blue curves [17]. E'_D and E''_D correspond to respective dissociation energies. R'_e and R''_e correspond to equilibrium inter-atomic distances corresponding to the two electronic states.

For an incident electromagnetic wave $\mathbf{E} = \boldsymbol{\varepsilon} \cdot |\mathbf{E}|$, the transition probability for absorbing transition is:

$$\begin{aligned} \mathcal{P}_{ik} = & \varepsilon_0 E^2 \cdot |M_{ik}^{el}(R_e)|^2 \times \left| \int \psi_{\text{vib}}^{v_i} \cdot \psi_{\text{vib}}^{v_k} \cdot dR \right| \\ & \times \left| \int Y_{J_i}^{M_i^*} \hat{\boldsymbol{\varepsilon}} \cdot \hat{\mathbf{p}} Y_{J_k}^{M_k} \sin \vartheta d\vartheta d\varphi \right|^2 \end{aligned} \quad (2.7)$$

where the Franck–Condon factor

$$FCF(v_i, v_k) = \left| \int \psi_{\text{vib}}^{v_i} \cdot \psi_{\text{vib}}^{v_k} \cdot dR \right| \quad (2.8)$$

and the Hönl–London factor is

$$HL(J_i, J_k) = \left| \int Y_{J_i}^{M_i^*} \hat{\epsilon} \cdot \hat{p} Y_{J_k}^{M_k} \sin \vartheta d\vartheta d\varphi \right|^2 \quad (2.9)$$

Equation 2.8 determines the selection rules involving different energy levels with different v quantum number. Transitions between different rotaional levels with vibrational levels v' and v'' respectively are allowed only if $\Delta J = \pm 1$ [17]. For homonuclear dimer system, transitions can happen only between rotational-vibrational levels of different electronic states [17].

Transitions in Cs_2 that cover the wavelength spectrum 0.9-1.4 μm belong to $X^1 \Sigma_g^+ \rightarrow A^1 \Sigma_u^+$ band. We will lock the laser to one of the ro-vibrational transitions in this *band system* [17].

2.2 Laser Spectroscopy

As we saw in section 2.1, the band system belonging to $X^1 \Sigma_g^+ \rightarrow A^1 \Sigma_u^+$ contains transitions near 1064 nm, thus this can be used to frequency stabilize 1064 nm laser. Doppler free spectroscopy of Cs_2 is precursor to extraction of error signal that is suitable for frequency stabilization. Sub-Doppler signals are essential for locking as this provides a narrower and larger error signal making it robust for frequency stabilization. In this section we will understand spectral-line broadening and principles behind SAS.

2.2.1 Spectral line broadening

Spectral lines are never monochromatic, there's a natural linewidth associated due to spontaneous emission [7]. This can also be illustrated as a consequence of uncertainty principle [18]. But extraction of absorption lines with a narrow linewidth involves special techniques.

This is because, absorption spectra is always fraught with Doppler background due to thermal motion of atoms or molecules being probed. Spectral lines in this case are said to be Doppler broadened [7]. Molecules in a vapour cell follow Maxwell-Boltzmann distribution for velocity. This causes different Doppler shifts for molecules with different velocities. Doppler background can be suppressed by sub-Doppler spectroscopy techniques like Saturation Absorption Spectroscopy [19]. Nevertheless, sub-Doppler spectral lines do not yield lines at their natural linewidth. They are always limited by collisional broadening (also called pressure broadening) and power broadening [7] [19]. Power broadening is reduced by keeping the pump beam (section 2.7 power at lowest possible level such that, sub Doppler signals are good enough and at the same time power broadening is minimal. Collisional broadening can be reduced by maintaining vapour cell at low vapour pressures.

2.2.2 Saturation Absorption Spectroscopy

Saturation Absorption Spectroscopy or SAS is a non-linear spectroscopy technique that exploits the condition when absorption of radiation by molecules under examination depends non-linearly on incident radiation (figure 2.3). This occurs upon optical pumping of two electronic levels involved.

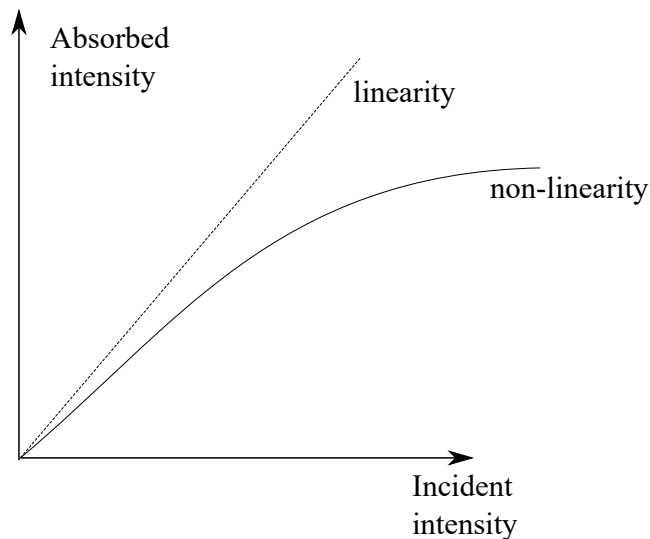


Figure 2.3: Deviation of absorption curve from linearity as the saturation intensity for a two level system is approached [19].

Hole-burning

Consider a vapour cell with molecules exhibiting a Doppler profile with central frequency ω_o . Pump beam of frequency ω , with intensity close to saturation intensity [19], passing through this vapour cell excites only molecules moving with velocity $\mathbf{v}$ such that the Doppler shifted frequency $\omega' = \omega - \mathbf{k} \cdot \mathbf{v}$ in the moving molecules frame of reference falls within the laser linewidth. This saturation of inhomogeneous line profile burns a hole in the population of ground state molecules as shown in figure 2.4. This is called *Bennet hole*. This can't be detected by just one laser. Saturation of the two level system, that burns a Bennett hole (as in figure 2.4) causes the absorption profile to broaden (figure 2.5). Thus detection of this hole is not possible with one laser beam passing through the vapour cell.

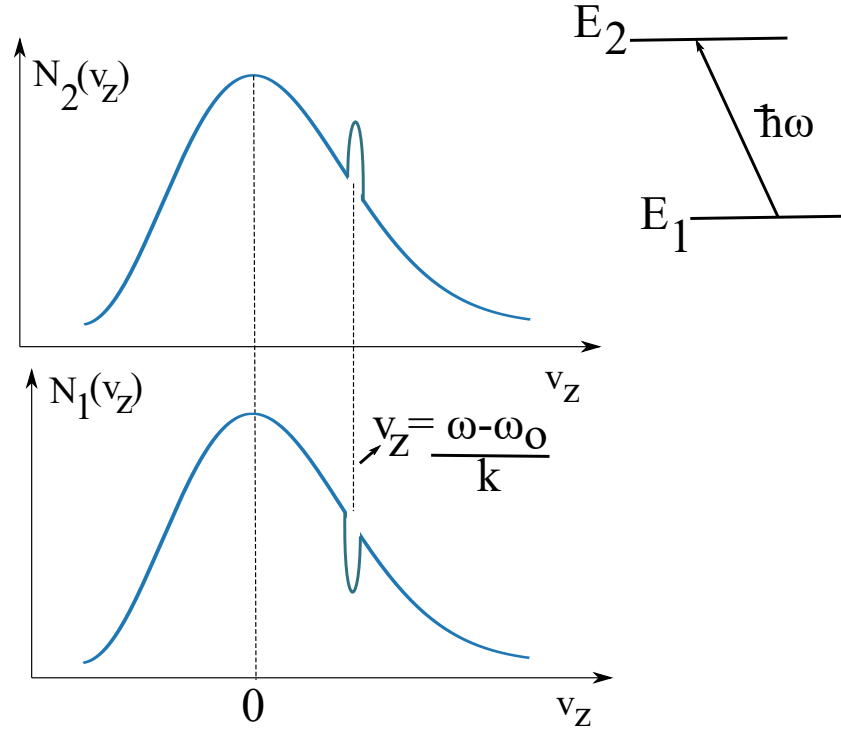


Figure 2.4: Here, N_2 and N_1 correspond to population density of energy levels E_2 and E_1 respectively. Hole-burning in population profile of N_1 when a pump beam of frequency ω and momentum vector $\mathbf{k}$ illuminates the vapour cell [19]. This is called *Bennet Hole* and the corresponding peak in N_2 is called *Bennet peak*.

This hole can be detected only when two laser beams are used. A weak probe beam of frequency ω_2 passing through a vapour cell along with a pump beam of frequency ω_1 as shown in the figure 2.6.

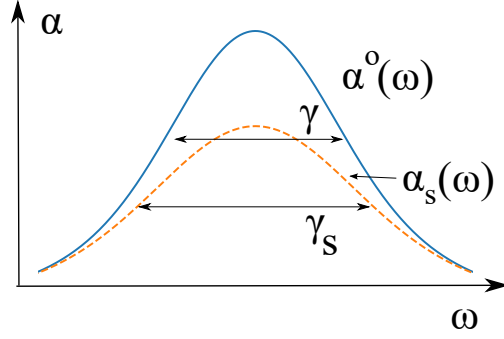
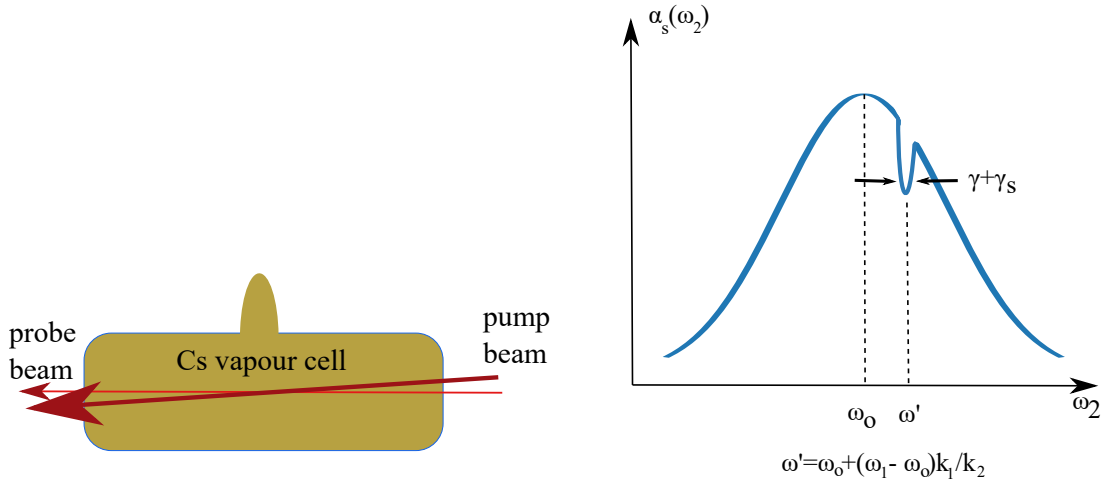


Figure 2.5: Absorption profile for the two level system (indicated in figure 2.4) when it's not saturated or not illuminated by a pump beam (blue solid curve $\alpha^o(\omega)$), and when it's saturated or illuminated by pump beam (orange dashed curve $\alpha(\omega)$) as detected by the pump beam upon scanning through the Doppler profile. Here γ and γ_s indicate widths of unsaturated and saturated absorption profiles respectively.



(a) Probe and pump beams co-propagating through a Cs vapour cell to enable detection of *Bennet dip*. (b) Bennett dip in the absorption spectra detected by the probe beam.

Figure 2.6: Bennett hole (figure 2.4) detected by a probe beam appears as a dip in the absorption spectrum. Probe beam is weaker than pump beam by at least a factor of ten. Here, ω_1 ($\mathbf{k}_1$) and ω_2 ($\mathbf{k}_2$) correspond to frequencies (wave-vectors) of pump and probe beams respectively. Here γ and γ_s indicate unsaturated homogeneous absorption width of probe beam and width of the saturated dip due to the strong pump beam respectively.

Lamb-Dip and Saturation Absorption Spectroscopy

When probe beam and pump beams from the same laser pass through the cell in opposite directions as in figure 2.7, a dip occurs in the Doppler absorption profile (figure 2.8). This sub-

Doppler feature that corresponds to a transition is called *Lamb dip* [19]. Intuitively, probe and pump beams interact with the same set of molecules only when they are tuned to the central resonance frequency of the Doppler transitions. This is a sub-Doppler spectroscopy technique and the width of the sub-Doppler profile or Lamb dip is limited by collisional broadening and power broadening (caused by the saturating pump beam).

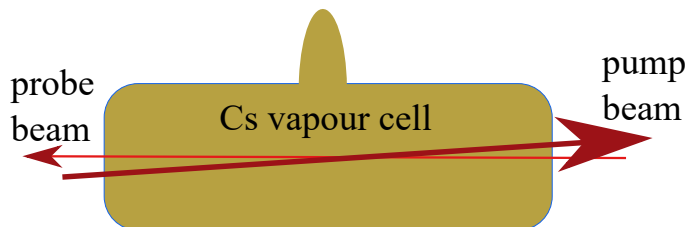


Figure 2.7: Pump and probe beams emerging from the same laser and anti-propagating in a vapour cell to facilitate sub-Doppler spectroscopy.

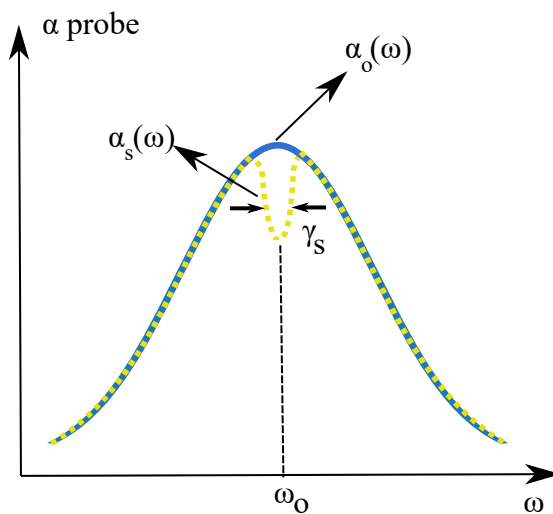


Figure 2.8: Lamb dip detected at a resonance frequency ω_o when the laser frequency is scanned. Here, yellow dotted curve $\alpha^S(\omega)$ indicates saturated absorption curve as detected by a probe beam and blue solid curve indicates unsaturated Doppler absorption curve $\alpha^o(\omega)$ [19]. γ_S is width of the saturated dip due to the strong pump beam.

Saturation absorption spectroscopy and subsequent locking of laser to a molecular transition involves performing sub-Doppler spectroscopy as described above followed by extracting an error signal. A technique called Frequency-Modulation spectroscopy [20] is employed to

perform this task. The optics setup for this task can be found in figure 6.1 of chapter 6. Probe beam entering the vapour cell is phase modulated at a radio frequency and pump beam is aligned such that it propagates anti-parallel to the probe beam. The extent of alignment directly determines the amount of stray Doppler broadening in the sub-Doppler signal. In reality, there is always some stray Doppler broadening. Pump beam is chopped using a chopper. Downstream the vapour cell, probe beam is then demodulated and sent to a lock-in amplifier that gets its reference from the chopper controller. Chopping the pump beam using an optical chopper helps remove Doppler background and amplify sub-Doppler error signals. A suitable error signal from one of the absorption lines is fed back to the laser, specifically to the piezo mounted on laser cavity crystal, and this helps achieve frequency stabilization and reduce laser frequency drift. In the next chapter we will present experimental details of pursuing SAS and frequency stabilization.

Chapter 3

Experimental Details

Now that we have briefly been through literature of SAS in the previous chapter, in this chapter we will look at the experimental details of the efforts made towards achieving absolute frequency stabilization of the laser. We will start by discussing about the laser that will be stabilized. We will discuss steps taken in developing Cs vapour cells (section 3.2). Three vapour cells were made during the course of the project. We will also present details about characterization of one of the vapour cells using an 852 nm diode laser (section 3.4).

3.1 The Laser

The laser we use during this project has an output wavelength of 1064 nm (Make: [Coherent](#), Model: [Mephisto](#), Power: 1W). This consists of a monolithic Nd:YAG (neodymium-doped yttrium aluminum garnet; $\text{Nd:Y}_3\text{Al}_5\text{O}_{12}$) crystal in NPRO (Non-Planar Ring Oscillator) configuration which behaves as a lasing medium (figure 3.1). and also as an optical resonator. The facets of the crystal are fabricated in such a way that polarization rotation from total internal reflections along with that due to externally applied magnetic field forces the lasing medium to impose unidirectional propagation of laser beam inside the crystal. An NPRO crystal thus reduces the number of optical elements in the laser oscillator system. An 808 nm laser diode is used to facilitate optical pumping of the gain medium. The laser crystal is temperature stabilized and provides slow temperature tuning and fast piezoelectric scanning of frequency. It's a single mode laser ($M^2 \leq 1.1$ [6]) operating at TEM_{00} mode [21]. It has

a narrow spectral linewidth of about 1 kHz range over a 100 ms time frame. It offers a mode-hop free scanning range of upto 7 GHz. Frequency drift of the laser measured over a ten minutes time duration is 30 MHz (can be found in figure 3.2).

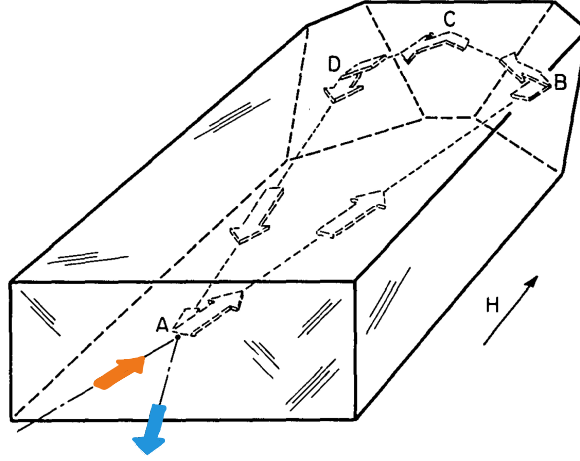


Figure 3.1: Schematic of an NPRO crystal as a monolithic laser resonator . Here the orange arrow indicates 808 nm input beam on facet A used for pumping the gain medium; blue arrow indicates 1064 nm output beam and dotted arrows indicate beam undergoing total internal reflection in the Nd:YAG monolithic crystal and exiting the crystal represents 1064 nm output beam. Facets A, B, C and D are those involved in creating the cavity system. Total internal reflection on these facets occurs such that the optical path followed is not in a plane, hence called Non-Planar Ring Oscillator (NPRO) laser. Magnetic field (H) is applied so that the crystal also behaves as a Faraday isolator thus ensuring unidirectional propagation of light inside the resonator (figure adapted from Fig. 1 of [22]).

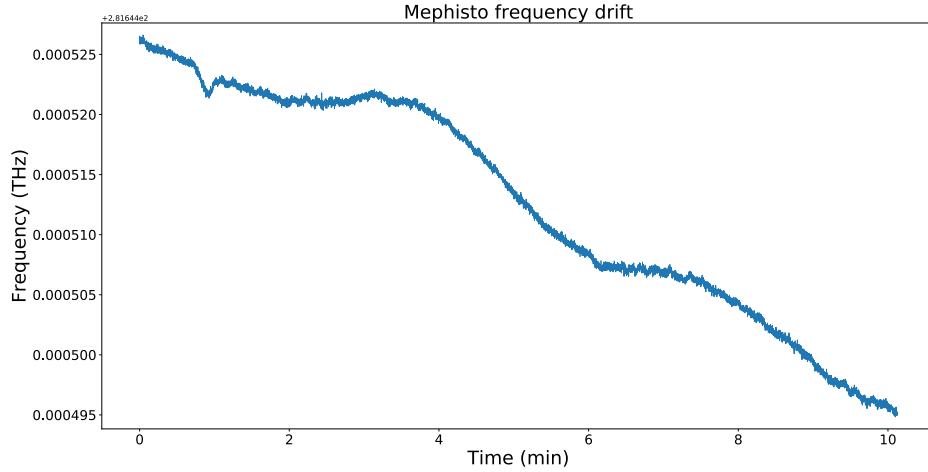


Figure 3.2: Frequency drift of the laser measured by a HighFinesse wavemeter (WSU-844) with an accuracy of about 10 MHz.

The Free-Spectral Range (FSR) of this laser is 2 GHz. Thus from the equation $\text{FSR} =$

$c/2L$ where c is the speed of light in the medium and L is the round trip resonator length [21], the round-trip resonator length is 7.5 cm, which gives us an idea of the littleness of the lasing medium as compared to the 20cm $\times$ 15 cm $\times$ 11 cm large laser head. The laser offers a slow temperature frequency scanning range of about 30 GHz with only two to three mode-hops and a fast piezo frequency scanning range of 130 MHz.

3.2 Development of vapour cell

3.2.1 Glass manifold

Good quality borosilicate glass was used to prepare glass manifold (figure 3.3) with the help of a local glass blower ¹.

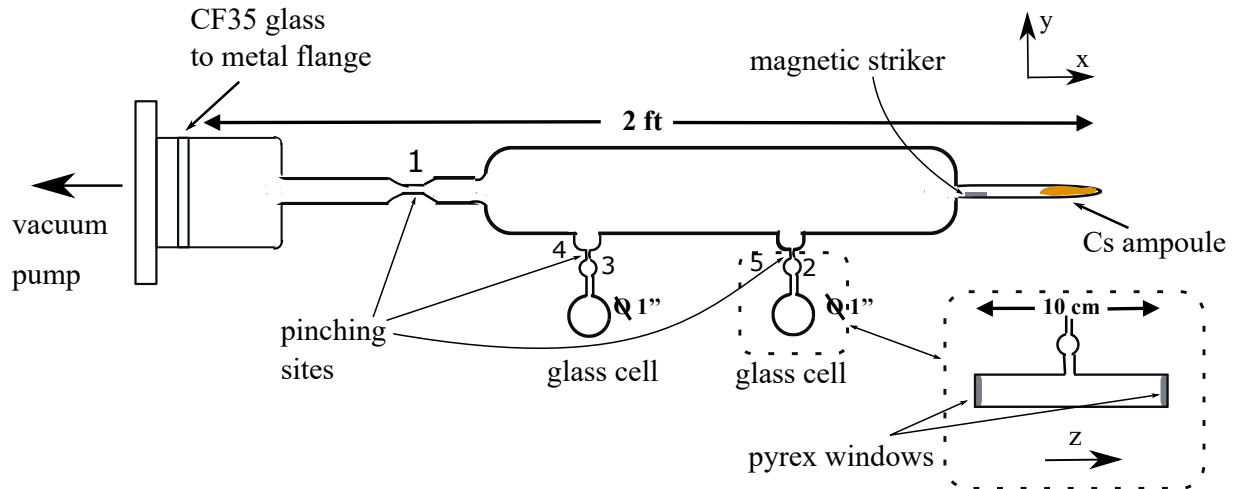


Figure 3.3: Borosilicate glass manifold design to prepare vapour cells. Inset shows cell which is detached from the manifold upon further processing to form vapour cell.

The two cylindrical glass cells are of 10 cm in length each. High quality pyrex windows/optical flats procured from [Applied Optics Inc.](#) were fused on to the ends of the by the glass blower (inset in figure 3.3). The windows are of 1" diameter and 2 mm thickness, with standard $\frac{\Lambda}{10}$ polishing. They have to be fused carefully avoiding any deformations lest the

¹Mr. Suresh K. Nair, Balewadi, Pune

cell disturbs collimation of incident laser beam. Locations (1), (4) and (5) in figure 3.3 are pinching sites that help detach certain sections of the manifold from the parent manifold. Bulbs (2) and (3) are made to provide enough surface area to maintain or control the temperature of the stem which is cold point of the vapour cell. CF35 glass to metal flange (3.3) fused at the head of the manifold facilitates connection of the manifold to vacuum pump station. 1 gram Cs ingot (≥ 99.5 trace metals basis) procured from Sigma Aldrich was used to prepare two cesium vapour cells. This borosilicate glass ampoule is fused to the rear side of the manifold. The ampoule consists a breakable seal that isolates Cs from atmosphere. A magnetic striker placed in front of the ampoule is used to break the seal once a vacuum of about 10^{-8} is reached. Since Cs is very reactive, the magnetic striker is made by sealing a ferromagnetic metal in a small glass cell. This avoids potential degradation of the vapour cell due to reaction of Cs with impurities in the metal striker.

3.2.2 Cleaning of the manifold

Sonication and thorough rinsing of the manifold is a must to remove moisture, grease and other impurities accumulated in the glass during glass blowing. The manifold is initially sonicated with normal grade toluene and acetone. Toluene has the ability to dissolve greasy impurities. The manifold is then rinsed with Dil. HCL, HPLC grade acetone, HPLC grade toluene, and ethanol. The glass manifold is then heated thoroughly using a hot air gun to ensure that no liquid remnant from cleaning is present. This is to avoid any load on the vacuum pumping system. The manifold must be closed with aluminium foil to avoid dust particles from entering, until further use.

3.2.3 Preparing Vacuum

Vacuum pump system

Making a cesium vapour cell requires high vacuum of the order of 10^{-8} torr. The vacuum pump system in lab consists of scroll pump and turbomolecular pump. Scroll pump reduces pressure of the cell to about 10^{-2} torr and turbomolecular pump initiates pumping to further evacuate the cell to lower orders of magnitude. Angle valve attached to the glass to metal CF35 flange helps preserve vacuum during detaching of vapour cell from the vacuum pump

station post to evacuation. The manifold is connected to the vacuum pump and to initiate pumping. Once pressure inside the manifold reaches a certain saturation point, baking is done to reduce the base pressure further.

Baking

It is wrapped completely with aluminium foil. The system is then wrapped with heating tapes with the help kapton (heat resistant) tape. Now the system is ready to be baked. This facilitates degassing of adsorbed impurities, thus accelerates the process of pumping. It also reduces the base pressure. Heating was started when vacuum pressure reached a saturation point. Heating was done by gradually increasing the temperature at steps of 40 °C to 150 °C over a period of two days and then kept at this temperature for about a day under continuous vacuum pumping. One thermocouple is placed on the manifold to monitor temperature. Once the manifold reaches its lowest base pressure, the manifold is allowed to cool down gradually. This is done to ensure the glass to metal flange doesn't experience high temperature gradients lest the connection is disrupted. After the manifold reaches room temperature, heating tape and aluminium foil around the manifold are removed.



Figure 3.4: Cesium glass manifold. Cesium can be seen as golden coloured liquid spread through out the manifold.

Preparing cesium vapour cells

The magnetic striker in (figure 3.3) is used to break the seal of the ampoule. Care should be taken not to let cesium flow into cells lest the cells are inundated with cesium. Desired

amount Cs is then directed to the two vapour cells with the help of hot air gun and ice bath. The cells are placed in an ice bath while rest of the manifold is heated with a hot air gun. The gun should be kept at a safe distance from the glass structure lest abrupt local heating causes glass to crack. The vapour cells are then pinched off at locations (3) and (5) using oxyflame. Cesium vapour cells are now ready for spectroscopy!

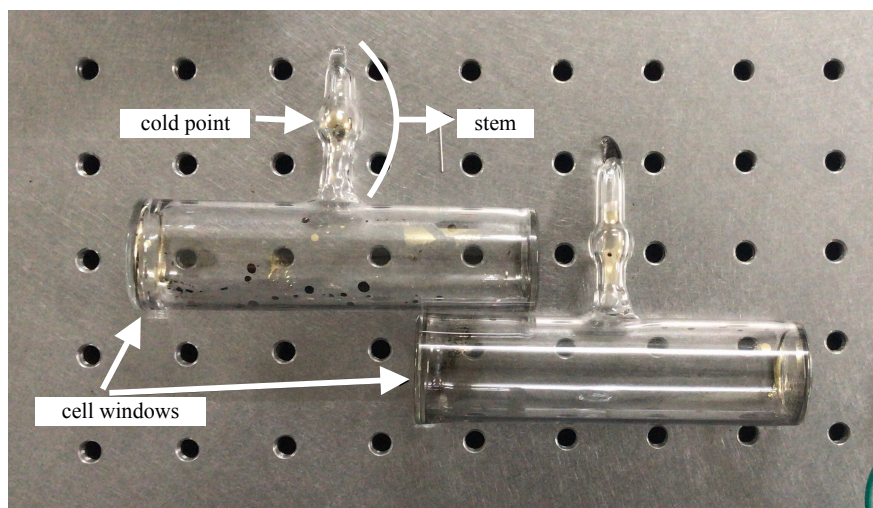


Figure 3.5: Vapour cells created after pinching off from the [glass manifold](#). Here the cell windows are formed the optical flats that are used to preserve laser beam shape. The stem of the cells contains the coldest part of the cell.

3.3 Heating System

As we know, Cs_2 molecules are formed at temperatures above 150°C . This requires controlled heating of the cell. The cell was wrapped with kapton tape and several layers of aluminium foil to maximize thermal contact between glass and the enclosure. One should ensure minimal air gap between the layers of aluminium foil to avoid inefficient heating. A high quality nichrome wire is wound around stem of the cell. This part of the cell is maintained at the lowest temperature and is called the cold point. The windows and the stem are heated by two separate variacs.

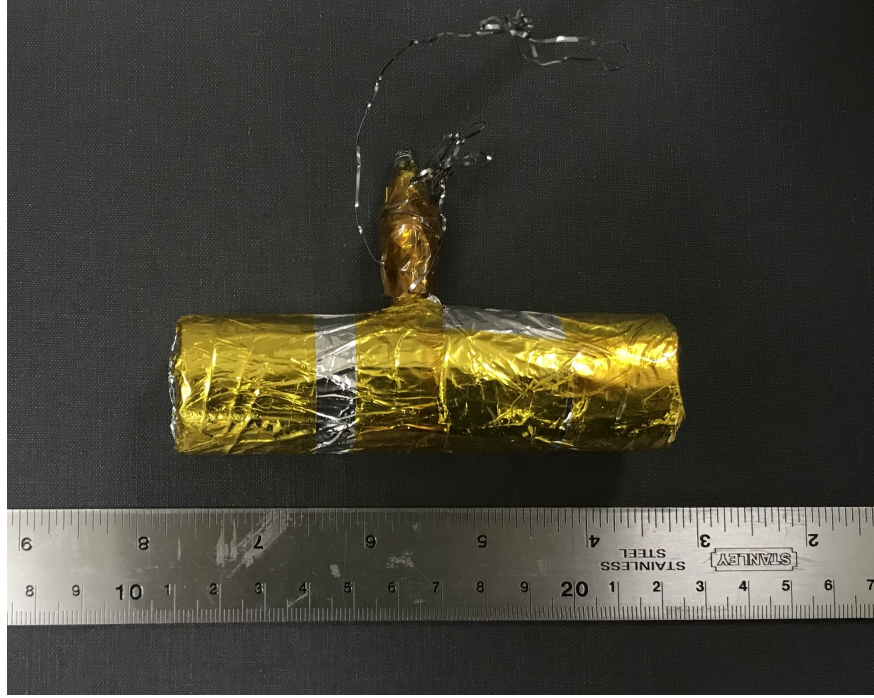


Figure 3.6: Vapour cell wrapped with multiple layers of aluminium foil. Nichrome wire is wound around the stem to facilitate its independent heating. Yellow material on the cell is kapton tape used to keep in place the aluminium foil wrapped around the cell.

An aluminum enclosure (figure 3.7) was designed² to house the vapour cell. The Cs cell snugly fits into these cap-like enclosures. 10 mm diameter holes on the faces of these caps enabled passage of laser beam through the cell. The resultant assembly ensured uniform heating of the cell.

Locally purchased heating bands of about $170\ \Omega$ resistance was placed around the windows. The windows are at relatively elevated temperature than the rest of the cell and this ensures that Cs does not deposit on the windows. All the Cs eventually accumulates in the cold point region. Aluminium foil is wrapped around the window region covering the heating bands, such that shiny surface is facing inside. This ensures minimal convection heat loss (figure 3.8). The system is mounted on a [kinematic prism mount](#) from Thorlabs. This gives freedom to adjust pitch, yaw and height of the vapour cell. The cell is placed on calcium silicate platform to thermally insulate from the platform. The top portion of the cell is wrapped with aluminium silicate glass wool, thus ensuring thermal insulation from the atmosphere. Kapton tape is used to hold the wool in place.

²Made by Fourvac Technologies, Pune.

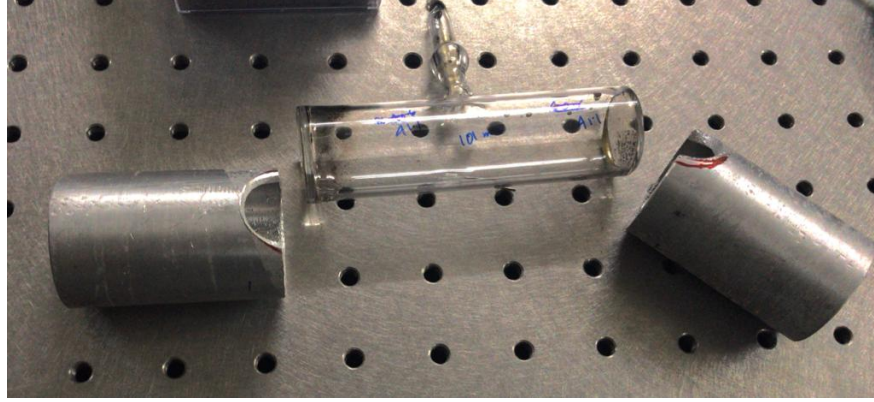


Figure 3.7: Aluminium enclosure caps for the vapour cell. They are designed so that the cell snugly fits into the caps. Cell shown in figure 3.6 goes into these caps such that contact between the cell and the enclosure is significant and uniform.

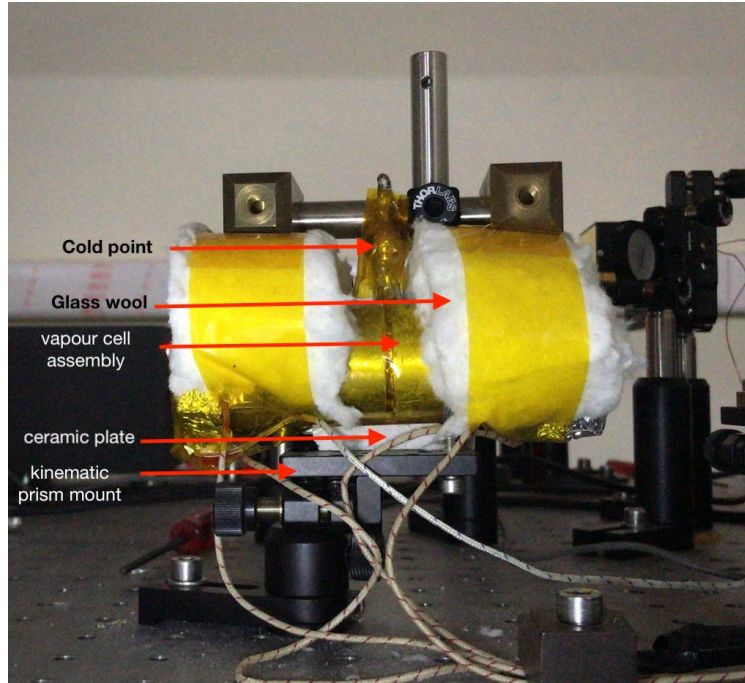


Figure 3.8: This figure shows the vapour cell assembly with heating bands placed on the windows. The whole ensemble is wrapped with glass wool to insulate from the atmosphere. The system is mounted on a kinematic prism mount that is positioned on optical table. In addition, two thermocouples are attached (not shown in figure), one on either of the windows and one on the cold point. Kapton tape or heat tape (yellow) is used to hold components of the assembly in place.

A [PID](#) in ON-OFF mode in conjunction with a solid state relay was used to heat the vapour cell in a controlled manner.

3.4 Characterization of vapour cell

Now that we have the vapour cell ready, the next step is to characterize the cell using an 852 nm diode laser. We exploit the D₂ line of Cs atoms to complete this task. Only first of the vapour cells was characterized and other two cells were not characterized due to obstacles faced due to the pandemic.

3.4.1 Setting up 852 nm laser diode

An 852 nm³ tunable External resonator Diode Laser (ECDL) is used to characterize the vapour cell. Laser diode is coupled to an external resonator that incorporates a diffraction grating and a collimating lens, this assembly is called the Littrow configuration [7]. Here, the grating acts as end mirror of a linear resonator. Wavelength of the emitted laser beam can be tuned by rotating the diffraction grating about a pivot point (figure 3.9).

$$2 \times \sin\theta_{in} = m \times \frac{\Lambda}{2} \quad (3.1)$$

First order diffracted beam is sent back along the incident beam to the diode through the anti-reflective coating on the diode laser's surface for feedback purpose. We used a grating of groove density 900 lines/mm.

3.4.2 Cesium D-Line

Cesium ([Xe]6s¹) is a chemical element with single valence electron. Spin-orbit coupling causes electron excitation pathway from ground state of cesium to the excited state ($L = 0 \rightarrow L = 1$) to split into two: $6^2S_{1/2} \rightarrow 6^2P_{3/2}$ and $6^2S_{1/2} \rightarrow 6^2P_{1/2}$. These correspond to fine structure doublet of cesium. Coupling between orbital angular momentum $\mathbf{L}$ and Spin angular momentum $\mathbf{S}$ of the valence electron gives rise to energy level splitting owing to different values of $\mathbf{J}$ defined as [23]:

³Element14

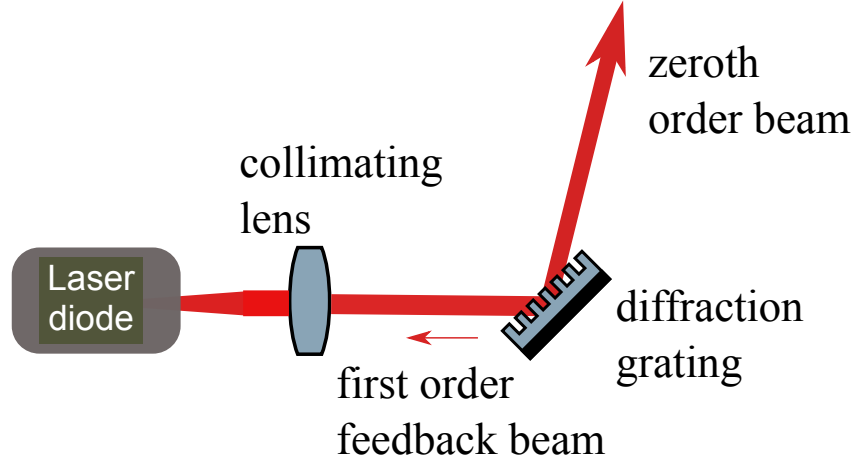


Figure 3.9: Schematic diagram of a diode laser in Littrow configuration. The red arrows indicate laser beam path.

$$\mathbf{J} = \mathbf{L} + \mathbf{S} \quad (3.2)$$

$$|L - S| \leq J \leq |L + S| \quad (3.3)$$

Thus single excitation pathway $L=0 \rightarrow L=1$ splits into fine structure $L = 0, J = \frac{1}{2} \rightarrow L = 1, J = \frac{1}{2}$ and $L = 0, J = \frac{1}{2} \rightarrow L = 1, J = \frac{3}{2}$ or $6^2S_{1/2} \rightarrow 6^2P_{3/2}$ (D_2) and $6^2S_{1/2} \rightarrow 6^2P_{1/2}$ (D_1). Here the excited energy level split into two states bases on different values angular momentum number $\mathbf{J}$.

Coupling between $\mathbf{J}$ and nuclear angular momentum $\mathbf{I}$ leads to further splitting of energy levels called Hyperfine splitting. The governing total atomic angular momentum $\mathbf{F}$ is given by:

$$\mathbf{F} = \mathbf{I} + \mathbf{J} \quad (3.4)$$

Similar to equation (3.3),

$$|J - I| \leq F \leq |J + I| \quad (3.5)$$

Now, the ground state of cesium corresponds to $J = \frac{1}{2}$, $I = \frac{7}{2}$ so $F = 3$ or $F = 4$. And the fine structure excited state $6^2P_{1/2}$ splits into states that take values $F = 3, 4$ while $6^2P_{3/2}$ splits into states that can take values $F = 2, 3, 4$, or 5 . While energy levels are shifted according

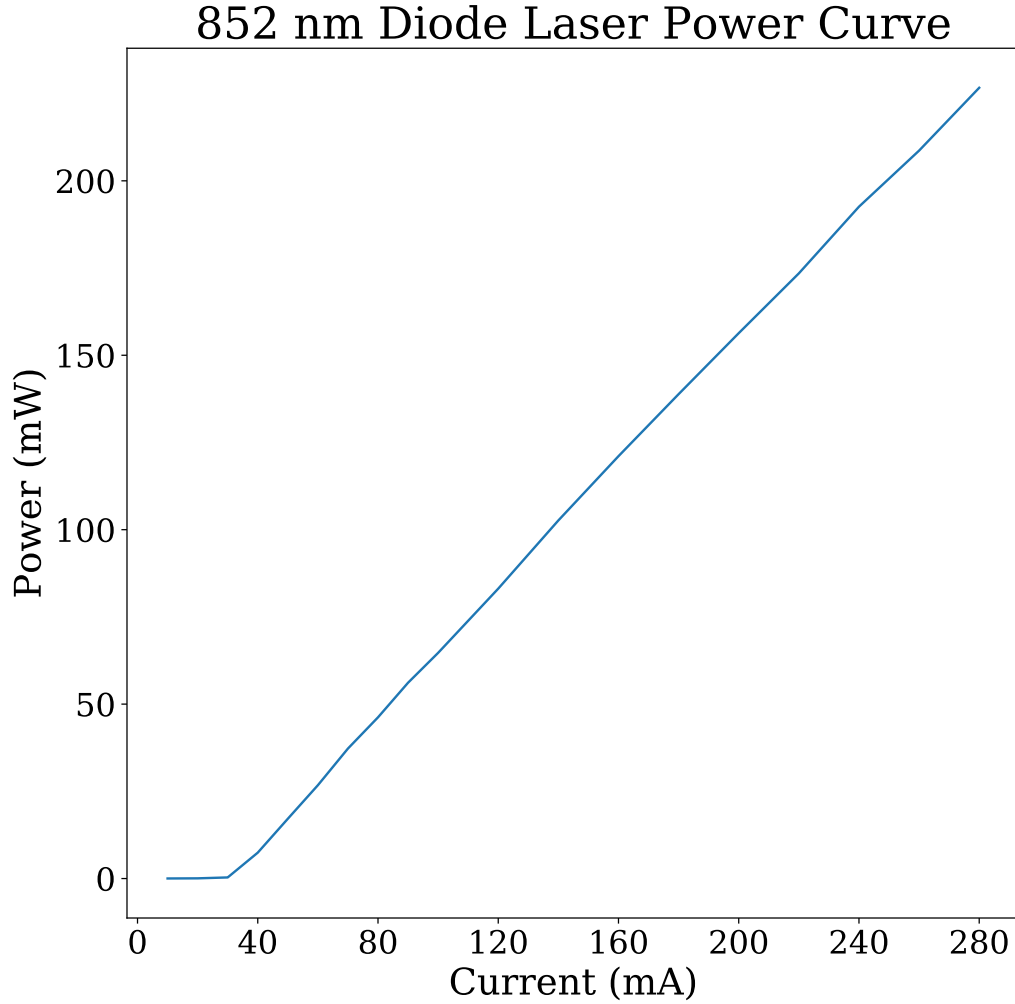


Figure 3.10: 852 nm diode laser power curve. The threshold is where the laser begins to lase. Here the threshold current is about 35 mA.

to J values in fine structure, splitting owes to different F values in hyperfine structure.

The D lines in cesium correspond to frequencies 335.116048 THz (D_1) and 351.725718 THz (D_2). Energy level diagram for (D_2) transition can be found in figure 3.11. Transitions between these hyperfine structures is used to characterize the vapour cell.

As described in section 3.4.1, an 852 nm laser diode is used to detect fluorescence as-

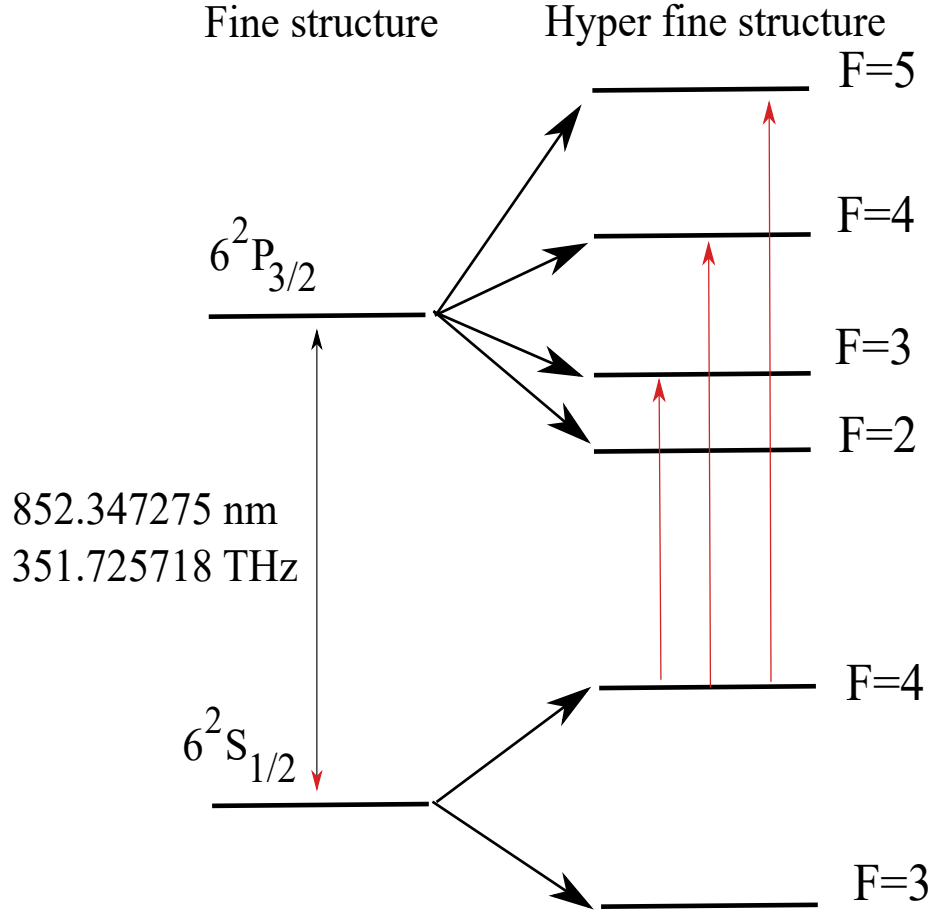


Figure 3.11: Cesium D₂ transition hyperfine structure, with frequency splittings between the hyperfine energy levels [23]. The red arrows indicate transitions involved in fluorescence in figure 3.12 and absorption dips in figure 3.13.

sociated with hyperfine transitions. Fluorescence detected by an IR camera is shown in figure 3.12. This corresponds to the transitions Cs $6^2S_{1/2}(F = 4) \rightarrow 6^2P_{3/2}(F = 3, 4, 5)$ (figure 3.13) [24]. This is a Doppler profile, the hyperfine transitions are not resolved. This transition will be used to characterize the vapour cell⁴.

⁴Here the first vapour cell was used.

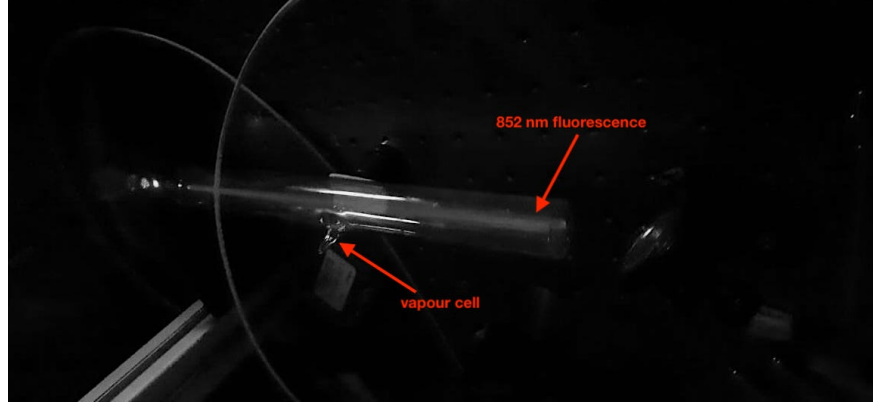


Figure 3.12: Cs fluorescence pictured with an IR camera. Fluorescence was detected at 852.35763 nm at room temperature.

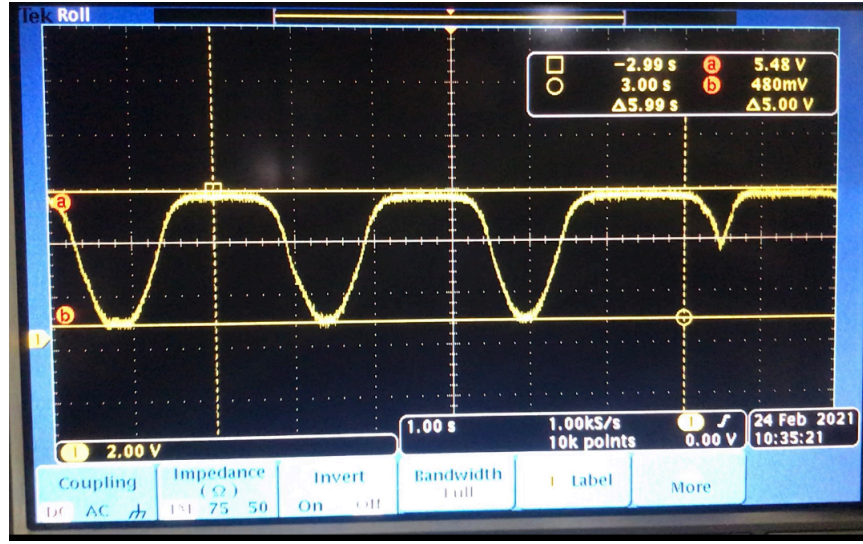


Figure 3.13: Cs $6^2S_{1/2} (F=4) \rightarrow 6^2P_{3/2} (F=3, 4, 5)$ absorption dip seen on oscilloscope as the piezo is scanned through about 300 MHz with a central frequency of 351.72144 THz

3.4.3 Vapour density estimation

Cs follows the following vapour pressure model in liquid phase [23]:

$$\log_w P_v = 8.22127 - \frac{4006.048}{T} - 0.00060194T - 0.19623 \log_{10} T \quad (3.6)$$

and the following model in solid phase:

$$\log_{10} P_v = 219.48200 + \frac{1088.676}{T} - 0.08336185T + 94.88752 \log_{10} T \quad (3.7)$$

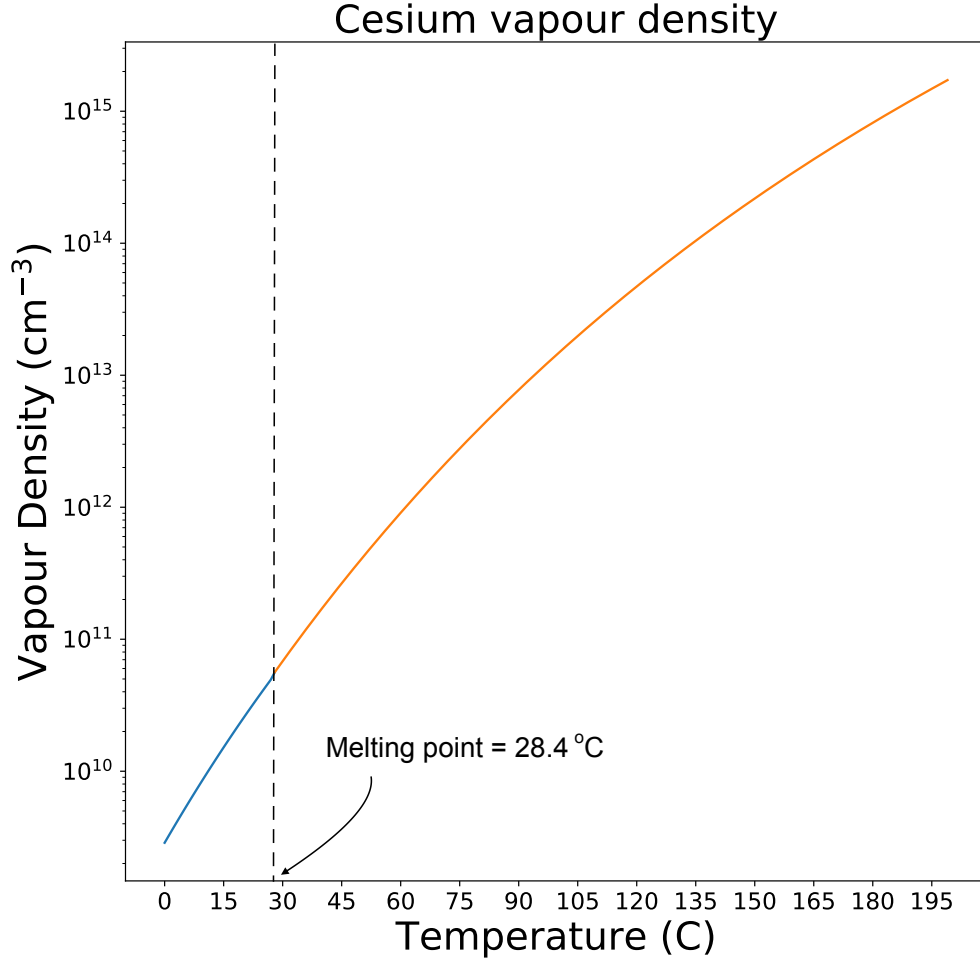


Figure 3.14: Vapour density of cesium from equations (3.6) and (3.7). Here, the blue curve corresponds to the latter equation while the red curve corresponds to the former equation.

where the vapour pressure P_v is in torr and temperature T is in kelvin (figure 3.14). The above equation in conjunction with the ideal gas equation $P = nk_bT$ (n : number density, k_b : boltzmann constant, T : temperature in kelvin, P : pressure in pascals) gives a rough estimate of density of Cs in a vapour cell. A more rigorous way to estimate vapour density is to utilize Beer-Lambert law.

$$-\log_e \frac{I}{I_o} = \alpha \quad (3.8)$$

where α is the absorption coefficient, I is the transmitted intensity, I_o is the incident

intensity and l is the absorption path length. A photodiode is placed downstream the vapour cell to measure the transmitted intensity. It should be noted that since a photodiode measures power but equation (3.8) demands intensity, laser beam should be collimated so that power measured upstream and downstream the vapour cell can be directly substituted into equation. Absorption coefficient α is given by

$$\alpha = [Cs] \times \sigma \times l \quad (3.9)$$

where $[Cs]$ is number density of cesium, σ is the resonant scattering cross section and l is length of absorption media or the vapour cell. Scattering cross section σ for π -polarized light is $2.313 \times 10^{-9} \text{ cm}^2$ [23]. Substituting this into equation (3.9) gives value for $[Cs]$.

Temp. (K)	Temp. (C)	log P	P (torr)	P (pascal)	[Cs] from the model (cgs)
463	180	-1.233	0.059	7.805	$1.22E + 15$
403	120	-2.473	0.003	0.449	$8.07E + 13$
373	90	-3.248	0.001	0.075	$1.46E + 13$
307	24	-5.530	0.000	0.000	$9.29E + 10$

Table 3.1: Table shows Cs vapour density ($[Cs]$) in cm^{-3} derived from empirical relations (3.7) and (3.6). Here α is absorption coefficient.

Temp. (K)	Temp. (C)	% Transmission of laser light	α	[Cs] from data (cgs)
463	180	4.65	3.068	$8.88E + 06$
403	120	5.71	2.863	$8.29E + 06$
373	90	10.00	2.303	$6.66E + 06$
307	24	57.00	0.562	$1.63E + 06$

Table 3.2: Table shows Cs vapour density ($[Cs]$) in cm^{-3} calculated from Beer-Lambert law (equations (3.8) and (3.9)). Here α indicates absorption coefficient.

Tables 3.1 and 3.2 show that $[Cs]$ values derived from experiment do not concur with those derived empirically. Let's try to locate the reason for this discrepancy. It should be noted that there is significant amount of Cs in the vapour cell. The scattering cross-section for the Cs D-line with frequency detuning from the resonance frequency is [23]:

$$\sigma = \frac{\sigma_0}{1 + 4(\Delta/\Gamma)^2 + (I/I_{\text{sat}})} \quad (3.10)$$

where Δ is frequency detuning from the resonance frequency, Γ is spectral width of the transition line. This indicates that for a intensity beam, even a deviation from the resonance

frequency of 3Γ would lead to reduction of the cross-section and also α (from equation (3.9)) by a factor of 37. So, there is no inaccuracy in the calculations that could lead to such a discrepancy. On the other hand, the vapour cell was baked at 150 °C for about two days during evacuation. But before characterizing the cell, it was persistently heated to 200 °C. This could have led to significant out-gassing of contaminants adsorbed on the glass, thus leading to decrease in partial vapour pressure of Cs. This in-turn leads to decrease in Cs number density thus justifying the number density as derived from the experiment. Moreover, equations (3.6) and (3.7) may not be apt for use in degraded vacuum conditions.

Now that we have prepared the vapour cell, the next step is setting up the optics to perform spectroscopy of Cs₂ molecules.

3.5 Optical Setup

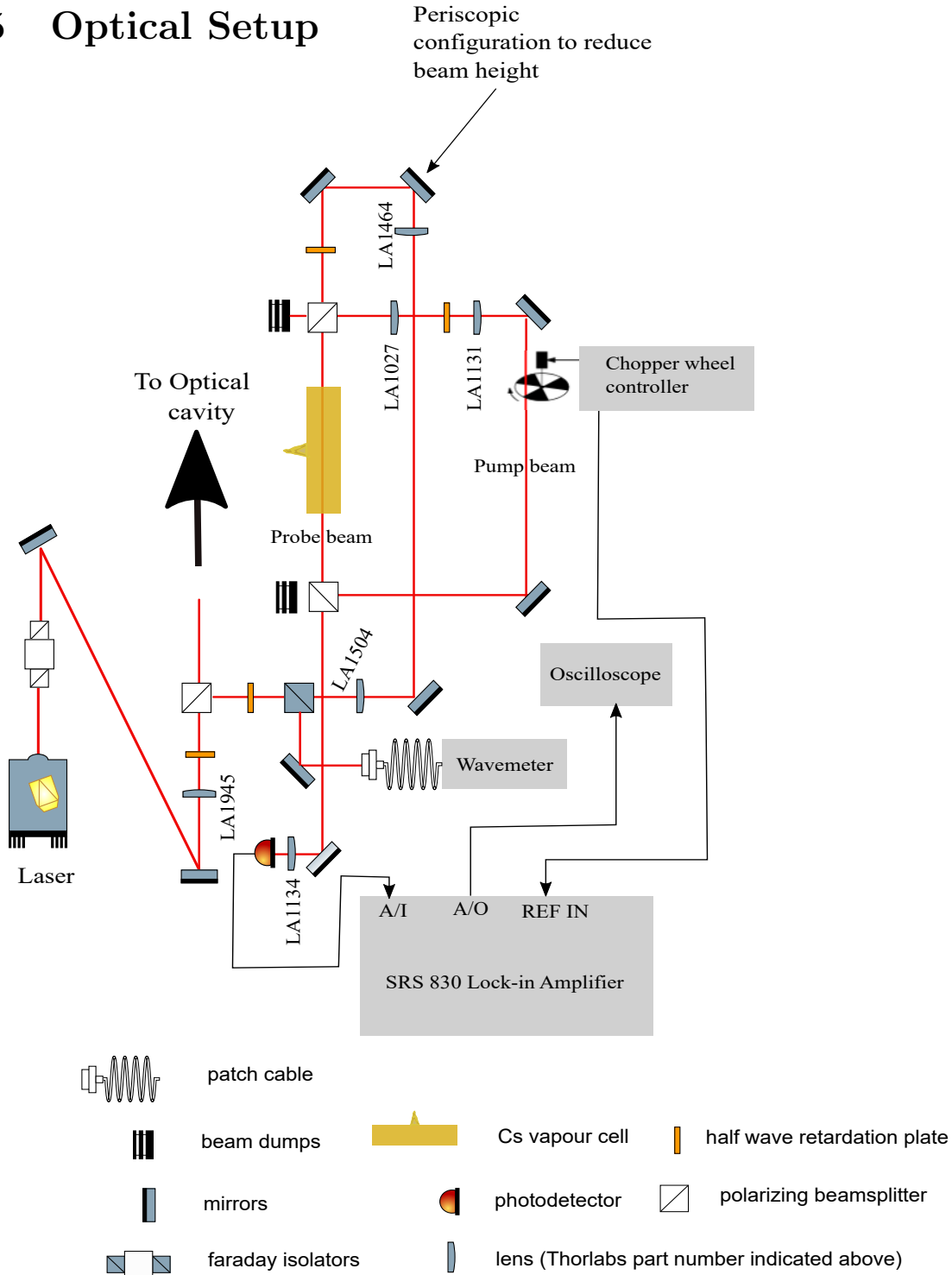


Figure 3.15: This diagram shows optics setup for SAS. A small amount of power from the input beam is picked up to read wavelength of the laser. A probe beam (lower intensity) sent through the vapour cell while a pump beam (higher intensity) is sent anti-parallel to it and overlapping it. Certain combination of lenses are used to optimize pump-probe beams' overlap. The probe beam is detected by a photodiode as the laser frequency is temperature scanned. Signal from the photodiode is sent to a lock-in amplifier to detect SAS signals.

Optics setup for Cs₂ spectroscopy is shown in figure 3.15. This setup shows optics to enable lock-in detection of Lamb dips. Lenses from Thorlabs- LA1509 ($f = 100$ mm) and LA1464 ($f = 1000$ mm) - are used to collimate the beam to a spot size of 1.9 mm. This beam was split at the PBS facing the front end of the vapour cell into probe (p-polarized) and pump (s-polarized) beams. Combination of lens LA1951 ($f = 25$ mm) and LA1131 ($f = 50$ mm) is used to collimate the pump to a spot size of 3.5 mm. Size of the pump beam is larger than that of the probe beam which ensures good overlap of the probe and pump beam. The pump beam is chopped by a chopper at a frequency of few hundreds of Hz. Output from the detector fast photodiode was sent to the Lock-in AN/IN port while the Lock-in received REF/IN from the chopper controller. Output from the Lock-in was observed on an oscilloscope. This procedure is a precursor to Frequency-Modulation (FM) spectroscopy technique [20], which further involves phase modulating the probe beam and demodulating it to extract error signal. This error signal is then used to lock the laser to vapour cell a vapour cell transition. Beam dumps and black out papers were placed at appropriate locations to extinguish stray or scattered pump light from reaching the photodiode. The aforementioned beam sizes gives us a pump power of about 48 mW for saturation to happen [20]. Probe power is kept below 1 mW to avoid unwanted saturation effects. One of the PBSs is used to split the beam power such that a few tens of μ W of power is picked off to enable wavelength reading with the wavemeter.

3.6 Data Acquisition

3.6.1 LabView Program

A LabView program was written to interface with the wavemeter software and acquire Cs spectrum data with laser frequency scan in real time.

3.6.2 Python Program

A python program was written that enabled processing of data acquired by the LabView software. Cs₂ spectrum signal was acquired over at least 10 traces and averaged using this program.

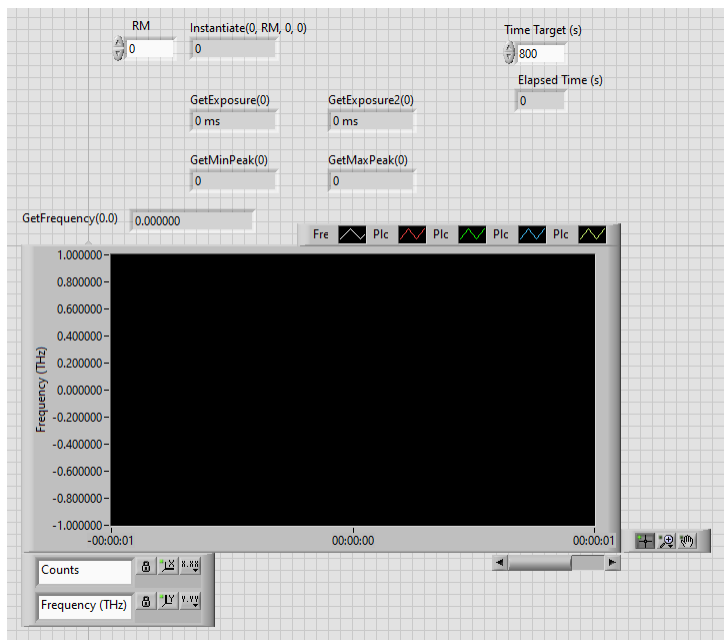


Figure 3.16: LabView program block diagram. The graph here shows frequency scan plot as measured by the wavemeter.

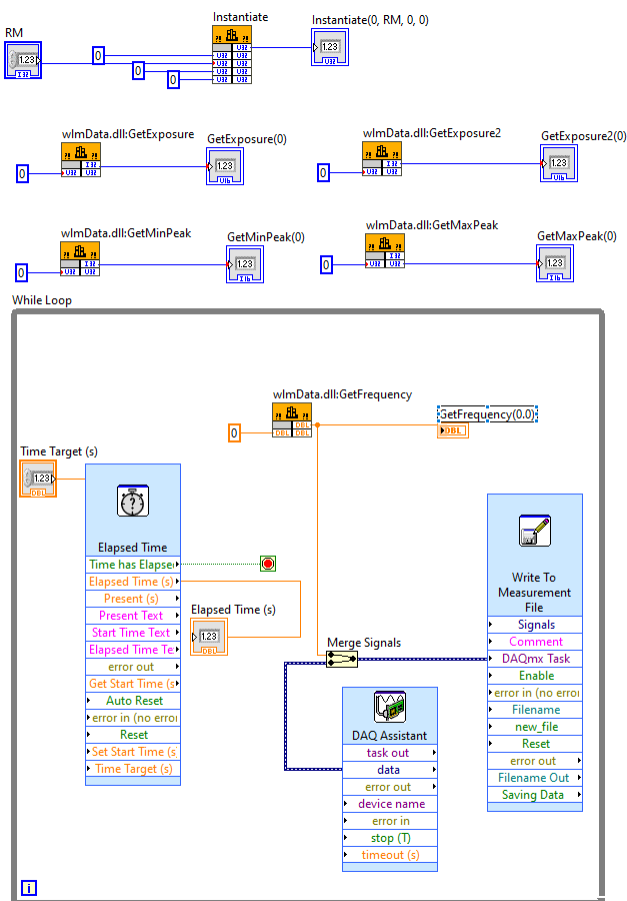


Figure 3.17: LabView program block diagram. Here, LabView software communicates with the wavemeter with the help of a .dll library. The DAQ acquisition VI was run in 1 Sample (on Demand) mode. The while loop ran till 10-20 traces of frequency-temperature scans were completed.

```

1 import pandas as pd
2 import numpy as np
3 import matplotlib.pyplot as plt
4 from matplotlib.ticker import FormatStrFormatter
5 from scipy.constants import c
6
7 ## Loading data file saved by the LabView program
8 file= '15'
9 df =pd.read_csv(r'C:\Users\Admin\Documents\Thejas\Cs_2\doppler data\DAQ\
    Cs2_{}.lvm'.format(file), sep= "\t", header= None)
10 df.rename(columns = {0:"Time",1:"Freq",2:"Signal"}, inplace="False")
11 del df["Time"]
12
13 ## Grouping signal values from several traces according to their frequency
    values and taking mean of signal values within every group to give us
    average frequency vs. signal data over several traces
14 grouped_df = df.groupby("Freq")
15 mean_df = grouped_df.mean()
16 mean_df = mean_df.reset_index()
17 print (mean_df)
18 print (mean_df['Freq'].min(), mean_df['Freq'].max())
19
20 ## Offset correction
21 mean_df['Signal']= mean_df['Signal']-0.016288
22 mean_df.to_csv(r'C:\Users\Admin\Documents\Thejas\Cs_2\doppler data\DAQ\
    Cs2_{}.csv'.format(file), index= True)
23
24 ## Plotting frequency vs. photodiode signal
25 df =pd.read_csv(r'C:\Users\Admin\Documents\Thejas\Cs_2\doppler data\DAQ\
    Cs2_{}.csv'.format(file), usecols= ['Freq', 'Signal'])
26 x=df['Freq'].tolist()
27 y=df['Signal'].tolist()
28 fig = plt.figure(figsize=(20,7) )
29 ax = fig.add_subplot(111)
30 ax.set(ylabel = 'Photodiode Signal (V)', xlabel='Frequency (THz)', title='
    Cs\u2082 Doppler lines')
31 plt.xlabel('Frequency (THz)',fontsize=18)
32 plt.ylabel('Photodiode Signal (V)', fontsize=18)
33 plt.title('Cs\u2082 Doppler lines', fontsize=20)
34 ax.yaxis.set_major_formatter(FormatStrFormatter('%.6f'))
35 ax.xaxis.set_major_formatter(FormatStrFormatter('%.6f'))

```



```
36 ax.scatter(x,y)
37 start, end = ax.get_xlim()
38 ax.xaxis.set_ticks(np.arange(start, end, 0.001))
39 fig.savefig(r'C:\Users\Admin\Documents\Thejas\Cs_2\doppler data\DAQ\Cs2_
    {}.jpg'.format(file))
```

Chapter 4

Cs₂ Spectroscopy Results

Figure 4.1 indicates Doppler spectrum of Cs₂ for an arbitrary frequency range. The Doppler features become prominent at 140 °C indicating that Cs → Cs₂ between 130 °C and 140 °C. The curves here slant downwards with scanning frequency, that alludes to a systematic decrease in power with temperature scan of the laser frequency measurement conditions:

1. Photodiode gain: 0 dB
2. Incident laser power: 1 mW

Figures 4.2 and 4.3 are Doppler profiles for the full temperature-frequency scan range of the laser's frequency. Run 1 was acquired with the second vapour cell, while Run 2 was acquired with the third vapour cell. The frequency range in both the spectrum is slightly different. Between the two runs, a higher than advised voltage was applied to the laser piezo scan. This could have displaced the piezo sitting on the laser crystal that facilitates frequency scan, thus causing shift in the laser frequency scan range as observed in the second run. The curves in these figures are normalized with respect to the change in responsivity of the photo-detector with laser frequency.

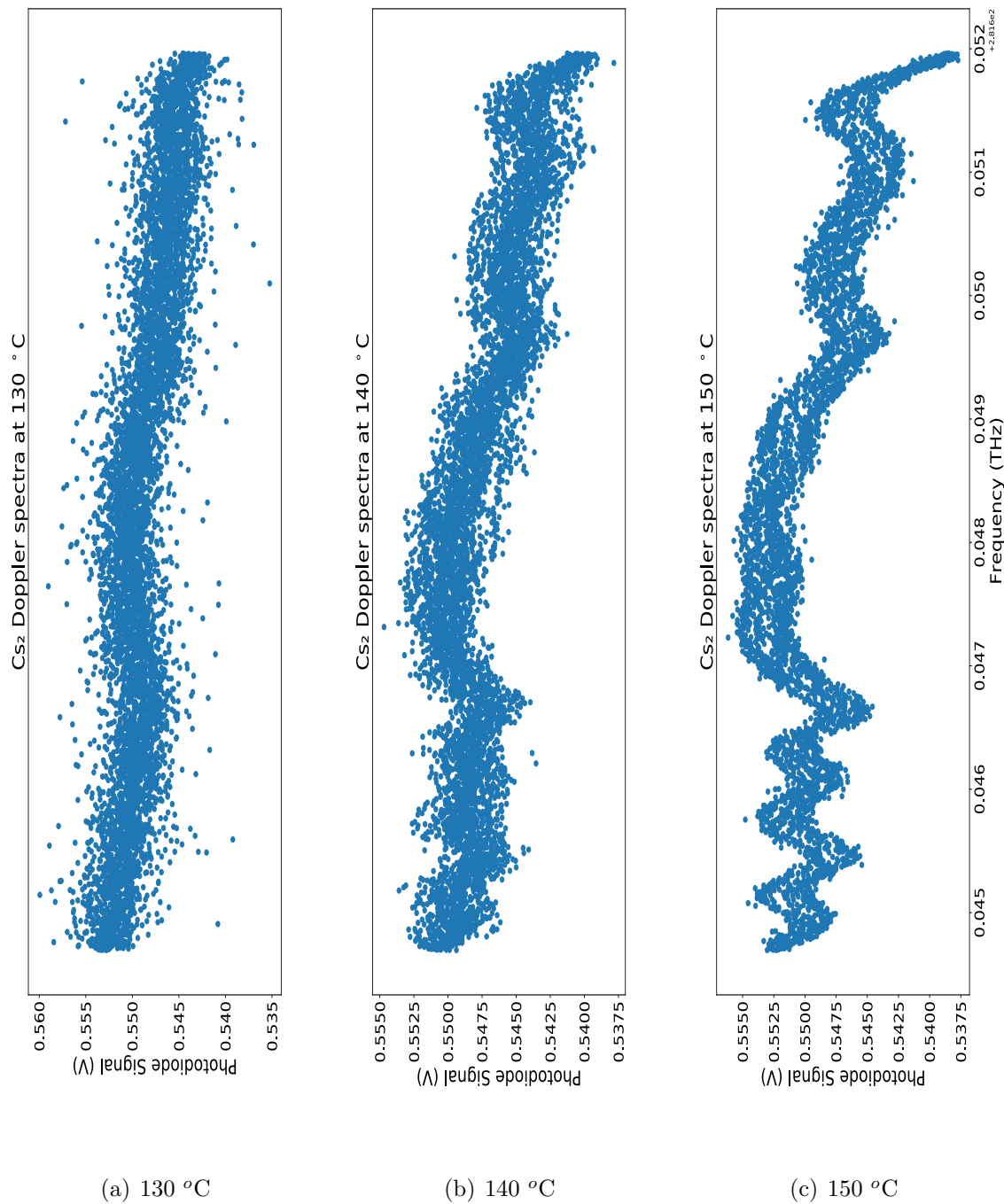


Figure 4.1: Doppler spectrum of Cs_2 indicating molecular formation between 130 and 140 °C. Here all the graphs share the same frequency axis. 281.6 THz should be added to the markings on the frequency axis to get the right readings.

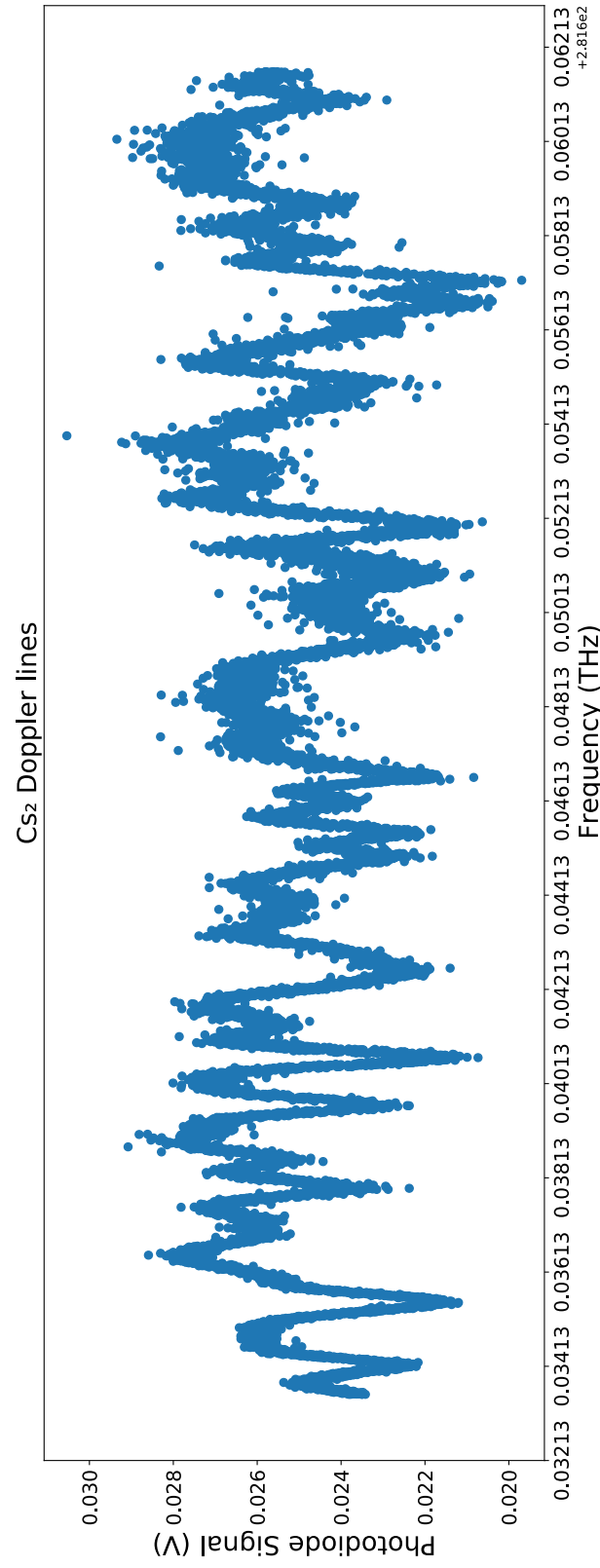


Figure 4.2: Cs₂ Doppler spectrum over the full temperature scan range of the laser at a vapour cell temperature of 200 °C - Run 1. 281.6 THz should be added to the markings on the frequency axis to get the right readings.

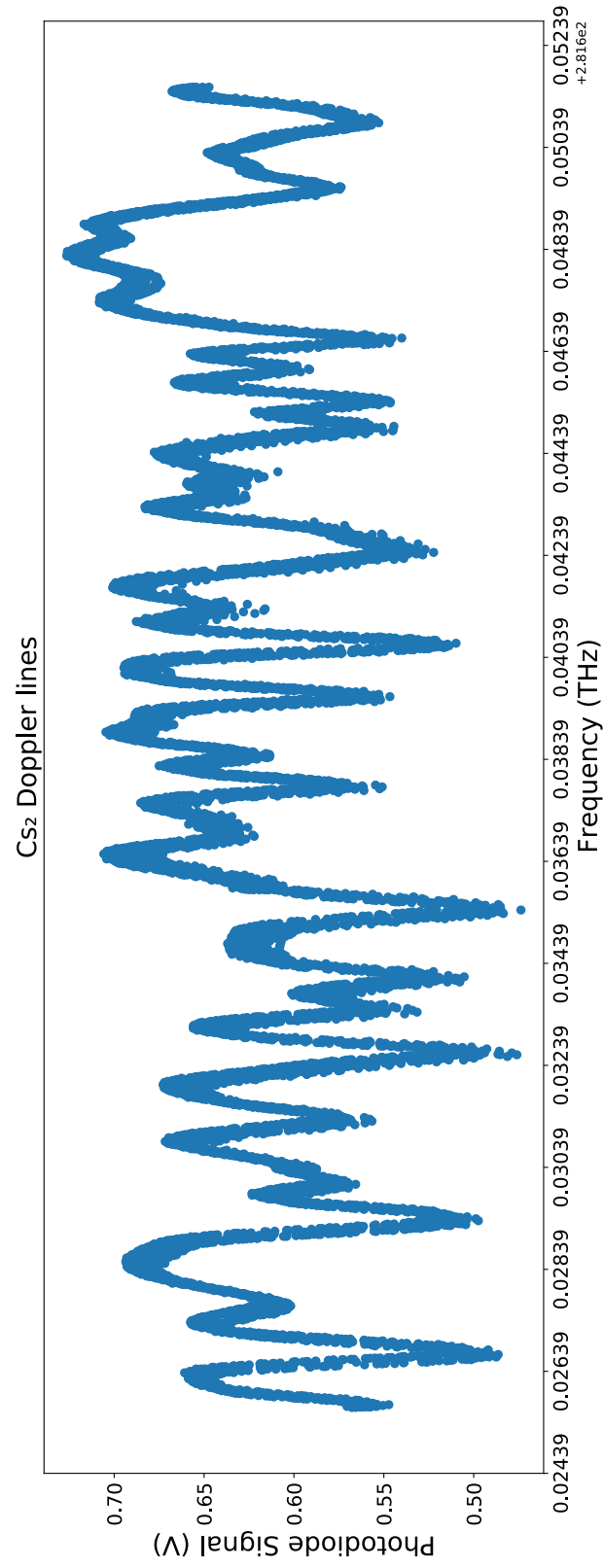


Figure 4.3: Cs₂ Doppler spectrum over the full temperature scan range of the laser at a vapour cell temperature of 200 °C - Run 2. 281.6 THz should be added to the markings on the frequency axis to get the right readings.

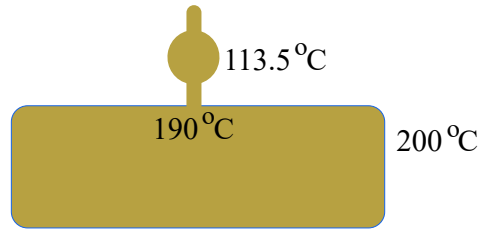


Figure 4.4: This figure indicates temperature of the cell for [Run 1](#) and [Run 2](#). Temperature of the windows is 200 °C, that of the cold point is 113.5 °C and that of the cell body is 190 °C.

Experimental conditions for [Run 1](#):

- Photodiode Gain: 20 dB
- Temperature of the windows: 200 °C
- Incident laser power: 10.6 μ W.

Experimental conditions for [Run 2](#):

- Photodiode Gain: 10 dB
- Temperature of the windows: 200 °C
- Incident laser power: 500 μ W.

Chapter 5

Experimental Challenges

Absolute Frequency Stabilization

Several obstacles were encountered during the project. All three vapours cells prepared during the course of the project were lost due degradation of vacuum. This will be presented and understood in detail in the section [5.1](#). We also faced a challenge of keeping Cs from depositing on vapour cell windows (section [5.2](#)). Another obstacle faced was collisional broadening of SAS signals. This could not be rectified due to the immediate loss of the vapour cell under examination. This will be presented in detail in section [5.3](#).

5.1 Degradation of vapour cells

During the project, two vapour cells were lost. The cause of this is not clearly known. The first vapour cell failed to show any Cs_2 Doppler signal. Upon disassembling the vapour cell system, it was noticed that whitish-black spots (figure [5.1](#) & [5.2](#)) had developed on the cell. This alludes to contamination of cesium. The cell might have developed micro-cracks upon heating thus contaminating vacuum inside the cell. Nevertheless, significant amount of cesium was still found in the vapour cell (figure [5.3](#)).

The second vapour cell stopped showing Doppler signal about 12 hours after increasing the temperature of the cell windows to 250 °C. Loss of Cs_2 Doppler absorption happened

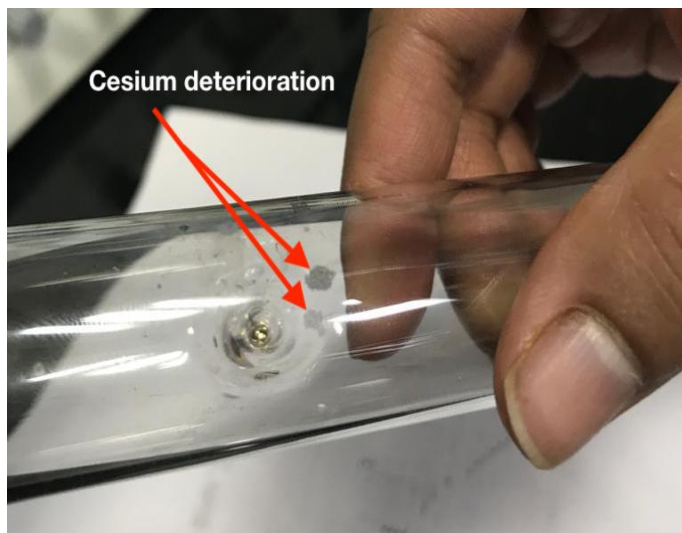


Figure 5.1: Deterioration of first cesium vapour cell. Here, whitish-black spots indicate probable degradation of vacuum inside the cell.

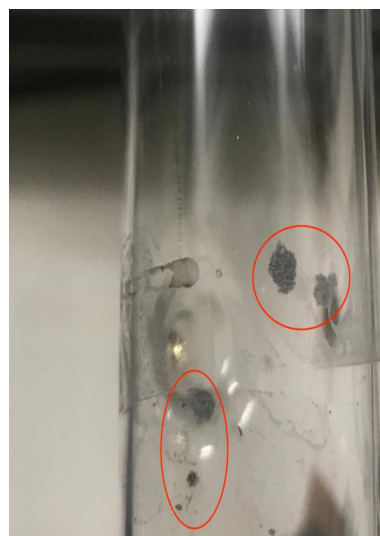


Figure 5.2: Close up of the deterioration region.

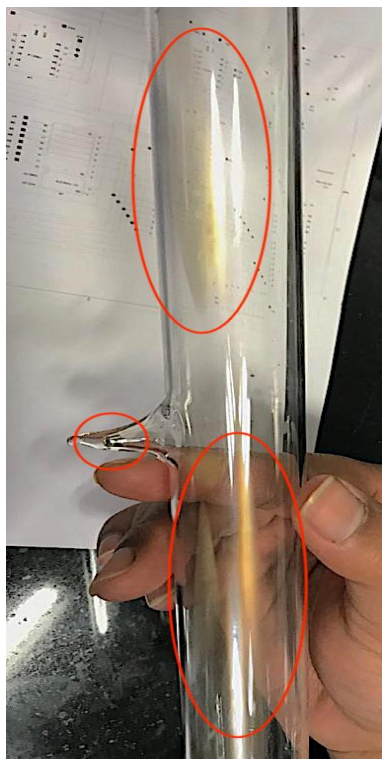


Figure 5.3: Pristine cesium in deteriorated (first) vapour cell.

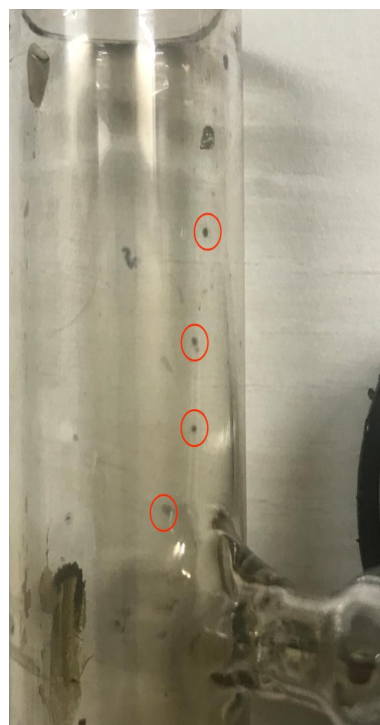


Figure 5.4: Deterioration of second cesium vapour cell. Encircled are whitish-black spots, similar to that in figure 5.1.

gradually over a period of a couple of minutes. Upon disassembling the system, tiny black spots similar to that developed on the first vapour cell were noticed (figure 5.4). Despite this, plenty of pristine cesium was found in the cell. However, Doppler signal never showed up.

5.2 Cs deposition

There was no issue of deposition of cesium on windows in case of the first vapour cell as the amount of cesium installed in the cell was minimal. While the amount of cesium was in excess in the other two vapour cells. Moreover, the latter cells were smaller than the first cell by almost a factor of two. So deposition of cesium had to be circumvented before performing spectroscopy.

The cold point of the cell is exposed to the atmosphere in our experiments. But due to smaller size of the latter cells, glass mass separating the windows and the cold point was not enough to maintain low temperatures at the cold point. This means increased vapour pressure at the cold point held back more cesium from depositing and increased the chances of deposition on the windows. Increasing the temperature of the windows did not help as this does little to the temperature difference across the cell. Moreover, attack of alkali metals on glass at elevated temperatures forced us operate the cell below 250 °C.

Cs was held away from the windows in the second vapour cell by removing insulation covering from the body of the cell that is portion proximal to the cold point of the cell. This reduced the temperature of the cold point significantly enough to prevent Cs deposition on the windows. The third vapour cell has more cesium than the first cell, but of the same size. So keeping cesium at bay from the windows was harder. A fan from computer CPU was used to lower the temperature of the cold point by a few tens of degrees (figure 5.5). This did not completely solve the issue, as there was still some Cs depositing at the centre of one of the windows of the cell. A hollow copper rod of inserted on the stem of the cell. This behaved as heat sink for the cold point and performed very efficiently, even when the cell windows were heated to 200 °C, the cold point remained at 45 °C. A high quality nichrome wire was wound around the rod to facilitate independent heating the cold point powered by a separate variac (figure 5.6).

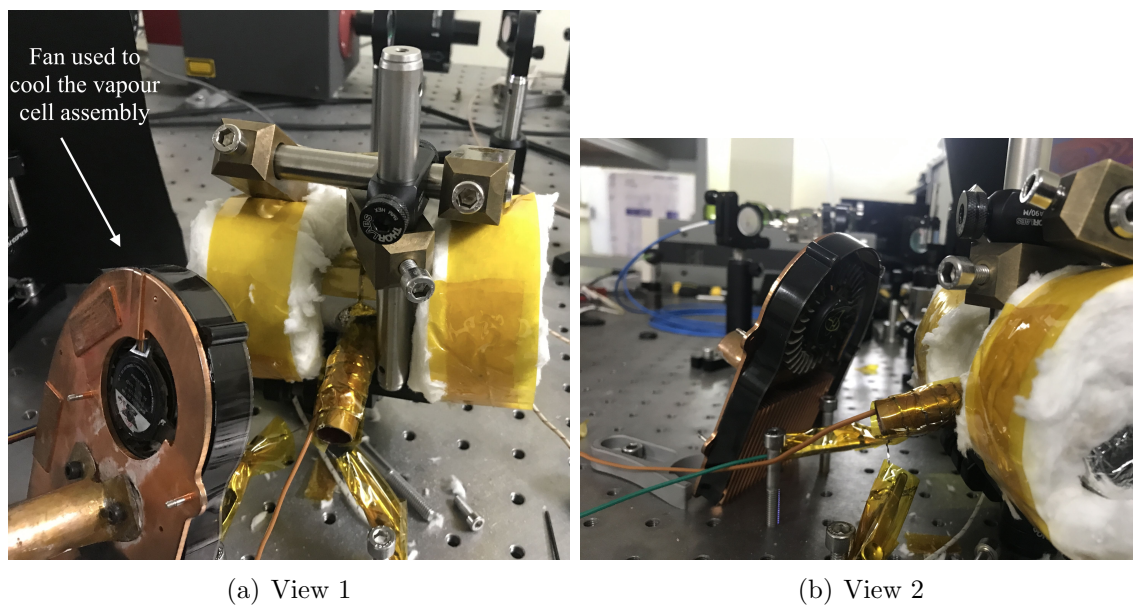


Figure 5.5: Fan used to cool the cold point further is placed facing the cell assembly.

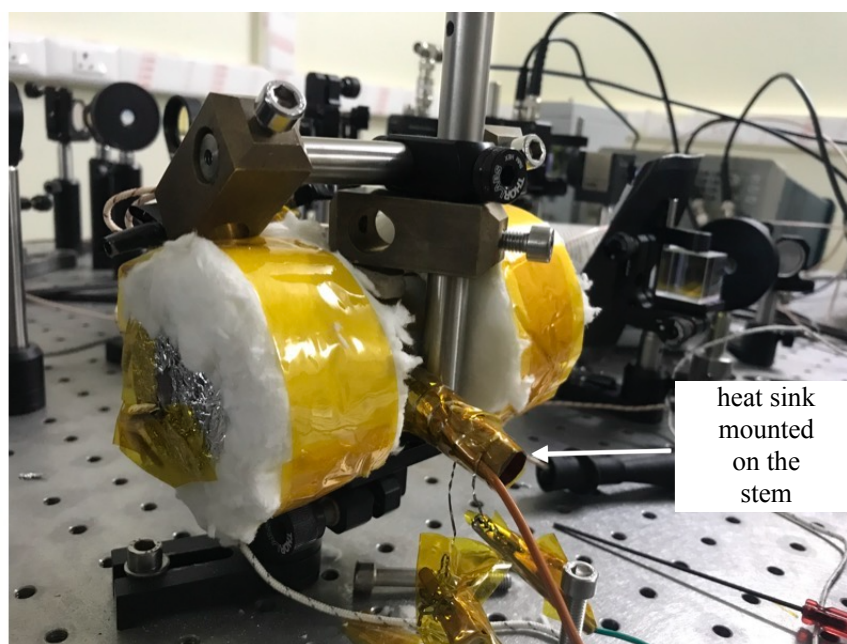


Figure 5.6: Mounting a hollow copper tube on the stem to provide heat sink for the cold point.

However, in the former scenario (figure 5.5) the deposition problem was circumvented by aligning the pump probe beam at an angle through the vapour cell such that both the beams evade the deposition region as indicated in figure 5.7.

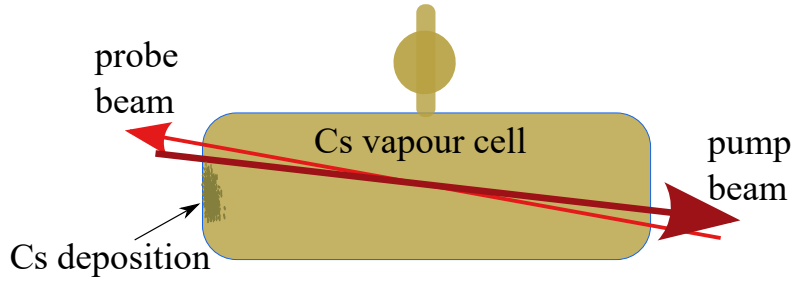


Figure 5.7: Pump probe alignment to evade Cs deposition on windows.

5.3 Collision broadening of SAS signals

In figure 3.15, lock-in detection is done to extract Lamb dips since we could not observe unconditioned Lamb dip signal. Lock-in signals for increasing pump power is shown in figure 5.8. The saturation power for the SAS setup is 48 mW. As we see in these figures, the lock-in signal shows very broad peaks. This observation is in contrary to a supposed SAS or Sub-Doppler signal which are narrow. To ensure and confirm that these indeed correspond to SAS signal and not a bogus signal like stray pump falling on the photodiode, we performed a few steps:

1. Blocking only the pump beam: This rendered a null signal. The pink trace (refer figure 5.8) was flat and almost zero.
2. Blocking only the probe beam: This rendered a flat signal with a non-zero offset. We tried isolating the photodiode from stray pump power by mounting iris on the photodiode and Mounting beam blocks to prevent scattering of stray light from metal surfaces.

From these oscilloscope images, it is seen that the lock-in signals increase with pump power or saturation. Since, depth of Lamb dip increases with increase in saturation (equivalently pump power) [19], it can be deduced that the lock-in signal corresponds to SAS signal. Since, sub-Doppler spectroscopy has negligible Doppler broadening when pump and probe beams are on top of each other, broadening of Lamb dips in figure 5.8 can be attributed to power broadening or collisional broadening [19] [7]. It is inferred from figure 5.8 that power broadening due to excess saturation is not the cause for broadening since the lock-in signals are very broad even at low pump powers. To infer if collisional broadening is the underlying

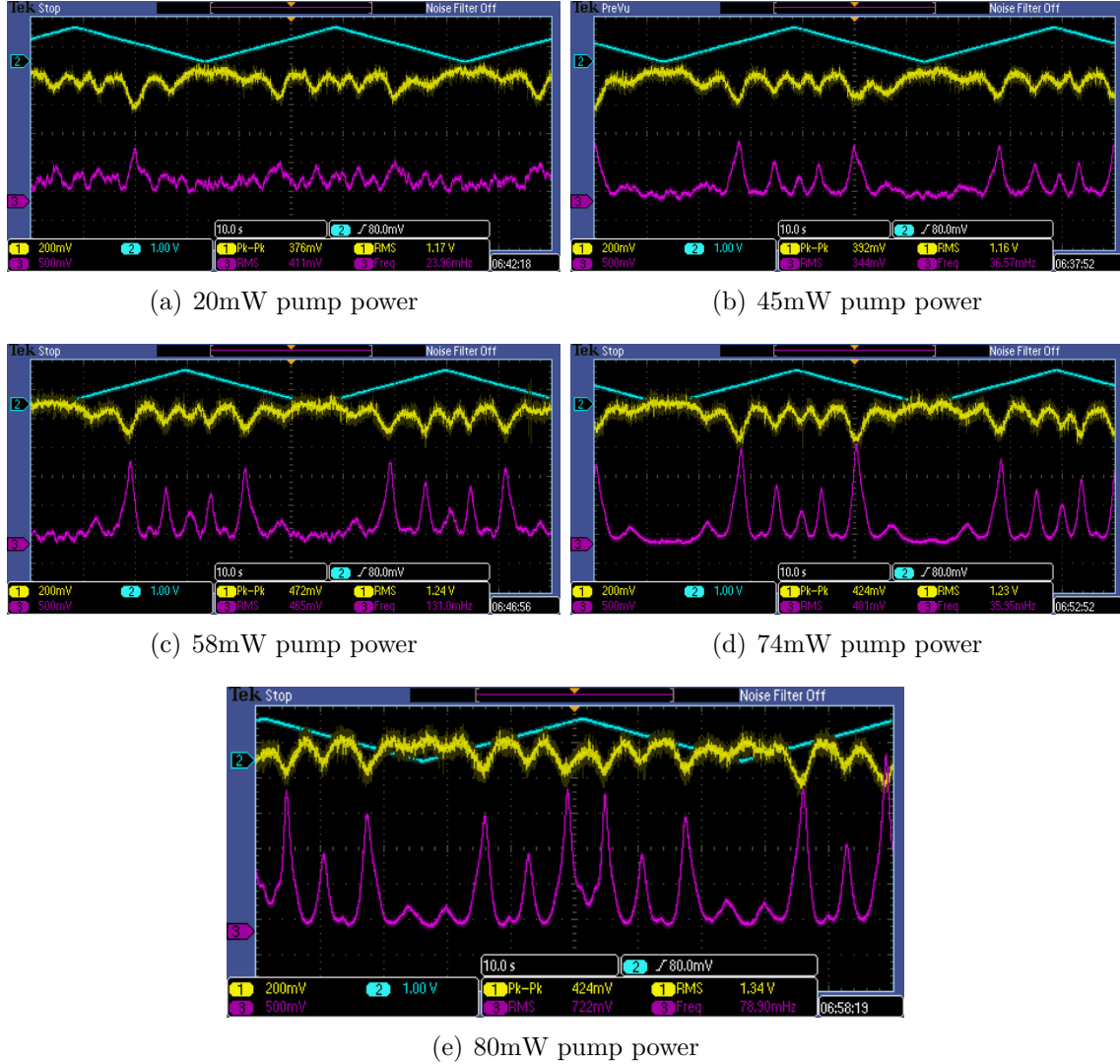
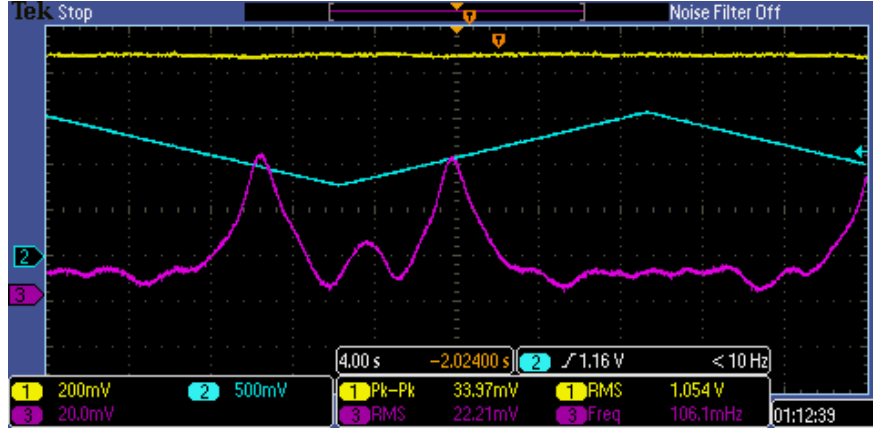


Figure 5.8: SAS signal for various pump power as detected by Lock-in amplifier (refer figure 3.15). Here, the yellow trace corresponds to Doppler signal, blue trace corresponds to temperature scan of laser frequency, and the pink trace corresponds to lock-in signal.

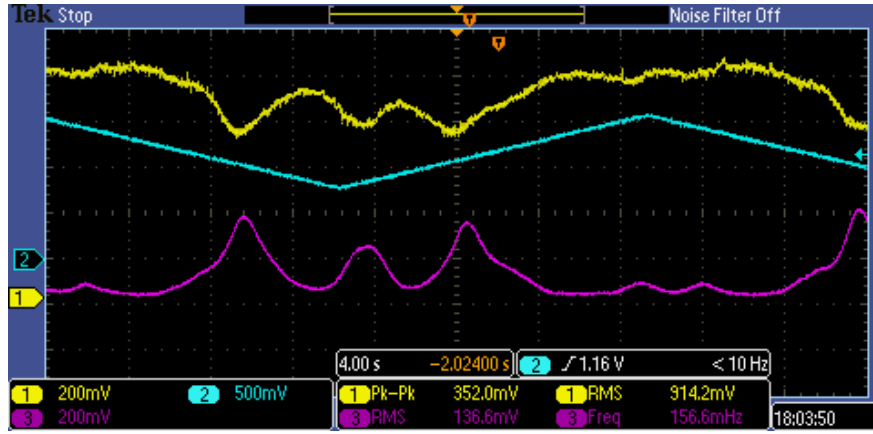
Probe power: $750 \mu\text{W}$, chopper frequency: 430 Hz, Lock-in time constant: 300 ms.

cause for significant broadening observed here, we maintained the pump power at 77 mW and observed the SAS signals at 155 °C and 200 °C (figure 5.9). Decreasing the temperature of the cold point of the cell decreases the vapour pressure inside the cell.

From images in figure 5.9, it is inferred that collisional broadening was not affected much upon decrease in temperature or vapour pressure inside the vapour cell. An apt procedure would be to characterize the cell with an 852 nm laser diode in situ and control the determine



(a) Temperature of the windows is 155 °C and the cold point's temperature is ≈ 90 °C.



(b) Temperature of the windows is 200 °C and the cold point's temperature is ≈ 115 °C.

Figure 5.9: Oscilloscope images of SAS signals: Yellow trace indicates Doppler signal, blue trace indicates temperature scan of laser frequency, and the pink trace indicates lock-in signal. Pump power is at 77 mW.

a suitable cold point temperature to perform Cs_2 spectroscopy. Unfortunately, we couldn't do further experiments with the vapour cell as it was destroyed/contaminated.

In principle, SAS signals in figure 5.8 can not be used to perform laser frequency stabilization of the laser since the error signal synthesised will be broad and locking to this will not provide any advantage over using the Doppler signals.

Now that we have seen preliminary results from the experiment and some of the obstacles faced during our attempt to achieve the goal, in the next chapter we will see how we can improve the system and take steps such that we can achieve absolute frequency stabilization.

Chapter 6

Future Directions

Absolute Frequency Stabilization

All three vapour cells prepared during the course of the project were lost probably to deterioration of vacuum inside the cells. As we saw this in detail in section 5.1 of previous chapter, this could be due to inadequate baking. We could not bake to the desired level because the glass to metal flange can not handle temperatures more than 150 °C. So, we have purchased ready-made Cs vapour cell (borosilicate cell) from Thorlabs ([GC25075-CS](#)) which is on its way. Heating assembly for the cell will be the same as before. We will perform lock-in detection of Lamb dips and we also expect negligible collision broadening as the amount of Cs on these high quality vapour cells is less than 10 mg unlike excess cesium in the home-made cells. Subsequently, probe beam will be phase modulated at radio frequency, and demodulated signal from the fast photodiode will be sent to AN/IN port of the lock-in amplifier while REF/IN signal arrives from the chopper chopping the pump beam (refer figures 6.1 and 3.15). This will give an error signal, robust enough to perform frequency stabilization of the laser. We will then have the laser's frequency drift measured over a ten minutes' time frame, to conclude the experiment.

In [Introduction](#) we introduced stable laser systems (section 1.2) and studied frequency noise. We also described how we will be attempting to address frequency noise of the laser in two stages. Now that we have presented our experimental efforts on absolute frequency stabilization, in the next chapter we will describe the same on relative frequency stabilization.

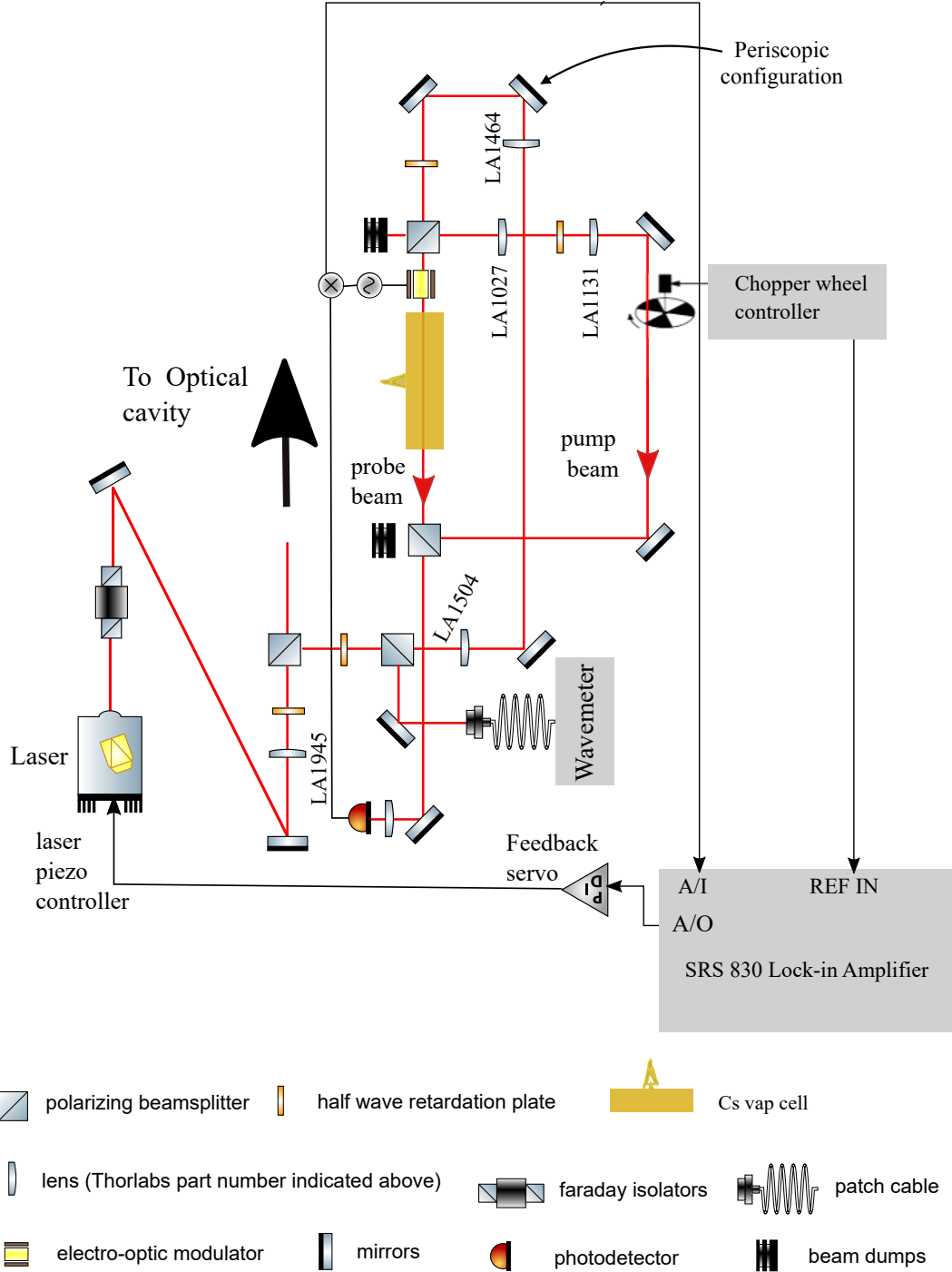


Figure 6.1: This optical diagram shows the Frequency-Modulation spectroscopy technique to do absolute frequency locking of a laser (adapted from Fig. 3 of [20]). A small amount of power from the input beam is picked up to read wavelength of the laser. A probe beam (lower intensity) sent through the vapour cell while a pump beam (higher intensity) is sent anti-parallel to it and overlapping it. Certain combination of lenses are used to optimize pump-probe beams' overlap. The probe beam is detected by a photodiode as the laser frequency is temperature scanned. Signal from the photodiode is demodulated, and is sent to a lock-in amplifier to detect SAS signals. As a result, the error signal is sent to the laser device's piezo for frequency stabilization.

Chapter 7

Optical Resonators

Optical resonator is a set of mirrors arranged together such that laser light repeatedly reflects between them and allows single-mode oscillation of laser beam. This is a resonance condition thus the term *optical resonators*. These resonators also behave as frequency filters dictated properties of the mirrors used, and they behave as spectrum analyzers as the resonator length is scanned. It is also called an optical cavity as the mirrors form an enclosed region where large power laser light is circulates. [21]. In this chapter we will briefly present essential literature on optical resonator(s) that is used to perform relative frequency stabilization. After introducing Gaussian beams we will discuss about higher order modes and their origin (section 7.2). We will also understand the efficacy of optical cavities in laser stabilization and describe associated experimental aspects like PDH locking (subsection 7.3.1).

Laser light waves can form standing wave and travelling wave inside an optical resonator. Stable two-mirror resonators form standing wave cavities, while higher number of mirrors make up travelling wave cavities. Figure 7.1 shows different forms of optical cavities. In this project we have attempted to use a triangular resonator to frequency stabilize the 1064 nm laser.

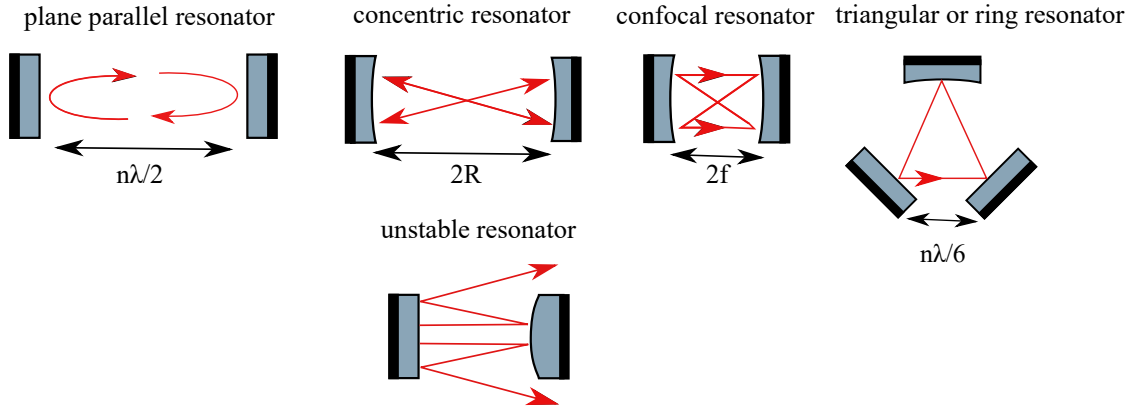


Figure 7.1: Different geometries of optical resonators. Red arrows indicate laser light path.

7.1 Stability of resonators

Laser beam bounces back and forth between mirrors such that, this configuration is equivalent to a periodic sequence of transmission lenses. This periodic sequence can be stable or unstable, and forms a stable configuration under certain conditions given by equation (7.1).

$$0 < \left(1 - \frac{L}{R_1}\right) \left(1 - \frac{L}{R_2}\right) < 1 \quad (7.1)$$

where L is resonator round trip length, R_1 and R_2 are radius of curvature of mirrors forming the optical resonator. Stability diagram in figure 7.2 shows graphical representation of equation (7.1).

It is essential to build a stable optical resonator to perform laser frequency stabilization.

7.2 Modes of propagation

The plane wave approximation of a light beam considers dynamics of a laser beam in one dimension or the direction of propagation. A more detailed description considers shape of the beam and its geometry. Although laser beams are similar to plane waves in many aspects, their intensity distribution is non-uniform, concentrated around the axis of propagation and have curved wavefronts. [21]. In this scenario, electric field can be treated as sum of different

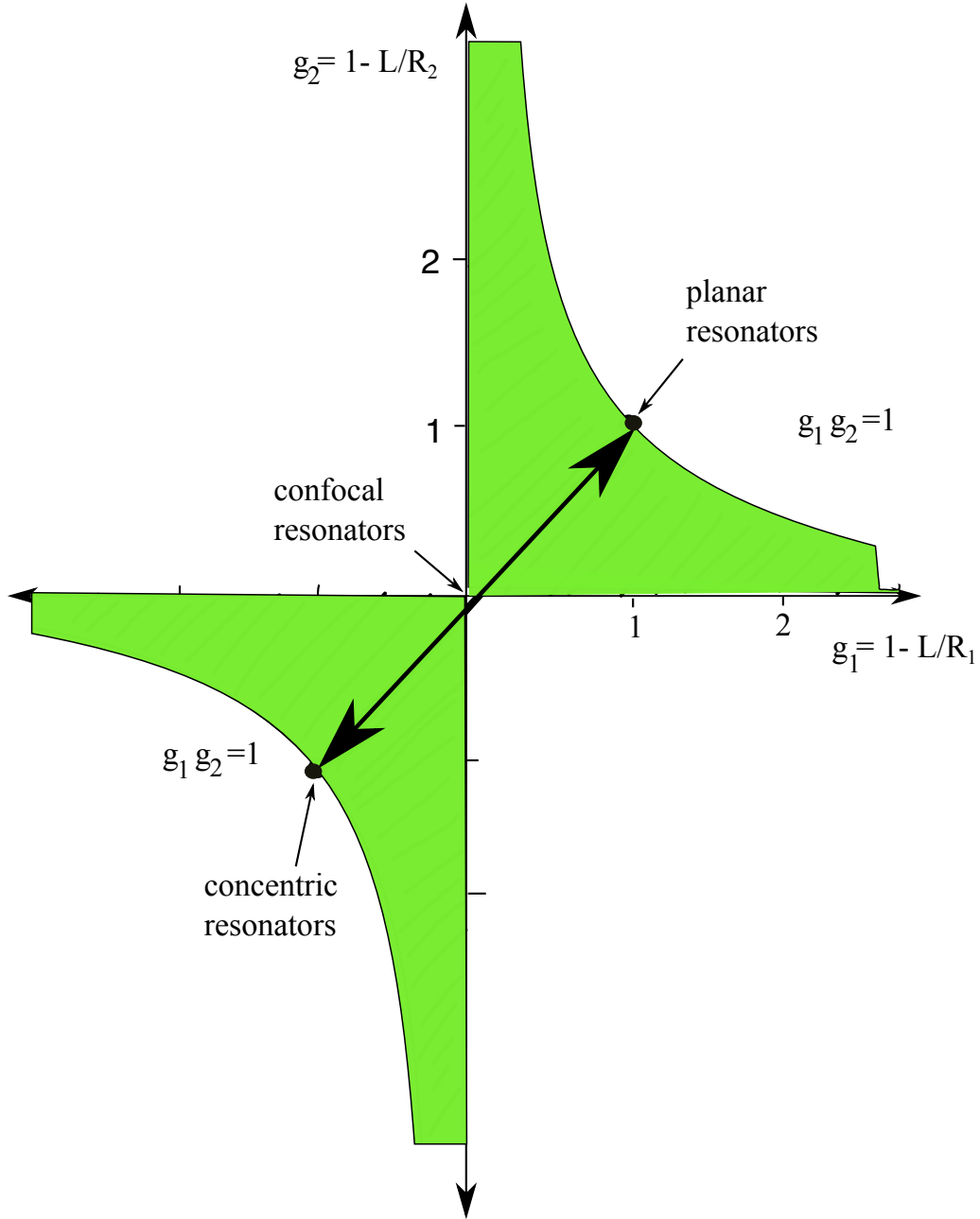


Figure 7.2: Stability diagram for a two mirror optical resonator [5]. This is a graphical representation of equation (7.1). Here the x and y axes are $\left(1 - \frac{L}{R_1}\right)$ and $\left(1 - \frac{L}{R_2}\right)$ respectively, where R_1 and R_2 are radius of curvatures of the mirrors forming the resonator. The green shaded region represents all the stable cavities while the unshaded white region represents unstable cavities.

spatial modes and different frequencies, completing the picture of a light beam. Most lasers emit Gaussian beams due to the way they are constructed using stable optical cavities with

spherical mirrors. When the stability condition (7.1) is satisfied, the phase fronts of these modes exactly match the geometry of mirror surfaces.

All electromagnetic (EM) waves follow the general Helmholtz wave equation $\nabla^2 \mathbf{E} - \frac{1}{c^2} \ddot{\mathbf{E}} = 0$ where $\mathbf{E}$ is the electric field. Any paraxial field assumes the form: form for the scalar electric field as:

$$E(x, y, z) = u(x, y, z) \exp(-i k z) \quad (7.2)$$

Feeding this back into Helmholtz equation $\nabla^2 \mathbf{E} - \frac{1}{c^2} \ddot{\mathbf{E}} = 0$ where $\mathbf{E}$ gives following equation for geometry of the beam $u(x, y, z)$.

$$(\partial_x^2 + \partial_y^2) u(x, y, z) - 2i k \partial_z u(x, y, z) = 0 \quad (7.3)$$

Equation (7.3) is called the paraxial wave equation. Any solution to this equation would describe an EM wave. A certain set of solutions that form a complete, orthogonal, infinite, countable set are called Transverse Electromagnetic modes (TEM). This Gaussian basis assume the form of Hermite-Gaussian (HG) for rectangular symmetry or Laguerre Gaussian (LG) for cylindrical symmetry. Either of these basis solutions can be used to span all solutions to the paraxial wave equation [21].

7.2.1 Gaussian Mode or TEM₀₀

An ideal laser emits beam in pure Gaussian form or the fundamental mode given by:

$$u_{00}(x, y, z) = \sqrt{\frac{2}{\pi}} \frac{1}{w(z)} \exp(i \Psi(z)) \exp\left(-i k \frac{x^2 + y^2}{2R_C(z)} - \frac{x^2 + y^2}{w^2(z)}\right) \quad (7.4)$$

Here the radial intensity profile of a beam, of power P is given by:

$$I(r) = \frac{2P}{\pi w^2(z)} \exp(-2r^2/w^2) \quad (7.5)$$

where $r^2 = x^2 + y^2$ and $w(z)$ is the beam size. Figure 7.3 shows axial cross section schematic of a propagating Gaussian beam. As depicted in figure 7.3 Rayleigh range (equation (7.6)) separates the beam propagation axis into two regimes: far field and near field.

$$z_R = \frac{\pi w_0^2}{\lambda} \quad (7.6)$$

λ is wavelength of the laser beam. Beam size $w(z)$ along the propagation direction is given

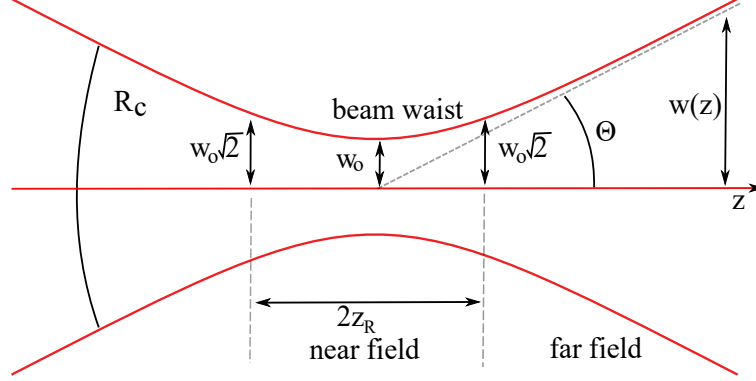


Figure 7.3: Profile of Gaussian beam along z -axis or the axis of propagation. Here, θ corresponds to beam divergence angle, w_0 is the beam waist, $w(z)$ is the spot size at a point z on the optic axis, R_c is the radius of curvature of phasefront, z_R is the Rayleigh range (adapted from Fig. 80 of [10]).

by:

$$w(z) = w_0 \sqrt{1 + \left(\frac{z - z_0}{z_R} \right)^2} \quad (7.7)$$

And the radius of curvature of the phasefront is given by:

$$R_C(z) = z - z_0 + \frac{z_R^2}{z - z_0} \quad (7.8)$$

In equation (7.4) Ψ is the *Gouy Phase* [5] given by:

$$\Psi(z) = \arctan \left(\frac{z - z_0}{z_R} \right) \quad (7.9)$$

This is an additional axial phase shift experienced by a Gaussian beam relative to a plane wave. From the above equation it is realized that phase velocity and the separation between the phase fronts of a Gaussian beam is higher than that for a plane wave [5].

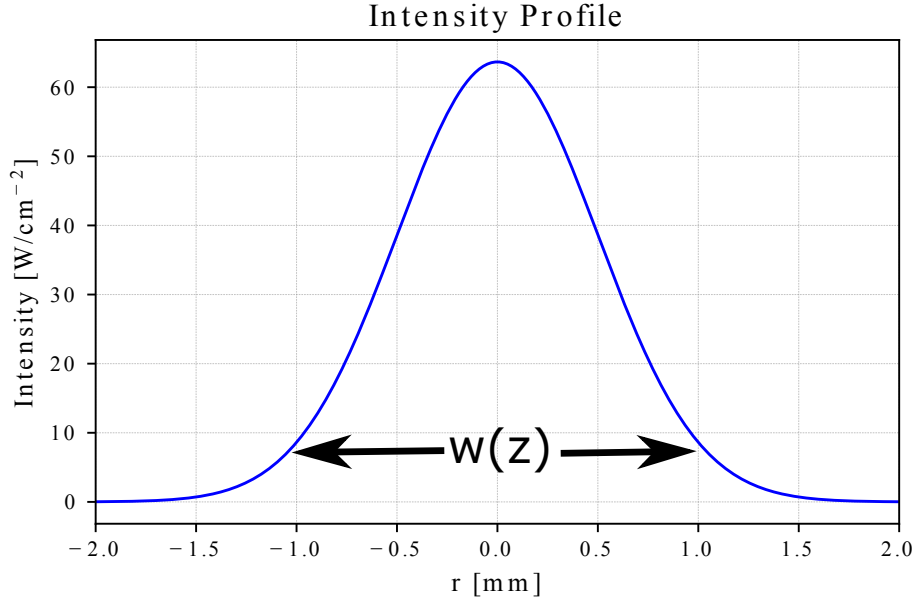


Figure 7.4: Lateral cross section of a Gaussian beam. Here the spot size/ beam size $w(z)$ is defined at $\frac{1}{e^2} \times$ intensity peak.

7.2.2 Higher Order Modes (HOMs)

Laser beams are never ideally Gaussian in shape. Imperfections in optical systems like misalignment from the optical axis, imperfect mode-matching [21], distorted optical element surfaces among others tend to excite Higher Order Modes of a laser beam. These higher order modes are next to fundamental Gaussian mode that make up the complete orthogonal set of solutions (HG or LG) as described in one of the previous sections.

Higher order HG modes are given by [21]:

$$u_{nm}(x, y, z) = u_n(x, z)u_m(y, z) \quad (7.10)$$

where

$$u_n(x, z) = \left(\frac{2}{\pi}\right)^{1/4} \left(\frac{\exp(i(2n+1)\Psi(z))}{2^n n! w(z)}\right)^{1/2} \times H_n\left(\frac{\sqrt{2}x}{w(z)}\right) \exp\left(-i\frac{kx^2}{2R_C(z)} - \frac{x^2}{w^2(z)}\right) \quad (7.11)$$

Here $H_n(x)$ is a Hermite polynomial. First few Hermite polynomials are:

$$H_0(x) = 1, \quad H_1(x) = 2x, \quad H_2(x) = 4x^2 - 2, \quad H_3(x) = 8x^3 - 12x$$

We observe in equation (7.11) that the solutions to spatial geometry of the HOM Gaussian beam are a product of two hermite polynomials of order m and n and a Gaussian function. The indices m, n define the mode order $m + n$ of a HOM.

The gouy phase shift for a TEM_{mn} is given by:

$$\varphi = (n + m + 1)\Psi(z). \quad (7.12)$$

And, higher order LG modes are given by [10]:

$$\begin{aligned} u_{p,l}(r, \phi, z) = & \frac{1}{w(z)} \sqrt{\frac{2p!}{\pi(|l| + p)!}} \exp(i(2p + |l| + 1)\Psi(z)) \\ & \times \left(\frac{\sqrt{2}r}{w(z)} \right)^{|l|} L_p^{|l|} \left(\frac{2r^2}{w(z)^2} \right) \exp \left(-ik \frac{r^2}{2q(z)} + il\phi \right) \end{aligned} \quad (7.13)$$

Here, r, ϕ, z are cylindrical coordinates around the optical axis. p and l are radial and azimuthal mode indices respectively. $L_p^{|l|}$ is associated Laguerre polynomial given by:

$$L_p^{|l|}(x) = \frac{1}{p!} \sum_{j=0}^p \frac{p!}{j!} \binom{|l| + p}{p - j} (-x)^j$$

The gouy phase shift for these HOMs is given by [21]:

$$\varphi = (2p + l + 1)\Psi(z). \quad (7.14)$$

From equations (7.12) and (7.14) we see that, HOMs have larger phase velocity due to larger Gouy phase shift. So, an optical resonator can essentially behave as a mode spectrum analyzer, separating all the modes depending on their Gouy phase shifts about round-trip inside the resonator.

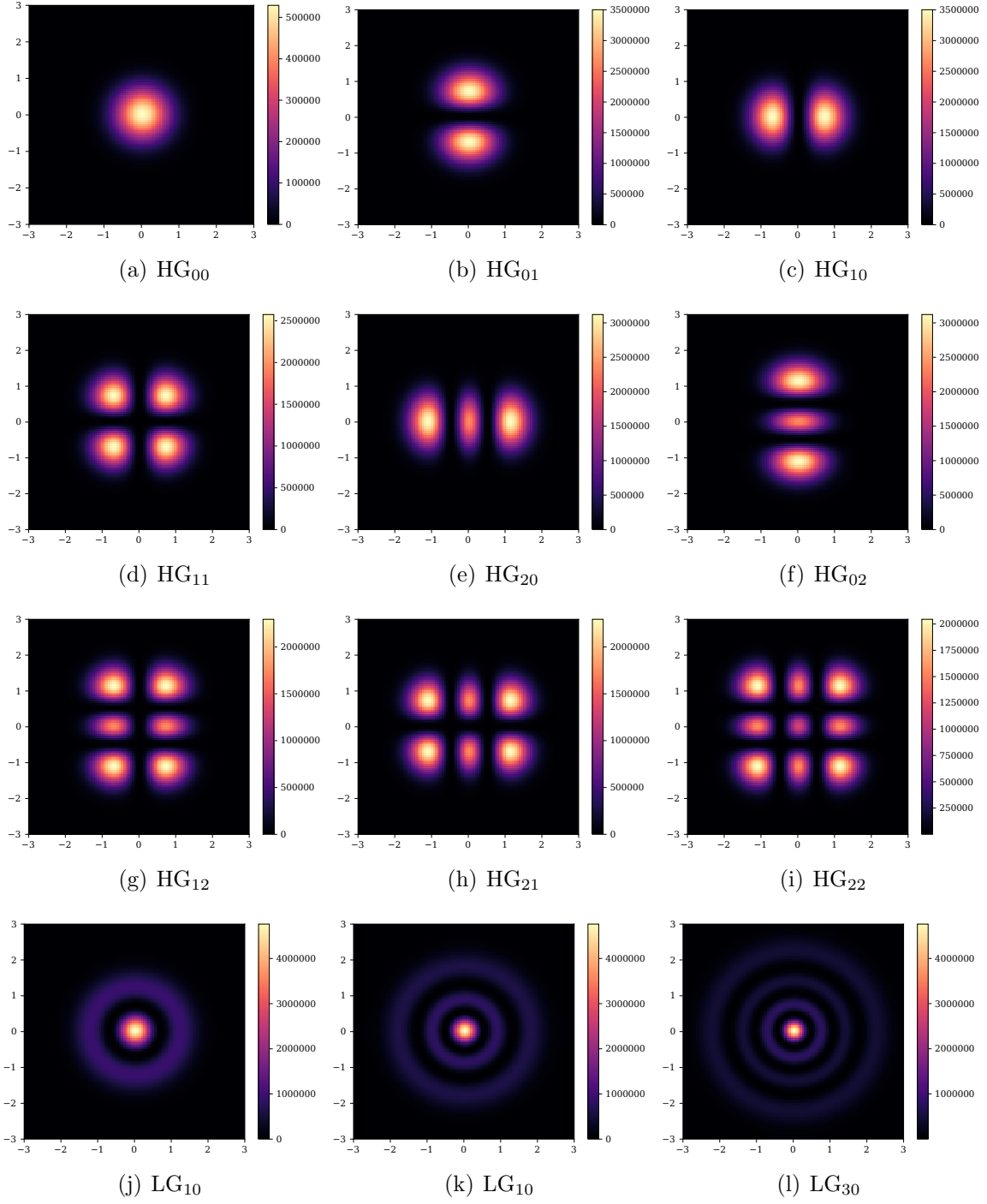


Figure 7.5: Intensity plots of first few orders of TEM modes corresponding to equations (7.11) and (7.13).

7.3 Optical cavities for laser stabilization

Ever since, its development in 1899 [25], Fabry-Perot cavities [21] [5] have shown promising applications in precision metrology like gravitational wave detection [26], and laser linewidth reduction [27] among many others. A laser diode is sensitive to optical feedback, so emission of the diode is stabilized by feeding back laser light into the diode through the anti-reflective coating as described in section 3.4.1 [28]. But in the advent of cheap feedback electronics, active stabilization of lasers using electronically feeding back error signal has become a commonplace [29]. In this section we will look at a technique called Pound-Drever-Hall locking that is used to lock a laser to a cavity (or vice-versa).

7.3.1 The Pound Drever Hall (PDH) technique

Phase Modulation

Phase modulation of a light beam (figure 7.6) is changing the phase of an EM wave sinusoidally as described in the following equation:

$$E = E_0 \exp(i(\omega_0 t + m \cos(\Omega t))) \quad (7.15)$$

This induces frequency sidebands to the laser line profile as is indicated in figure 7.7.

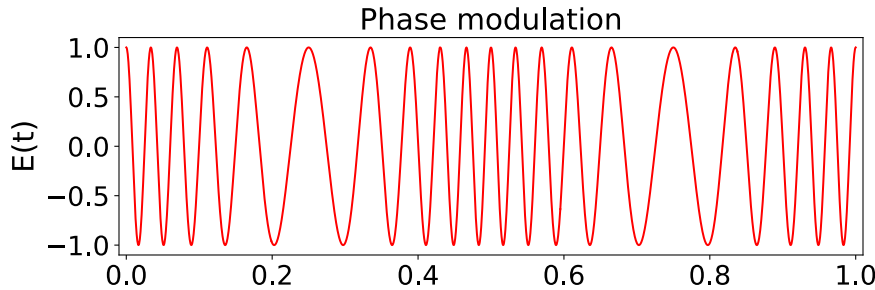


Figure 7.6: This figure illustrates electric field amplitude with time (x-axis) of a phase modulated electromagnetic wave (adapted from Fig. 17 of [10]).

Equation (7.15) can be written in terms of Bessel functions of the first kind $J_k(m)$ as:

$$E = E_0 \exp(i\omega_0 t) \sum_{k=-\infty}^{\infty} i^k J_k(m) \exp(ik\Omega t) \quad (7.16)$$

where

$$J_k(m) = \left(\frac{m}{2}\right)^k \sum_{n=0}^{\infty} \frac{\left(-\frac{m^2}{4}\right)^n}{n!(k+n)!} = \frac{1}{k!} \left(\frac{m}{2}\right)^k + O(m^{k+2}) \quad (7.17)$$

for small modulation indices [10]. Upon considering only the first sideband and assuming $m \ll 1$ the above equation reduces to [10]:

$$E = E_0 \exp(i\omega_0 t) \times (J_0(m) - iJ_{-1}(m) \exp(-i\Omega t) + iJ_1(m) \exp(i\Omega t)) \quad (7.18)$$

and with

$$J_{-k}(m) = (-1)^k J_k(m)$$

we obtain

$$E = E_0 \exp(i\omega_0 t) \left(1 + i\frac{m}{2}(\exp(-i\Omega t) + \exp(i\Omega t))\right) \quad (7.19)$$

This equation is a first order approximation in m . The power transfer from carrier to sidebands is not seen in this equation, but a second order approximation in m makes this conspicuous:

$$E = E_0 \exp(i\omega_0 t) \left(1 - \frac{m^2}{4} + i\frac{m}{2}(\exp(-i\Omega t) + \exp(i\Omega t))\right) \quad (7.20)$$

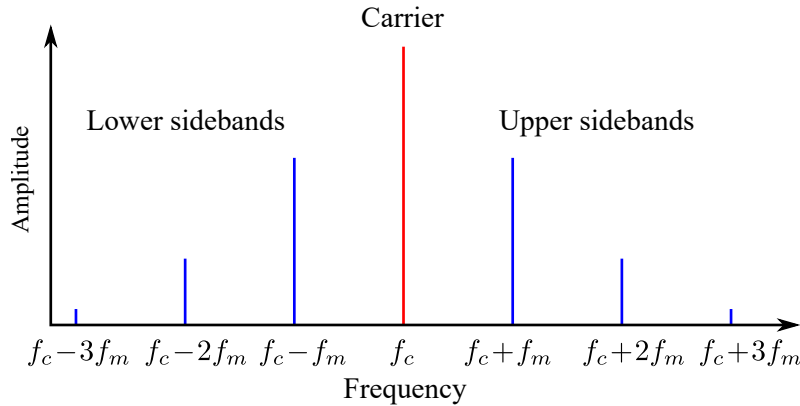


Figure 7.7: First few sidebands corresponding to equation (7.7). Here, f_c is the carrier frequency or the frequency of the laser and f_m is the modulation frequency.

Electro-Optic Modulator (EOM)

EOMs are used to phase modulate laser beams. An EOM works in the basis of electro-optic effect [30], it is the change in refractive index of a material upon application of a static or low frequency electric field. When change in refractive index depends linearly on the applied electric field, the phenomenon is called linear electro optic effect or Pockels effect. When an RF (radio frequency) electric field is applied across the electro-optic material in an EOM and a laser light with polarization axis aligned parallel to the applied E-field passes through the crystal, there is a phase modulation in the output laser beam (figure 7.8). The modulation depth or modulation frequency is same as the frequency of the applied RF voltage (or the E-field) and modulation index depends on the amplitude of the applied voltage. An EOM is essentially a voltage controlled phase shifter for light.

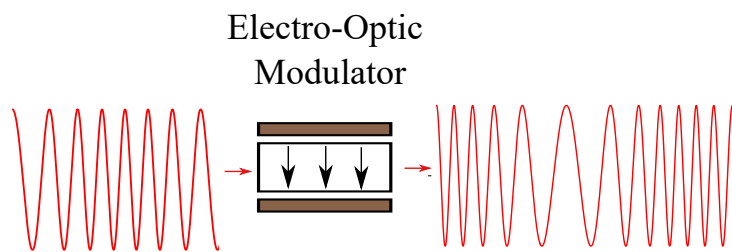


Figure 7.8: EOM modulating an incident light wave upon application of transverse electric field (black arrows). The input beam is a pure sinusoidal wave while the output beam is phase modulated.

Some commonly used crystals for EOMs are Lithium niobate (LiNbO_3), Rubidium Titanyl Phosphate or RTP (RTiOPO_4).

PDH error signal and locking

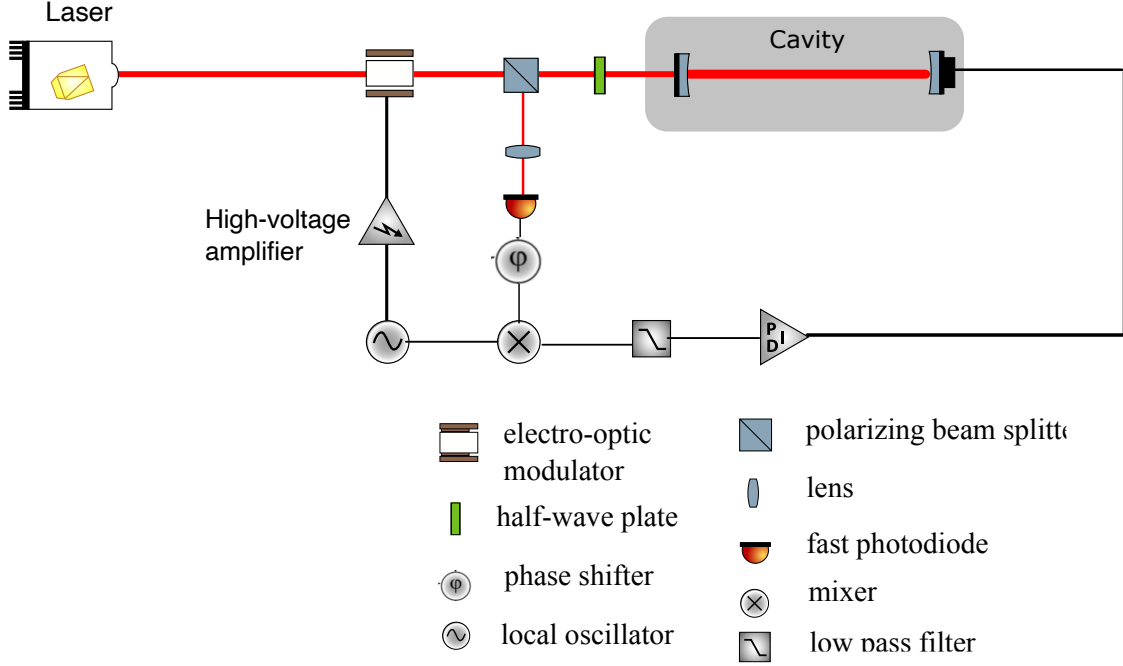


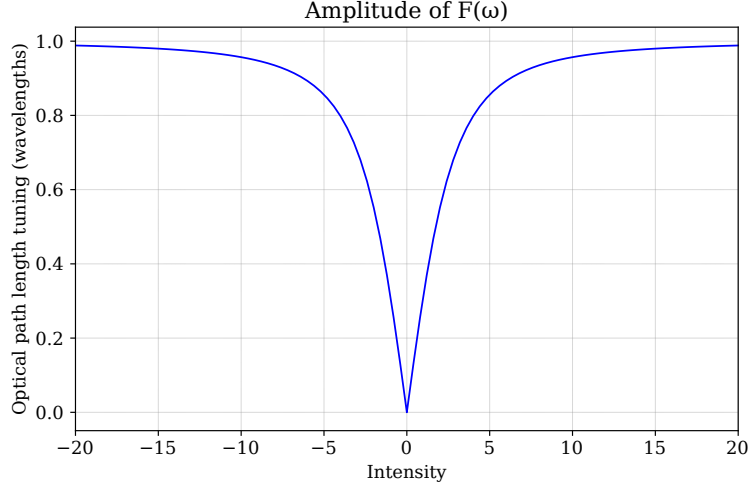
Figure 7.9: Schematic of the Pound-Drever-Hall locking technique. This figure illustrates how a PDH error signal is fed back to piezo controlling the resonator optical path length, thus locking cavity to laser. Similarly, the error signal could be diverted and fed back to laser cavity piezo thus achieving locking of laser to the resonator.

The PDH technique [3] allows extraction of error signal from a resonator or a laser that can be fed back to the controller to stabilize the system. The process of feeding back error signal elicited from a laser (through a control servo) to a resonator piezo (or vice-versa) is called locking as this enable one to follow the other. A phase modulated laser beam is incident on an optical resonator. Reflection coefficient for a symmetric lossless resonator is given by:

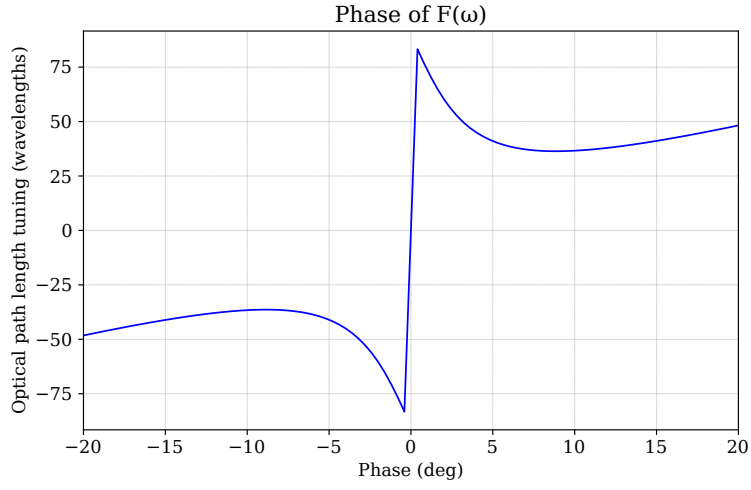
$$F(\omega) = E_{\text{ref}}/E_{\text{inc}} = \frac{r \left(\exp \left(i \frac{\omega}{\Delta\nu_{\text{fsr}}} \right) - 1 \right)}{1 - r^2 \exp \left(i \frac{\omega}{\Delta\nu_{\text{fsr}}} \right)} \quad (7.21)$$

where r is reflection coefficient for each mirror and $\Delta\nu_{\text{fsr}} = c/2L$ is free-spectral range (FSR) of the resonator ($2L$ corresponds to cavity round-trip length.).

From equation (7.19) we see that sidebands have a definite phase relationship with the



(a) Amplitude



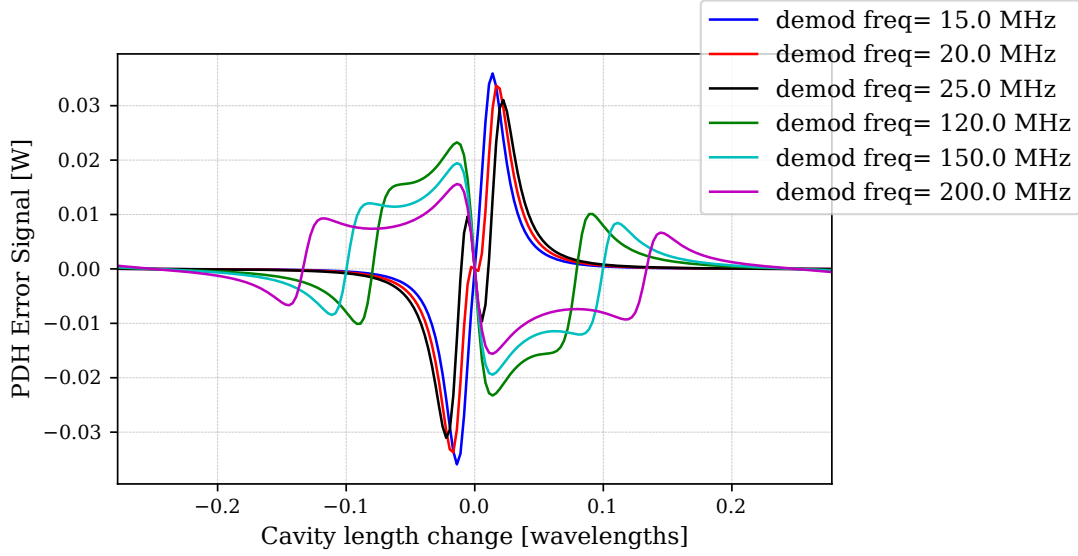
(b) Phase

Figure 7.10: Resonance curves for the reflection coefficient $F(\omega)$ in equation (7.10) for a symmetric lossless optical resonator with $R_1=R_2=0.9$ and round-trip length of 3000 m.

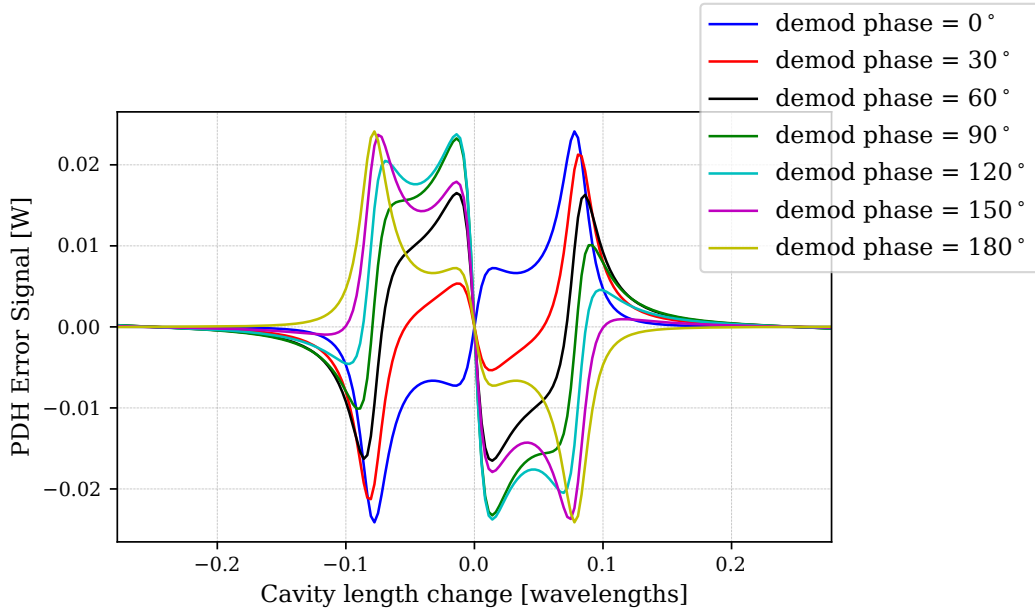
carrier beam. At resonance, the incident carrier resonates with the resonator and total reflected power at the carrier frequency is zero. While, a significant amount (depending on modulation depth) of sideband power is reflected off the resonator. When the laser frequency jitters, a small amount of carrier is reflected off the resonator (figure 7.10). This reflected carrier beam interacts with the reflected sidebands such that the resultant signal when passed through a PDH servo gives rise to an error signal. A PDH servo involves a mixer that mixes the reflected RF signal from the photodiode with the local oscillator signal (that drives the EOM). This is passed through a low pass filter to yield only a DC error signal. The whole

scheme is called *demodulation* (figure 7.9).

Error signals for a fast modulation (larger modulation depth compared to FWHM of the resonator) and slow modulation (slower modulation depth) are shown in figure 7.11.



(a) Error signals corresponding to different modulation depth.



(b) Error signals corresponding to different demodulation phases.

Figure 7.11: The above error signals are for a symmetric lossless resonator with FSR of 749.481 MHz and FWHM of 38.857 MHz.

The main advantage of this technique is that the phase of $F(\omega)$ (figure 7.10) is asymmetric about resonance, so the error signal elicited based on this model would differentiate between different sides of the resonance curve. This makes it propitious for more robust locking. A usual protocol is to lock the resonator to error signal corresponding to one of the error signals.

Now that we have seen adequate literature on optical resonators and their use in laser stabilization, details on the experimental efforts made to pursue stabilization of the 1064 nm laser (section 3.1) will be presented in the next chapter.

Chapter 8

Experimental Details

In this chapter we will describe the the experimental efforts made towards relative frequency stabilization of the 1064 nm [laser](#).

The triangular optical resonator understood and developed during the course of the project was originally designed by GW physicists at Caltech West Bridge Labs (Courtesy: Prof. Rana Adhikari, Caltech, USA and Dr. Evan Hall, MIT, USA) as a pre-mode cleaner (or PMC) [1] for spatial filtering and reduction of beam pointing fluctuations.

This chapter will start with basic properties of this resonator and subsequent sections will present efforts made to setup, align and perform mode-matching.

8.1 Development of high finesse optical resonator

The triangular optical resonator understood and developed during the course of the project was originally designed by GW physicists at Caltech West Bridge Labs as a pre-mode cleaner [1] for spatial filtering and reduction of beam pointing fluctuations. It is also proposed to work as a passive frequency stabilization resonator and reduce RF power noise. But, due to unavailability of suitable mirrors for the resonator, we resorted to High Finesse *Supermirrors* from Newport. High finesse cavities are used for frequency stabilization, however, the

stainless steel resonator developed here can not be utilized for this purpose due to deviations in optical path length caused by temperature fluctuations. This can be avoided by actively stabilizing temperature of the resonator spacer.

8.1.1 Resonator parameters

Mirrors used for the triangular resonator depicted in figure 8.1 are of high reflectivity with $R > 99.97\%$ (High Finesse mirrors). The **front mirrors** are planar while the **back mirror** has a radius of curvature of -1000 mm.

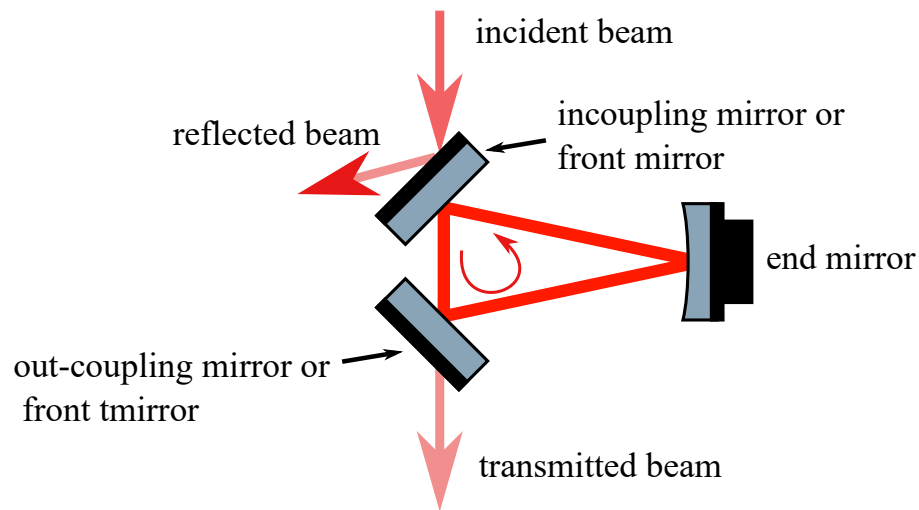


Figure 8.1: This figure shows the schematic of the triangular optical resonator developed during this project. Red arrows here indicate laser beam path.

The resonator parameters are listed below:

- Roundtrip resonator length = 247.96 mm
- Distance between front mirrors = 29.4 mm
- Angle of incidence on front mirror = 41.6°
- Angle of incidence on back mirror = 6.8°
- resonator finesse = 6980

- FWHM = 173 kHz (resonator pole: 87 kHz)
- FSR = 1.2 GHz
- $w_{(0x)} = 333.4 \mu\text{m}$, $w_{(0y)} = 334.8 \mu\text{m}$
- Position of beam waist inside the resonator is 14.68 mm from the front mirror where the laser beam is incident on and is located at the base of the triangular resonator (refer figure 8.3).

8.1.2 Resonator spacer

Mirrors are mounted on a stainless steel spacer with circular clamps and screws (figure 8.2). For an input (phase modulated) light of power 100 mW, at resonance circulating power inside the resonator is 140 W, transmitted light power is 41 mW while power of light reflected of the resonator is 1.5 mW.

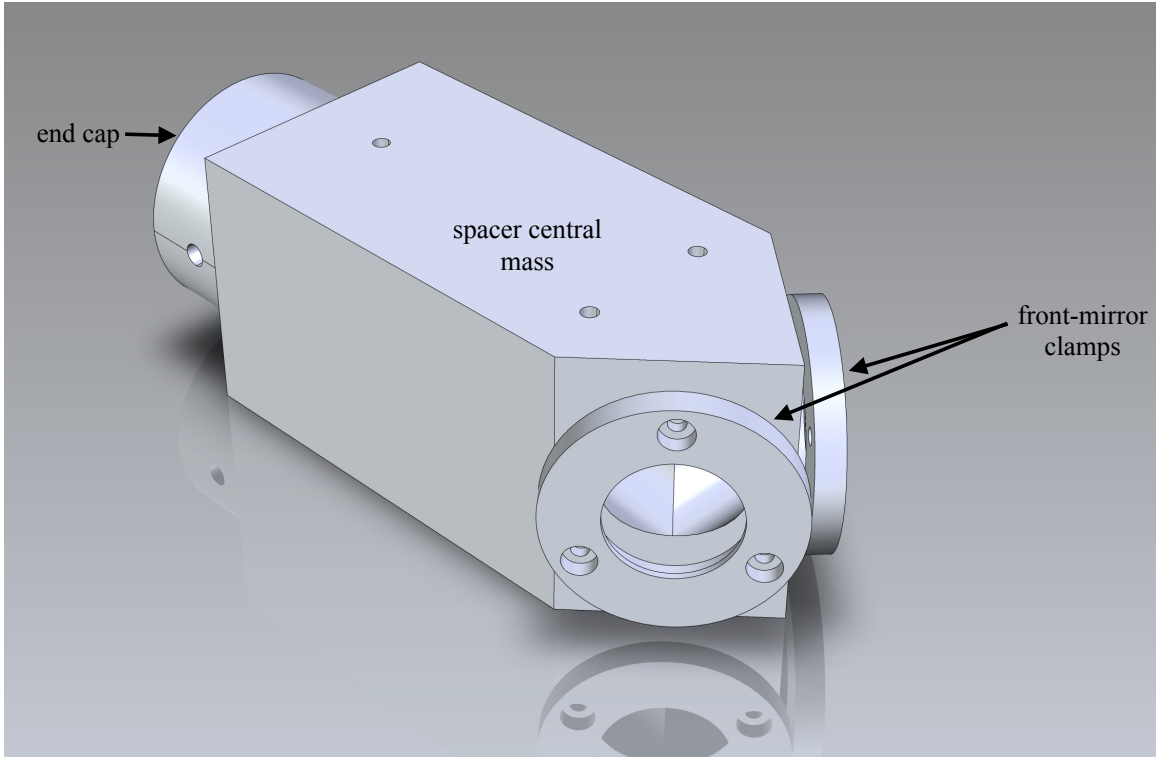


Figure 8.2: CAD drawing of the resonator spacer. The spacer is made of stainless-steel. The mechanical design is composed of end-cap, front-clamps and the spacer-central mass.

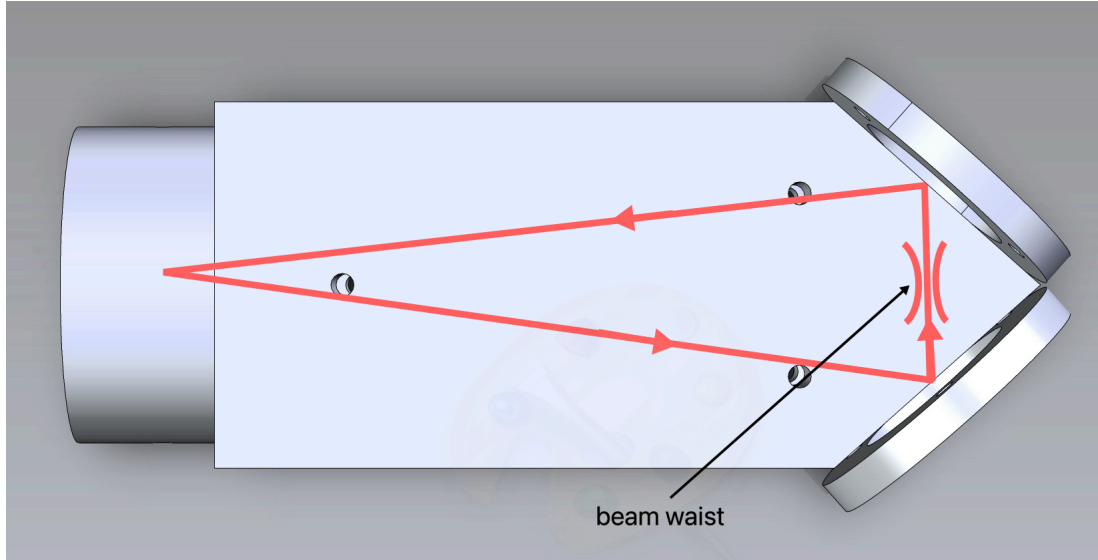
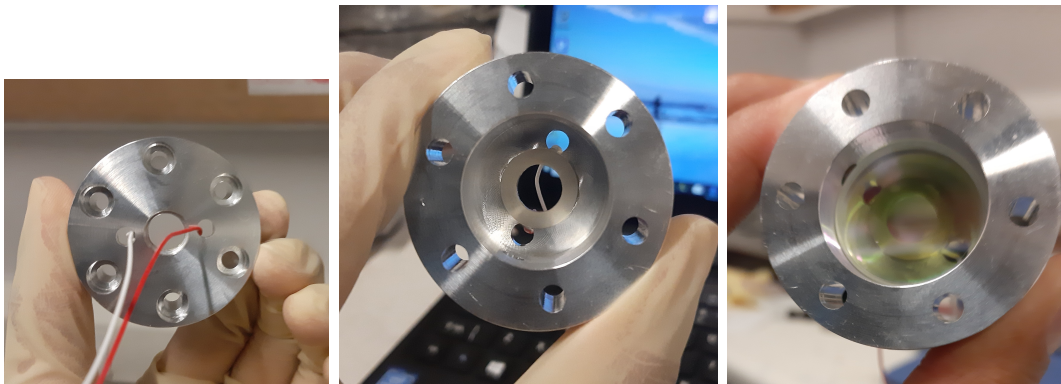


Figure 8.3: Top view of the resonator spacer. Red arrows indicate circulating laser beam.

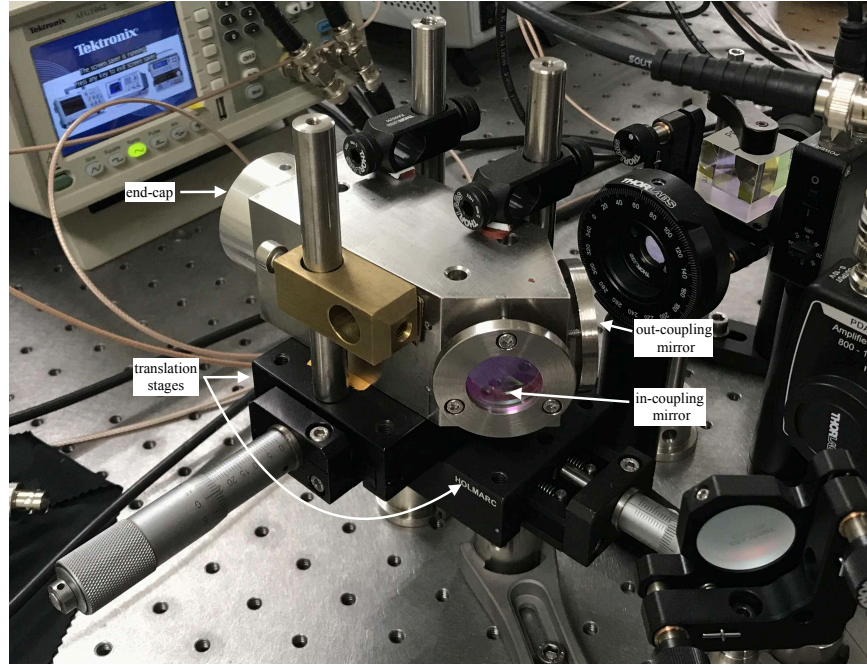
8.1.3 Mounting the resonator

The resonator is mounted on two perpendicularly placed translation stages (refer figure 8.5), which are then mounted on two 25 mm pedestal posts that are fixed using fork-clamps.

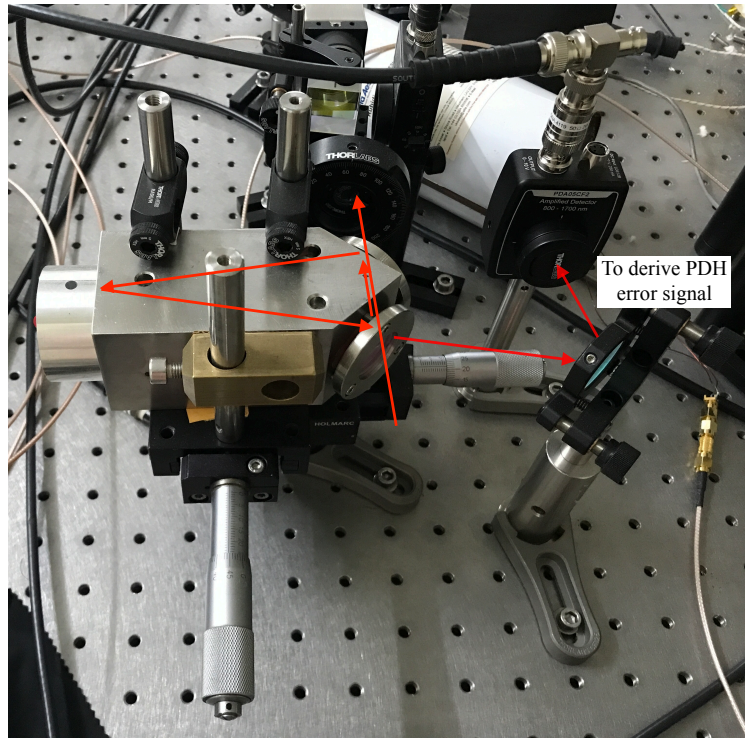


(a) Piezo glued onto the end cap. (b) Wires from behind the end cap used to drive the piezo. (c) Back mirror glued onto the piezo.

Figure 8.4: Mounting a ring piezo between the back mirror and the end cap to facilitate tuning of the resonator's optical path length.



(a) Front view



(b) Top view

Figure 8.5: Optical resonator mounted on two translation stages mounted on top of each other at 90° with respect to each other to enable micro movement of the cavity in the horizontal plane. Right angle clamps were used to hold the cavity in place. The red arrows indicate laser beam path.

A [Noliac 2124](#) piezoelectric ring actuator was glued onto the back mirror using suitable Loctite superglue (figure [8.4](#)). The front mirrors are mounted on a rubber ring (refer figure

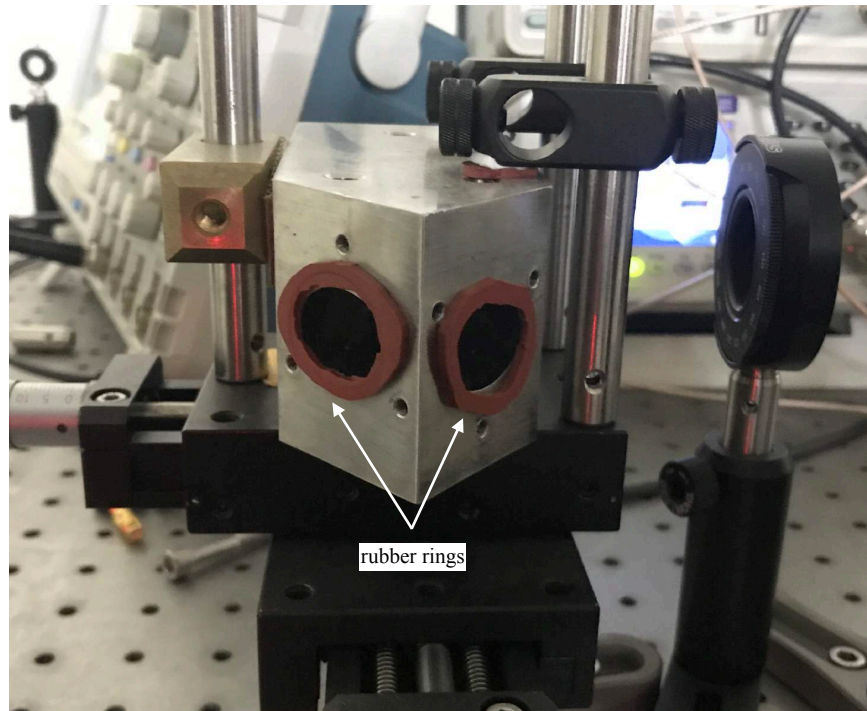


Figure 8.6: Rubber rings glued onto the metal spacer before clamping the front mirrors on them.

[8.6](#)) to allow yaw and pitch movements of these mirrors that is otherwise not possible when they are directly clamped on to the spacer. Of course, this completely renders the whole idea of using a rigid cavity spacer futile. But, this is done to compensate for any inaccuracies in spacer geometry. The rubber rings are cleaned in a soap solution and dried thoroughly before bringing them in contact with the mirrors' HR surface.

8.2 Optical setup

An optical isolator is used right in front of the laser head to prevent back scattered light from entering the laser head and causing unwanted laser noise. Four lenses [LB1945](#) ($f = 200$ mm), [LA1608](#) ($f = 75$ mm), [LB1904](#) ($f = 125$ mm) and [LA1229](#) ($f = 175$ mm) are used to perform mode-matching (section [8.2.2](#)). The EOM is driven by a resonant circuit ~ 19.6 MHz. Two input coupling mirrors are used to steer laser beam entering the resonator.

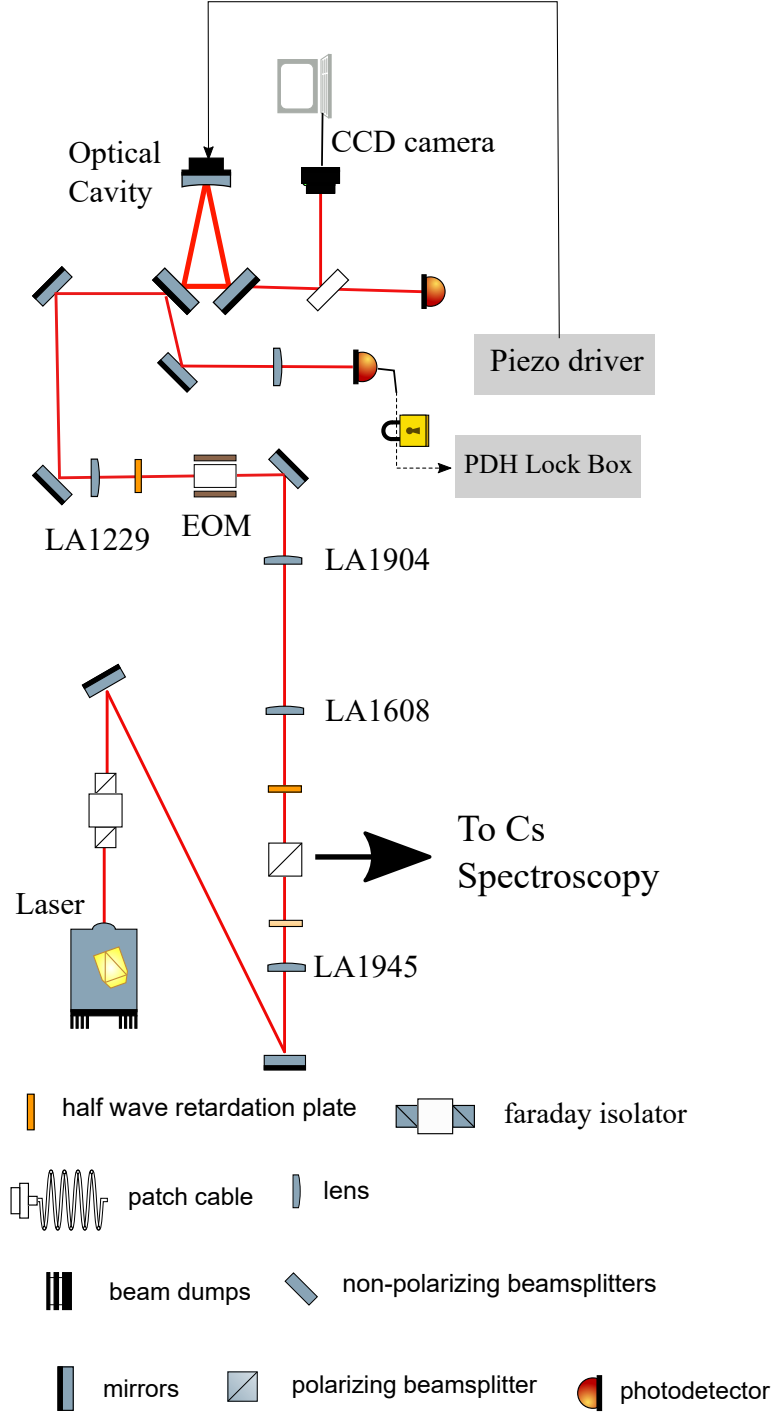


Figure 8.7: This figure shows optical layout for the experiment. Lenses of focal lengths 200 mm, 75 mm, 125 mm and 175 mm (in the direction of laser beam propagation) are used to modify beam profile and perform mode-matching. An EOM is used to phase modulate the laser beam at 20 MHz. The two mirrors upstream and closest to the optical cavity (after LA1229 lens) are used to steer and walk the beam through the cavity; these are called *beam steering* mirrors.

Beam exiting the resonator is split for observation of the modes using a CCD camera and a photodiode. Beam reflected off from the input coupling mirror of the resonator is reflected off from a mirror into a fast photodiode to perform PDH locking. A lens is used to focus beam on the small aperture of this fast photodiode.

Modes of an optical cavity are those TEM modes that sustain after every round-trip of the circulating laser beam, and these are called cavity *eigenmodes*. Any mode structure that is assumed by the circulating beam inside the resonator (cavity mode spectrum, for example figure 9.1) can be mathematically described as a weighed sum of the resonator's eigenmodes. Thus these eigenmodes are as a basis for the cavity modes. These cavity modes are excited by the input laser beam. Consider an input laser beam that is purely HG_{00} . This is the fundamental mode or HG_{00} thus one of the eigenmodes of the input beam. If this beam resonates perfectly with the optical resonator, then the HG_{00} mode of the input beam couples into the fundamental mode (HG_{00}) of the optical resonator. This can be imagined as basis transformation from the eigenmodes of the input beam to that of the resonator. But in reality, the input beam is *misaligned* or *mode-mismatched* with the resonator, and this leads to imperfect resonance [10]. In the next two sections we will describe our efforts to align and perform mode-matching of the resonator.

8.2.1 Alignment

Following mounting the resonator firmly on the optical table and mounting the optics as shown in figure 8.7, laser beam is aligned to the resonator such that the resonator modes are visible on a CCD camera placed downstream the cavity. Alignment is done systematically as follows:

1. Firstly, it is ensured that the input beam traverses a constant height and is horizontally aligned. Two irises are placed at constant height and aligned along a line of screw holes in the optical bench. These irises are used to align the laser beam vertically and horizontally.
2. The spacer with back mirror and the out-coupling mirror is mounted on the optical table such that the input beam hits the center of the out-coupling mirror. An IR viewing card is held inclined at the position of the input coupling mirror such that the

beam reflected off the back mirror and the input beam are visible simultaneously on the card. If these beam spots are not visible then the the direction of the reflected beam is slightly adjusted with the screws that hold the out-coupling mirror. These two beam spots should coincide. (*Optional step: First mount the out-coupling mirror and then ensure that the beam reflected off this mirror arrives almost at the center of the end mirror by observing the beam before hole on the back side of the spacer*)

3. The piezo scanning is turned on. The in-coupling mirror is clamped and the screws holding this mirror are fine-adjusted to tune the yaw and pitch of the mirror. We look for scanning of modes in the camera while the mirror's position is adjusted.
4. The piezo scanning is done across a few FSRs - this is ensured by observing repetition of modes in the same scan in the oscilloscope. While the modes are visible on the ccd camera and photodiode, the beam steering mirrors are used to steer and walk the beam such that HOMs are suppressed and transmission of TEM₀₀ mode (fundamental mode) is maximized. Subsequently, *mode-matching* (section 8.2.2) is done to further improve the transmission of TEM₀₀ mode. Further optimization is done by observing dips in the power of the reflected beam on the oscilloscope as the piezo is scanned.

8.2.2 Mode-matching

We have seen in section 7.2 that the geometry of an optical resonator defines the modes and beam propagation parameters inside the resonator. Thus it is essential that a laser beam injected into an optical resonator has Gaussian (or HOMs) propagation properties that match with that of the resonator. This condition is called *mode-matched*. The Gaussian mode is favoured over all other modes due to its less lossy compared to all other HOMs [21]. Mode-matching by transforming a Gaussian beam using thin lenses. From section 8.1.1 we know that a Gaussian mode inside the resonator has a beam waist of $\approx 333 \mu\text{m}$, so the input laser beam should be transformed using lenses such that it has the same beam waist at the location of the cavity's beam waist. [JamMt](#), was used to perform mode-matching. From figure 8.7 it is seen that four lenses are used to assist us in mode-matching. During data acquisition it was seen that the input beam was mode-mismatched with the cavity to a large extent. Spot size of the input beam at the location of the beam waist inside the cavity is $\sim 660 \mu\text{m}$. And the input beam focuses at a distance of about 14 cm from the cavity waist with waist size of about $560 \mu\text{m}$. Mode-mismatches lead to coupling of input HG₀₀ mode

laser beam into higher order LG modes while beam circulates in the cavity [10]. This leads to unwanted HOMs (Laguerre Gaussian) in the transmitted beam. Efforts will be made in the future to improve mode-matching (chapter 6).

8.3 FINESSE simulation

[FINESSE](#) is an interferometer simulation program. FINESSE is used from Jupyter notebooks, using its Python wrapper [PyKat](#). Below is a program written during the course of this project to closely simulate the optical resonator that is developed here. Performing a FINESSE simulation of the resonator enables to better understand the resonator dynamics. The cavity parameters mentioned in section 8.1.1 are a result of this simulation.

```

1 from pykat import finesse          # import the whole pykat.finesse package
2 from pykat.commands import*        # import all packages in pykat.commands
3 import numpy as np                 # for basic math/sci/array functions
4 import matplotlib.pyplot as plt    # for plotting
5
6 basekat = finesse.kat() # initialising Finesse
7 f_mod = 19.6*(10**6)
8 # Writing FINESSE code to simulate the Optical resonator.
9 basekat.parse("""
10 l laser 1 0 n1
11
12 #phase modulation
13 s smod 1 n1 nmod1
14 mod eom {f} 0.0000001 1 pm nmod1 nmod2
15 s s0 1 nmod2 nBSs
16
17 # front mirror 1 modelled as a realistic beam splitter
18
19 bs BS 0.015 0.985 0 41.6 nBSs nBSw nBSi1 nBSi3
20 s sBS1 0.007 1.44963 nBSi1 nBSi2
21 s sBS2 0.007 1.44963 nBSi3 nBSi4
22 bs BSAR1 0.9997 0.0003 0 26.7 nBSi2 nBSrefl1 nBSn nBSAR
23 bs BSAR2 0.9997 0.0003 0 26.7 nBSi4 dump nBSe dump
24 #s sBS3 0.007 1.44963 nBSrefl1 nBSrefl2
25 #bs BSrefl 0.015 0.985 0 41.6 dump nBSrefl_leak dump nBSrefl2
26

```



```

27 # base of the tri cavity
28 #increase by 4mm
29 s base 0.0294 nBSn nBSsf2
30
31 # front mirror 2 .....
32
33 bs BSf2 0.9997 0.0003 0 -41.6 nBSsf2 nBSwf2 nBSi1f2 nBSi3f2
34 s sBS1f2 0.007 1.44963 nBSi1f2 nBSi2f2
35 s sBS2f2 0.007 1.44963 nBSi3f2 nBSi4f2
36 bs BSAR1f2 0.015 0.985 0 -26.7 nBSi2f2 dump nBSnf2 nBSARf2
37 bs BSAR2f2 0.015 0.985 0 -26.7 nBSi4f2 dump nBSef2 dump
38
39 # tri arm length
40 #increase by 2mm
41 s arm1 0.10926 nBSwf2 nBSsb
42
43 # Back mirror.....
44
45 bs BSb 0.9997 0.0003 0 -6.8 nBSsb nBSwb nBSi1b nBSi3b
46
47 #s sBS1b 0.00638 1.44963 nBSi1b nBSi2b
48 #s sBS2b 0.00638 1.44963 nBSi3b nBSi4b
49 #bs BSAR1b 0.015 0.985 0 -4.7 nBSi2b dump nBSnb nBSARb
50 #bs BSAR2b 0.015 0.985 0 -4.7 nBSi4b dump nBSeb dump
51
52 attr BSb Rc 1
53
54 # tri arm length
55 #increase by 2mm due to viton gaskets placed to allow freedom on mirror
    position.
56 s arm2 0.1093 nBSwb nBSAR
57
58 # cavity parameters
59
60 cav pmc BSAR1 nBSn BSAR1 nBSAR
61
62 """.format(f=f_mod)) # Parsing the FINESSE-code
63
64 detectors_kat= finesse.kat()
65 detectors_kat = basekat.deepcopy()
66

```

```

67 # Parsing necessary detectors to the above FINESSE code
68 detectors_kat.parse("""
69 pd circ nBSn
70 pd tran nBSnf2
71 pd refl_prompt nBSw
72 pd refl_leak nBSrefl
73 ad ad_refl_prompt 0 nBSw
74 ad ad_refl_leak 0 nBSrefl
75 ad inp 0 nBSs*
76 ad refl_side 11.5M nBSw
77 ad refl_carr 0 nBSw
78 ad circ 0 nBSn
79 trace 2
80 cp pmc x stability
81 pd1 PDH {f} 0 nBSrefl
82 xaxis BSb phi lin -0.1 0.1 500
83 """.format(f= f_mod))
84 kat.verbose=False
85 out = amplitude_kat.run() # Running the FINESSE-simulation, storing output
    in out.

```

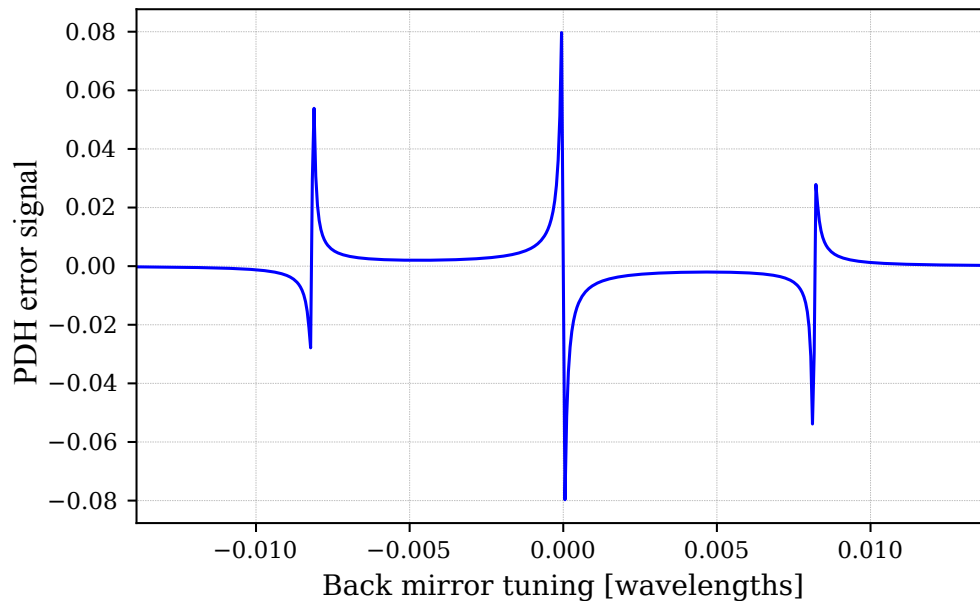


Figure 8.8: Error signal derived from FINESSE simulation of the resonator. Modulation (and demodulation) frequency: 19.67 MHz. The x-axis here indicates cavity length tuning in FINESSE parlance; back mirror tuning of 0.005 wavelengths (i.e. 1064 nm) corresponds to a length change of about 5 nm.

Now that we have seen experimental details of our efforts to develop a high finesse optical resonator, in the next chapter we will look at a few observations pertaining to the frequency stabilization of the 1064 nm laser.

Chapter 9

Optical Resonator Results

In this chapter we will present preliminary results of the optical resonator set-up that was described in the previous chapter. The first section will include images of cavity modes captured using a CCD camera and their positions in the resonator mode spectrum. In a later section we will show the PDH error signal derived from the cavity and laser beam phase modulation sidebands. These observations can be used to improve the cavity alignment and mode-matching further. The error signal can be used to lock the cavity to the laser or vice-versa (refer chapter [11](#)).

9.1 Cavity modes

As shown in figure [8.7](#) in the previous chapter, a p-polarized input laser beam is coupled into the cavity. A non-polarizing beam splitter is placed upstream the cavity to split the beam between a photo-diode and a CCD camera. The piezo is scanned such that the cavity length is tuned across multiple FSRs. Figure [9.1](#) shows that the incoming laser beam couples into two other modes (besides few others with negligible power) apart from the fundamental mode. Images of these modes captured with a CCD camera is presented in figure [9.2](#).

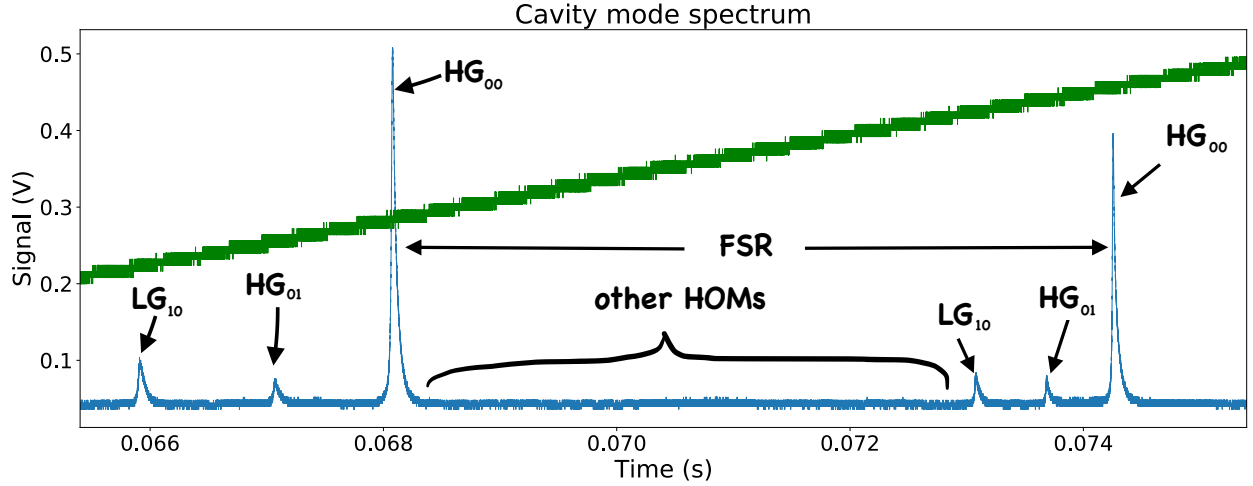
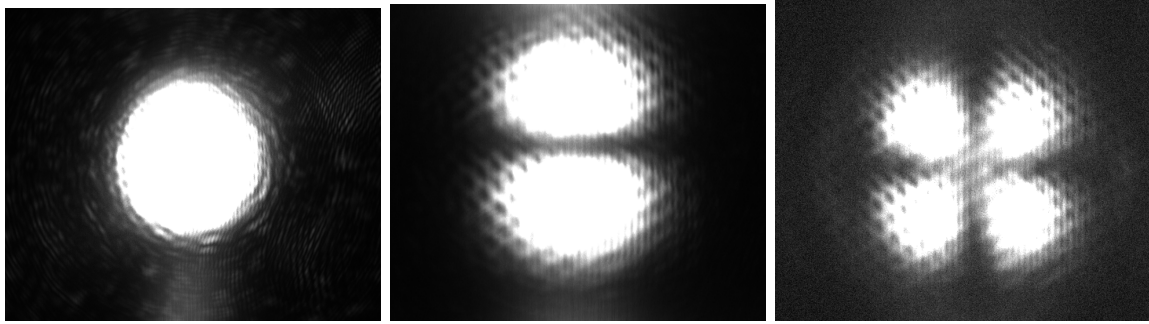
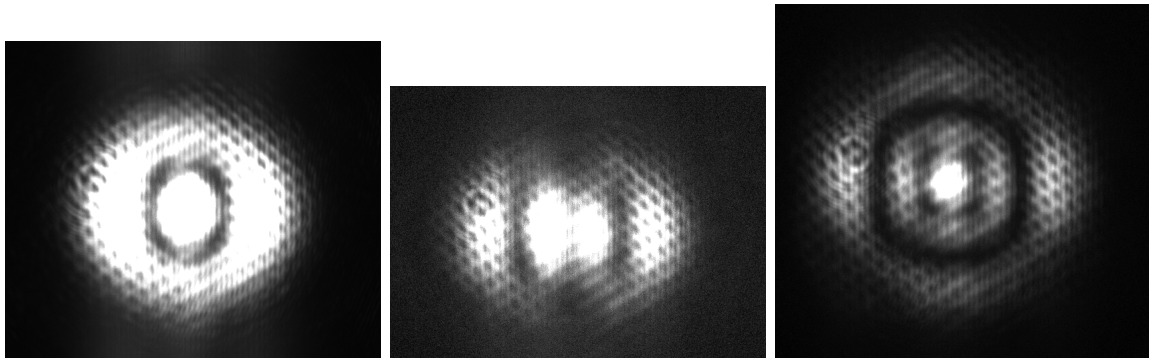


Figure 9.1: Here, solid blue curve shows resonator's mode spectrum or eigenmodes of the triangular resonator resonating inside and the cavity. Solid green curve is the piezo scan voltage with time. This curve has been scaled by a factor of 0.1 for illustration purpose. For the solid blue curve the y-axis corresponds to photodiode voltage (at 50 dB gain) and for the green curve y-axis depicts the voltage input to the piezo driver that drives the piezo (after a gain factor of 15). The mode-spacing varies as the piezo scans subsequent FSRs, thus the piezo scan is non-linear. Also, the amplitude of the fundamental mode is different for consecutive FSRs. This could be due to limited response speed of the photodiode used; this can be avoided by using a fast photodiode.

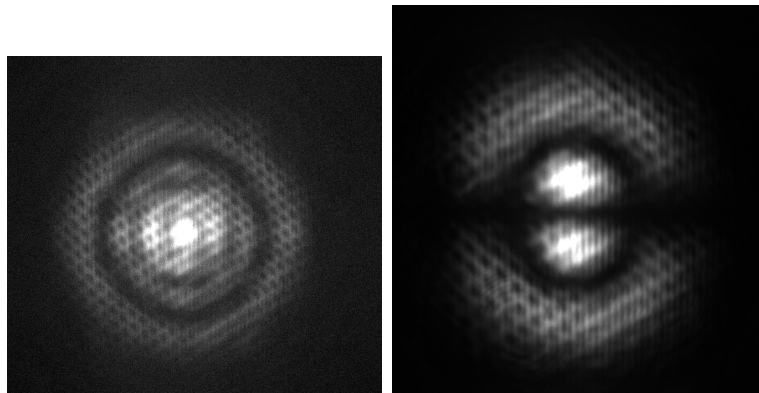
From the images in figure 9.2 we observe that apart from HG_{00} there are other HOMs. We know that even if the input laser beam has a good beam quality parameter ($M^2 < 1.1$) such that the input beam is mostly oscillating in the fundamental mode, a slight misalignment or mode-mismatch can cause coupling into HOMs (refer chapters 10 & 11 of [10]). In subsection 8.2.2 we saw that mode-matching is not good-enough since the input beam waist position is far away from the cavity waist position. Improvements to this will be made in the future (refer chapter 11). Laguerre-Gaussian modes LG_{10} , LG_{20} and LG_{30} are a result of mode-mismatch [10]. The presence of second and third order LG modes agrees with the degree of mode-mismatch as described in subsection 8.2.2 and suggests that mode-matching could still be improved. We also observe HG_{01} and HG_{11} modes. While HG_{01} results from a slight misalignment or even beam jitter, presence of HG_{11} , although negligible in power, suggests that cavity alignment has room for improvement.



(a) HG_{00} or the fundamental mode. (b) HG_{01} - a first order Hermite Gaussian mode. (c) HG_{11} - a second order mode.



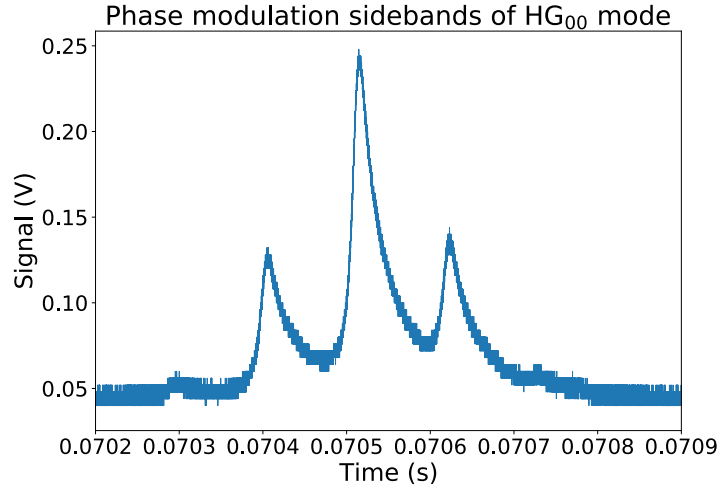
(d) LG_{10} - a first order Laguerre Gaussian mode. (e) LG_{11} - a second order Laguerre Gaussian mode. (f) LG_{20} - a second order Laguerre Gaussian mode.



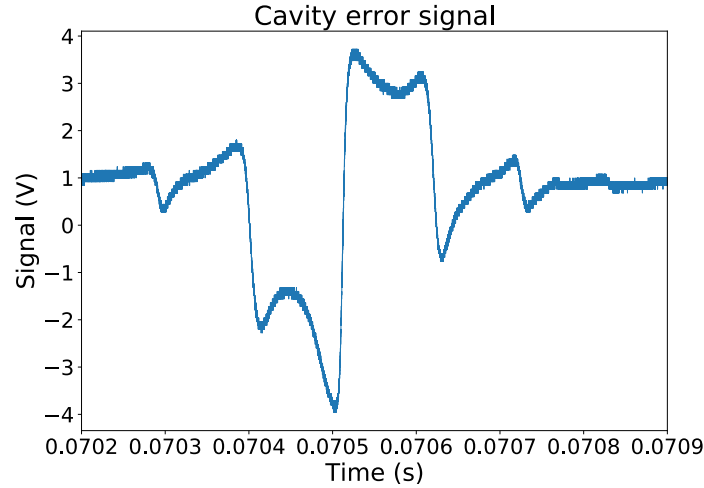
(g) LG_{30} - a third order Laguerre Gaussian mode. (h) IG_{31} - a fourth order Ince-Gaussian mode.

Figure 9.2: Transverse Electromagnetic Modes (TEM) of the resonator captured with a CCD camera. The intensity of the modes have been scaled for illustration purpose.

9.2 Error signal



(a) Phase modulation sidebands of HG00 eigenmode with modulation frequency of 19.67 MHz. Here we observe asymmetry between the left and right wings of the spectral shapes, this could be due to non-linearity in scanning of the piezo.



(b) PDH error signal corresponding to the sidebands in the above figure (a).

Figure 9.3: Figure (a) is the photodiode signal with gain set at 50 dB. Power in the carrier is 0.25 V and that in sideband is 0.13 V. Thus the modulation index is about 0.9 ([EOM manual](#)). Figure (b) shows the error signal as a result of demodulation at 19.67 MHz of the beam reflected off the in-coupling mirror of the resonator. Subsequently, the demodulated signal is passed through an active low pass filter to render the error signal above.

Chapter 10

Experimental Challenges

Relative Frequency Stabilization

Several challenges were faced during the course of development of the optical resonator. The stainless steel spacer fabricated to make up the cavity has imperfections. This posed a challenge to cavity alignment. Another obstacle faced was dust on the optics. This poses serious issues like power loss and finesse reduction. Dust also damages high finesse mirrors permanently. Piezo calibration is another challenge faced during data acquisition. In figure [9.1](#) we see that the x-axis is time and not frequency. It is more appropriate to plot cavity modes with frequency as this will shine light on some cavity parameters like Gouy phase shift, transverse mode spacing, cavity pole and FSR. In this chapter we will present these obstacles in detail and present some measures taken towards circumventing them.

10.1 Spacer imperfections

As observed in figure [8.2](#), the solid spacer was designed to hold the front mirrors directly onto the mirrors by circular clamps. But this posed problems during resonator alignment. Imperfections in spacer dimensions caused deviations from ideal optical path length and angle of incidence of the resonator. To overcome this, rubber rings were placed between the

HR coated surface of the mirrors and the steel spacer. The rubber rings were cleaned with soap solution and dried thoroughly before use. They were glued onto the steel surface using superglue. This arrangement allowed some freedom on pitch and yaw of the front mirrors. This enabled alignment of the resonator quicker and easier.

However, these rubber rings impinged impressions/markings on the outer surface region of the HR coated side of the mirrors. This contaminated the HR coating in those regions, also contributing to reduction of resonator finesse.

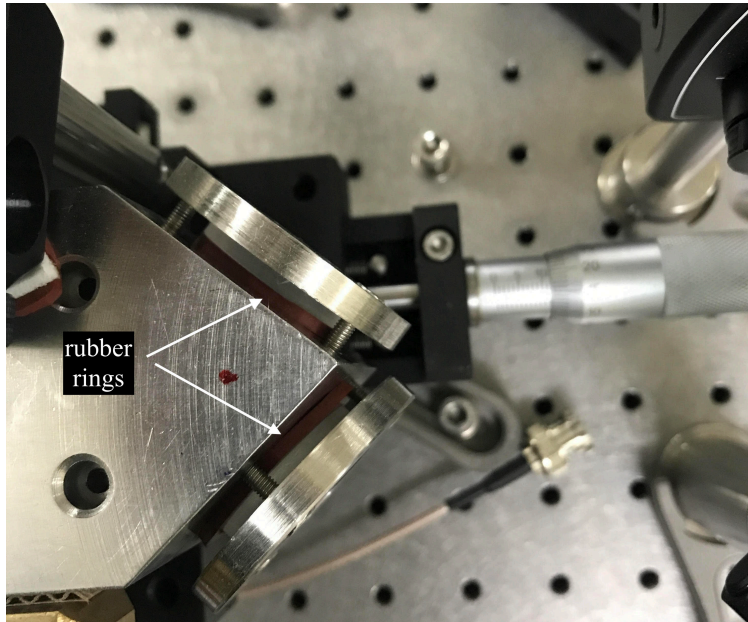


Figure 10.1: Thoroughly cleaned rubber rings used to facilitate movement of cavity's front mirrors.

10.2 Finesse reduction

Accumulation of dust on the resonator system (figure 10.2) further contributed to reduction in resonator finesse. This was cleaned by using a low pressure filtered dry nitrogen nozzle. This seemingly improved cavity transmission and reduced cavity pole by about a few factors.

Now that we have seen preliminary results from this experiment and obstacles faced during the attempt towards relative frequency stabilization, let us look at future directions

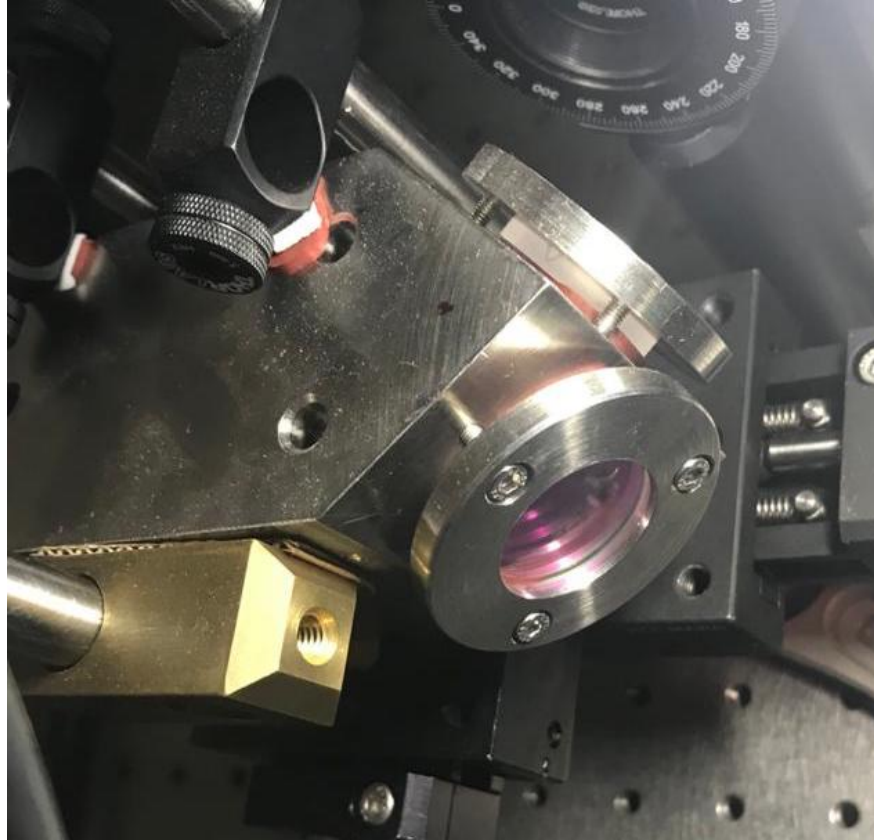


Figure 10.2: Dust accumulation on the resonator.

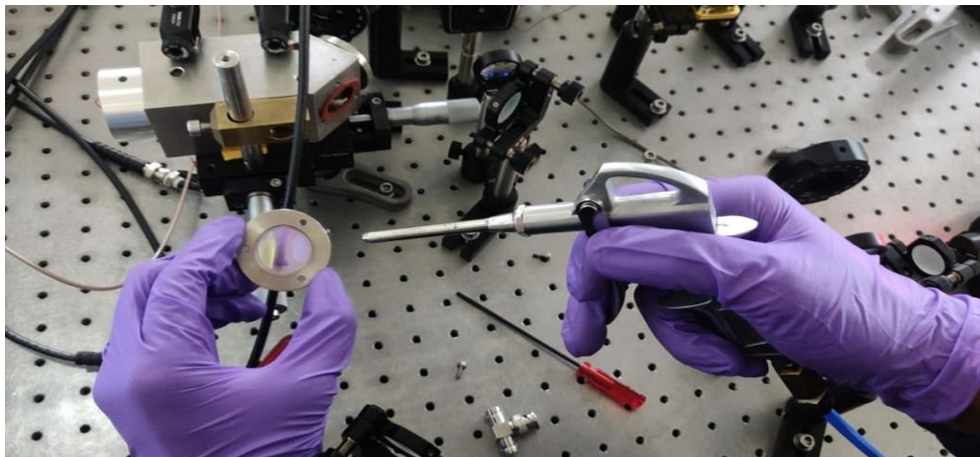


Figure 10.3: Cleaning the resonator system using a low pressure filtered dry nitrogen nozzle.

in the next chapter. Let us understand in detail how the system can be improved, and how we can move towards achieving the goal.

Chapter 11

Future Directions

Relative Frequency Stabilization

From the graphs in chapter 9 we realize that the x-axis in these plots is time. But presenting these graphs as a function of cavity length tuning in terms of cavity FSR as the x-axis is desirable. Moreover the piezo scan is non-linear and has hysteresis, thus disabling us from using a linear relationship between cavity length tuning from the piezo driving voltage to deduce cavity parameters. Our efforts to acquire cavity mode spectrum as a change in laser frequency failed because the laser piezo scan range is only 65 MHz which is much smaller than cavity FSR (1.2 GHz), and the temperature frequency scan has a bandwidth lower than 1 Hz. Efforts will be made in the future to calibrate the piezo and plot the mode-spectrum as a function of cavity length tuning (or the FSR).

The main purpose of development of the resonator is to frequency stabilize the laser. But the present cavity design incorporates a stainless steel cavity spacer. This invites thermal noise coupling into the system as cavity length fluctuations. This would inflict phase noise in the error signal derived from the cavity thus making the frequency stabilization harder to achieve. Efforts will be made in the future to incorporate an INVAR (FeNi36) or titania-silicate glass (ULE 7972) spacer that would inflict negligible thermal noise due to their low thermal expansion properties.

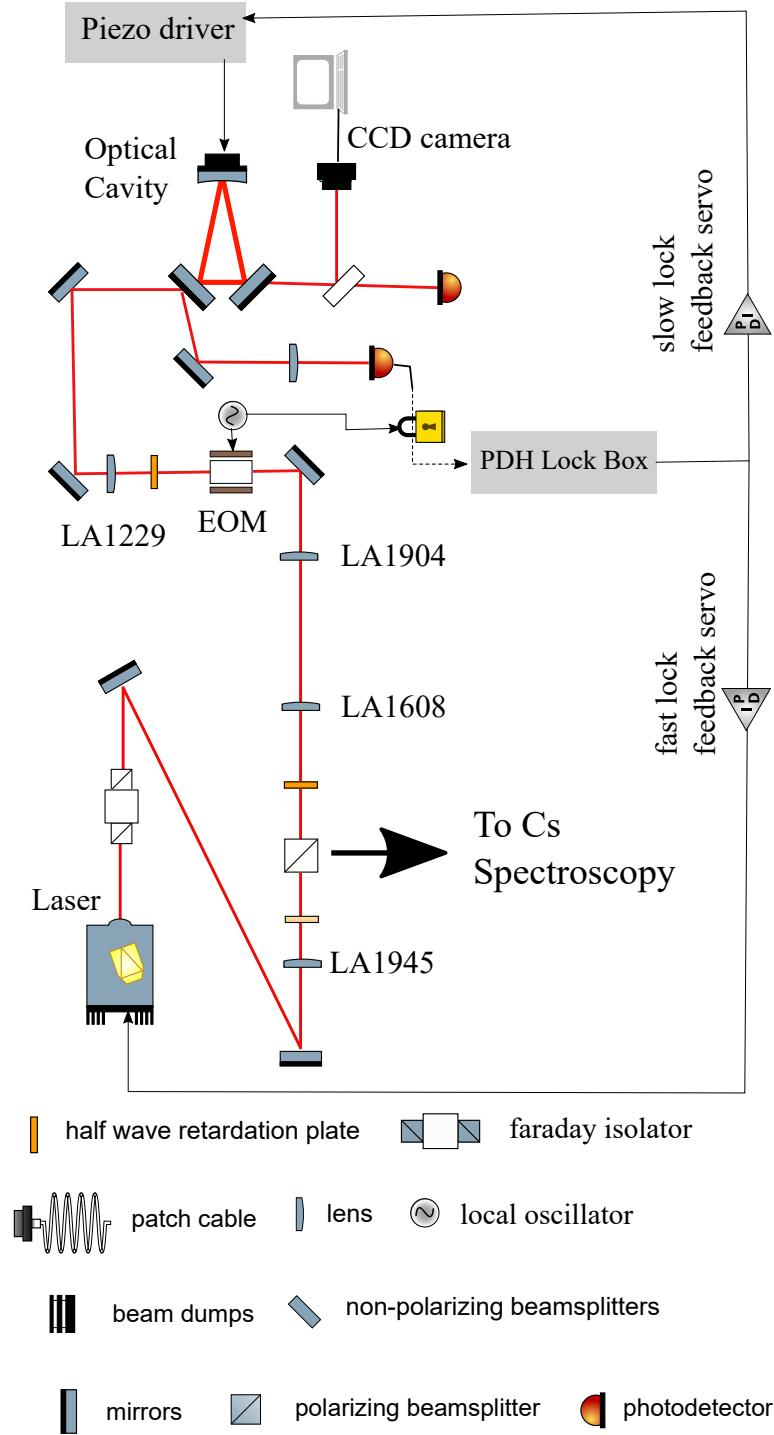


Figure 11.1: Optics setup for relative frequency stabilization and mode-cleaning (adapted from Fig. 1 of [31]).

Furthermore, graphs and data presented in chapter 9 are preliminary results pertaining to relative frequency stabilization. Efforts will be made in the future to improve mode-matching and alignment. Efforts will be made to improve mode-matching using less number of lenses unlike the current setup (figure 8.7).

Initially, we will build an analog PID to lock the cavity to the laser. Subsequently, a spectrum analyzer will be used to study the frequency and power noise downstream the optical resonator since the cavity is designed to suppress RF power fluctuations and perform passive frequency stabilization [1][12].

Upon incorporating the aforementioned changes or improvements, stabilization scheme shown in figure 11.1 will be attempted. In this setup the cavity is designed and developed such that it can be exploited for relative frequency stabilization as well as mode-cleaning of the laser beam. A slow locking servo will lock the cavity to the laser while a fast locking servo will lock the laser to the cavity. The former is aimed at enhancing the spatial and temporal profiles of the laser's fundamental mode and the latter is aimed at performing frequency stabilization of the laser.

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