Resource Dependency In Complex Networks

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by

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Certificate

This is to certify that this dissertation entitled "Resource Dependency In Complex Networks" towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by Nishanga P at Indian Institute of Science Education and Research under the supervision of Dr. Snehal M Shekatkar, DST INSPIRE faculty, Department of Scientific Computing, modelling and simulation, Savitribai Phule Pune University , and Prof. M S Santhanam, Professor, Department of Physics, Indian Institute of Science Education and Research during the academic year 2022-2023.

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This thesis is dedicated to my Parents and my Sister

Declaration

I hereby declare that the matter embodied in the report entitled **Resource Dependency In Complex Networks** are the results of the work carried out by me at the Department of Physics, Indian Institute of Science Education and Research, Pune, under the supervision of Dr. Snehal M Shekatkar and Prof. M S Santhanam, and the same has not been submitted elsewhere for any other degree.

Nishanga P

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Abstract

This thesis delves into the intricate domain of complex networks, specifically emphasising dependency networks where edges signify diverse forms of vertex interdependencies. The exchange of resources within these systems is pivotal to their functionality. The central objective of this investigation is to scrutinize the influence of heterogeneity in production capacities across network vertices and the network structure on network performance. A surplus resource distribution model is employed to study this impact, with simulations on networks characterized by homogeneous and heterogeneous degree distributions. The study aims to enhance our understanding of complex networks and provide insights into the interplay of heterogeneity and degree correlation on network performance evaluated by fitness. The thesis' exploration of resource exchange dynamics within complex networks provides a broader perspective on the ramifications of resource sharing and its capacity to instigate social transformations and enhance resource allocation.

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Chapter 1

Introduction

Network science has rapidly evolved in recent decades, offering a powerful lens to understand and analyze complex systems. Complex systems have numerous interconnected components and have traditionally defied easy analysis due to their intricate and multifaceted nature. At its core, network science involves representing and modelling these intricate real-world systems as complex networks. Network/graph is a fundamental concept in graph theory, consisting of nodes/ vertices and edges/links that interconnect in a particular pattern or in a random way that is unique to the system under study.

The advent of network science has bestowed upon researchers and analysts a formalism that permits the exploration of real-world systems that might otherwise remain inscrutable. Its applications span a vast spectrum of disciplines, including biology, physics, computer science, economics, and sociology, to name just a few. Through the lens of network science, we can discern hidden patterns, identify pivotal nodes or entities within these networks, and study the flow of information or resources within these intricate systems. As such, the study of networks has illuminated novel insights into the structure and dynamics of complex systems, rendering it an engrossing and highly relevant area of research for this thesis.

Complex networks have emerged as a robust framework for investigating complex systems characterized by many individual components or entities. In these systems, individual component's continued operation and survival hinge upon the availability of one or more crucial resources. Humans require sustenance in the form of food and water while manufacturing companies rely on materials to maintain their production processes. These components often possess a resource threshold, below which their performance deteriorates, or they face the risk of collapse[13][11]. To secure these vital resources, components within complex systems must engage in resource allocation and exchange, a process facilitated through intricate networks of interdependence. These relationships among components form a web, where each element generates diverse resources and redistributes excess quantities to others in need. Traditional examples of such networks include trade networks, where resources are exchanged for currency or other valuable assets[10][7], and supply chains within the manufacturing sector. Additionally, distribution networks play a crucial role in the flow of resources from one vertex to another, exemplified by power grids[3], gas pipelines[4], and river networks[14][13].

These networks, underpinning the interplay of resources and dependencies among components, belong to a specific category known as dependency networks. Within this category, edges represent various forms of resource dependency, and we scrutinize this interdependence in this thesis. In particular, our focus lies on the impact of heterogeneous production capabilities within these systems, which is brought about by making the production capacities a function of their vertex property. To analyze this, we employ a Surplus-distribution model [19] on two types of degree distributions: heavy-tailed and peaked. Poisson and power-law degree distributions are chosen for this purpose, as they provide valuable insights into the intricate dynamics of complex networks.

Sharing resources can challenge the conventional idea of collective well-being. When an individual augments the resources they distribute to a specific partner from a fixed pool, it necessitates a decrease in the resources allocated to others[17][20]. This zero-sum characteristic in social exchange systems means that the overall benefit of a group cannot increase unless there is an augmentation in the available resources for exchange or, in certain cases, through taxation and redistribution [20][16]. Here comes the notion of heterogeneity in resource production in a network. The subsequent sections of this thesis will provide a finer analysis of this model, aiming to elucidate the consequences associated with heterogeneous resource production in dependency networks. Our central inquiry revolves around the question of whether, given a specific network topology and a constant distribution of production across the entire network, it is feasible to enhance network fitness by adjusting the average production levels across nodes. Additionally, we will investigate how variations in production levels affect network behaviour within this context. Throughout these investigations, we have primarily concentrated on the network's degree distribution. However, it is worth considering that other properties might influence network fitness. Therefore, we examine the complex interplay of the coefficient of assortativity of a network and the resource inequality.

In this pursuit, we aim to contribute to network science by deepening our understanding of complex systems and their resource dynamics, with implications for various applications and industries. We endeavour to solve challenges associated with resource allocation and dependencies within complex systems.

1.1 Scope of the Thesis

In Chapter 2, we provide a concise overview of key definitions and concepts in network science that are essential for a comprehensive understanding of the thesis. In Chapter 3, we present a detailed explanation of the surplus resource distribution model and an examination of the statistical characteristics of the selected probability distributions used for simulating resource production. Chapter 4 is dedicated to an in-depth exploration of the results on the effects of resource production fluctuations in scenarios where vertex production capacities are uniform. Additionally, this chapter includes a discussion on the analytically derived constraints on fitness and wastage for various resource generators. Chapter 5 focuses on the outcomes of introducing heterogeneity in production capacities within the network, with a particular emphasis on the factors influencing network fitness. We also see how networks with the same degree distribution but different degrees of assortativity show different fitness values.

Chapter 2

Definitions and concepts in network science

In our quest to comprehend the dynamics of complex systems and the resource dynamics within dependency networks, we must first lay a solid foundation. In this chapter, we go through the fundamental concepts and principles of network science that underpin our exploration of these complex structures.

2.1 Networks

As said, the networks introduced so far are mathematically known as graphs[21]. In its simplest form, a network is a collection of vertices that are connected through edges. On a formal note, a graph/network is a widely studied combinatorial structure with N the set of **vertices** and $E \subseteq N \times N$ the set of **edges**[18] [12] It is used to model real-world scenarios with vertices representing objects and edges representing some relation between said objects[21].

These edges can be directed or undirected. Also known as digraphs, **directed** networks have edges with a specific direction. Each edge in a directed network is represented by an ordered pair of vertices, indicating the direction from one vertex (the source) to another (the destination). In contrast, **undirected** networks have edges that do not have a specific direction.[18][5] For example, edges are directed in a predator-prey network[18] but are undirected in a social network of a group of friends.

Beyond this, each edge can also be associated with different weights, quantifying the significance, cost, or distance associated with the connection between the corresponding vertices in a **weighted** graph. For example, these weights can quantify the strength of a relationship in a social network or the current flowing through a power transmission line in a power grid[5]. In an **unweighted** graph, all edges are treated equally and don't carry additional information or numerical values. Unweighted graphs are suitable for scenarios where the relationships between objects are binary or qualitative. We deal with only undirected and unweighted networks in this thesis.[5][18]

2.2 Representation of networks

From a mathematical point of view, we can represent a network through an **adjacencymatrix** [5]. An adjacency matrix, **A**, is a binary square matrix representing a finite graph[12]. For a network of N nodes, **A** is an $N \times N$ matrix where,

$$A_{ij} = \begin{cases} 1, & \text{if i and j are connected} \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

for an undirected and unweighted graph[15]. In this case, the adjacency matrix is symmetric,i.e. $A_{ij} = A_{ji}$ [15]. But $A_{ij} = 1$ for a directed network suggests that node i is directed to node j. In the case of weighted graphs, $A_{ij} = w_{ij}$ and, generally, w_{ij} usually lies between 0 and 1.[5][18]

2.3 Degree distribution

A relevant information that can be deduced from the adjacency matrix is the **degree**, k_i of a vertex i [15][21]. It is defined as the number of links/edges attached to a vertex i, i.e., the number of nearest neighbours of vertex i [21]. It can be written as,

$$k_i = \sum_{j=1}^{N} A_{ij} \tag{2.2}$$

Often, the degree of a node is used to quantify the importance of a node. A node with more connections is considered more crucial to the network.[5]

Each edge within an undirected network possesses two endpoints, which implies that in a network with a total of m edges, there are a combined total of $2m$ endpoints for those edges. Remarkably, this quantity is also equal to the summation of the degrees of all the nodes in the network.[18], so

$$2m = \sum_{i=1}^{N} k_i = \sum_{ij} A_{ij} \tag{2.3}$$

Statistical measures like degree distribution often give a more appropriate description for vast systems like social networks. The degree distribution, denoted as $p(k)$, represents the probability distribution of degrees across all nodes within a network. Alternatively, it signifies the probability that a node chosen at random will exhibit a degree of k. [18][5]

A relevant information that can be extracted from the degree distribution is the average degree, $\langle k \rangle$ of that network [15].

$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i = \sum_{k} k p(k) \tag{2.4}$$

The degree distribution is a simplified measure that characterizes one aspect of a network's structure. A network can be **homogeneous** and **heterogeneous** based on degree distributions. The homogeneous exhibits a fast decaying tail, e.g., Poisson distribution, and the other exhibits a heavy tail, e.g., Powerlaw distribution. We look into two such network models in the below section [1]. The degree distribution corresponding to these networks is given in the below figure 2.1

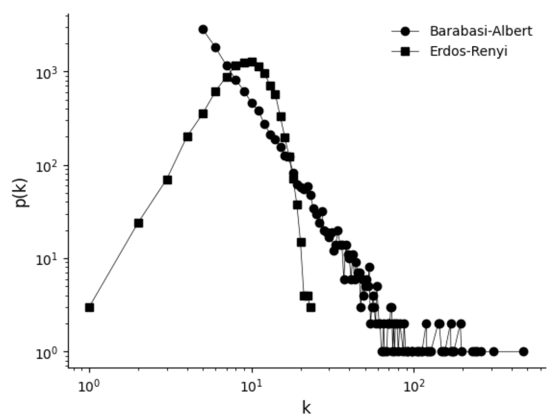


Figure 2.1: A log-log plot of degree distribution for ER and SF topologies consisting of 10000 nodes and average degree, $\langle k \rangle = 10$. Obtained by averaging over 100 realizations.

2.4 Degree correlation and assortativity

Degree correlation assesses the relationship between the degrees of nodes when they are interconnected. One approach to gauging the degree of correlation for a node involves evaluating the average degree of its neighbouring nodes [2].

$$k_{nn}(k_i) = \frac{1}{k_i} \sum_{j=1}^{N} A_{ij} k_j \tag{2.5}$$

where k_i and k_j are the degree of node i and its neighbour j, respectively. Networks with similar degree distributions can exhibit distinct structures arising out of different correlations, leading to the emergence of diverse network classes.

- Neutral network

  In the context of a neutral network, the degree correlation function remains constant and is defined as follows:

  $$k_{nn} = \frac{\langle k^2 \rangle}{\langle k \rangle} \tag{2.6}$$

  Therefore, the relationship between $k_{nn}$and the node's degree k appears as a horizontal line when graphed. In case we choose to model the degree correlation function using an exponential equation:

  $$k_{nn} = Ak^r \tag{2.7}$$

  where A is a constant. Then, $r = 0$ for neutral networks.

- Assortative network

  In networks exhibiting assortativity, nodes with higher degrees tend to form connections with other nodes with high degrees selectively. As a result, the average degree of their nearby neighbours increases. Consequently, in assortative networks, the function k_{nn} shows a positive association with the degree k, leading to a situation where $r > 0$.

- Disassortative network

  In these types of networks, the hub preferably links to lower degree nodes, and if modelled by equation 2.7, we have $r < 0$.

2.5 Network models

In the realm of network science, understanding the fundamental principles governing the structure and behaviour of complex systems is of greatest importance. To this end, network models provide an invaluable lens through which we can explore and decipher the intricacies of real-world networks. Two prominent paradigms that have left an indelible mark in this field are the Erdos-Renyi (ER) and Scale-Free (SF) networks.

2.5.1 Erdos-Renyi (ER) network

Among the many types of networks researchers have explored, the Erdos-Renyi (ER) network is a fundamental and influential archetype. This network model offers valuable insights into the basic principles of network theory. While it may appear deceptively simple at first glance, the ER network serves as a cornerstone for understanding the properties and behaviour of more complex real-world networks.

In their first article[8], Erdos' and Renyi's definition characterizes a random graph as a configuration comprising N uniquely labelled nodes linked by a set of m randomly selected edges, drawn from the total of $\frac{N(N-1)}{2}$ potential edges. [6]. Altogether, there exist $\binom{\frac{N(N-1)}{2}}{m}$ possible graphs, constituting a probability space where each configuration is equally likely. This encompasses the random graph model $G(N,m)$, where a graph is selected randomly from the entire set of graphs with N nodes and m edges. Another variation is the $G(N,p)$ model, where each edge is created with a probability p[18][5].

In the limit of $N \to \infty$, i.e., for a large network, the probability distribution for such a network with an average degree $\langle k \rangle$ is given by a Poisson distribution.

$$p(k) = \frac{\langle k \rangle^k e^{-\langle k \rangle}}{k!}$$

where $\langle k \rangle = np$.

This model fits into the class of homogeneous networks as most node degrees are concentrated around the mean degree. These types of graphs where connections between nodes are formed randomly are called random graphs and are often used as a baseline model to compare against real-world networks.[1]

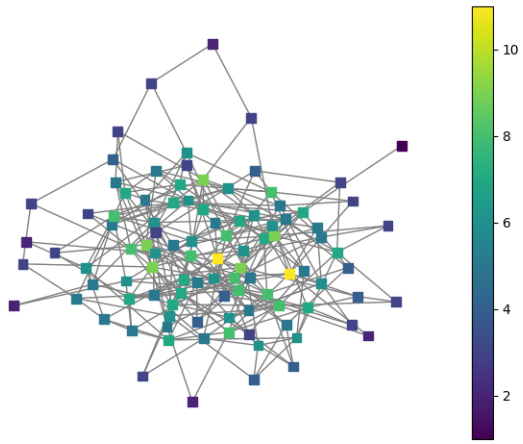


Figure 2.2: Illustration of an ER network of 100 nodes and average degree 5. The colour scale indicates the vertex degree.

2.5.2 Scale free (SF) network

Scale-free networks, a captivating concept in the realm of network theory, have captured the imagination of scientists and researchers for their ubiquitous presence in complex systems. These networks, characterized by a distinct and intriguing structural pattern, play a pivotal role in diverse fields such as social interactions, the World Wide Web, and biological systems. Hence giving rise to their significance in modelling real-world phenomena.
A scale-free graph exhibits a power-law degree distribution.

$$p(k) \sim k^{-\gamma}$$

where k is the node degree and γ is the scale-free exponent. A skewed degree distribution characterizes this. It has a heavy-tailed distribution where the network comprises mostly lower-degree and a few higher-degree nodes[18].
Barabasi-Albert (BA) algorithm is a preferential attachment model developed for generating a scale-free network. The BA model incorporates two important general mechanisms[1][5]:

- growth: The model initiates with a limited number of nodes and subsequently incorporates nodes individually, step by step.

- preferential attachment: Every fresh node establishes connections with m pre-existing nodes, and the likelihood of forming these connections is linked to the degree of the pre-existing nodes. Consequently, nodes with greater degrees have a higher probability of receiving new connections, thereby facilitating the emergence of hubs.

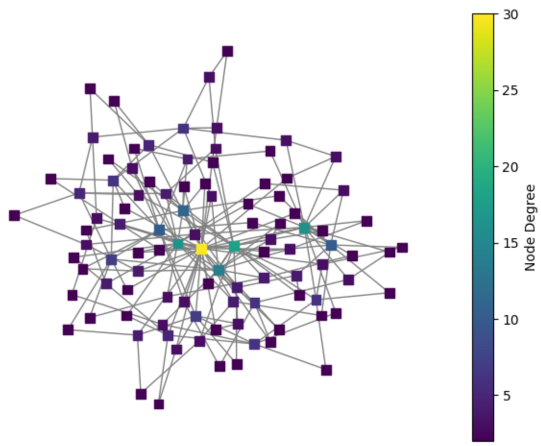


Figure 2.3: Illustration of an SF network of 100 nodes and average degree 5. The colour scale indicates the vertex degree.

2.6 Centrality

Centrality is a fundamental concept in network analysis that seeks to quantify the importance, influence, or prominence of individual nodes within a network. Different centrality metrics address this question from various perspectives, allowing us to capture different aspects of importance[18][5][1]. Here are some common centrality measures:

- Degree
  As said earlier, a node with more connections is considered important to a network's performance in certain cases. It is one of the simplest centrality measures[9].

- Eigen-vector
  Eigenvector centrality quantifies a node's importance based on the importance of its neighbours. A node connected to other highly central nodes is considered more influential.
  The eigenvector centrality corresponds to the largest eigenvalue λ obtained by solving $Ax = \lambda x$ where A is the adjacency matrix[5][18].

- Closeness

  The shortest or geodesic path in a network refers to the most concise route between a specific pair of nodes, namely, the path that covers the fewest number of edges. The shortest distance or geodesic distance between two nodes is the length of the shortest path in terms of the number of edges [1]. Closeness centrality serves as a metric to assess how rapidly a node can connect with all other network nodes. This metric is determined by taking the reciprocal of the sum of the shortest path lengths between the node in question and all the other nodes present in the graph.

$$C_C(i) = \frac{1}{\sum_j d_{ij}}$$

- Betweenness

  Betweenness centrality identifies nodes that serve as connectors or intermediaries between other nodes. It quantifies the count of shortest paths between pairs of vertices that traverse through a specific node.[5].

$$C_B(v) = \sum_{s \neq t \neq v} \frac{n_{st}(V)}{n_{st}}$$

  Where $n_{st}(v)$ represents the count of shortest paths between s and t that traverse through the node v, while n_{st} signifies the total number of shortest paths between the same pair of vertices.

Chapter 3

Surplus Distribution Model

In this thesis, the surplus distribution model takes centre stage as a pivotal tool utilized in simulations. This chapter thoroughly examines the model and the associated evaluation parameters, specifically fitness and wastage. These metrics are fundamental to evaluating the network's performance, with which we can gauge the efficiency and effectiveness of different allocation, production or distribution strategies.

3.1 Surplus distribution model

Consider a network composed of n nodes and m edges. Each edge depicts the flow of resources between its vertices. Each vertex might require different resources for survival, but here, we confine our focus to a single resource category that each vertex can generate.[11]. The model is agnostic to whether the network's connections are directional or not; however, all the simulations presented in this paper focus on undirected networks. At each discrete time step $t = 0, 1, 2...$, each vertex stochastically produces some amount of resource denoted by $X_i(t)$. The variable X_i follows a probability distribution with distinct parameters denoted as β_i, which determine the characteristics of the distribution [11]. Additionally, each vertex i requires a certain threshold amount,R_i, of resources at each time step.[13]
A vertex j has a surplus if $X_i(t) > R_i$ and is distributed among its neighbours. The surplus is

denoted by:

$$S_j(t) = \begin{cases} X_j(t) - R_j, & \text{if } X_j(t) > R_j \\ 0, & \text{otherwise} \end{cases} \tag{3.1}$$

The fraction of surplus received is linearly proportional to the node deficit denoted by:

$$D_i(t) = \begin{cases} R_i - X_i(t), & \text{if } X_i(t) < R_i \\ 0, & \text{otherwise} \end{cases} \tag{3.2}$$

The total amount of resources on a vertex at time t is given by,

$$X_i^{tot}(t) = \begin{cases} X_i(t) + \sum_{j=1}^{n} A_{ij} S_j(t) \frac{D_i(t)}{\sum_{l=1}^{n} A_{lj} D_l(t)}, & \text{if } X_i(t) < R_i \\ R_i, & \text{otherwise} \end{cases} \tag{3.3}$$

where **A** is the adjacency matrix.
Or one could possibly model the time evolution of resources using a simple difference equation of the following form,

$$X_i(t + \Delta t) = X_i(t) + y(x_i - R_i) F(\sum_j A_{ij} S_j) - y(R_i - X_i) G(\sum_j A_{ij} D_j) \tag{3.4}$$

where y(x) = 1 if x ¿ 0 and 0 otherwise.
F is resources received from neighbours with surplus resources, and G is resources sent to neighbours deficient in resources.

An essential characteristic of the model under consideration pertains to its assumption that resources possess a limited lifespan of just one unit of time. This assumption finds relevance in various real-world contexts, for instance, in the case of perishable items like agricultural or dairy products, where the resources in question are subject to rapid decay and deterioration over a short duration. Consistent with this, we assume that if the total amount $X_i^{tot}(t)$ available to vertex i at time t is less than its threshold R_i, vertex i completely depletes it during that time period. However, if $X_i^{tot}(t)$ is equal to or greater than R_i, vertex i only consumes the quantity R_i, and any surplus amount, $X_i^{tot}(t) - R_i$, is discarded at time t without carrying over to time $t+1$[13].
Another assumption is that each vertex possesses solely the topological details concerning its neighbours, such as their degrees, without any awareness of the specific production quantities associated with them at any given time. This aspect holds significant implications as we develop

strategies to enhance the network's resilience[13].

3.2 Network fitness

The fitness of a network is defined by the following:

$$f_i(t) = \min\left(X_i^{tot}(t)/R_i, 1\right) \tag{3.5}$$

If X_i^{tot} takes the probability distribution $Q_i(x)$, the expected value of fitness of a vertex is given by:

$$\langle f_i \rangle = \int_0^{R_i} \frac{x}{R_i} Q_i(x)dx + \int_{R_i}^{\infty} Q_i(x)dx \tag{3.6}$$

The distribution $Q_i(x)$ linked to vertex i relies not only on $p(x : \beta_i)$ but also on the vertex's topological attributes such as its degree, centrality, clustering coefficient, and so on. This implies that even when all the parameters β_i of the generating distribution are identical, the distribution $Q(x)$ and, consequently, the anticipated fitness differ across vertices [13].

Also, it is important to note that the functions $Q_i(x)$ and $f_i(t)$ are time-independent in this model[13]. This means that the behaviour and characteristics of the system described by the model do not change over time. The functions $Q_i(x)$ and $f_i(t)$ remain constant throughout the simulation or analysis. By removing the time dependency, the model simplifies and focuses solely on the relationships and interactions between the vertices and their resources without considering the temporal aspect. This can be advantageous in certain scenarios where the system's behaviour is not influenced by time or when time is not a relevant factor in the analysis. Overall, the time independence of $Q_i(x)$ and $f_i(t)$ in this model allows for a more streamlined and simplified representation of the network dynamics, focusing on the essential aspects of resource production, sharing, and deficits among the vertices.

The average expected fitness of the network is given by:

$$F = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \langle f_i \rangle \tag{3.7}$$

For a finite network,

$$F(t) = \frac{1}{n} \sum_{i=1}^{n} f_i(t) \tag{3.8}$$

Since n is finite, the above quantity would fluctuate with time. However, for a higher n, the fluctuations were reduced. Also, since $Q_i(x)$ is time-independent, a more precise estimate of fitness is obtained by incorporating the time average as shown below:

$$F = \frac{1}{nT} \sum_{t=0}^{T} \sum_{i=1}^{n} f_i(t) \tag{3.9}$$

3.3 Resource wastage

Once the surplus amounts have been allocated to neighbouring vertices, if the total amount X_i^{tot} on vertex i exceeds its threshold R_i, its fitness reaches its maximum value 1. Nevertheless, from the X_i^{tot} total, only the R_i portion is utilized by the vertex, while the remaining amount, denoted as $W_i = (X_i^{tot} - R_i)$, goes unused. The wastage per unit of time per vertex can be denoted as:

$$w = \lim_{n\to\infty} \lim_{T\to\infty} \frac{1}{nT} \sum_{t=1}^{T} \sum_{i=1}^{n} w_i(T) H(w_i(T)) \tag{3.10}$$

Here, $H(x)$ is a heavy side step function that equals 1 if $x \geq 0$, and 0 otherwise.

3.4 Resource generators

Resource generators are the probability distributions used for the stochastic generation of resources at a vertex. The distribution's mean is defined as the vertex's production capacity. Now, in the subsections, we look into the resource generators used for the simulations.

3.4.1 Exponential distribution

$$p_e(x) = \begin{cases} \frac{1}{\beta} e^{-\frac{x}{\beta}}, & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{3.11}$$

Here β is the average of the distribution. The higher the β, the higher the production of resources at a vertex. For this particular distribution, the mean and standard deviations are the same. This hampers its ability to accurately represent cases where variability in fluctuation sizes should be distinct from the average production. However, the Gaussian and Pareto generators serve this purpose.

3.4.2 Truncated Gaussian distribution

Gaussian distribution is defined over the real line. Since the resource production cannot be negative, the negative tail of the usual Gaussian distribution is removed. The probability distribution function is given by:

$$p_g(x; \mu, \sigma) = \frac{1}{\psi(\mu, \sigma)} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{3.12}$$

Where,

$$\psi(\mu, \sigma) = \int_0^\infty \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx \tag{3.13}$$

In the normal Gaussian distribution, the mean is μ and is independent of standard deviation σ. But for the asymmetric truncated Gaussian distribution, the mean is a function of both μ and σ and is given by:

$$\langle x \rangle = \frac{1}{\psi(\mu, \sigma)} \int_0^\infty x \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx \tag{3.14}$$

Thus, while increasing σ, we lower the value of μ so that the mean remains fixed. Calculating this integral analytically is difficult. Thus, we numerically calculate the value of μ for varying σ.

3.4.3 Pareto distribution

Pareto distribution is defined over [x$_m$, ∞).

$$p_P(x; x_m, \alpha) = \frac{\alpha x_m^\alpha}{x^{\alpha+1}} \tag{3.15}$$

The expectation value for this distribution is:

$$\langle x \rangle = \frac{\alpha x_m}{\alpha - 1} \tag{3.16}$$

From 3.16, we can see that the mean exists only for $\alpha > 1$. The standard deviation is calculated below.

$$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \tag{3.17}$$

$$\langle x^2 \rangle = \alpha x_m^\alpha \left[\frac{x^{2-\alpha}}{(2-\alpha)} \right]_{x_m}^{\infty} \tag{3.18}$$

From 3.18, it is clear that for Pareto distribution, standard deviation exists only for $\alpha > 2$. Thus 3.17 becomes:

$$\sigma = \sqrt{\frac{\alpha x_m^2}{(\alpha-1)^2(\alpha-2)}} \tag{3.19}$$

where

$$\alpha = 1 + \sqrt{1 + \left(\frac{\langle x \rangle}{\sigma}\right)^2} \tag{3.20}$$

To introduce variability in the fluctuations, we manipulate the parameter σ while keeping $\langle x \rangle$ constant. With each specific σ value, we derive a distinct α value from this equation. Subsequently, by employing equation 3.16, we can determine the corresponding x_m for the given combinations of σ and $\langle x \rangle$, facilitating the simulation of the model. As σ increases, both x_m and α decreases.

Chapter 4

Impact of fluctuations in resource production

Inevitable variations in produced resource quantities emerge when resource production is stochastic in a system[13]. Within this chapter, we explore how these fluctuations in resource production influence resource dependencies in complex networks while ensuring uniform production capacities across network vertices. We limit our analysis to solely investigating the network's degree distribution. Specifically, our focus is on examining how variations in resource production impact the network's overall fitness, particularly for two common types of degree distributions: peaked and heavy-tailed. To address this, we employ a simulation of the surplus distribution model using truncated Gaussian and Pareto resource generators applied to both scale-free (SF) and Erdos-Renyi (ER) network topology. It is important to make sure that the average degree of both these networks the same for a fair comparison. As mentioned in the previous chapter, when utilizing both truncated-Gaussian and Pareto generators, we incorporate $\langle x \rangle$ as a model parameter, which can be set to an appropriate value, and subsequently, we manipulate the degree of fluctuation by adjusting σ to observe its impact on the network's performance.

The simulation outcomes presented are obtained by averaging over 100 random realizations of size 1000 and an approximate average degree of 10. Here, we assume a uniform average production of $\langle x \rangle$ for all the vertices. We also assume that the thresholds R_i have the same value $R = 1$ for all the vertices.

4.1 Effect of generators

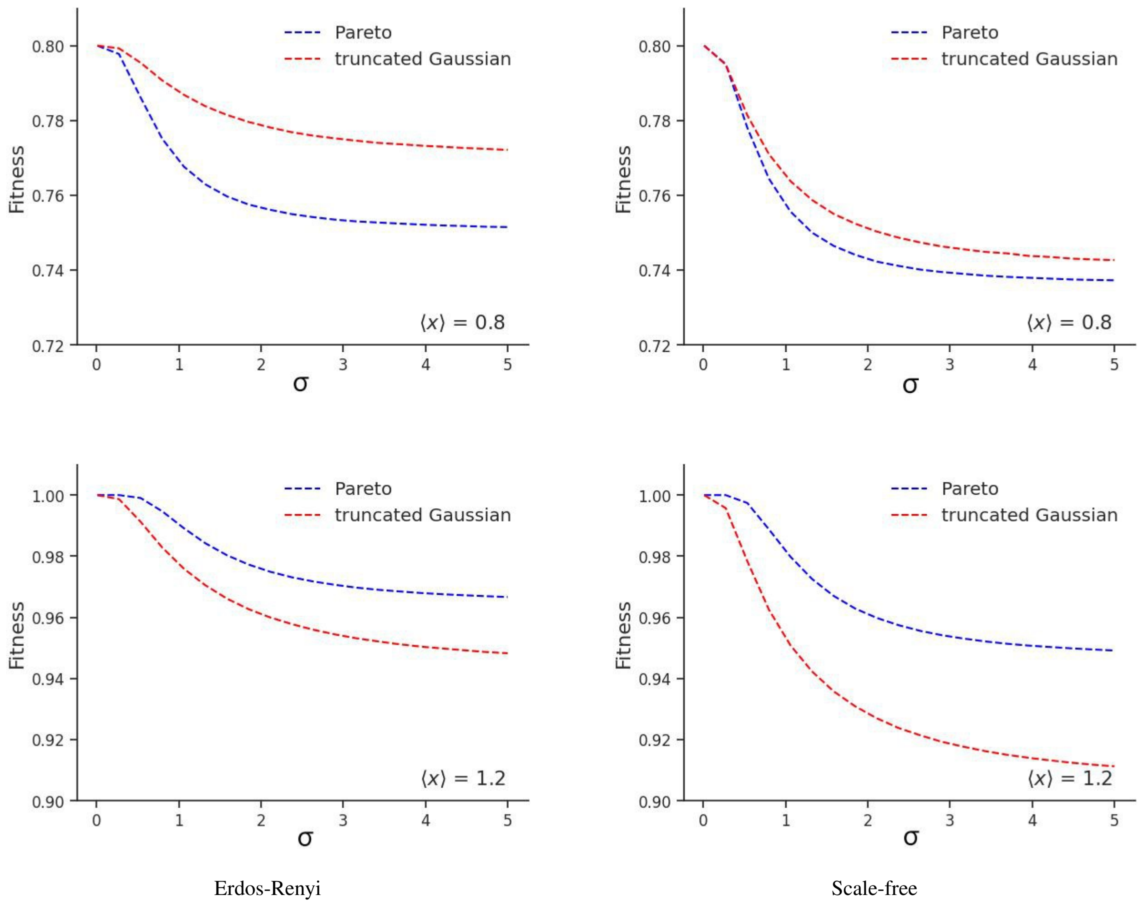


Figure 4.1: Comparison of fitness curves for ER and SF network. The columns on the left and right correspond to differentiating ER and SF networks, while the top and bottom rows highlight scenarios with $\langle x \rangle < R$ and $\langle x \rangle > R$, respectively.

It is evident from figure 4.1 that the network fitness worsens with increasing fluctuation size irrespective of the generators used.

As the standard deviation gradually increases from zero, a wider range of resource quantities, both exceeding and falling short of the mean value, are produced. However, it is important to note that when resource quantities surpass the threshold, R, they all result in the same fitness score of 1. On the contrary, resource quantities lower than R tend to dominate the overall average across the network, causing a decline in the network's overall fitness.

Nevertheless, the depicted plots reveal an interesting pattern: as σ increases, it does not indefinitely reduce the network's fitness. Instead, there is a point beyond which further increases in σ do not lead to a decrease in network fitness, and the fitness value stabilizes as σ approaches infinity Also, it can be seen that the Pareto generator performs better when $\langle x \rangle > R$, but the Truncated Gaussian generator outperforms the Pareto generator when $\langle x \rangle < R$.

4.2 Lower bound for fitness

Let us examine a scenario wherein vertices are restricted from redistributing their surplus to other vertices. In this context, the fitness metric denoted as F_L serves as the lower bound of fitness in any network configuration where surplus sharing is permitted.
It is apparent in this case that $Q_i(x)$ in 3.5 can be substituted with $p(x:\beta_i)$.

$$F_L = \int_0^{R_i} \frac{x}{R_i} p(x:\beta_i)dx + \int_{R_i}^{\infty} p(x:\beta_i)dx \tag{4.1}$$

4.2.1 For Gaussian generator

$$F_L = \frac{1}{\psi(\mu,\sigma)} \int_0^{R_i} \frac{x}{R_i} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx + \int_{R_i}^{\infty} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx \tag{4.2}$$

For varying σ, we calculate corresponding μ by keeping $\langle x \rangle$ a constant. Thus, we compute the integral numerically.

4.2.2 For Pareto generator

From 3.16 we can see that:

$$x_m = \left(1 - \frac{1}{\alpha}\right)\langle x \rangle \tag{4.3}$$

Case 1: $\langle x \rangle < R$.
This implies, $x_m < R$ for all α Hence,

$$F_L = \int_{x_x}^{R_i} \frac{x}{R_i} \frac{\alpha x_m^\alpha}{x^{\alpha+1}} dx + \int_{R_i}^{\infty} \frac{\alpha x_m^\alpha}{x^{\alpha+1}} dx \tag{4.4}$$

$$F_L = \frac{\alpha x_m^\alpha}{(1-\alpha)}(R^{1-\alpha} - x_m^{1-\alpha}) + x_m^\alpha R^{-\alpha} \tag{4.5}$$

From 3.20, in the limit of large σ, α tends to 2. In this limit, using 3.16 in the above, we get:

$$F_L = \langle x \rangle + \frac{\langle x \rangle^2}{R}\left(\frac{1}{4R} - \frac{1}{2}\right) \tag{4.6}$$

Case 2: $\langle x \rangle > R$

Here, x_m can be greater than or less than R depending on the value of α. So, there exist a critical point α_c, below which $x_m < R$ and above which $x_m > R$. The $x_m = R$ at this transition point. So,

$$\alpha_c = \frac{\langle x \rangle}{\langle x \rangle - R} \tag{4.7}$$

From 3.20, we get the transition value of σ to be:

$$\sigma_c = \sqrt{\frac{\langle x \rangle(\langle x \rangle - R)^2}{2R - \langle x \rangle}} \tag{4.8}$$

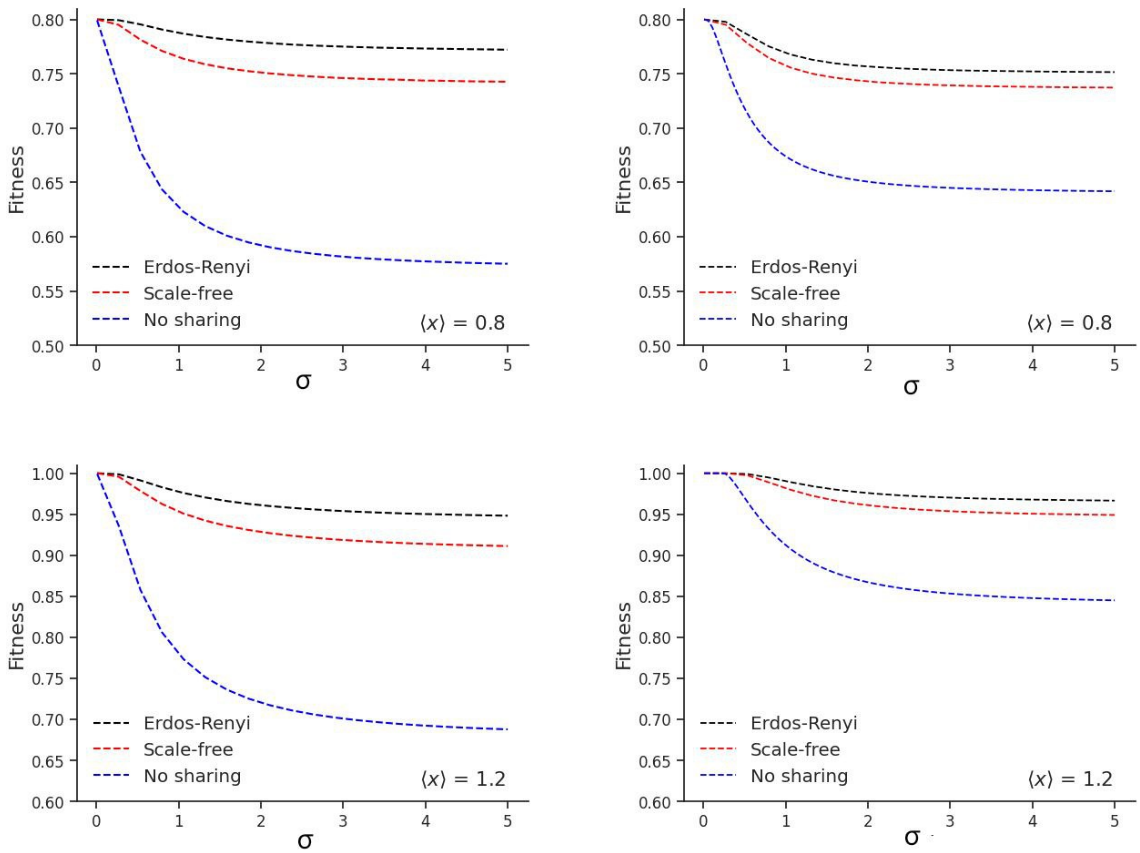
Hence for $\sigma < \sigma_c$, $x_m > R$, and F_L = 1.

Therefore, it is evident that in contrast to the scenario where $\langle x \rangle < R$, we observe two regions where distinct behaviours manifest. The network remains fully fit within the range where$\sigma < \sigma_c$. However, as soon as σ surpasses σ_c, it transitions into a partially fit state.

4.3 Effect of topology

We analyze the variation of F in the context of both ER and SF networks in figure 4.2. Different panels represent various combinations of $\langle x \rangle$ and generator. When we alter the network topology within a specific combination of $\langle x \rangle$ and generator, it affects the numerical characteristics of F but not its fundamental nature. Additionally, regardless of the chosen combination, the ER network consistently outperforms the SF network.

In each subplot of figure 4.2, a lower bound for the fitness for a particular generator depicts how the network's fitness when resources are not shared changes with σ. The plots also illustrate that the overall shape of F is not significantly distinct from F_L. In other words, sharing primarily scales the curve up. Also, it is reasonable to note that sharing smoothens the transition of the Pareto generator at σ_c.



Truncated Gaussian　　　　Pareto

Figure 4.2: Comparison of fitness curves for ER and SF networks when $\langle x \rangle > R$. The no-sharing curve corresponds to the analytically obtained lower bound of the fitness. The columns on the left and right represent distinct generators, while the upper and lower rows pertain to cases of $\langle x \rangle < R$ and $\langle x \rangle > R$, respectively

4.4 Upper bound for wastage

Similar to the case of the lower bound of fitness, the maximum amount of resource wasted occurs when the surplus resources are not distributed. Hence, it is possible to get an upper bound on wastage in this case.

$$w = \lim_{n\to\infty} \lim_{T\to\infty} \frac{1}{nT} \sum_{n} \sum_{T} w_i(t) H(w_i(t))$$

$H(a)$ denotes the Heaviside step function defined as 1 for $a \geq 0$ and 0 otherwise.

4.4.1 For Gaussian generator

$$W = \frac{1}{\psi(\mu,\sigma)} \int_{R_i}^{\infty} (x-R) \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx \tag{4.9}$$

As mentioned earlier, this integral cannot be computed analytically. Hence, for a fixed $\langle x \rangle$, we vary σ and μ and numerically calculate the integral.

4.4.2 For Pareto generator

As seen earlier, the Pareto generator exhibits two distinct regimes on either side of its critical point. We know that α and x_m decrease with increasing σ.
Case 1: $\sigma \leq \sigma_c$ and $x_m > R$

$$W = \int_{x_m}^{\infty} (x-R)p(x)dx = \int_{x_m}^{\infty} (x-R)\frac{\alpha x_m^{\alpha}}{x^{\alpha+1}} dx \tag{4.10}$$

Case 2: $\sigma > \sigma_c$ and $x_m < R$

$$W = \int_{R}^{\infty} (x-R)p(x)dx = \int_{R}^{\infty} (x-R)\frac{\alpha x_m^{\alpha}}{x^{\alpha+1}} dx \tag{4.11}$$

So, the wastage of resource per vertex is obtained by evaluating 4.10 and 4.11:

$$W = \begin{cases} \langle x \rangle - R, & \text{if } \sigma \leq \sigma_c \\ \langle x \rangle^{\alpha} \left(1 - \dfrac{1}{\alpha}\right)^{\alpha} \dfrac{R^{(1-\alpha)}}{\alpha - 1}, & \text{if } \sigma > \sigma_c \end{cases} \tag{4.12}$$

A comparison of resource wastage for Truncated Gaussian and Pareto generators is illustrated in figure 4.3. Just like in the context of network fitness, we observe that the qualitative changes in the resource wastage curve for both ER and SF networks resemble the case where the vertices are restricted from sharing the surplus resources. The sole impact of sharing is to diminish the magnitude of the no-sharing curve. As the plots show, although wastage strongly depends on $\langle x \rangle$ and the generator used, the ER network always leads to lesser wastage than the SF network. The reason for this lies in the fact that the degree distribution of the ER network is more evenly balanced, resulting in a more uniform distribution of generated resources when compared to the SF network.

As we observed in the preceding section, this characteristic makes the ER network generally more adaptable than the SF network.

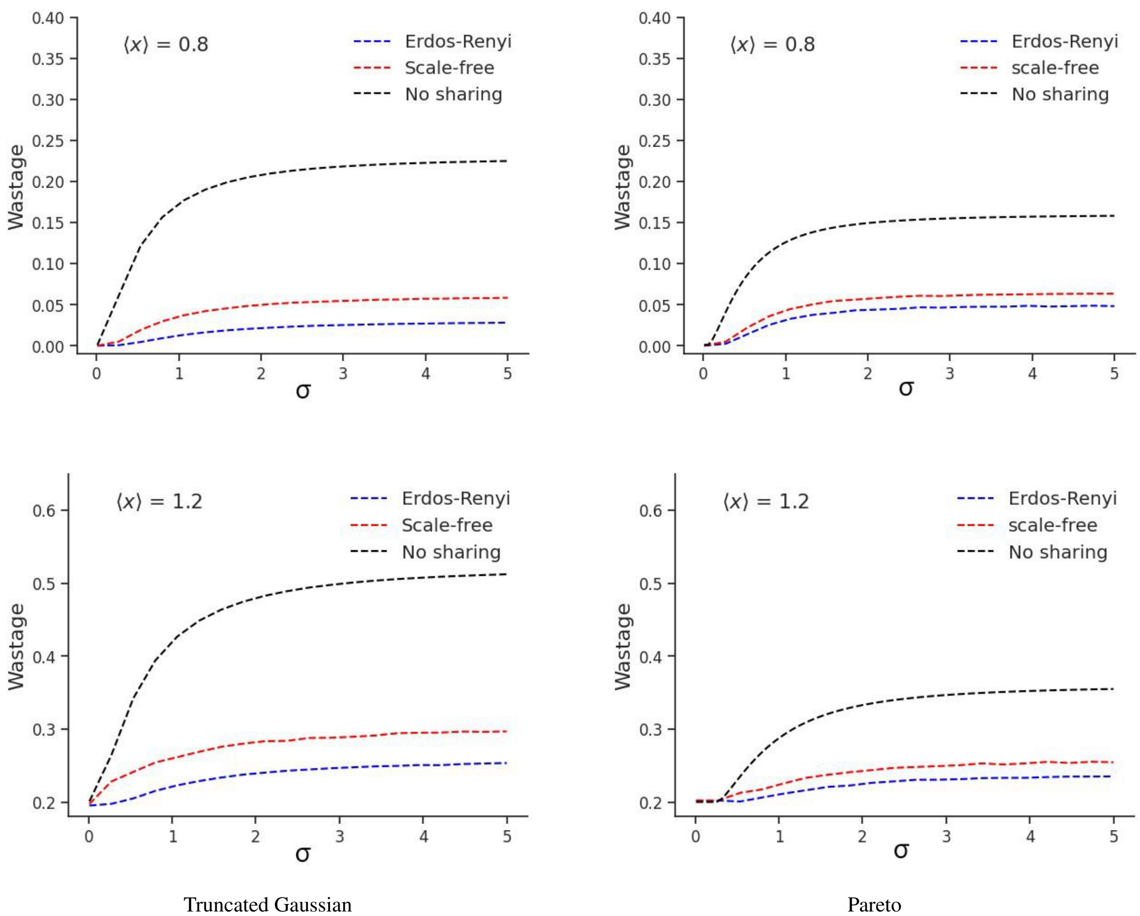


Figure 4.3: Comparison of resource wastage for different generators. The no-sharing curve(black) corresponds to the analytically obtained upper bound of the wastage. The columns on the left and right represent distinct generators, while the upper and lower rows pertain to cases of $\langle x \rangle < R$ and $\langle x \rangle > R$, respectively

4.5 Degree Fitness

The central measure of network fitness involves calculating the average fitness value across all network vertices. However, it is crucial to acknowledge that variations in network topology result in distinct fitness levels among these vertices. This distinction becomes evident through the application of equation 3.3, which reveals that a vertex's fitness is approximately proportional to its degree, leading to high-degree vertices consistently exhibiting greater fitness compared to their low-degree counterparts. The concept of degree fitness corroborates this observation, representing the average fitness of nodes with a degree of k. In figure 4.4, scatter plots depict the relationship between fitness and k for different combinations of σ and the generators within the SF network framework. These visual representations clearly demonstrate that irrespective of the chosen parameters, fitness rapidly increases with increasing degree. Additionally, for relatively small degree values, degree fitness attains its maximum value of 1. While the specific numerical trends in the plots vary across different combinations of generators and average production values, a consistent observation is that fitness generally rises as the degree increases.

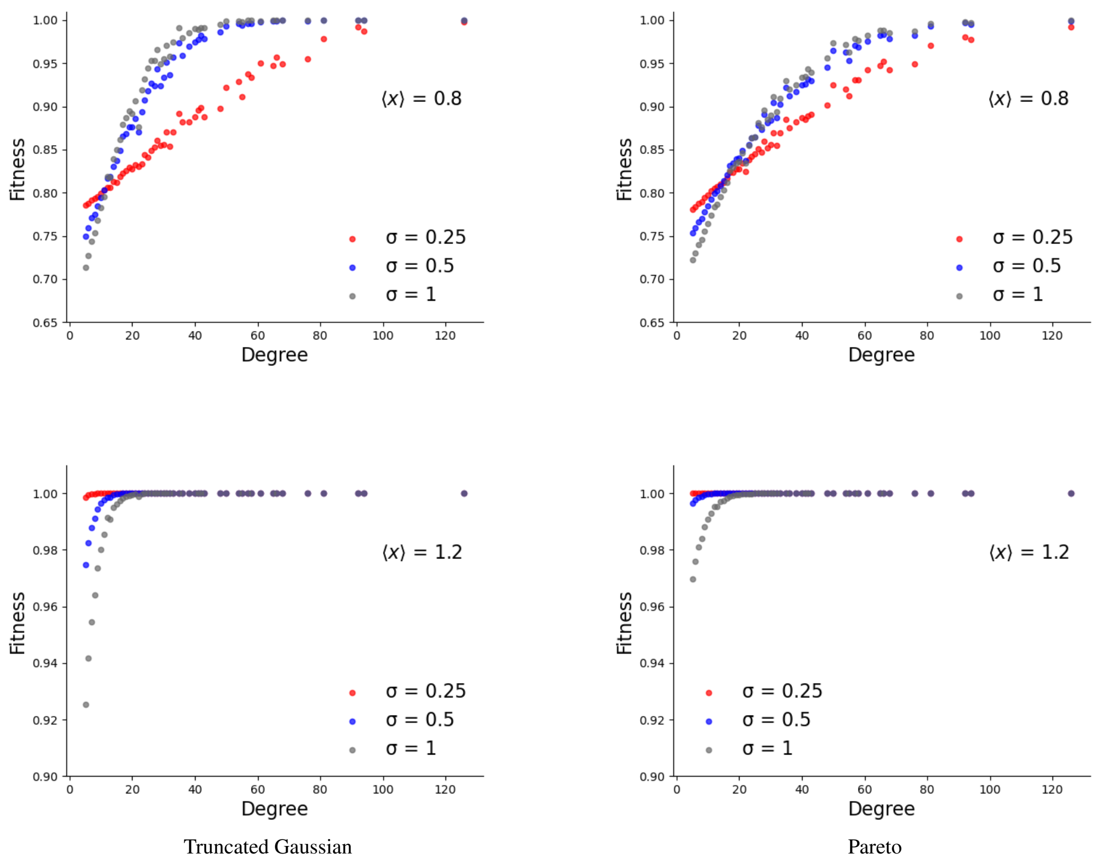


Figure 4.4: Comparison of degree for different generators. The columns on the left and right represent distinct generators, while the upper and lower rows pertain to cases of $\langle x \rangle < R$ and $\langle x \rangle > R$, respectively.

Chapter 5

The interplay of assortativity and production capacities in dependency networks

This chapter addresses the impact of heterogeneity in the production capacity of network vertices on network fitness. We investigate resource production based on vertex degree and other centrality metrics to uncover the key parameters and factors that are crucial in determining network fitness. Furthermore, our inquiry is multifaceted, encompassing an assessment of these aspects within disassortative, assortative, and neutral networks. We are particularly interested in understanding how the degree of assortativity within these network types interacts with the varying production capacity scenarios and impacts network fitness. This multi-dimensional approach provides a holistic perspective on the intricate relationship between production capacity, network structure, and overall network performance, shedding light on the nuanced interplay of these factors.

Understanding the impact of different parameters on the network's fitness is pivotal for theoretical insights and practical applications in fields as diverse as social networks, transportation systems, and ecological communities. By establishing the connections between vertex centrality and resource production, we can potentially optimize resource allocation strategies, identify critical nodes, and enhance such networks' overall resilience and efficiency.

All the results presented are for a network size of 1000 and are averaged over 100 realizations unless mentioned otherwise. We also assume that the thresholds R_i have the same value $R = 1$ for all the vertices.

5.1 Impact of degree-based resource heterogeneity

To accomplish this, let us assume that the production capacity denoted by β_i of a vertex, based on its degree, follows a proportional relationship with a real number θ. The outcomes in the preceding chapter of homogeneous production capacity pertain to the specific scenario where $\theta = 0$. If $\theta > 0$, vertices with higher degrees have a higher production capacity; the opposite holds if $\theta < 0$.
In the case of homogeneous production capacity, each vertex has an average production denoted as $\langle x \rangle$, resulting in a total production of $n\langle x \rangle$ in the network, where n represents the network size. To ensure a fair comparison, we keep the total average production at the same level. Therefore, we must establish the following relationship:

$$\beta_i = \frac{k_i^\theta}{\sum_{i=1}^n k_i^\theta} n\langle x \rangle \tag{5.1}$$

5.1.1 Exponential resource generator

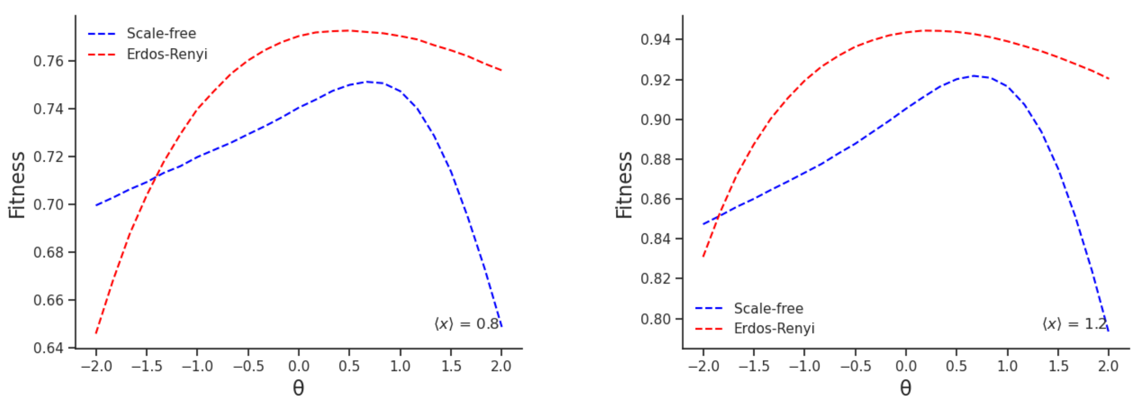
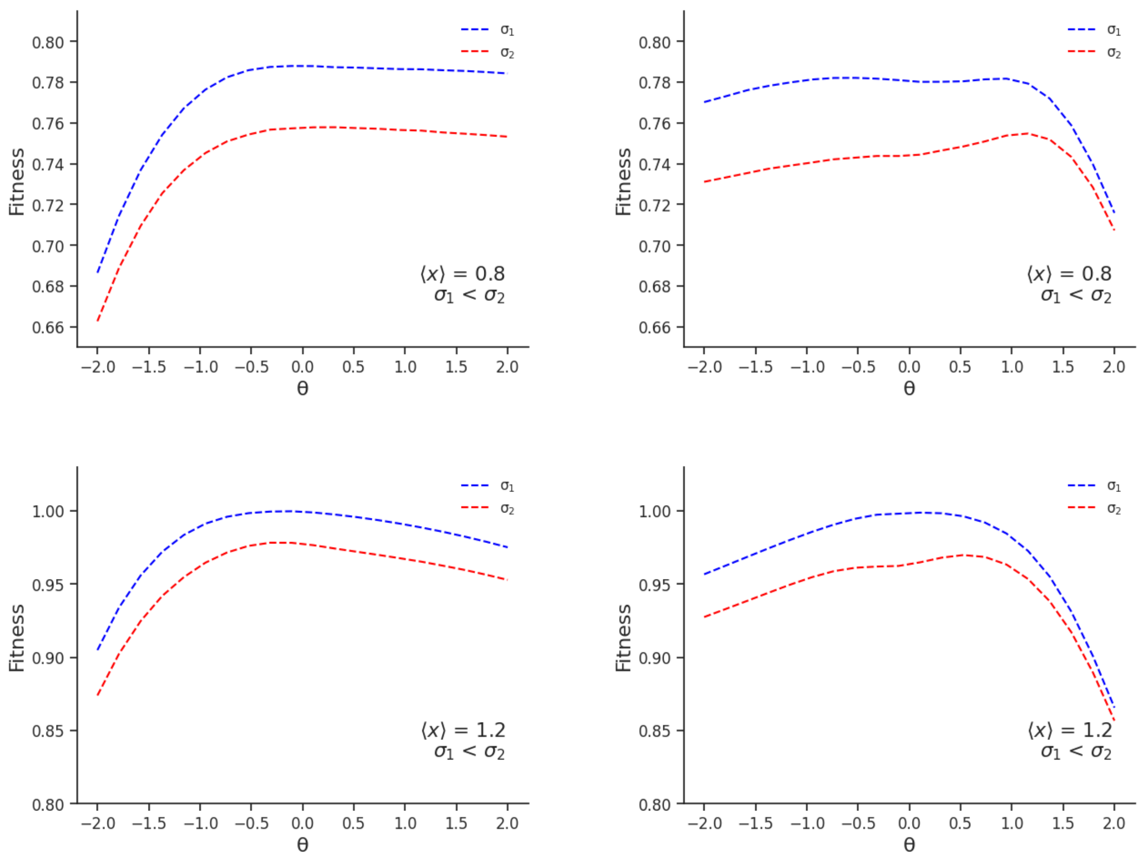


Figure 5.1: Comparison of fitness values for exponential resource generator over ER and SF networks when the production capacities across the vertices were made as a function of vertex degree to the heterogeneity parameter θ. The columns on the left and right pertain to $\langle x \rangle < R$ and $\langle x \rangle > R$, respectively.

From the figure 5.1 above, it can be seen that a certain level of heterogeneity is beneficial for SF networks. However, a uniform production capacity over the vertices ensures better fitness in the case of ER networks. An increase in average production doesn't seem to dictate the level of heterogeneity θ_{max} that provides maximum fitness to the network. As we have seen earlier, the exponential distribution's mean and standard deviation σ are the same. The fluctuations in production across the vertices are not constant in this case. To gain insight into the effect of fluctuations, we repeat the same with the Pareto resource generator. We look into this in the following section.

5.1.2 Pareto resource generator



Erdos-Renyi Scale-free

Figure 5.2: Comparison of fitness values for Pareto resource generator over ER and SF networks when the production capacities across the vertices were made as a function of vertex degree to the heterogeneity parameter θ. The left and right columns correspond to ER and SF networks, while the Top and Bottom rows correspond to $\langle x \rangle < R$ and $\langle x \rangle > R$, respectively.

Figure 5.2 reflects the admissible range of heterogeneity for ER and SF networks when the resource generator takes Pareto distribution. It is evident from figure 5.2 that some level of heterogeneity is often helpful in enhancing network fitness for scale-free networks. Given that the SF network is predominantly composed of nodes with lower degrees, it is possible that introducing a certain degree of heterogeneity could potentially improve the network's overall fitness. A nearly homogeneous production capacity at vertices ensures better performance since the degree distribution is nearly uniform for an ER network.

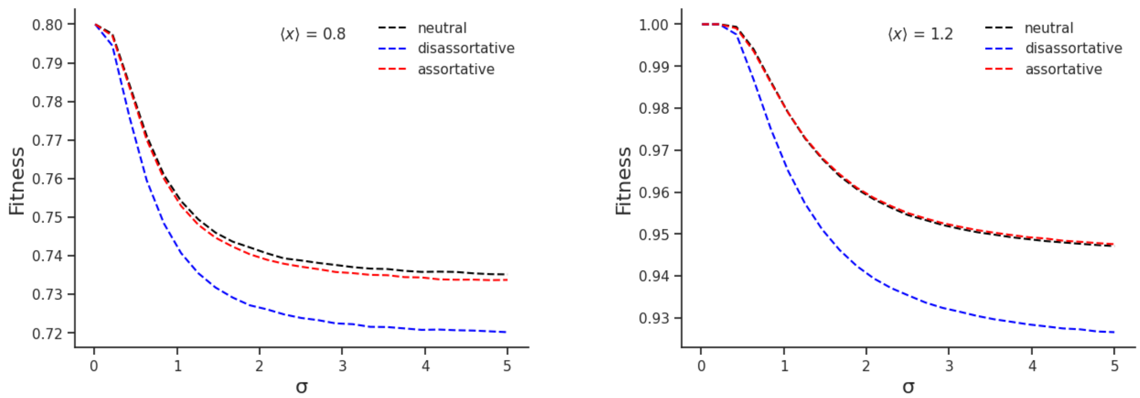
As seen earlier, increasing the fluctuation size lowers the network fitness but doesn't seem to alter θ_{max} for which the network shows maximum fitness. Thus, θ_{max} is found to be independent of the fluctuation size. However there is a shift in θ_{max} with change in the average production values. This isn't investigated in detail.

For lower values of θ, the fitness of Erdos-Renyi networks is lower than that of scale-free networks. Increasing production at lower degree vertices leads to inefficient sharing of surplus amounts. The surplus amounts get distributed only to a few neighbours, and since the fraction of smaller degree nodes in the SF network is larger compared to the ER graph, the lowering in fitness is not as notable as in that of ER topology.

5.2 Impact of degree assortativity

In an Erdos-Renyi(ER) network, the degree distribution is typically not assortative or disassortative by design, as ER networks are characterized by a Poisson degree distribution, which means that nodes have roughly equal probabilities of connecting to any other node in the network. Hence, for studying the impact of assortativity, we considered only scale-free degree distribution.

Figure 5.3 illustrates a scenario wherein the resource production at network vertices was uniform for the Pareto resource generator within a scale-free network. In this context, all graphs maintained an identical degree sequence while varying in terms of degree assortativity. Specifically, the networks were categorized as neutral with an assortativity coefficient close to 0, assortative with an assortativity coefficient of approximately 0.35, and disassortative exhibiting an assortativity coefficient of around -0.35.



(a) $\langle x \rangle < R$ **(b)** $\langle x \rangle > R$

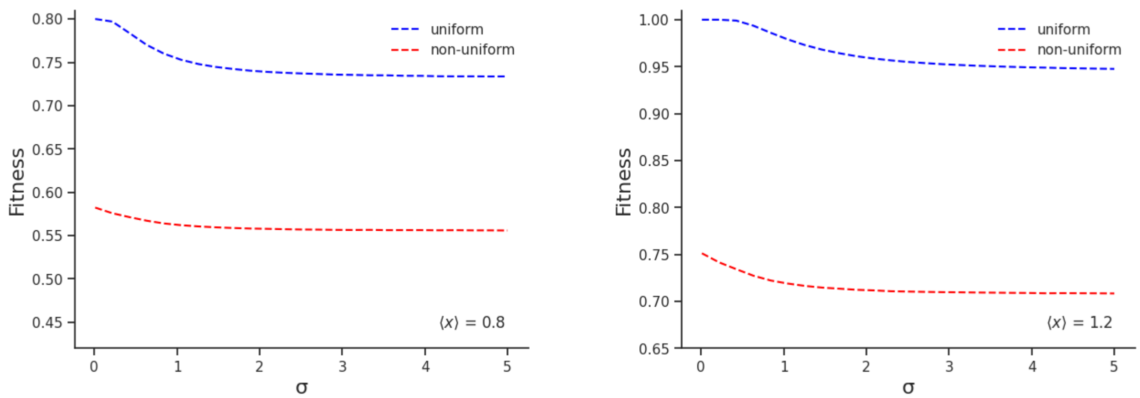
Figure 5.3: Fitness comparisons of the Pareto resource generator in uniformly equipped SF networks with varied assortativity Coefficients. The distinction between the two columns lies in the conditions where $\langle x \rangle < R$ or $\langle x \rangle > R$.

An intriguing observation emerges from this analysis: regardless of the specific assortativity coefficients, fluctuations within the network exert a consistently adverse influence on network fitness, as seen in the previous chapter. This implies that fluctuations or variations in resource production are generally detrimental to the overall performance or robustness of the network.
Furthermore, it is worth noting that disassortativity does not confer a fitness advantage in the context of uniform production capacity across network vertices. In other words, network configurations that exhibit a tendency for nodes to connect to dissimilar nodes do not exhibit superior fitness when compared to their assortative or neutral counterparts.

5.3 Interplay of assortativity and resource heterogeneity

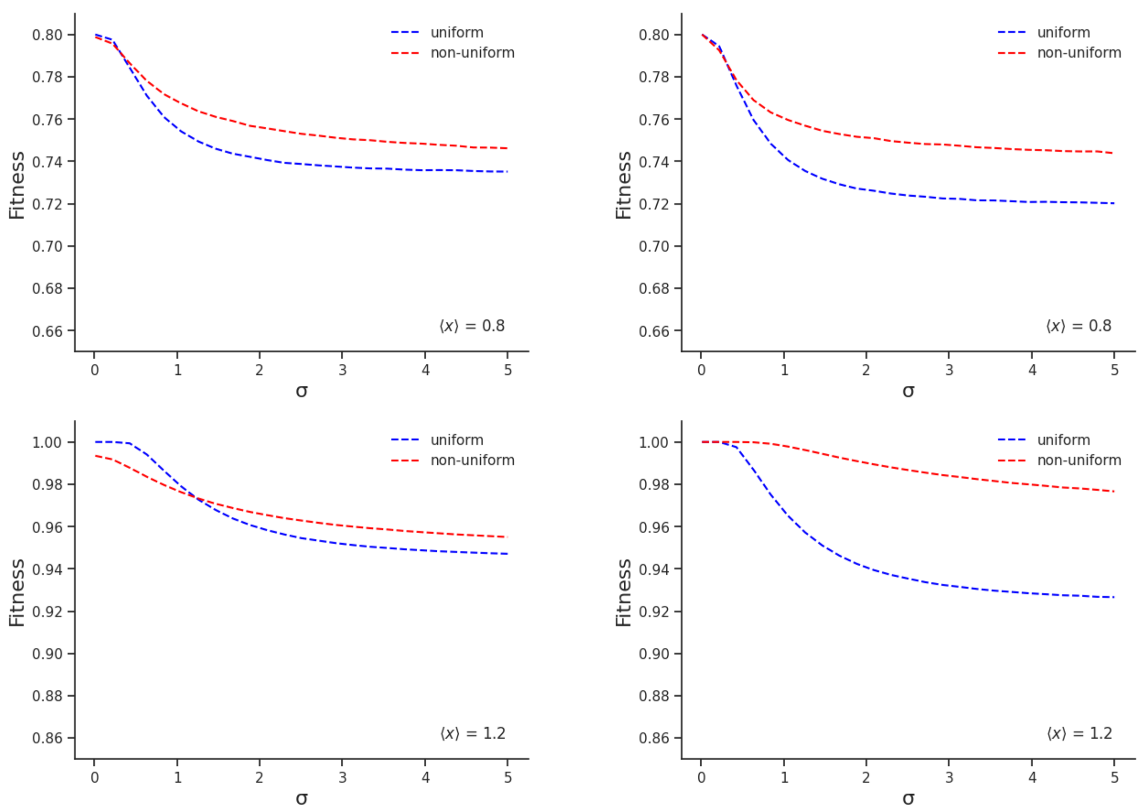
In Figure 5.4, production capacities were aligned proportionally with the vertex degree. An intriguing observation is that, within an assortative network, the introduction of production capacity heterogeneity is associated with a significant reduction in network fitness.
The underlying reason for this phenomenon may be attributed to the assortative nature itself. In assortative networks, nodes with similar degrees tend to connect with each other. When production capacities align with vertex degrees, it accentuates the disparity in resource production among nodes, potentially leading to inefficiencies or imbalances within the network. Consequently, this production capacity heterogeneity within an assortative network can hinder its overall fitness or

performance.



(a) $\langle x \rangle < R$ **(b)** $\langle x \rangle > R$

Figure 5.4: Fitness comparisons of the Pareto resource generator in assortative SF network with uniform production capacities. The distinction between the two columns lies in the conditions where $\langle x \rangle < R$ or $\langle x \rangle > R$.



(a) neutral **(b)** disassortative

Figure 5.5: Fitness comparisons of the Pareto resource generator in neutral and disassortative SF network with non-uniform production capacities. The distinction between the two rows lies in the conditions where $\langle x \rangle < R$ or $\langle x \rangle > R$.

Observations in the figure 5.5 reveal a noteworthy pattern. While disassortative networks exhibited lower fitness than neutral or assortative networks when production capacity was uniform, a distinct transformation occurs when resource production aligns with node degrees. Under this degree-based resource production scheme, fitness experiences a remarkable increase and outperforms the neutral network counterpart.

The rationale behind this shift in network performance can be attributed to the interplay between disassortative mixing and resource allocation. In disassortative networks, nodes tend to connect with dissimilar nodes, fostering diversity in resource sharing. When production capacities correlate with node degrees, this diversity ensures that resources are distributed effectively across the network, potentially enhancing collective welfare.

Consequently, it can be inferred that the combination of disassortative network structures and resource inequality, as evidenced by degree-based resource production, contributes to improving collective well-being within deficit-based sharing networks.

To enhance our understanding of how vertices of networks with different assortativity respond in homogeneous and heterogeneous cases, we have grouped vertices into two categories: 'low degree' for those with degrees below the network's average and 'high degree' for the rest. We define n_s as the fraction of vertices belonging to the mentioned categories that successfully procured the required amount of resources from its neighbours when it was in deficit during the simulation. The figure 5.9 below represent how n_s varies for different combinations of assortativity and production.

The impact of resource production heterogeneity on network fitness varies depending on the network's connectivity pattern. In disassortative networks, where dissimilar degree vertices connect, having heterogeneous production levels benefits lower degree vertices by enabling resource sharing while limiting higher degree vertices' ability to do the same. However, this setup benefits from the larger fraction of lower degree vertices in scale-free networks, ultimately improving overall fitness. In neutral networks, heterogeneity in resource production enhances the ability of lower degree vertices to share resources without compromising higher degree vertices' capabilities, resulting in increased fitness. In contrast, assortative networks, where similar degree vertices are more likely to connect, experience a significant reduction in lower degree vertices' ability to procure resources through sharing when heterogeneity is introduced. This is because the heterogeneity leads to lower production levels among lower degree vertices, which hampers network performance. Therefore, we can conclude that the potential of resource production heterogeneity to enhance network fitness depends on the network's connectivity pattern and is not solely determined by the degree sequence.

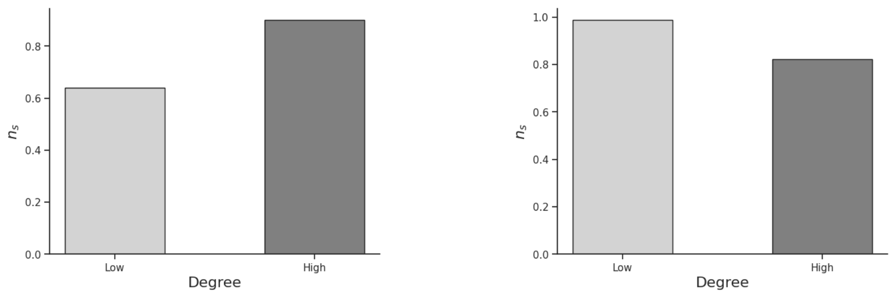


Figure 5.6: disassortative

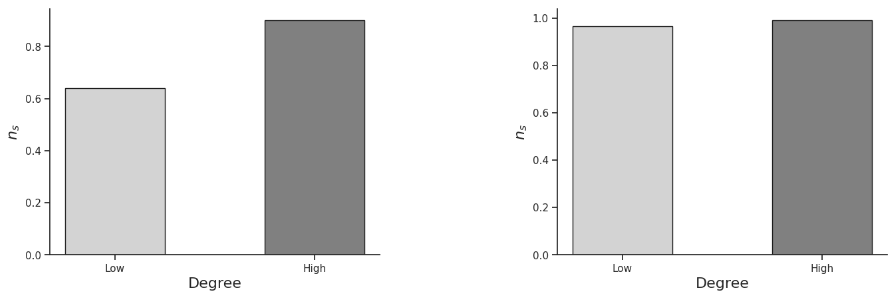


Figure 5.7: Neutral

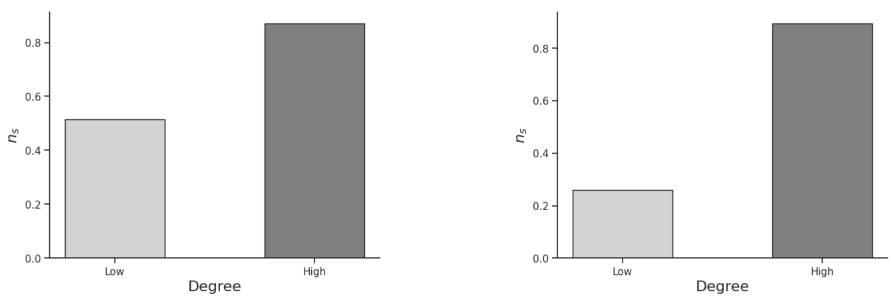


Figure 5.8: assortative

Figure 5.9: Comparison of the fraction of vertices that were successful in procuring the resources through sharing for Pareto resource generator in disassortative, neutral and assortative SF networks. The distinction between the columns lies in the conditions where production is homogeneous and heterogeneous. All the graphs correspond to the case when average production $\langle x \rangle = 0.8$

Chapter 6

Conclusion

The principal revelation elucidated throughout this thesis underscores that the average resource production does not exclusively dictate the overall performance of a resource dependency network. Instead, fluctuations, assortativity, and heterogeneity in production capacities emerge as pivotal determinants in network performance. Fluctuations in resource production is has a detrimental effect on the overall network fitness. This holds irrespective of whether the average production per vertex falls below or surpasses the threshold resource requirement for individual vertices.

The central finding within this thesis revolves around the profound impact of disassortative mixing and degree-based heterogeneity in average production capacities on a resource dependency network governed by the surplus distribution model. Specifically, it unveils that when a fixed total average production is considered, these two factors synergize to bolster the collective well-being within the network significantly.

6.1 Limitations and future directions

This thesis predominantly focuses on Erdos-Renyi (ER) and scale-free (SF) networks, while acknowledging that these network models may not fully capture the diversity of real-world networks. Real-world networks can exhibit a wider range of structures, and future research should consider more network types for a comprehensive understanding.

The study primarily explores random resource production hence the stochastic nature, which may not fully represent the deterministic dynamics found in many real resource dependency networks. Additionally, real network structures can be dynamic, evolving over time. Future research should consider incorporating deterministic elements and dynamic network structures to better mirror real-world scenarios.

This thesis assumes that resources have a fixed lifetime of one unit of time and are shared only with immediate neighbours. However, real networks may exhibit different resource lifetimes and more complex sharing dynamics, where resources can traverse multiple vertices. In which case other centrality measures like closeness, betweenness etc could possibly bring about significant impact. Future investigations should address these complexities to provide a more realistic depiction of resource-dependent networks.

To address these limitations, future research should prioritize the utilization of real-life data and modelling techniques that can capture the intricacies of resource dependency networks. This approach would provide a more accurate and comprehensive understanding of the dynamics and performance of such networks.

In summary, while this thesis highlights the importance of fluctuations, assortativity, and heterogeneity in production capacities in resource dependency networks, it is crucial to expand the scope of research to encompass a wider variety of network structures, dynamics, and real-life data for a more comprehensive analysis of network performance.

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