Saturation Aided Non-Linear Absorption in a Double Λ System of EIT

A thesis submitted towards
partial fulfilment of the requirements
of the BS-MS dual degree programme

By

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Certificate

This is to certify that this dissertation entitled *"Saturation Aided Non-Linear Absorption in a Double Λ System of EIT"* towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by **Miss Sayali Ganesh Shevate**, Reg.no.2012031, at Raman Research Institute (RRI), Bangalore under the supervision of Dr. Andal Narayanan, RRI, Bangalore during the academic year 2016-17.

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Declaration

I hereby declare that the matter embodied in the report entitled "Saturation Aided Non-Linear Absorption in a Double Λ System of EIT" are the results of the work carried out by me at Raman Research Institute, Bangalore under the supervision of Prof. Andal Narayanan and the same has not been submitted elsewhere for any other degree.

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Miss Sayali Ganesh Shevate

Abstract

Dark states are atomic dressed eigen states which decouple from interaction with EM waves even on resonance. The most simple dark state production requires at least three atomic levels interacting with two EM fields typically one stronger and another weaker. A classic atomic level configuration of a three-level interaction which involves two ground states and one excited state structure goes under the name of a Lambda system. The resultant transparency effect which is quantum in origin is the Electromagnetically Induced Transparency (EIT) effect.

Introduction of additional number of levels leads to multiple formation of dark states. In this thesis we study dark state formation in a double-Λ system of EIT. Here two dark states are observed corresponding to two excited states. In particular, we experimentally study the competing interaction between the two dark states where the atomic levels are chosen in the D2 manifold of ^{85}Rb involving $5S_{1/2}$ ground states and $5P_{3/2}$ excited states.

The specific question we address is how in a suitable geometry of beam configurations the quantum EIT effect is enhanced by the presence of a classical hole-burning saturation absorption phenomena. Saturation absorption aided EIT results in an enhanced non-linear absorption in a non-EIT beam by the same atom due to suppression of linear absorption.This is the main focus of our study.

In parallel I also present a theoretical work, where we observe the evolution of form-stable coupled excitations in a closed-Lambda system of EIT. These coupled excitations are known as polaritons. The formalism for undertaking the study is done and this is what is reported.

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Chapter 1

Introduction

1.1 Introduction to Electromagnetically Induced Transparency(EIT)

Electromagnetically Induced Transparency(EIT) is a Quantum phenomenon through which an otherwise opaque medium to a resonant laser field becomes transparent to that resonant frequency. Such a state of the system is called as Dark State. This can be given a semi-classical treatment where atom is treated quantum mechanically and the fields can be dealt classically. EIT is observed in dilute atomic media where sharp energy levels are present. First theoretical proof of EIT was given by Harris et. al. [1] and experimentally observed by Bolter et. al. in strontium vapor [2] and by Field et. al. in lead vapor [3].

Creation of Dark States [4] has been extensively used as a fundamental tool to control coherences among atomic energy levels. As a consequence, the interference of absorption and emission pathways originating from these energy levels takes place to give rise to phe-

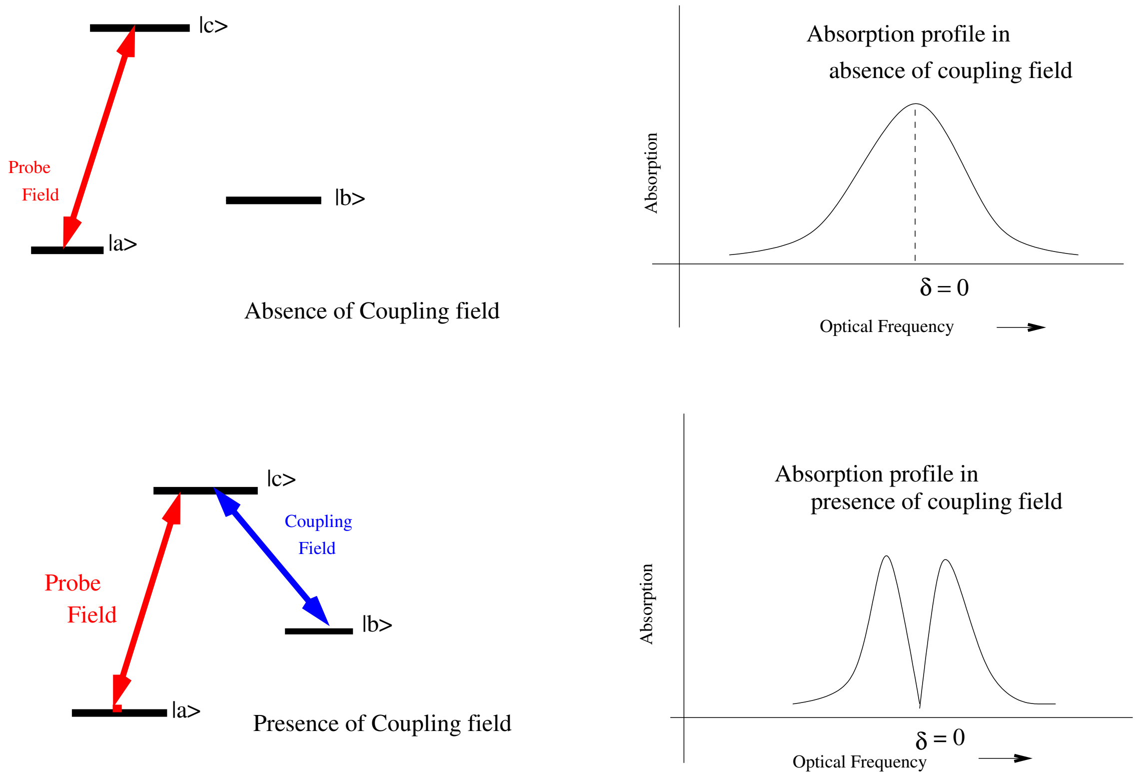

Figure 1.1: The absorption profile of probe beam intensity scanned over a range of frequencies when passed through a dilute atomic medium in presence and absence of coupling field where ω_p is the resonant probe field where $\delta = \omega_{ac} - \omega_c$
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nomenon like Electromagnetically Induced Transparency [5-8] or Lasing Without Inversion [9]. Multiple dark states get created when system with multiple levels interact with multiple fields [10]. Those multiple dark states then interact among each other [11]. Example of such multilevel systems are N-system [12] and double Λ system [13] where two interacting dark states get created with three fields. These systems which involve non-degenerate ground state configurations have been studied thoroughly in the reference of enhancement of four-wave mixing signals [14-15].

Our work involves double-Λ system consisting of two degenerate ground states and two non-degenerate excited states. Here, two dark states are formed in this double-Λ system when it interacts with three laser fields having proper polarizations. These dark states interact at every detuning to give a resultant state. As this interaction is coherent, it results in partial or full cancellation of effects produced by individual dark states. When each of the dark states attains same transition strength with respect to corresponding excited state, the maximum interaction between the two ground states is observed [16].

This thesis also talks about the parallel theoretical work I am involved in. This work is based on form-stable coupled excitations of light and matter associated with the propagation of quantum field in EIT media [17]. The system under consideration is Δ system of EIT. A microwave-driving field couples the two lower energy states of a Λ-energy level system of atomic media thereby transforming it into a Δ system [6]. As double-Λ-system is mapable to Δ system, the results from this theoretical work can be compared and verified with that of the above experimental results. The aim is to find the form of polariton when we introduce a microwave field to Λ energy level configuration to couple the ground state and metastable state, the transition which is otherwise dipole forbidden. The microwave field makes the transition to be allowed through magnetic dipole. The probe field used here, is quantum mechanical in nature. Polaritons are mixtures of photonic and Raman-like matter branches [17]. By adiabatically changing a coherent field, dark state polariton can be stopped and reaccelerated in a way that its shape and quantum state is preserved. During this process, quantum state of light is transferred to collective atomic excitations [17] and vice versa.

Such access to the control of the propagation of quantum light through dark state polaritons can be applicable for the generation of squeezed or entangled states, reversible quantum memories for light waves,etc. [18-20]. This technique can be proved as a very good method for quantum information processing where quantum information is stored in collective excitations of matter [17]. The concept of dark state polariton can also be a powerful tool in the study of quantum scattering of dark state polaritons in optical lattices and quantum phase gates for photons [17]. In this theoretical work, we have managed to obtain a raw form of polariton analytically in atomic Δ- system, details of which will be presented in Ch.3.

1.2 EIT Schemes

The energy level configurations where EIT can be observed, are as shown in fig. 1.2. The state $|b\rangle$ is a metastable ground state in Λ system. Hence, atoms in Λ-system have a very low decay rate from state $|b\rangle$, the EIT effect is much stronger in the Λ-system than the other configurations [21]. In other configurations, the state $|b\rangle$ is an excited state from where atoms decay via spontaneous emission. Hence, main focus of this theory will be on Λ-type configuration. Let $|a\rangle \rightarrow$ Ground state, $|b\rangle \rightarrow$ Metastable ground state, $|c\rangle \rightarrow$ Excited state

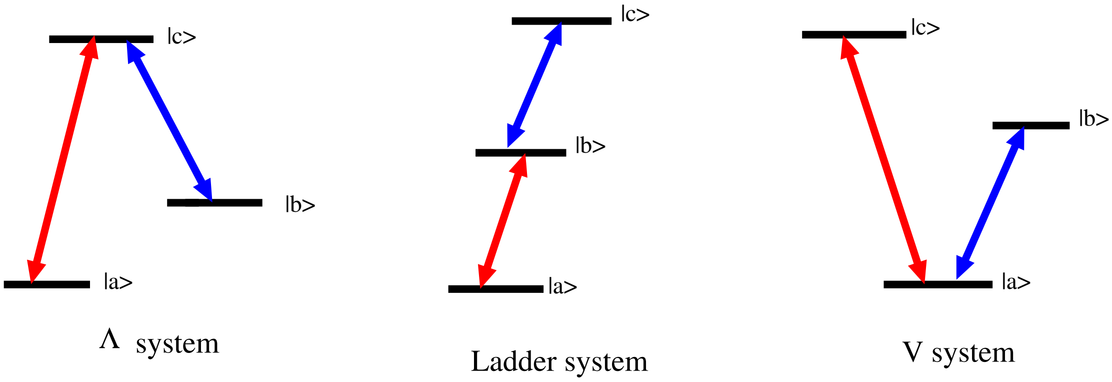


Figure 1.2: Energy level schemes where EIT can be observed

Energies of states: $E_n = \hbar\omega_n$ where n = a, b, c and ω_n = corresponding frequencies. Atoms at $|a\rangle$ and $|b\rangle$ can get excited to $|c\rangle$ but transitions between $|a\rangle$ and $|b\rangle$ is considered to be dipole forbidden. Therefore, the corresponding resonant frequencies between the relevent energy levels are: $\omega_{ac} = \omega_c - \omega_a$ and $\omega_{bc} = \omega_c - \omega_b$.

1.3 Bare-atom Hamiltonian:

In the absence of any external field, the energy eigenvalues and eigenstates are $\hbar\omega_n$ and $|n\rangle$ respectively. By completeness and orthogonality, $\sum |n\rangle \langle n| = 1$ and $\langle n|m\rangle = \delta_{nm}$. Hence the bare-atom hamiltonian can be expressed as

$$H_0 = \begin{pmatrix} \hbar\omega_a & 0 & 0 \\ 0 & \hbar\omega_b & 0 \\ 0 & 0 & \hbar\omega_b \end{pmatrix} \tag{1.1}$$

Spontaneous emission causes the system to get settled in the lowest energy state $|a\rangle$

1.4 Applied Fields and Interaction Hamiltonian:

Let E be the applied field such that E = $\epsilon_{co}\cos(\omega_{co}t - k_{co}.r) + \epsilon_p \cos(\omega_p t - k_p.r)$ where ϵ_{co} is tuned to that of $|b\rangle \rightarrow |c\rangle$ and ϵ_p is tuned to that of $|a\rangle \rightarrow |c\rangle$. Here, ω_{co} is a coupling field and ω_p is a probe field. This implies that $\omega_{co} \approx \omega_{bc}$ and also $\omega_p \approx \omega_{ac}$, r = Atomic Diameter, k= $2\pi/\lambda$. As r $\ll \lambda$, k·r comes out to be very small quantity and hence can be neglected. Hence,

$$E = \epsilon_{co}\cos(\omega_{co}t) + \epsilon_p \cos(\omega_p t) \tag{1.2}$$

The perturbed hamiltonian will become H = H_0 + H_1 where H_1 = -q**E**·**d**.

$$H_1 = -qEd\cos(\theta)$$

where θ is an angle between the direction of external field E and the separation vector d. This implies that atom behaves as electric dipole in the presence of external field with charge q and separation vector d. This atomic dipole vector aligns itself in the direction of external field. Hence,$H_1 = -qEd$ as the dipoles are aligned along the beam direction. For simplicity of notations, the term qd can be clubbed under the standard notation μ where μ can be defined as dipole moment operator = qd. Hence, H_1 becomes H_1 = -μE . The elements of μ are $\langle n|\,\mu\,|m\rangle$. $\implies$

$$H_1 = -E\begin{pmatrix} 0 & 0 & \mu_{ac} \\ 0 & 0 & \mu_{bc} \\ \mu_{ca} & \mu_{cb} & 0 \end{pmatrix} \tag{1.3}$$

As transition from $|a\rangle \rightarrow |b\rangle$ is dipole forbidden, the elements μ_{ab} and μ_{ba} are zero. Dipole operators couple levels of opposite parity. That is why the diagonal elements are zero.

1.5 Rotating Wave Approximation:

To see the effect of electric field, the above hamiltonian can be transformed into the interaction picture of the unperturbed system using following unitary matrix:

$$U(t) = e^{iH_0t/\hbar} = \begin{pmatrix} e^{i\omega_a t} & 0 & 0 \\ 0 & e^{i\omega_b t} & 0 \\ 0 & 0 & e^{i\omega_c t} \end{pmatrix} \tag{1.4}$$

This implies

$$U(t)H_1U^+(t) = E\begin{pmatrix} 0 & 0 & \mu_{ac}e^{-i\omega_{ac}t} \\ 0 & 0 & \mu_{bc}e^{-i\omega_{bc}t} \\ \mu_{ca}e^{i\omega_{ac}t} & \mu_{cb}e^{i\omega_{bc}t} & 0 \end{pmatrix} \tag{1.5}$$

As E $= \epsilon_{co}\cos(\omega_{co}t) + \epsilon_p\cos(\omega_p t)$. This implies

$$E = \frac{\epsilon_{co}}{2}\left(e^{i\omega_{co}t} + e^{-i\omega_{co}t}\right) + (\epsilon_p/2)\left(e^{i\omega_p t} + e^{-i\omega_p t}\right) \tag{1.6}$$

$\Longrightarrow$

$(UH_1U^+)_{13} =$ -$\mu_{ac}((\epsilon_{co}/2)(e^{i\omega_{co}t} + e^{-i\omega_{co}t}) + (\epsilon_p/2)(e^{i\omega_p t} + e^{-i\omega_p t}))e^{-i\omega_{ac}t}$

$(UH_1U^+)_{23} =$ -$\mu_{bc}((\epsilon_{co}/2)(e^{i\omega_{co}t} + e^{-i\omega_{co}t}) + (\epsilon_p/2)(e^{i\omega_p t} + e^{-i\omega_p t}))e^{-i\omega_{bc}t}$

$(UH_1U^+)_{31} =$ -$\mu_{ca}((\epsilon_{co}/2)(e^{i\omega_{co}t} + e^{-i\omega_{co}t}) + (\epsilon_p/2)(e^{i\omega_p t} + e^{-i\omega_p t}))e^{i\omega_{ac}t}$

$(UH_1U^+)_{32} =$ -$\mu_{cb}((\epsilon_{co}/2)(e^{i\omega_{co}t} + e^{-i\omega_{co}t}) + (\epsilon_p/2)(e^{i\omega_p t} + e^{-i\omega_p t}))e^{i\omega_{bc}t}$

Rotating wave approximation helps to average out rapidly oscillating terms. Hence, the exponential terms having large imaginary arguments can be ignored. Using $\omega_{co} \approx \omega_{bc}$ and $\omega_p \approx \omega_{ac}$, we obtain,

$$(UH_1U^+)_{13} = \text{-}\mu_{ac}(\epsilon_p/2)(e^{i\omega_{pt}-i\omega_{ac}t})$$

$$(UH_1U^+)_{13} = \text{-}\mu_{ac}(\epsilon_p/2)(e^{i\omega_p t-i\omega_{ac}t})$$

$$(UH_1U^+)_{23} = \text{-}\mu_{bc}(\epsilon_{co}/2)(e^{i\omega_{co}t-i\omega_{bc}t})$$

$$(UH_1U^+)_{31} = \text{-}\mu_{ca}(\epsilon_p/2)(e^{-i\omega_p t+i\omega_{ac}t})$$

$$(UH_1U^+)_{32} = \text{-}\mu_{cb}(\epsilon_{co}/2)(e^{-i\omega_{co}t+i\omega_{bc}t})$$

Transforming back to Schrodinger Picture, the interaction hamiltonian becomes,

$$H_1 = U_+(U(t)H_1U^+(t))U = -(\frac{1}{2})\begin{pmatrix} 0 & 0 & \epsilon_p\mu_{ac}e^{i\omega_p t} \\ 0 & 0 & \epsilon_c\mu_{bc}e^{i\omega_{co}t} \\ \epsilon_p\mu_{ca}e^{-i\omega_p t} & \epsilon_c\mu_{cb}e^{-i\omega_{co}t} & 0 \end{pmatrix}$$

The dipole moment operators are:

$$\mu_{ac} = \mu_{ca}^* = |\mu_{ac}|e^{i\phi_p} \text{ and } \mu_{bc} = \mu_{cb}^* = |\mu_{bc}|e^{i\phi_{co}}$$

$\Longrightarrow$

$$H = H_0 + H_1 = (\hbar/2)\begin{pmatrix} 2\omega_a & 0 & -\Omega_p e^{i\omega_p t+\phi_p} \\ 0 & 2\omega_b & -\Omega_{co}e^{i\omega_{co}t+\phi_{co}} \\ -\Omega_p e^{-(i\omega_p t+\phi_p)} & -\Omega_{co}e^{-(i\omega_{co}t+\phi_{co})} & 2\omega_c \end{pmatrix}$$

Here, Rabi frequencies are defined as follows:

$$\Omega_p = \epsilon_p |\mu_{ac}|/\hbar$$

and

$$\Omega_{co} = \epsilon_{co} |\mu_{bc}|/\hbar$$

1.6 Time and Phase Independent Frame:

To see EIT contribution clearly, the time and phase part is removed using the transformation by following unitary matrix:

$$\mathrm{G} = \begin{pmatrix} e^{-(i\omega_p t+\phi_p)} & 0 & 0 \\ 0 & e^{-(i\omega_{co} t+\phi_{co})} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If $|n\rangle$ are eigenstates of full hamiltonian H,then $|n'\rangle = \mathrm{G}\ |n\rangle$ are the eigenstates of corotating hamiltonian H'

$$H'\,|n'\rangle = i\hbar \tfrac{\partial |n'\rangle}{\partial t}$$

$$= i\hbar \tfrac{\partial (G|n\rangle)}{\partial t}$$

$$= i\hbar (\tfrac{\partial G}{\partial t}\,|n\rangle + G\tfrac{\partial |n\rangle}{\partial t})$$

$$= (i\hbar \tfrac{\partial G}{\partial t} G^{+} + GHG^{+}) G \left| n \right\rangle$$

$$\implies H' \left| n' \right\rangle = (i\hbar \tfrac{\partial G}{\partial t} G^{+} + GHG^{+}) \left| n' \right\rangle$$

$$\implies H' = (i\hbar \tfrac{\partial G}{\partial t} G^{+} + GHG^{+})$$

$$\implies H' = (\frac{\hbar}{2}) \begin{pmatrix} 2\omega_p & 0 & 0 \\ 0 & 2\omega_{co} & 0 \\ 0 & 0 & 0 \end{pmatrix} + (\frac{\hbar}{2}) \begin{pmatrix} 2\omega_a & 0 & -\Omega_p \\ 0 & 2\omega_b & -\Omega_{co} \\ -\Omega_p & -\Omega_{co} & 2\omega_c \end{pmatrix}$$

$$\implies H' = (\frac{\hbar}{2}) \begin{pmatrix} 2(\omega_p + \omega_a) & 0 & -\Omega_p \\ 0 & 2(\omega_{co} + \omega_b) & -\Omega_{co} \\ -\Omega_p & -\Omega_c & 2\omega_c \end{pmatrix}$$

Adding $-\hbar(\omega_w + \omega_p)I$ in above matrix, we obtain,

$$H' = (\frac{\hbar}{2}) \begin{pmatrix} 0 & 0 & -\Omega_p \\ 0 & 2(\omega_{co} + \omega_b - \omega_a - \omega_p) & -\Omega_{co} \\ -\Omega_p & -\Omega_{co} & 2(\omega_c + \omega_a + \omega_p) \end{pmatrix}$$

Here,

$$\Delta_p = \omega_{13} - \omega_p = \omega_3 - \omega_1 - \omega_p$$

$$\Delta_{co} = \omega_{23} - \omega_{co} = \omega_3 - \omega_2 - \omega_{co}$$

$$\implies H' = (\frac{\hbar}{2}) \begin{pmatrix} 0 & 0 & -\Omega_p \\ 0 & 2(\Delta_p - \Delta_{co}) & -\Omega_{co} \\ -\Omega_p & -\Omega_{co} & 2\Delta_p \end{pmatrix}$$

1.7 Dressed State Picture

: For $\Delta_p = \Delta_{co} = \Delta$, the characteristic equation

$$det|H' - \lambda I| = 0$$

yields the eigenvalues as:

$$\lambda_0 = 0$$

$$\lambda_- = \hbar\omega_- = \frac{\hbar}{2}(\Delta - \sqrt{\Delta^2 + (\Omega_p)^2 + (\Omega_{co})^2}$$

$$\lambda_+ = \hbar\omega_+ = \frac{\hbar}{2}(\Delta + \sqrt{\Delta^2 + (\Omega_p)^2 + (\Omega_{co})^2}$$

The corresponding eigenstates of dressed system are:

$$|0\rangle = \frac{\Omega_{co}\,|a\rangle - \Omega_p\,|b\rangle}{\sqrt{(\Omega_p)^2 + (\Omega_{co})^2}}$$

$$|-\rangle = -\frac{\Omega_p\,|a\rangle + \Omega_{co}\,|b\rangle}{\Delta - \sqrt{\Delta^2 + (\Omega_p)^2 + (\Omega_{co})^2}} + |c\rangle$$

$$|+\rangle = \frac{\Omega_p\,|a\rangle + \Omega_{co}\,|b\rangle}{\Delta + \sqrt{\Delta^2 + (\Omega_p)^2 + (\Omega_{co})^2}} - |c\rangle$$

As $|0\rangle$ does not have any contribution from $|c\rangle$ in it, atoms remain in this state and does not take part in processes involving the excited state like absorption and emission So, this state $|0\rangle$ is then called as dark state as the probability of it being in $|c\rangle$ is zero. For $\Delta = 0$ which means for on resonant laser field, the eigenstates of dressed state system become,

$$|-\rangle = \frac{1}{\sqrt{2}}(-\frac{\Omega_p\,|a\rangle + \Omega_{co}\,|b\rangle}{\sqrt{(\Omega_p)^2 + (\Omega_{co})^2}} + |c\rangle)$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(-\frac{\Omega_p\,|a\rangle + \Omega_{co}\,|b\rangle}{\sqrt{(\Omega_p)^2 + (\Omega_{co})^2}} - |c\rangle\right)$$

1.8 Dark State:

For smaller probe amplitude compared to that of pump beam, the dark state will then have a form $|0\rangle \approx |a\rangle$ because $\Omega_P \ll \Omega_{co}$. Here, we see that the dark state is a stationary state of the bare atom hamiltonian as well as the dressed state system. It has been decoupled from the other two. Due its time independence, atoms in this state remain in the same state and they never excite. Therefore, probe beam will experience zero absorption.

1.9 Bright State:

By taking linear combination of non-dark states, we obtain,

$$|A\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2}} = |c\rangle$$

$$|B\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}} = \frac{\Omega_p\,|a\rangle + \Omega_{co}\,|b\rangle}{\sqrt{(\Omega_p^2 + \Omega_{co}^2)}}$$

Both are non-stationary states. The strong pump field will make atoms oscillate back and forth between these two states. As $|B\rangle$ accounts for all the transitions to and from the excited state $|A\rangle$, it is termed as bright state. Again, using, $\Omega_p \ll \Omega_{co}$,$|B\rangle \approx |b\rangle$.
Hence, in the presence of both the electric fields, the strong beam will be absorbed by the atoms in $|b\rangle$ and get excited to $|c\rangle$.

Chapter 2

Experimental Details

The goal of this experiment is to locate the region where the destructive interaction between dark states take place in double-Λ configuration of EIT. This double-Λ system consists of Zeeman levels of ^{85}Rb D2 transition. The theory covered in previous chapter is based on general EIT. The zeeman levels here get splitted by applying proper magnetic field. Details will be discussed further in this chapter.

2.1 Rubidium and Energy Level Structure:

Due to simpler structure of energy levels and availability of optical equipments, Rb gas has been chosen for this experiment. Also,a whole lot of work has been done with Rb so far so that we could easily get large material to refer to and compare our results with past research. Please refer fig. 2.2.

The transition used here is D2 transition in ^{85}Rb. It involves ground state $|5S_{1/2}\rangle$. It is resonant with the wavelength 780.241 nm. There are several hyperfine levels within these states. Ground states are referred as F and excited states as F'. F=2 and F'=3 are the levels used in this experiment. These levels chosen due to prominent peak feature associated with their atomic transition on hyperfine absorption spectrum. When magnetic field is applied, the degenerate energy levels get shifted according to their different angular momenta. These angular momenta are denoted as m_f and m'_f for ground state and excited state respectively.

Two circularly polarised fields σ^+ and σ^- are applied to create Λ-type EIT scheme. σ^+ will increase angular momentum of atom by one quantum number. Hence, the atom at $m_f = 1$ will go to $m'_f = 2$. Similarly, σ^- will take atom from $m_f = 3$ to $m'_f = 2$. In this configuration, Λ system of EIT as well as V-system of EIT will also contribute to dark state. But as atoms tend to decay spontaneously, the Λ-system has greater contribution than V-system to EIT signal. Please refer fig. 2.1 for the full picture.

2.2 Preliminary Setup:

The setup involves diode laser which is locked at resonant frequency with the help of Saturation Absorption Spectroscopy. The beam is then passed through double pass configuration and properly polarised. It is then passed through ^{85}Rb enriched vapor cell which is surrounded by solenoid with μ-metal outside to prevent stray magnetic field from entering into cell. The beam coming out from cell gets detected by detector. The detector is connected to oscilloscope to see the EIT profile.

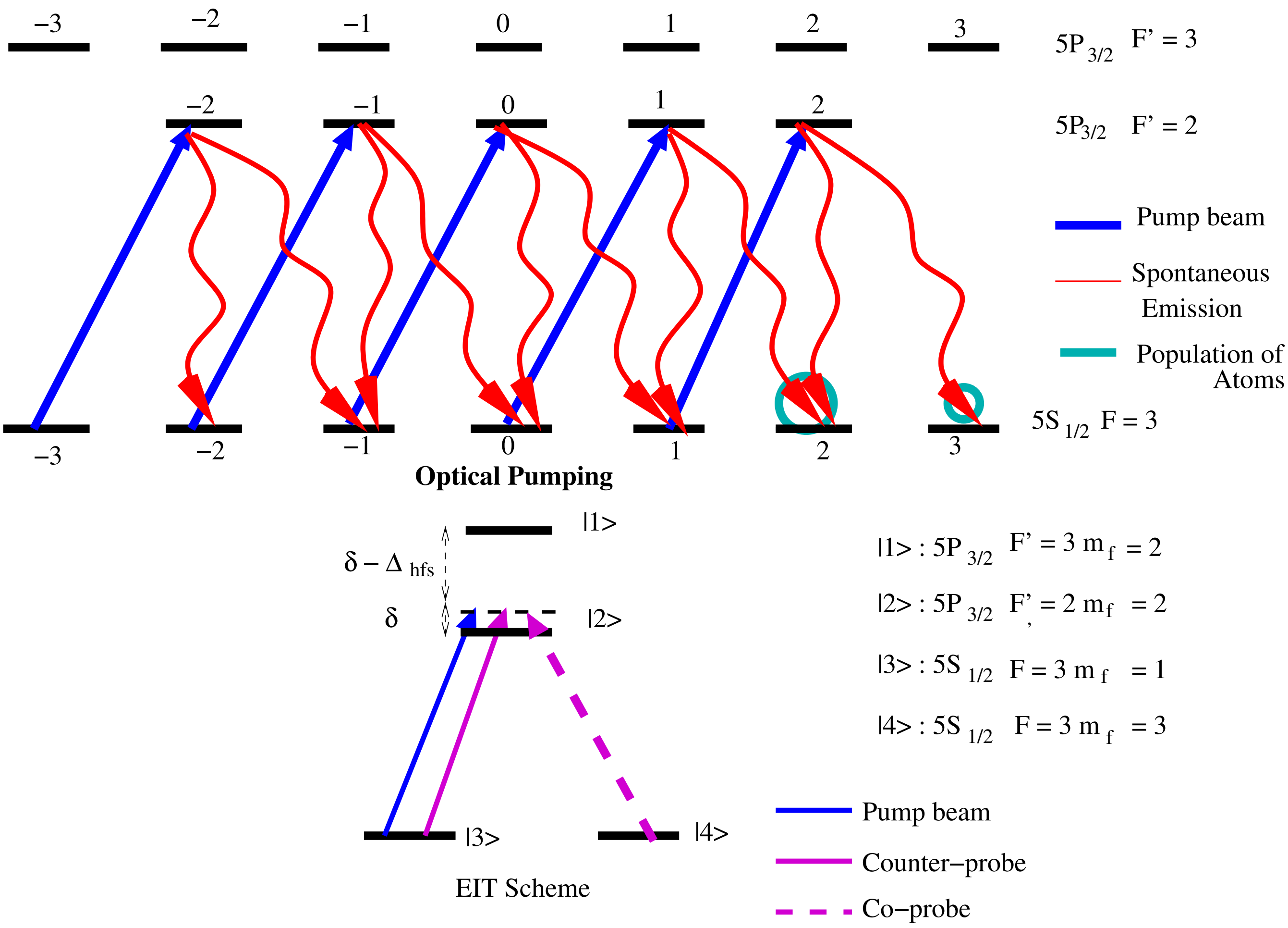


Figure 2.1: Optical Pumping and double-Λ EIT Scheme

2.3 Saturation Absorption Spectroscopy:

SAS is a technique through which hyperfine energy peaks of atom are observed on the background of doppler broadening. Due to Maxwellian distribution of velocity of atoms at room temperature, atoms with non-zero velocity will see the off-resonant laser light as on-resonance and emit the corresponding radiations. This introduces broadening of transmission profile in frequency regime which is termed as doppler broadening. But, in SAS, a laser beam with weak intensity along with highly intensed counter propagating pump beam are passed

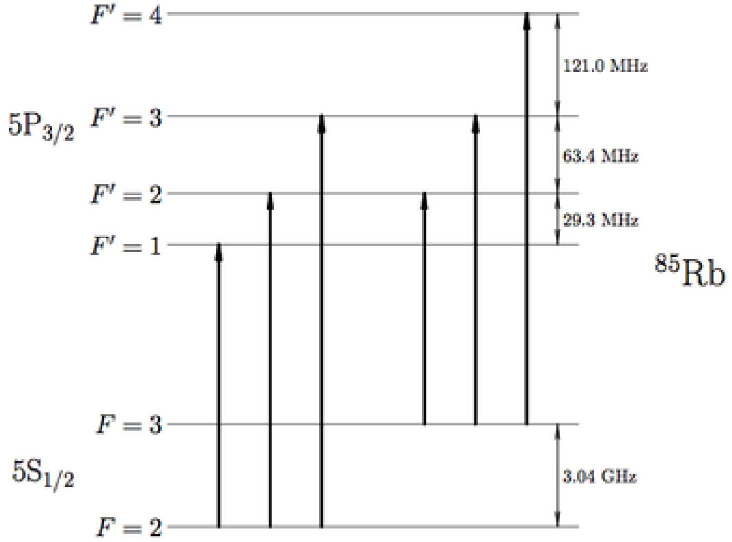


Figure 2.2: Energy level of ^{85}Rb D2 Transition (http://experimentationlab.berkeley.edu/MOT)

through vapor cell consisting of atoms. All the beams have same frequency and are being scanned. As weak probe and strong pump beam are counterpropagating to each other, they both see a particular common velocity class of atoms for which the beams are on resonance. The pump being highly intense, most of the atoms get absorbed by pump. It saturates the transition. Hence, weak probe will get transmitted with very little absorption. This results in crossover peaks along with main hyperfine peaks. Shown in fig 2.4 the energy level spectrum of ^{85}Rb obtained through SAS.

2.4 Laser and Laser Locking:

A stable and tunable laser source with narrow linewidth is necessary for this experiment. Therefore, we use "Toptica DL-pro" laser source with diode laser emitting 780 nm wavelength.

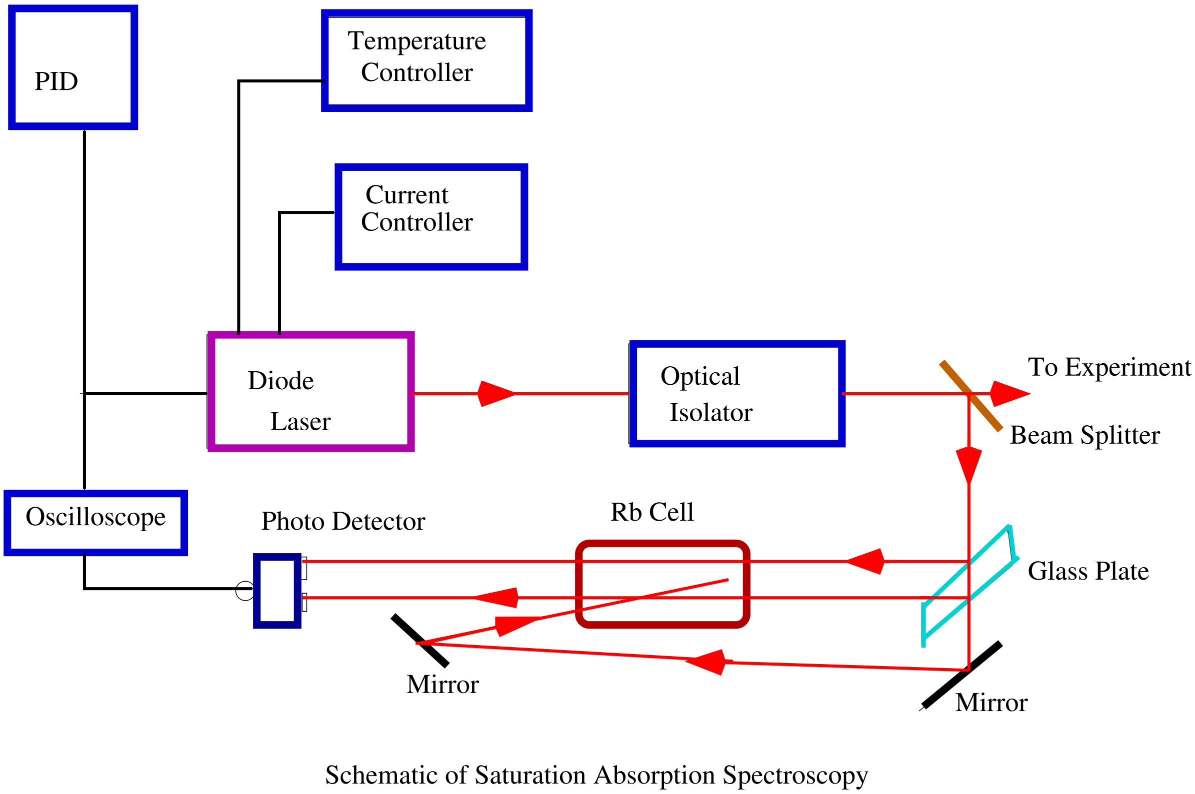


Figure 2.3: Schematic of Saturation Absorption Spectroscopy

Frequency stability of laser is very sensitive to temperature of surrounding as it drifts from desired frequency peak obtained from SAS even with small temperature fluctuations. Hence, there exists a feedback mechanism which corrects the laser frequency. This mechanism consists of generation of error signal using lockin amplifier. The modulation signal of this lockin amplifier is fed to the laser. To get an error signal with proper phase and amplitude, the frequency and amplitude of modulation and gain, need to be adjusted properly. PID generates a DC signal when the error signal is fed to PID. This DC signal is then fed back to piezo of laser. This corrects the laser frequency whenever it drifts from the desired frequency

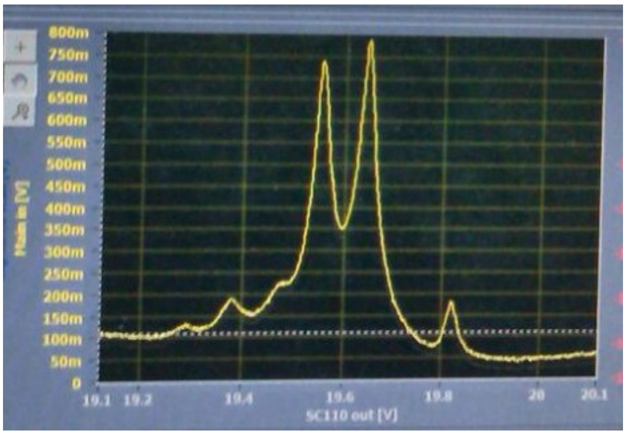


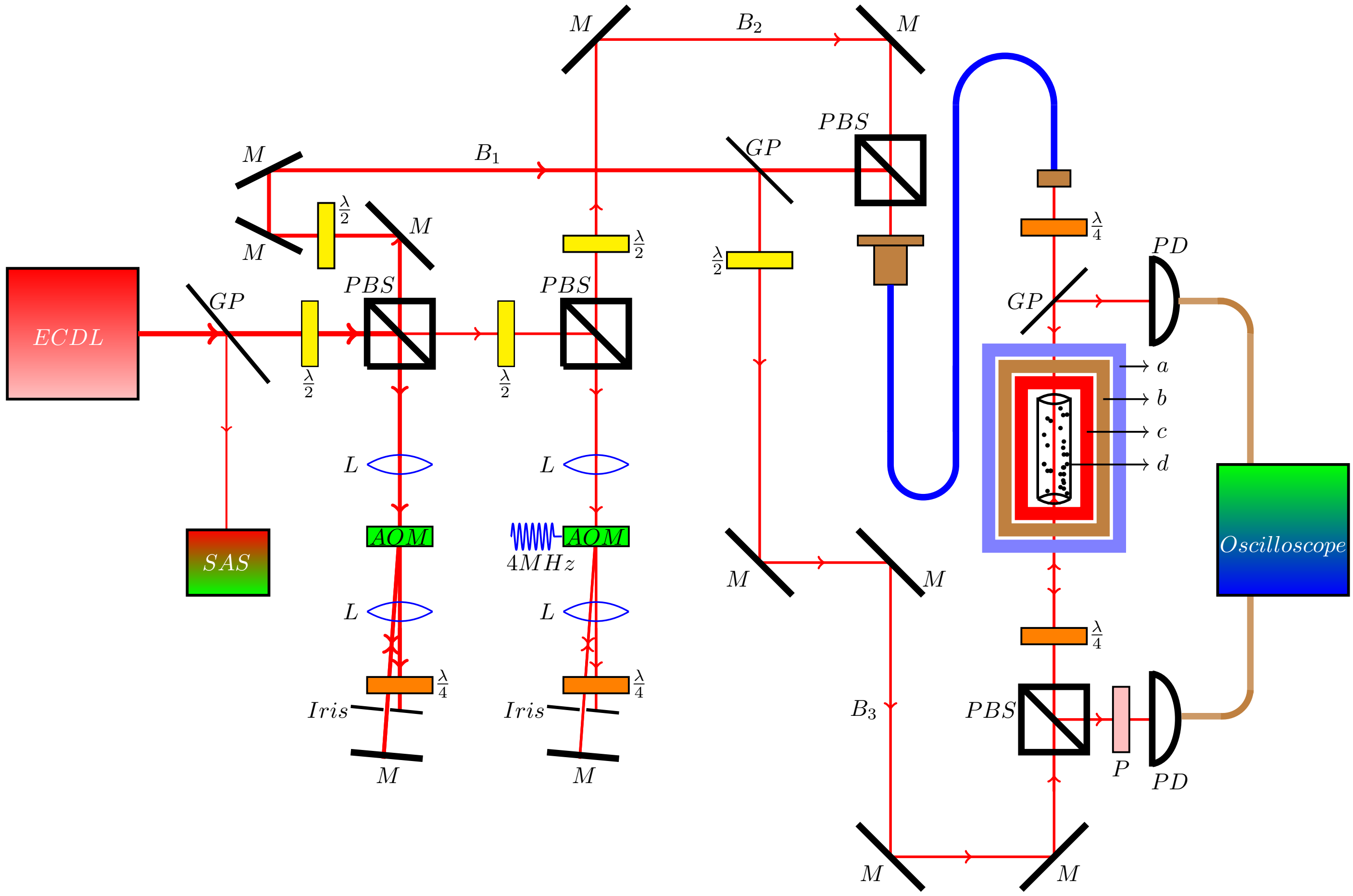
Figure 2.4: SAS Spectrum of D2 transition (cooling) of ^{85}Rb with the voltage values in the x-axis corresponds to frequency

peak. The frequency peak where we want to lock our laser, should be moved to center of scanning range by changing "offset". The frequency and amplitude of scan should be brought to "zero" by turning on PID.

2.5 Experimental set-up:

The beams B1,B2 and B3 are derived from single external cavity diode (ECDL)laser having linewidth around 1 MHz. The frequency of laser is tuned to D2 transition of ^{85}Rb with the help of Saturation Absorption Spectroscopy(SAS). These laser beams interact with ^{85}Rb atoms which are at room temperature in a vapor cell. These Rb atoms in vapor cell are present in their naturally found isotopic proportion. Laser frequency is being scanned over hyperfine levels of ^{85}Rb by a module starting from ground state $5S_{1/2}$F=3 to $5P_{3/2}$ $F' =$ 1,2,3,4.

As shown in above figure, B1 and B2 are co-propagating collinear beams. The B3 beam



PBS	→ *Polarizing Beam Splitter*	$\frac{\lambda}{4}$	→ *Quarter Wave Plate*	*PD*	→ *Photo Detector*
M	→ *Mirror*	$\frac{\lambda}{2}$	→ *Half Wave Plate*	*a*	→ μ − *Metal Sheet*
L	→ *Lens*	*P*	→ *Polarizer*	*b*	→ *Solenoid*
GP	→ *Glass Plate*	*ECDL*	→ *External Cavity Diode Laser*	*c*	→ *Cell Holder*
AOM	→ *Acousto Optic Modulator*	(blue line)	→ *Polarization maintaining optical fiber*	*d*	→ *Rb Vapour Cell*

Figure 2.5: Experimental set-up

counterpropagates to B1 and B2. B1 and B2 beams pass through AOMs. Each AOM is in double pass configuration. B3 beam is derived from B1 beam. B3 and B1 has same polarisation. The double pass shifts the frequency of all three beams down from ω_L(laser frequency) to ω_L-184 MHz. At any given ω_L-184 MHz, the B2 beam has been ramped over by 4 MHz of frequency range by providing triangular ramp to the corresponding AOM through the function generator. The detuning δ of B1, B2 and B3 is defined as the frequency difference of the laser beams from ω_{23} which is defined as the transition from ground state $5S_{1/2}$, F = 3 to $5P_{3/2}$, $F\prime = 2$. Since, each beam itself is shifted from ω_L as ω_L-184 MHz, $\delta = \omega_L$-184 MHz - ω_{23}. For $\delta = 0$, the B1, B2 and B3 beams are said to be on resonance with with $F\prime$ = 2. All the beam diameters are roughly in between 0.5mm to 1.5mm. beams B1 and B2 have power readings as 460 μW and 20 μW respectively and B3 has 8.3 μW.

With the help of polariser, half-waveplate and polarising prism, the polarsation purity has been maintained. The vapor cell containing ^{85}Rb atoms used in this experiment is maintained at around 37 degree celsius temperature with no buffer gas. It has been placed inside a solenoid with 3 layers of μ metal sheet surrounding it to prevent stray magnetic field from entering inside cell. The solenoid generates magnetic field around 1 G to 1.5 G based on the current flowing through it. This magnetic field direction will define the quantization axis. With respect to this axis, the beams B1 and B2 are orthogonally circularly polarised and B3 has the same polarisation as B1.

All the three beams pass through the vapor cell. The beam B3 is counterpropagating to the beams B1 and B2 as shown in fig.2.5. As mentioned above, the B2 beam has been ramped over by 4 MHz of frequency range by providing triangular ramp to the corresponding

AOM through the function generator. The outcoming B3 beam is detected at photo detector to observe the signal on oscillocope.

2.6 Results and Discussions:

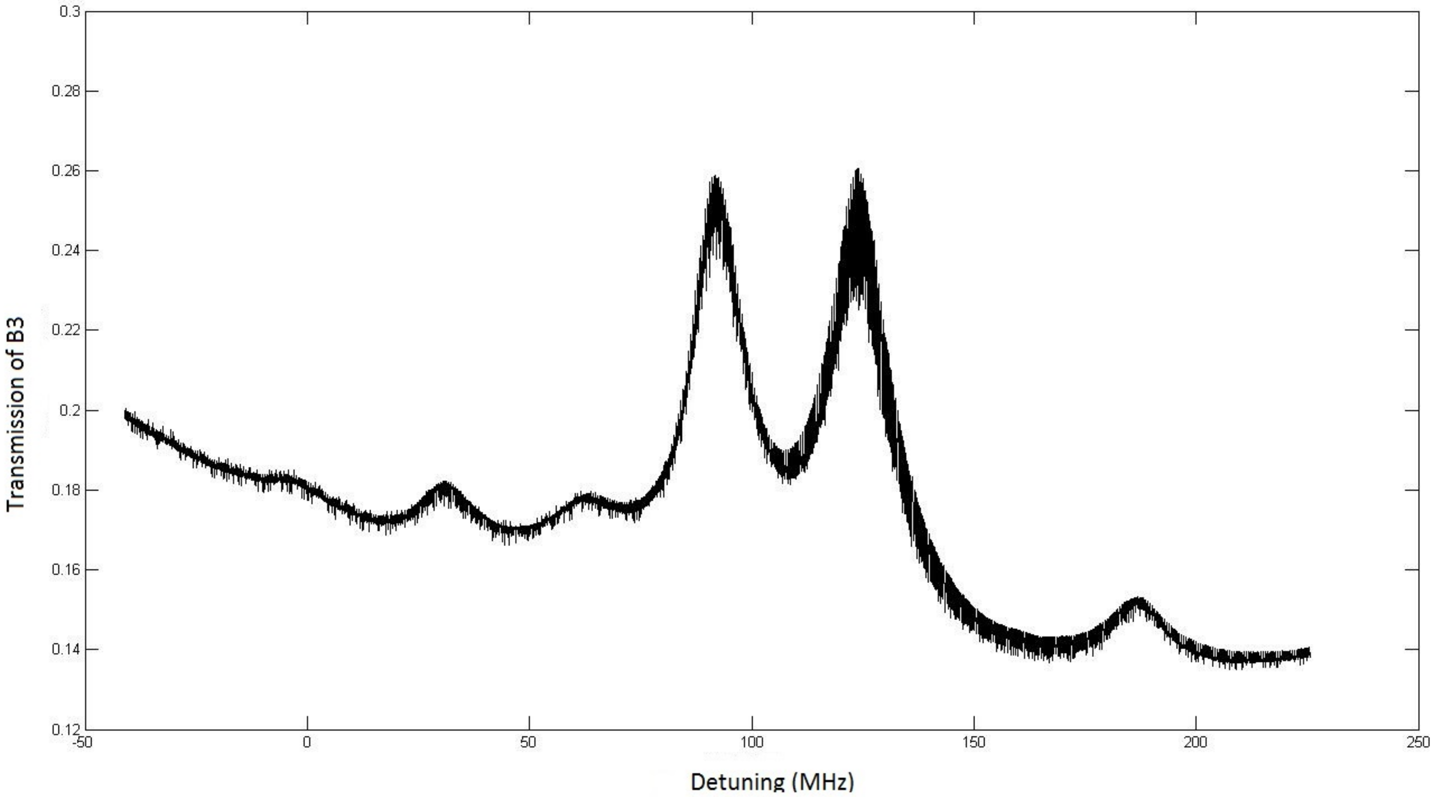


Figure 2.6: Transmission profile of beam B3 where X-axis represents Detuning in MHz and Y-axis denotes Transmission

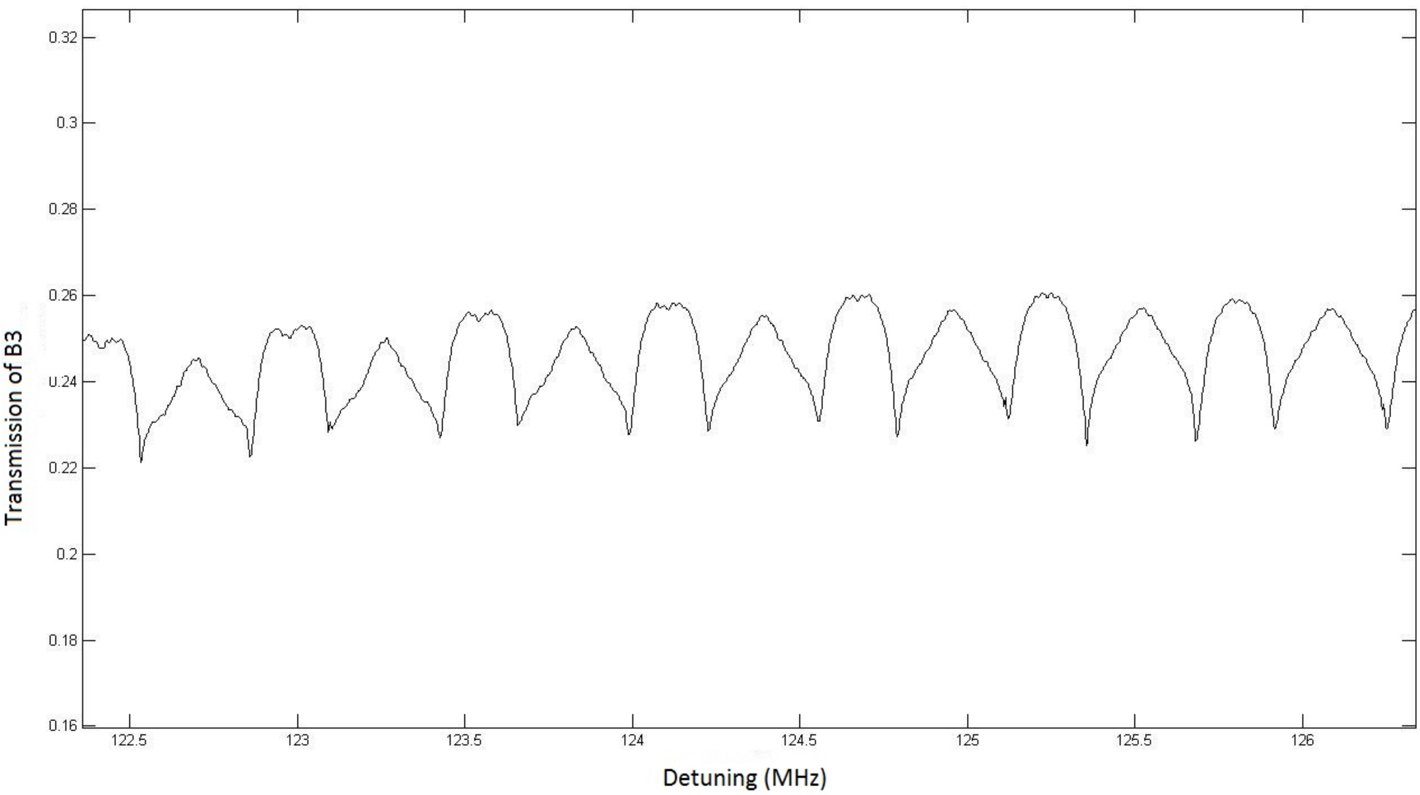


Figure 2.7: Zoom-in plot of absorption happening in transmission profile of B3 near cross-over where X-axis represents Detuning in MHz and Y-axis denotes Transmission

The transmitted beam B3 shows a slow variation due to changing ω_L, as the frequency of laser is scanned across the hyperfine transition from ground state $5S_{1/2}$ F=3 to $5P_{3/2}$ $F' =$ 1,2,3,4. As B1 and B3 are counterpropagating with same circular polarisation, they address same ground state $|3\rangle$. For $\delta = 0$, the optical pumping takes place from $|3\rangle$ to $|4\rangle$ due to powerful B1 resulting in increased absorption of weak B3 beam. For non-zero δ, B3 and B1 beams address the same set of levels. So, the B1 beam causes saturation effects which in turn results in increased transmission of B3. This can be compared with the usual spectrum

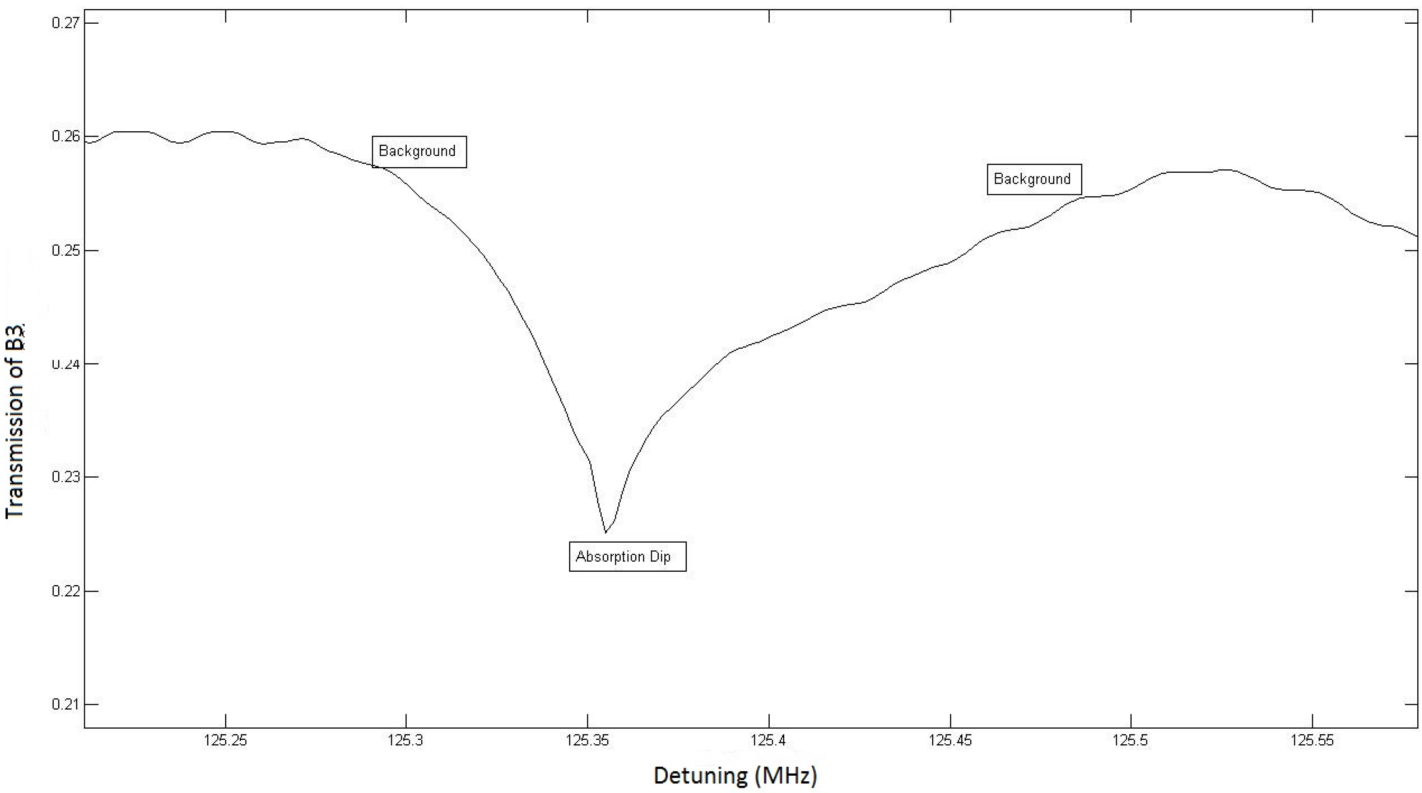


Figure 2.8: Non-linear absorption in the transmission profile of beam B3

obtained from the Saturation Absorption Spectroscopy of the reference cell.

In absence of B2 beam,B3 will show the saturation absorption profile as its geometry is counterpropagating to that of B1 laser. Now, B2 beam co-propagates along with B1 and also rapidly scans a frequency span of $\pm$ 4 MHz about the instantaneous and slowly varying frequency of ω_L. Therefore, for every round of scan, B1 and B2 satisfy two photon resonance condition giving rise to EIT. Please refer Fig. 2.9 for EIT in B2. Our experiment searches for references in B3 as B2 scans over its EIT profile.This is shown in Fig. 2.7 where we see sharp absorption dips riding over F'=3 to F' = 4 crossover saturation transmission occurring whenever B2 scans over a two photon resonance.

At given δ, the B2 beam scans a frequency width of 4 MHz centered around δ. When δ_{B1} and δ_{B3} are held fixed at δ, the laser beams B1, B2 and B3 all have equal detuning δ twice during each period of the scan. At these two positions, atom with velocity v satisfies the two photon resonance condition as

$$\Delta_{B1} - \Delta_{B2} = 0$$

$$\implies \Delta_{B1} = \delta_{B1} - k \cdot v$$

and

$$\Delta_{B2} = \delta_{B2} - k \cdot v$$

As a consequence, transmission of B2 beam increases due to EIT happening at these positions as shown in figure. At the same time, at two photon resonance condition, the atom absorbs the counter propagating B3 beam through three photon resonance [16]. This is a velocity selective resonant absorption due to three photon resonance condition as:

$$\Delta_{B1} - \Delta_{B2} + \Delta_{B3} = 0 \tag{2.1}$$

Taking into account the two photon resonance condition, this results into a resonant absorption condition for B3 as:

$$\delta - k \cdot v = 0 \tag{2.2}$$

This resonance has been observed to be riding on Saturation Absorption Spectroscopy

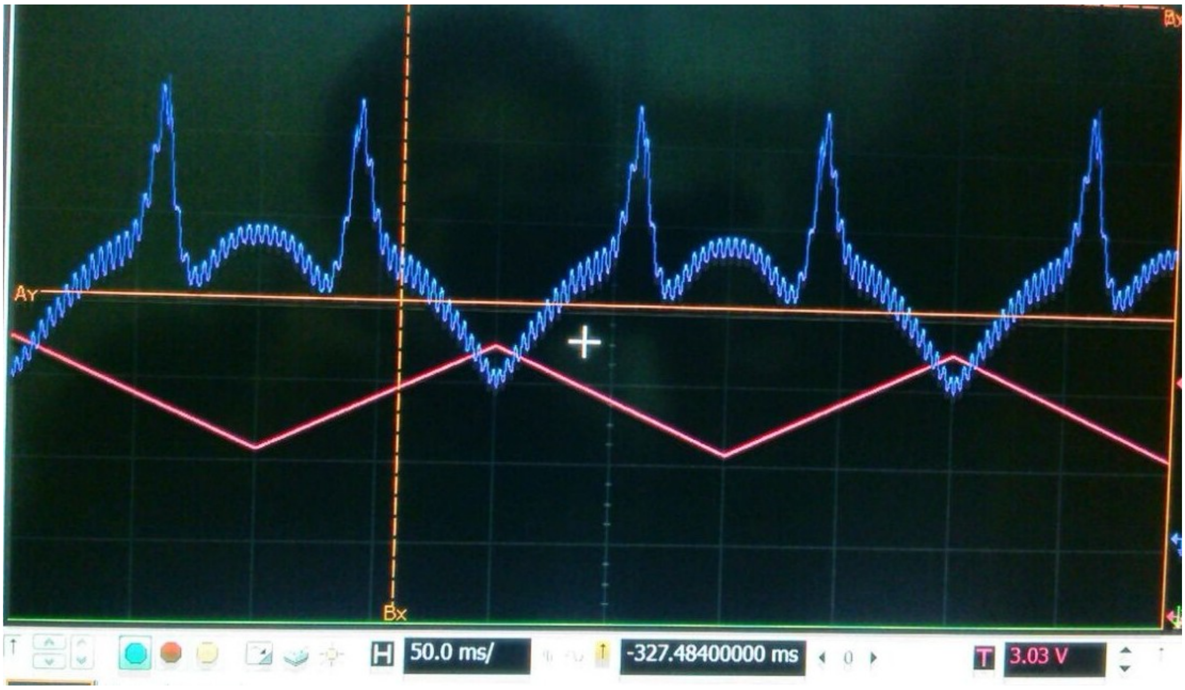


Figure 2.9: Profile of EIT seen in B2 beam as it is scanned twice over a two-photon resonance condition during one period of the scan which is shown in pink

(SAS) of B3. The dressed picture of our double-Λ system is given by [10]:

$$|0\rangle = |2\rangle - \frac{\Omega_{23}\Omega_{13}}{g^2}|1\rangle + \frac{\Omega_{23}(2\delta - \Delta_{hfs})}{g^2}|3\rangle$$

$$|-\rangle = \frac{1}{\Omega_0}[-j^+|1\rangle + \Omega_{13}(|3\rangle + \frac{\Omega_{23}}{\delta - j^+}|2\rangle)]$$

$$|+\rangle = \frac{1}{\Omega_0}[\Omega_{13}|1\rangle + j^+(|3\rangle + \frac{\Omega_{23}}{\delta - j^-}|2\rangle)]$$

where,

$$(\Omega_0)^2 = (\Omega_{13})^2 + (\frac{\delta - \Delta_{hfs}}{2} + \sqrt{\frac{\Omega_{13} + (\delta - \Delta_{hfs})^2}{4}})^2$$

$$j^{\pm} = \frac{\Delta_{hfs} - \delta}{2} \mp \sqrt{\frac{\Omega_{13} + (\delta - \Delta_{hfs})^2}{4}})^2$$

$$g^2 = \Omega_{13} - \delta_p(2\delta_p - \Delta_{hfs})$$

For our experimental setup,Ω_{13} and Ω_{23} are non-zero. hence, all the three bare atomic states $|1\rangle, |2\rangle, |3\rangle$ contribute to the dressed states $|+\rangle, |-\rangle \, and \, |0\rangle$. For proper strength and detuning of coupling and drive fields, the photons from B2 and B3 gets absorbed from atoms in ground state $|4\rangle$ to one of the three dressed states which happens only when three-photon resonance condition is satisfied i. e. $\delta - k \cdot v = 0$.

The plot shown in fig. 2.10 is termed as Contrast plot. With, the reference of Fig. 2.8, the contrast is defined as a ratio of maximum absorption to that of background. As it is observed in the fig. 2.10 that the contrast is getting enhanced around 100 MHz to 140 MHz. This happens because,the atom in this process, experiences EIT due to presence of Coprobe (B2) and SAS due to presence of Counter-probe beam (B3). Both the processes happen only in the presence of strong coupling beam B1. In the absence of B2 beam, B1 beam pumps the atoms to excited states. During EIT process in the presence of B2 beam the pumping action of B1 beam is suppressed. Hence, these large number of atoms, now, get absorbed by this counter-probe beam. We observe more contrast around crossover because number of atoms undergoing this process is more at crossover than at hyperfine levels. This phenomena can also be explained with the help of figure 2.11 : Let us consider an atom moving with positive velocity. It will be blue detuned with respect to counter-probe and red detuned with respect to pump. Hence, the atom will see pump on resonance with $|e_1\rangle$ and counter probe with $|e_2\rangle$. Pump, being highly intense, will absorb most of the atoms letting the counter-probe beam to pass through. Similarly, for negative velocity class of atoms, the pump will be on resonance with $|e_2\rangle$ and the probe will be resonance with $|e_1\rangle$. Now, in the presence of co-probe beam, atoms will undergo EIT. Hence, more and more number of atoms will be present in ground state. At this stage, counter probe can access these atoms

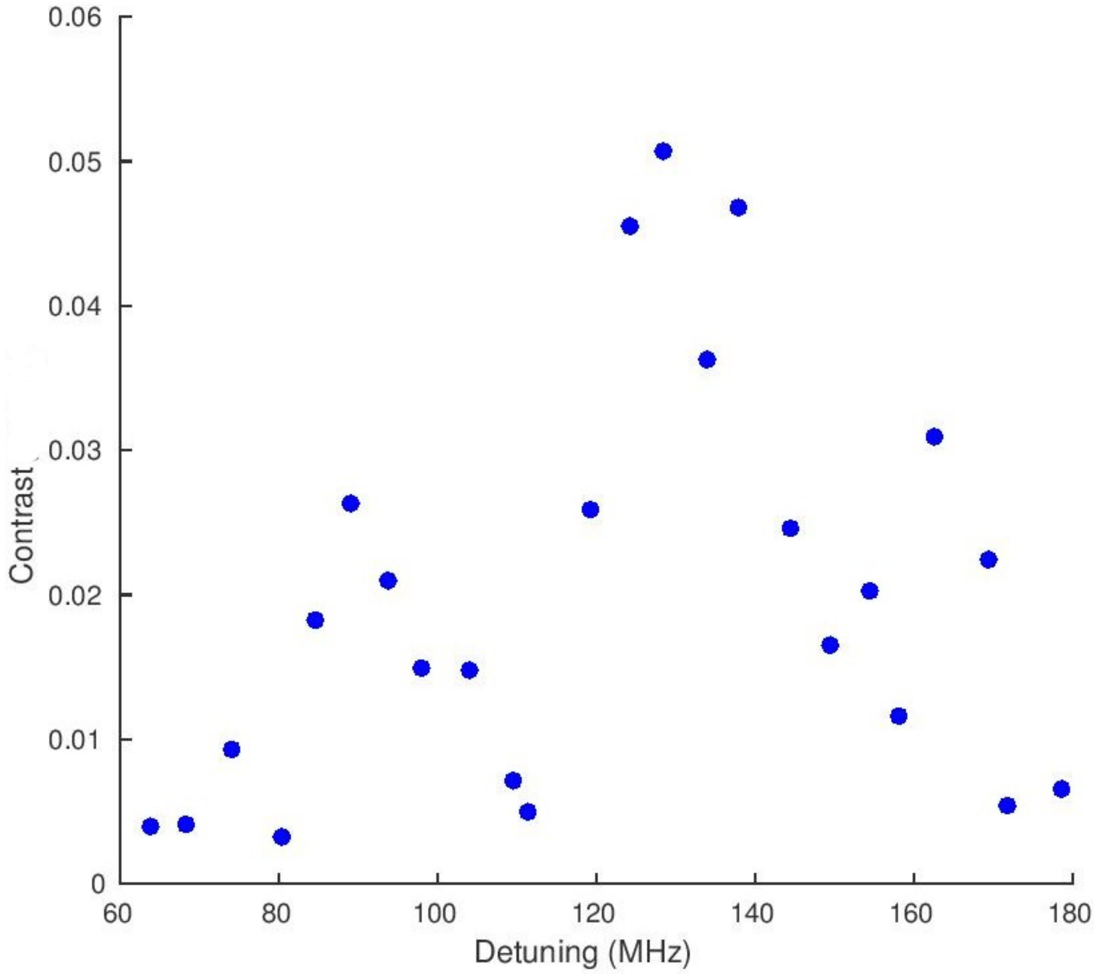


Figure 2.10: Contrast plot

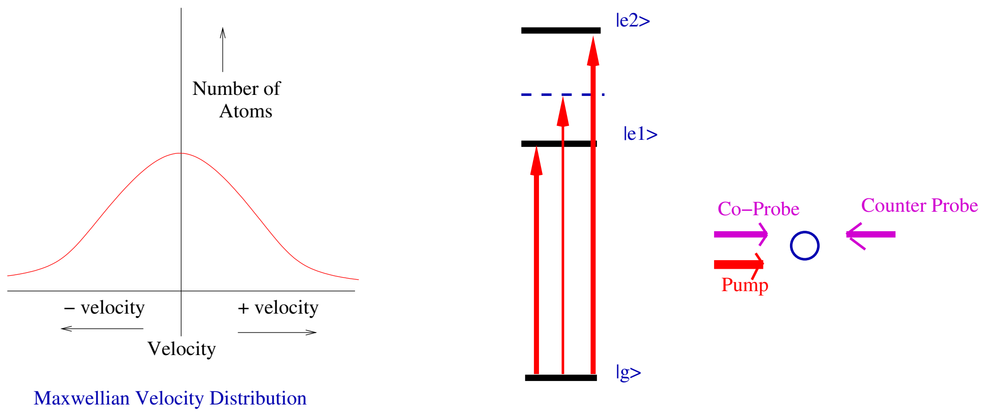


Figure 2.11: Behavior of atom at Crossover

and hence, we observe higher contrast at cross-over. The counter probe favours velocity selective absorption. At cross-over, the atom undergoes both EIT as well as SAS.These two processes together, suppress linear absorption ($\varkappa^{(1)}$) and enhances non-linear absorption ($\varkappa^{(3)}$) through three photon resonance. It is also clearly seen that the cross-over width in actual SAS is comparable to width of contrast peak which is 25 MHz.

2.7 Conclusion and Future Prospective

It is well known that, during EIT process, linear absorption is suppressed and non-linear absorption is seen at low intensities. In concluding remarks, we can say that SAS aids EIT through enhancement in non-linear absorption $\varkappa^{(3)}$ and suppression in linear absorption $\varkappa^{(1)}$. Also, the positions of laser detuning from 5P3/2 F' = 2 where we observe minimum contrast is the place where the two dark states of this double-Λ system interact maximally.

In the future experiments, we aim to experimentally study the transient effects of this system.

Chapter 3

Dark State Polariton in atomic Δ system

3.1 Introduction

This chapter talks about the parallel theoretical work. This work is based on form-stable coupled excitations of light and matter associated with the propagation of quantum field in EIT media [17]. The system under consideration is Δ system of EIT. A microwave-driving field couples the two lower energy states of a Λ-energy level system of atomic media thereby transforming it into a Δ system [17]. The aim is to find the form of polariton when we introduce a microwave field to Λ energy level configuration to couple the ground state and metastable state, the transition which is otherwise dipole forbidden. The microwave field makes the transition to be allowed through magnetic dipole. The probe field used here, is quantum mechanical in nature. Polaritons are mixtures of photonic and Raman-like matter branches [17]. By adiabatically changing the coherent field, dark state polariton can be

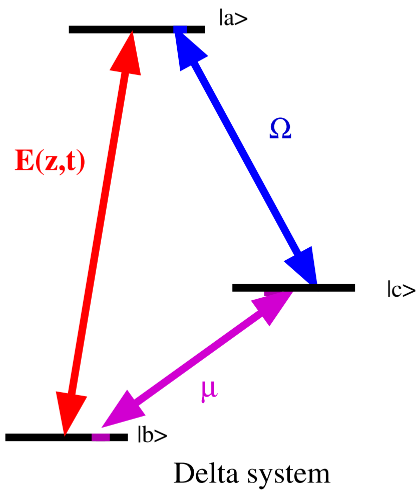


Figure 3.1: Δ system of EIT

stopped and reaccelerated in a way that its shape and quantum state is preserved. During this process, quantum state of light is transferred to collective atomic excitations [17] and vice versa.

3.2 What is polariton?

A microwave-driving field couples the two lower energy states of a Λ-energy level system of atomic media thereby transforming it into a Δ system [17]. Polaritons are form-stable coupled excitations of light and matter associated with the propagation of quantum fields in Electromagnetically Induced Transparency [19]. Being atomic Δ-system a closed system, it exhibits phase-sensitivity. Hence, it is a natural question to investigate the effect of phase-sensitivity on the propagation of polariton in Δ-system. Therefore, we are interested in Polariton Dynamics in atomic Δ-system.

3.3 Analytical Treatment

Let E(z,t) be a slowly varying dimensionless operator describing a quantum field as:

$$E(z,t) = \sum a_k(t)e^{ikz}e^{-i(\nu/c)(z-ct)} \tag{3.1}$$

which couples the ground state $|b\rangle$ to the excited state $|a\rangle$. $\nu = w_{ab}$ is the carrier frequency of the optical field. The excited state $|a\rangle$ is further coupled to $|c\rangle$ via a coherent control field with the slowly varying Rabi frequency Ω(t). Before applying any external field, all atoms are assumed to be in their ground state as $|b_j\rangle$.

Now, let's define a collective slowly varying atomic operator to describe the quantum properties of medium averaged over small volumes as:

$$\sigma_{\alpha\beta}(z,t) = (1/N_z)\sum_{j=1}^{N_z} |\alpha_j\rangle \langle\beta_j| \, e^{-i\omega_{\alpha\beta}t} \tag{3.2}$$

where N_z = number of atoms at position z and α , β are the corresponding energy levels of Δ configuration. Now that $|b\rangle$ is further coupled to $|c\rangle$ via a microwave field with Rabi frequency μ(t). The interaction between light and atom is governed by Hamiltonian:

$$V = -(N/L)\int dz(\hbar g\sum_k a_k e^{ikz}\sigma_{ab}(z) + \hbar\omega\sigma_{ac} + \hbar\mu\sigma_{cb}) - (\hbar g^*\sum_k a_k^+ e^{-ikz}\sigma_{ba}(z) + \hbar\omega^*\sigma_{ca} + \hbar\mu^*\sigma_{bc}) \tag{3.3}$$

Where, g= atom-field coupling constant

L= length in Z-direction

N= number of atoms in this volume

μ = Microwave field

The propagation equation for the operator corresponding to optical field is:

$$\frac{\partial E}{\partial t} + c\frac{\partial E}{\partial z} = igN\sigma_{ba}(z,t) \tag{3.4}$$

3.4 Evolution of Collective Atomic Operator

For the atomic evolution, using the set of Heisenberg-langevin equations,

$$\frac{\partial \sigma_{\mu\nu}}{\partial t} = -\gamma_{\mu\nu}\sigma_{\mu\nu} + (i/h)[V, \sigma_{\mu\nu}] + F_{\mu\nu} \tag{3.5}$$

Where, $\gamma_{\mu\nu}$: Transversal decay rate and $F_{\mu\nu}$: delta correlated Langevin operator, we obtained following equations as:

$$\dot{\sigma_{bc}} = -\gamma_{bc}\sigma_{bc} - i\mu(t) - i\Omega(t)\sigma_{ba} + F_{bc} \tag{3.6}$$

As γ_{bc} and F_{bc} are negligible, they can be ignored. Hence,

$$\sigma_{ba} = i(\dot{\sigma_{bc}}/\Omega(t)) - (\mu(t)/\Omega(t)) \tag{3.7}$$

$$\dot{\sigma_{ba}} = -\gamma_{ba}\sigma_{ba} + i\mu(t)\sigma_{ca} - igE - i\Omega(t)\sigma_{bc} + F_{ba} \tag{3.8}$$

using (3.7) and (3.8),we get,

$$\sigma_{ca} = (gE/\mu) - (\Omega\sigma_{bc}/\mu) - (i/\mu)(\frac{\partial}{\partial t} + \gamma_{ba})((i/\Omega)\frac{\partial}{\partial t} - (\mu/\Omega)) - F_{ba} \tag{3.9}$$

Using normalized time t'=t/T, where T is a characteristic time scale and expanding right hand side of (3.9) in powers of 1/T, we find in following in lowest non-vanishing order,

$$\sigma_{ca} = (gE/\mu) - (\Omega\sigma_{bc}/\mu) + (i\gamma_{ba}/\Omega) \tag{3.10}$$

Using similar arguments, we obtain,

$$(\frac{\partial}{\partial t} + \gamma_{ca})\sigma_{ca} = i\mu\sigma_{ba} + F_{ca} \tag{3.11}$$

$$\Longrightarrow \gamma_{ca}\sigma_{ca} = i\mu\sigma_{ba} \tag{3.12}$$

Using (3.9) and (3.10), we find,

$$\sigma_{bc} = (gE/\Omega) + (i\mu\sigma_{ba}/\Omega^2) + (i\mu^3/\Omega^2\gamma_{ca}) \tag{3.13}$$

Using (3.6) and (3.12), we obtain,

$$\sigma_{ba} = (ig/\Omega)\frac{\partial(E/\Omega)}{\partial t} - (\gamma_{ba}/\Omega)\frac{\partial(\mu/\Omega^2)}{\partial t} - (1/(\Omega\gamma_{ca})\frac{\partial(\mu^3/\Omega^2)}{\partial t} - (\mu/\Omega) \tag{3.14}$$

Using above equation, the propagation equation becomes,

$$(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z})E(z,t) = -(g^2N/\Omega)\frac{\partial(E/\Omega)}{\partial t}-(igN\sigma_{ba}/\Omega)\frac{\partial(\mu/\Omega^2)}{\partial t}-(igN/\Omega\sigma_{ca})\frac{\partial(\mu^3/\Omega^2)}{\partial t}-(igN\mu/\Omega) \tag{3.15}$$

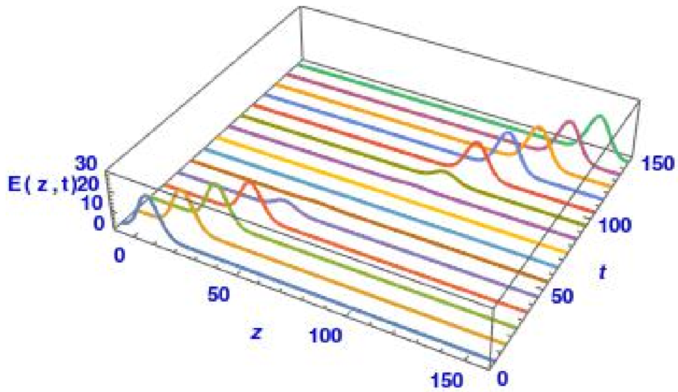


Figure 3.2: The coherent control of quantum pulse E(z,t) by controlling classical coupling field Ω(t)

As μ is considered to be very very small as compared to Ω, the higher order terms of (1Ω) can be ignored. It implies,

$$(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z})E(z,t) = -(g^2N/\Omega)\frac{\partial(E/\Omega)}{\partial t} - (igN\mu/\Omega) \tag{3.16}$$

3.5 Result and Discussion

3.5.1 Expression of Dark State Polariton for atomic Δ system

The quantum electric field can be controlled by controlling the coupling field $\Omega(t)$ shown in figure 3.1. For Λ-system of EIT, the dark state polariton has been found to be [16],

$$\Psi(z,t) = \cos(\theta(t))E(z,t) - \sin(\theta(t))\sqrt{N}\sigma_{bc}(z,t) \tag{3.17}$$

$$\cos(\theta(t)) = \frac{\Omega(t)}{\sqrt{\Omega^2(t) + g^2N}} \tag{3.18}$$

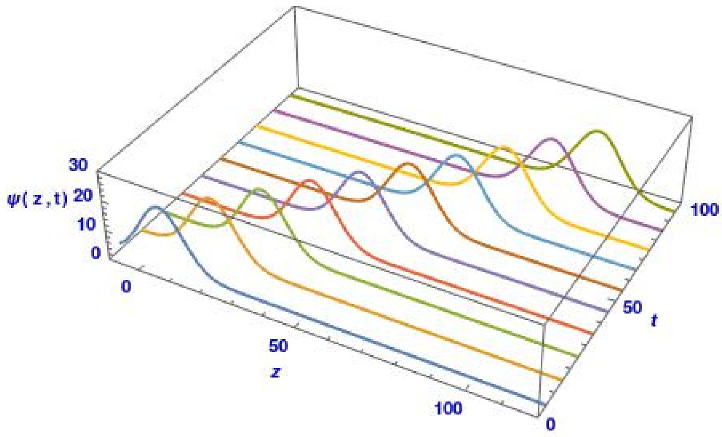


Figure 3.3: The shape-preserving propagation of polariton in Λ-EIT system

$$sin(\theta(t)) = \frac{g\sqrt{N}}{\sqrt{\Omega^2(t) + g^2 N}} \tag{3.19}$$

The polariton follows the homogeneous equation of motion as:

$$(\frac{\partial}{\partial t} + c\cos^2\theta\frac{\partial}{\partial z})\Psi(z,t) = 0 \tag{3.20}$$

which describes a shape preserving propagation as shown in fig. 3.2 with velocity,

$$v = v_g = c\cos^2(\theta(t)) \tag{3.21}$$

For Δ-system of EIT, equation(3.16) can be transformed as:

$$(\frac{\partial}{\partial t} + c\cos^2\theta\frac{\partial}{\partial z})(g\sqrt{N}\frac{E}{\Omega}\sin\theta + \cos\theta E) = -i\sqrt{N}\mu\sin\theta \tag{3.22}$$

Where gE/Ω can be replaced using (3.13), then the above equation will be:

$$(\frac{\partial}{\partial t} + c\cos^2\theta\frac{\partial}{\partial z})(\sqrt{N}\sigma_{bc}\sin\theta + \cos\theta E - i\sqrt{N}\sin\theta\frac{\mu^3}{\Omega^2\gamma_{ca}}) = -i\sqrt{N}\mu\sin\theta \quad (3.23)$$

$$\Longrightarrow \ (\frac{\partial}{\partial t} + c\cos^2\theta\frac{\partial}{\partial z})(\sqrt{N}\sigma\prime_{bc}\sin\theta + \cos\theta E) = -i\sqrt{N}\mu\sin\theta \quad (3.24)$$

and

$$\sigma\prime_{bc} = \sigma_{bc} - \frac{i\mu^3}{\Omega^2\gamma_{ca}} \quad (3.25)$$

Hence, we finally obtain

$$\psi\prime = \sqrt{N}\sigma\prime_{bc}\sin\theta - E\cos\theta \quad (3.26)$$

This can be referred to as the polariton in Δ-atomic system.

3.6 Conclusion and Future Prospective

In concluding remarks, we have shown that there exists the possibility of obtaining an expression for form-stable coupled excitations (polariton)between light and matter in the Δ system of EIT. With the help of the polariton, the quantum pulse of light can be coherently controlled through this media. It is possible to trap quantum light pulse and preserve its shape and state in stationary excitations of atoms. The reacceleration of the polariton converts it back to the photon pulse. This can be applicable for entanglement transfer between atoms and light and also for squeezing.

In future prospective, we intend to retain terms such as σ_{bb}, σ_{cc} , and σ_{aa} in Heisenberg-Langevin equation. After this modification,there will be 8 set of coupled equations. Hence, we would aim to solve the resultant system of equations numerically to investigate the dynamics of polariton.

Bibliography

[1] S. E. Harris, J. E. Field, and A. Imamoglu, Phys. Rev. Lett. 64, 10 (1990).

[2] K.-J. Boller, A. Imamoglu, and S. E. Harris, Phys. Rev. Lett. 66, 20 (1991).

[3] J. E. Field, K. H. Hahn, and S. E. Harris, Phys. Rev. Lett. 67, 22 (1991).

[4] G. Alzetta, A. Gozzini, L. Moi, G. Oriols, Nuovo Cimento, 36, 5 (1976)

[5] S.E. Harris, Phys. Today 50, 36 (1997)

[6] E. Arimondo, Progress in Optics, edited by E. Wolf (Elsevier, Amsterdam, 1996), Vol. 35, p. 257

[7] M.D. Lukin, Rev. Mod. Phys. 75, 457 (2003)

[8] M. Fleischhauer, A. Imamoglu, J.P. Marangos, Rev. Mod.Phys. 77, 633 (2005)

[9] M.O. Scully, M.S. Zubairy, Quantum Optics (Cambridge University Press, 1997), Chap. 7

[10] M.D. Lukin, S.F. Yelin, M. Fleischhauer, M.O. Scully, Phys. Rev. A 60, 3225 (1999)

[11] S.F. Yelin, V.A. Sautenkov, M.M. Kash, G.R. Welch, M.D.Lukin, Phys. Rev. A 68, 063801 (2003)

[12] H. Schmidt, A. Imamoglu, Opt. Lett. 21, 1936 (1996)

[13] G. Wasik, W. Gawlik, J. Zachorowski, Z. Kowal, Phys.Rev. A 64, R051802 (2001)

[14] B. Lu, W.H. Burkett, M. Xiao, Opt. Lett. 23, 804 (1998)

[15] S.E. Harris, L.V. Hau, Phys. Rev. Lett. 82, 4611 (1999)

[16] T.M. Preethi, M. Manukumara, and A. Narayanan, Eur. Phys. J. D 60, 389–395 (2010)

[17] M. Fleischhauer,M.D. Lukin, Phys. Rev. Lett. 84, 5094-5097 (2000)

[18] M. Fleischhauer, S. F. Yelin, and M. D. Lukin, Opt. Commun. 179, 395-410 (2000).

[19] M. D. Lukin, S. F. Yelin, and M. Fleischhauer, Phys. Rev.Lett. 84 , 4232 (2000).

[20] A. E. Kozhekin, K. Mølmer, and E. Polzik, quant-ph /9912014.

[21] W. W. Erickson, Electromagnetically Induced Transparency, Undergraduate thesis, Reed College (2012).