Eliciting Altruistic Responses by Incentivizing Strategic Choices: Implications for CSR Policy-making

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by

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Certificate

This is to certify that this dissertation entitled **Eliciting Altruistic Responses by Incentivizing Strategic Choices: Implications for CSR Policy-making** towards the partial fulfilment of the BS-MS dual degree programme at the Indian Institute of Science Education and Research, Pune represents study/work carried out by Abhay Chandran at Indian Institute of Science Education and Research under the supervision of Dr. Anisa Chorwadwala, Associate Professor, Department of Mathematics, and Dr. Vinay Ramani, Associate Professor, Department of Management Sciences, Indian Institute of Technology, Kanpur during the academic year 2023-2024.

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This thesis is dedicated to my Parents and Grandparents.

Declaration

I hereby declare that the matter embodied in the report entitled **Eliciting Altruistic Responses by Incentivizing Strategic Choices: Implications for CSR Policy-making** are the results of the work carried out by me at the Department of Mathematics, Indian Institute of Science Education and Research, Pune, under the supervision of Dr. Anisa Chorwadwala and Dr. Vinay Ramani and the same has not been submitted elsewhere for any other degree. Wherever others contribute, every effort is made to indicate this clearly, with due reference to the literature and acknowledgement of collaborative research and discussions.

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Abstract

In this thesis, we explore various facets of Corporate Social Responsibility (CSR) spending by firms. CSR regulation in India is governed by the Companies Act, 2013('Act'). Schedule VII of the 'Act' specifies the activities that qualify for CSR. As part of CSR spending, firms spend money on a variety of sectors - we classify this spending as *strategic* and *altrusitic* and develop a game-theoretic model of CSR spending by firms who produce different qualities of products as well as their pricing strategies. We address the following questions. First, what are the equilibrium outcomes when strategic CSR is not recognized and firms are allowed to undertake only altruistic CSR spending. Second, does allowing for strategic CSR lower the incentives for firms to undertake altruistic CSR spending? Surprisingly, we find that in contrast to the common perception that strategic and altruistic CSR spending are substitutes, allowing firms to undertake strategic CSR spending *increases* their altruistic CSR spending as well. In addition, profits as well as consumer surplus and hence social welfare increase. Next, we consider the situation where a regulator asks firms to spend a certain amount for CSR, similar to the 'Act' mandating firms to spend 2% of their profits on CSR. If a firm underspends (overspends) recommended CSR, it gets a penalty (reward). We find that the social welfare under regulation can be greater than or lower than under no regulation. Similarly, there are ranges of parameter values, where firms may be better off under the regulation as opposed to the case of no regulation. Finally, we ask whether firms with a higher perceived value of CSR spending overspend or underspend on CSR. To answer this question, we use data on firms' CSR expenditures to compute the perceived value or the k-value. A higher k-value signifies a higher tendency of a firm to overspend. We obtain k-value of around 6000 Indian firms and classify them into high, mid and low-profit groups. Low-profit firms generally have a higher perception of CSR. High-profit firms, whether overspending or underspending, don't have significant differences in their k-values.

Contents

Chapter 1

Introduction

1.1 What is Corporate Social Responsibility (CSR)?

'Corporate Social Responsibility(CSR) is a management concept whereby companies integrate social and environmental concerns into their business operations and interactions with their stakeholders. CSR is generally understood as being the way through which a company achieves a balance of economic, environmental and social imperatives ("Triple-Bottom-Line-Approach"), while at the same time addressing the expectations of shareholders and stakeholders' (UNIDO 2017).

1.2 CSR in India

CSR in India is governed by Section 135 of the Companies Act, 2013 ('Act') and Schedule VII of the Act and Companies (CSR Policy) Rules, 2014. It gives the criteria for eligibility of the companies that need to follow the regulations and recommends what activities they can do to contribute to society. According to Section 135 of the Companies Act 2013, every company having networth five hundred crores or more, or a turnover of rupees one thousand crores or more, or a net profit of rupees five crores or more during any financial year qualifies for CSR. They have to constitute a Corporate Social Responsibility committee of the Board consisting of 3 or more directors, out of which at least one director shall be an independent director(p. 33-34, Ministry of Corporate Affairs (2015)). This committee will recommend and monitor the company's CSR policy and the expenditure to be incurred on CSR activities.

CSR is a board-driven process. One of the key actions to be taken by the board is to ensure that in every financial year, the company spends at least two percent of the average net profits made during the three immediately preceding financial years on CSR. The Government of India(GOI) gave indicative activities that the company could undertake through Schedule VII of the Act. These include social factors such as eradicating poverty, hunger, and malnutrition, promoting healthcare, education, gender equality, etc. and contributing to the 'Swachh Bharat Kosh' set up by the Central Government to promote sanitation. The major environmental activities include environmental sustainability, ecological balance, and contribution to the 'Clean Ganga Fund' set up by the Central Government to rejuvenate the river Ganga. These activities range from the local to the national level, like training to promote sports(rural to Olympic), protection of national heritage, contribution to the Prime Minister's National Relief Fund(PMNRF), etc. (p.35-36, Ministry of Corporate Affairs (2015)).

From the National CSR portal of the Ministry of Corporate Affairs, we can see that in the Financial year(FY) 2020-21, major CSR was spent on Rural Development, PMNRF, Health and Sanitation; Environment, Animal Welfare, Conservation of Resources(EAC); Gender Equality, Women Empowerment, Old Age Homes, Reducing Inequalities(GWOR); Education, Differently Abled, livelihood(EDL). More than 1000 crores were spent on the above schemes. Encouraging sports, Swachh Bharat; Heritage, Art and Culture(HAC); Clean Ganga Fund and Slum developments were the other major development activities done through CSR(Ministry of Corporate Affairs 2023).

In FY2020-21, the total number of companies that account for CSR in India is 20840. Of these, 9935 companies spent more than prescribed for CSR, i.e., 48% of total companies, and 1511 companies(7%) spent exactly as prescribed for CSR. 55 % of companies are following the mandates recommended by the government. The data shows that 5888 companies(28% spent nothing on CSR. We can see that nearly half of the companies had overspent. From Ministry of Corporate Affairs (2019), some firms have spent almost 3 times their prescribed CSR. Also, the number of zero-spent companies has decreased over the years. There needs to be some motive for the firm to overspend other than the penalty incurred by the government.

In Chapter 3, we will ask what motivates the firm to overspend and derive a notion of

CSR perception. The penalty incurred in India for underspending the recommended CSR is the minimum of twice the unspent amount or one crore rupees(Ministry of Corporate Affairs 2021). The impact with the regulator will mainly examine how the threshold decided for CSR expenditure and penalty/ reward due to underspending/ overspending affects the firms and consumers.

1.3 Strategic and Altruistic CSR

CSR can be viewed as two different types according to its purpose. One is strategic CSR and the other one is altruistic CSR. Strategic CSR is the CSR contribution by the firms directed to consumers of their interest, which gives them direct business benefits. Altruistic CSR is the CSR contribution by the firms towards the overall population for the general welfare, i.e., they are not directed just toward their consumers so that it won't give them direct business benefit.

One example of strategic CSR is Jio Institute at Navi Mumbai, founded by Reliance Industries Limited(RIL) in 2018. Jio Institute is a RIL CSR initiative. The CSR contributions by RIL for Jio Institute during financial years were 303.57 cr(2017-18), 476.9 cr(2018-19), 229 cr(2019-20), 375 cr(2020-21) and 141.5 cr(2021-22) (Ministry of Corporate Affairs 2023). The total CSR expenditure for RIL during each financial year is 700 to 920 cr. Around one-fourth of RIL CSR goes to Jio Institute. However, to access education at Jio Institute, there is some cost,i.e., the cost of education. Also, it is better for a person in the same city or state than for someone on the other side of the country as there is travel cost. We can call all these costs accessibility costs. So, as we see, strategic CSR needs accessibility costs.

Scholarships given to students or investments in central/ state government activities like 'Clean Ganga' or charity work can be seen as examples of altruistic CSR. It is not directed towards specific consumers or done for just business benefit. Firms will benefit indirectly from these investments because consumers care about them. Socially concerned consumers will prefer these firms. The problem with altruistic CSR for profit-maximizing firms is that their altruistic CSR investment will benefit consumers buying products from their competitors. The benefit one firm is getting due to the actions of another firm is known as the spillover effect (d'Aspremont and Jacquemin 1988), which we will capture in our model. If a consumer population is concerned about social well-being, they will care more about altruistic CSR.

The model will also capture this by introducing a social concern parameter.

As the United Nations has ‘the 2030 Agenda for Sustainable Development’, let’s see how CSR can promote Sustainable Development Goals(SDGs). Fallah Shayan et al. (2022) proposed a comprehensive framework for CSR implementation addressing the SDGs’ contribution. They divided CSR into 3 dimensions: Environment, Society and Economy, where different SDGs are categorized into these 3 dimensions. In this paper, CSRs and SDGs are complementary because they promote environmental protection and socioeconomic development. Corporations benefit from investing in SDGs as they protect themselves from future risks. It helps in business sustainability, a stable economy, a functional society, environmental crisis prevention, labor market expansion, global resource management, universal market growth and human development. These SDGs’ universal value creation leads to CSR corporate value creation, such as better financial performance, public trust, credibility, reputation, customer satisfaction and loyalty and an overall increase in brand value.

According to Ministry of Corporate Affairs (2021), the spirit of the Act is to ensure that CSR initiatives are aligned with national priorities and enhance corporate sector engagement towards achieving Sustainable Development Goals (SDGs). A lawmaker or a regulator may prefer a firm to do more or only altruistic CSR as it benefits the general public. So, let’s consider if Schedule VII only allows altruistic CSR activities by not recognizing strategic CSR. We would like to know how it impacts the society.

In Chapter 2, we are not considering any regulation on how much a firm should spend on CSR, i.e., there is no regulator in the market. Many reasons motivate firms to do CSR despite no government mandate. Minor and Morgan (2011) theoretical framework says ‘CSR acts as a powerful form of reputation insurance when a firm suffers an adverse event.’ ‘When shareholders care about public goods and profits, and when managerial contracts reflect these concerns, managers redirect more profits toward public goods than shareholders would when acting separately. Shareholders are poorer but happier(Morgan and Tumlinson 2019).’ ‘Using panel data for U.S. firms from 2002 to 2011, firms with high financial or environmental risk can benefit from CSR as a risk mitigation strategy, and firms with high earnings stability can benefit from CSR as an efficiency enhancement strategy(Lu et al. 2021).’ ‘Larger firms, i.e., firms with greater free cash flow and higher advertising outlays, demonstrate higher levels of CSR(Borghesi et al. 2014).’ Lin et al. (2009) examined 1000 Taiwanese cases between 2002 and 2004 in which firms included their R&D expenditures as one of their business strategies for sustainable development and identified their charitable expenditures as contributions to CSR. They identified a positive relationship between CSR and financial performance. If gone to more specification, they found that while CSR does not positively impact short-term financial performance, it offers a remarkable long-term fiscal advantage.

1.4 Location Models

We need to consider location models to find the demand functions in our model. We can use these models to model consumer heterogeneity, i.e., consumers with different tastes and locations. 'Location' can be interpreted in 2 different ways: 1) the physical location of a particular consumer, and 2) the distance(or disutility) between the characteristics of a product produced by a firm in the market and the desired characteristics that the consumer wants from the product. There are mainly 2 models for product differentiation: 1)the Linear Hotelling Model(Hotelling 1929), which models linear product space, and 2) the Salop Circle Model(Salop 1979), which models circular product space. We will use the Salop circle in our model as it has some advantages over the linear Hotelling model. Consumers can be distributed continuously in the circular model, which will be beneficial for capturing some of the firms' locations. When firms are uniformly distributed along the linear model, firms at the end of the line segment will not have consumers on their other side, so this would affect the model in symmetric cases. This problem won't arise in the circular model as we can distribute all the firms equidistant from each other along the circle's circumference. In the Hotelling linear-city game, there is no equilibrium for the game where firms use both prices and location as strategies(Proposition 7.7, Shy (1996)), which is not the case in the circular model.

1.5 Research Questions and Analytical Approach

In Chapter 2, we will address the following research questions:

1. What are the implications for producer surplus, consumer surplus, and social welfare when the regulator recognizes firms' strategic CSR and when it doesn't recognize

strategic CSR?

2. How does the impact on firms' altruistic CSRs depend on whether the regulator recognizes strategic CSRs under CSR?

3. How do the equilibrium outcomes change depending on the firm's types?

4. How does one firm influence another firm's CSR investments?

Our Analytical approach would be as follows:

1. Prevailing competitive setting: A regulator recognizes firms' strategic CSRs distinctly from altruistic CSRs under CSR.

2. Benchmark competitive setting: A regulator does not recognize strategic CSRs under CSR in a competitive setting.

3. In the competitive setting, we will have a high-type firm and a low-type firm where the high-type's brand value is greater than the low-type's and their efficiency of investing is different.

4. Benchmark monopoly setting: strategic CSR is recognized under CSR in a monopoly setting.

    - This setting would be useful to obtain insights into the implications of positive externalities in a competitive setting.

5. Compare equilibrium outcomes under the prevailing and benchmark competitive settings.

    (a) Would the altruistic CSR level and the aggregate investment levels decrease (or increase) if the regulator permits strategic CSR under CSR, i.e., comparing the equilibrium outcomes under the prevailing competitive setting from that under the benchmark competitive setting?

    (b) Would producer surplus increase under the prevailing competitive setting from that under the benchmark competitive setting? Similarly, for consumer surplus and social welfare.

(c) Would the strategic and altruistic CSR levels decrease (or increase) under the prevailing competitive setting from those under the benchmark monopoly setting? What are the implications for producer surplus, consumer surplus, and social welfare?

In Chapter 3, we will ask the following:

1. How firms and consumers are affected if a regulator mandates firms to spend a certain amount on CSR?

2. What motivates firms to overspend the recommended CSR?

Our approach would be to introduce a parameter β_i which is the factor to which the profit of firm i is multiplied to get the recommended CSR spending of i and a parameter for penalty/ reward if the firm underspends/overspends its recommended CSR expenditure. These two will be treated as exogenous parameters in our model. We will compare: with regulator v/s without regulator for firms' CSR investments, product price and demand, profits; Consumer surplus and Social welfare.

1.6 Original Contributions

The thesis presents a game-theoretic model of firms' strategic and altruistic CSR efforts in a monopoly and a duopoly market. In Chapter 2, we will develop this model, and in Chapter 3, we will extend the model to include regulations on firms' CSR investments. The results, proofs, figures and tables in both chapters are entirely original. In Chapter 2, in the Monopoly section, we have the altruistic CSR investment, demand, price, profit, and CS [and hence SW] when strategic CSR is recognized are greater than the altruistic CSR investment, demand, price, profit, and CS [and hence SW] when strategic CSR is not recognized. Thus, a monopolist is incentivized to invest in strategic CSR. In Duopoly, strategic and altruistic CSR can act as substitutes or complements according to some market conditions given in a proposition. Consumer surplus and social welfare, when strategic CSR is recognized, always dominate the scenario of strategic CSR not being recognized. However, for producer surplus, we got that in some market ranges, strategic CSR recognized will dominate, and in the complement regions, strategic CSR not recognized will dominate. We have a proposition that gives the market conditions where a firm's CSR investment in a monopoly market will dominate when it is in a duopoly and vice versa.

In Chapter 3, we will derive the equilibrium firms' CSR investments, product prices and demand; consumer surplus and social welfare under regulation in a duopoly when strategic CSR is recognized. We will compare with regulator v/s without regulator for the above equilibrium outcomes. We get conditional results for all except consumer surplus, which are given as propositions, remarks and figures. For consumer surplus, with regulator always dominates without regulator. We derive a measure that captures firms' CSR perception from our model. We call it k-value where higher k-value means a higher tendency for the firm to overspend, hence, higher CSR perception. We obtain data from 6431 firms from the CMIE ProwessIQ database and calculate their k-value. We do statistical tests and analyses, which are our original work.

Chapter 2

A Game-Theoretic Model of Strategic and Altruistic CSR Efforts

In this chapter, we will develop a fundamental model, which will be extended in the next chapter, where we will also include a regulator whose action will be taken as an exogenous parameter. Here, we will study how consumers and firms are affected when strategic CSR is not recognized(only altruistic CSR is allowed) and when strategic CSR is recognized. We will see whether strategic and altruistic CSR acts as substitutes or complements. We look into firm(s)' CSR investments, product price, demand, profits; consumer surplus and social welfare in these two scenarios in monopoly and duopoly markets and compare. We will also compare monopoly v/s duopoly to understand how one firm influences another firm's action.

2.1 Model Building

Our Model considers monopoly and duopoly markets with consumers and firm(s) location given by Salop's Circle Model (Salop 1979) and consists of a regulator regulating the market.

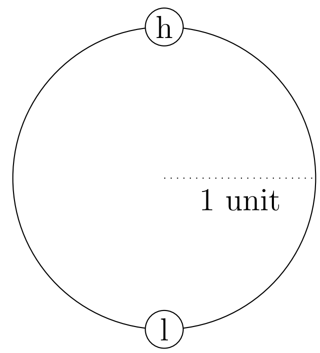


Figure 2.1: Salop Circle

Consider a circle with a radius of 1 unit. Consumers are uniformly distributed along the circle's circumference with length 2π. The 2 firms entering the market will be located equidistant from each other along the circle's circumference. Firms' symmetrically located across the circumference is an equilibrium as per the Salop Model(Salop 1979). WLOG let us consider one firm at the north pole and the other firm at the south pole of the circle. Let us call it h firm and l firm, respectively.

Let y_{1i} be the strategic CSR investment, y_{2i} be the altruistic CSR investment and p_i be the price of the product of the i-th firm. In our primary model, we are not considering with regulator situation. In both with regulator and without regulator, we have a 2-stage game The sequence of events in the market as follows:

1. The firm decides the strategic(y_{1i}) and the altruistic (y_{2i}) CSR investment.
2. The firm decides the price of its product(p_i).

The consumers decide which product to buy when it reaches the market.

2.2 Consumer Utility

When buying the product, consumers care about the firm's brand value and CSR investments. The strategic CSR investments by the firm directly benefit the consumer who will buy their product and the altruistic CSR investment by the firm benefits all. So, even if a consumer buys a product from one firm, the consumer benefits from the altruistic CSR investment made by another firm. Some consumers may care more about altruistic CSR investment than strategic CSR investment and some otherwise, which we will characterize with the parameter γ. The disutility for the consumers will be the price and the accessibility cost of the product.

Let us consider a consumer at a point P, which is at a distance of x unit along the circumference from firm h. Therefore, it is $\pi - x$ distance away from firm l.

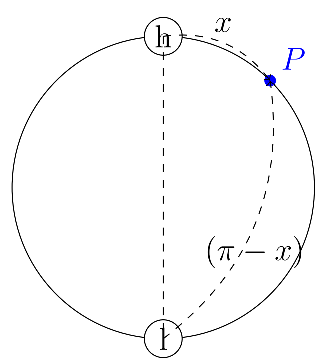


Figure 2.2: A consumer at point P on the circumference of the circle.

The utility of a consumer buying a product from firm h is,

$$u_h = \phi_h + a_h y_{1h} + \gamma(a_h y_{2h} + \theta_h a_l y_{2l}) - p_h - tx. \tag{2.1}$$

The utility of a consumer buying a product from firm l is,

$$u_l = \phi_l + a_l y_{1l} + \gamma(a_l y_{2l} + \theta_l a_h y_{2h}) - p_l - t(\pi - x). \tag{2.2}$$

$\phi_h > \phi_l > 0$, $\theta_i \in [0, 1)$.

Here ϕ_i is the brand value of firm i, a_i is the benefit factor for the consumers due to firm i's CSR investment, and γ is the social concern of consumers(i.e., the weight given to firm's altruistic CSR investment compared to it's strategic CSR investment by the consumers). θ_i is the spillover of other firm's altruistic CSR that benefits firm i. This may be due to the firm i's strategic location/ actions. When visiting a retailer, consumers incur travel costs at a (linear) rate of t per unit distance. These costs can include the opportunity cost of time, the real cost of travel, and the implicit cost of inconvenience (Balasubramanian 1998).

Symbol	Description
z^*	z is in Duopoly when strategic CSR is recognized setting.
$\hat{z}$	z is in Duopoly when strategic CSR is not recognized setting.
$\acute{z}$	z is in Monopoly when strategic CSR is recognized setting.
$\grave{z}$	z is in Monopoly when strategic CSR is not recognized setting.

Table 2.1: Notations used for Equilibrium solutions

2.3 Monopoly

2.3.1 Strategic CSR Is Recognized

First, we will consider a monopoly market where strategic CSR is recognized. Let the firm in the market be a high-type (h) or a low-type (l). Here, we will examine CSR investments, producer surplus, consumer surplus, and social welfare when strategic CSR is recognized in the market. Later, we will compare these indicators with situations where strategic CSR is not recognized.

Consumer utility

Consumer utility is derived from (2.1) or (2.2). Since there is only one firm in this market, the spillover effect is not there in consumers' utility.

$$u_i = \phi_i + a_i(y_{1i} + \gamma y_{2i}) - p_i - tx. \tag{2.3}$$

Firm's utility

$$\pi_i = (p_i - g_i)d_i - c_i(y_{1i}^2 + y_{2i}^2). \tag{2.4}$$

g_i is the manufacturing cost of the product produced by firm i, d_i is the demand of consumers for the product of firm i and c_i is the cost factor of firm i's CSR investment.

Proposition 2.3.1. *If there is no regulator and when* $t > \frac{1}{2}\left(\frac{\phi_i - g_i}{\pi} + \frac{(1+\gamma^2)a_i^2}{c_i}\right)$, *the equilib-*

rium price, demand of the product and the CSR investments made by the firm i are given by:

$$\acute{p}_i = \frac{1}{\left(\frac{2tc_i}{a_i^2} - \gamma^2 - 1\right)} \left[\frac{tc_i}{a_i^2}\phi_i + \left(\frac{tc_i}{a_i^2} - \gamma^2 - 1\right) g_i\right], \quad \acute{d}_i = \frac{2c_i(\phi_i - g_i)}{a_i^2\left(\frac{2tc_i}{a_i^2} - \gamma^2 - 1\right)}, \tag{2.5}$$

$$\acute{y}_{1i} = \frac{\phi_i - g_i}{a_i\left(\frac{2tc_i}{a_i^2} - \gamma^2 - 1\right)}, \quad \acute{y}_{2i} = \frac{\gamma(\phi_i - g_i)}{a_i\left(\frac{2tc_i}{a_i^2} - \gamma^2 - 1\right)}. \tag{2.6}$$

Proof. We use Subgame Perfect Nash Equilibrium(SPNE)(Osborne and Rubinstein 1994) to solve this sequential game. We will use the backward induction technique here. First, let's look at the product demand from the Salop Circle.

Consumers will buy the product iff $u_i \geq 0$. To find the demand function, $u_i = 0$, implies

$$\acute{x} = \frac{1}{t}[\phi_i + a_i(y_{1i} + \gamma y_{2i}) - p_i].$$

The indifferent consumer is at $\acute{x}$, i.e., the consumer is indifferent whether to buy or not buy the product. The consumers above the indifferent consumer will buy the product and those below will not. We have only considered a semi-circle part of the Salop circle. The same trend occurs in the other part of the circle. Hence, we are taking $2\acute{x}$ as the demand for the product. The demand,

$$d_i = 2\acute{x} = \frac{2}{t}[\phi_i + a_i(y_{1i} + \gamma y_{2i}) - p_i].$$

Let's use backward induction. We can start from the last step, where the firm will decide the price of its product.

Solving for the price,

$$\frac{d\pi_i}{dp_i} = 0.$$

We get,

$$p_i = \frac{1}{2}[\phi_i + a_i(y_{1i} + \gamma y_{2i}) + g_i].$$

Second-order condition,

$$\frac{d^2\pi_i}{dp_i^2} = \frac{-4}{t} < 0.$$

Solving for CSR investments simultaneously,

$$\frac{d\pi_i}{dy_{1i}} = 0, \quad \frac{d\pi_i}{dy_{2i}} = 0.$$

We get,

$$y_{1i} = \frac{\phi_i + \gamma y_{2i} - g_i}{\left(\frac{2tc_i}{a_i} - a_i\right)}, \quad y_{2i} = \frac{\phi_i + a_i y_{1i} - g_i}{\left(\frac{2tc_i}{\gamma a_i} - \gamma a_i\right)}. \tag{2.7}$$

Second order condition for the above,

$$t > \frac{(1+\gamma^2)a_i^2}{2c_i}.$$

Solving the equations in (2.7) simultaneously we get the equilibrium investment efforts $\acute{y}_{1i}$ and $\acute{y}_{1i}$. We then update price and demand to get respective equilibrium outcomes. We get equilibrium demand,

$$\acute{d}_i = \frac{2c_i(\phi_i - g_i)}{a_i^2\left(\frac{2tc_i}{a_i^2} - \gamma^2 - 1\right)}.$$

We need demand, $\acute{d}_i \in (0, 2\pi)$. Since $\acute{u}_i \geq 0$, we have $\acute{d}_i > 0$.
$\acute{d}_i < 2\pi$, implies, $\frac{2c_i(\phi_i - g_i)}{a_i^2\left(\frac{2tc_i}{a_i^2} - \gamma^2 - 1\right)} < 2\pi$.

$$\text{Therefore,} \quad t > \frac{1}{2}\left(\frac{\phi_i - g_i}{\pi} + \frac{(1+\gamma^2)a_i^2}{c_i}\right).$$

□

The equilibrium altruistic CSR investment by the firm is γ times its equilibrium strategic CSR investment. If the consumers are more socially concerned $\gamma > 1$, altruistic CSR investment by the company will be more than its strategic CSR investment.

Corollary 2.3.2. *If $a_i = 1$ and $g_i = 0$, there is no regulator and when $t > \frac{1}{2}\left(\frac{\phi_i}{\pi} + \frac{1+\gamma^2}{c_i}\right)$, the equilibrium price, the demand of the product; CSR investments made by the firm; Producer surplus, Consumer surplus and Social welfare; is given by:*

$$\acute{p}_i = \frac{tc_i\phi_i}{2tc_i - (1+\gamma^2)}, \quad \acute{d}_i = \frac{2c_i\phi_i}{2tc_i - (1+\gamma^2)}, \tag{2.8}$$

$$\acute{y}_{1i} = \frac{\phi_i}{2tc_i - (1+\gamma^2)}, \quad \acute{y}_{2i} = \frac{\gamma\phi_i}{2tc_i - (1+\gamma^2)}, \tag{2.9}$$

$$\acute{\pi}_i = \frac{c_i\phi_i^2}{2tc_i - (1+\gamma^2)} \quad \acute{CS} = t\left(\frac{c_i\phi_i}{2tc_i - (1+\gamma^2)}\right)^2, \tag{2.10}$$

$$\acute{SW} = \frac{c_i\phi_i^2}{2tc_i - (1+\gamma^2)}\left(1 + \frac{tc_i}{2tc_i - (1+\gamma^2)}\right). \tag{2.11}$$

Proof. We get the price, demand, CSR investments and constraint for t directly from Proposition 2.3.1 by substituting the values of a_i and g_i.
The profit of the firm is given by,

$$\begin{aligned}\acute{\pi}_i &= \acute{p}_i\acute{d}_i - c_i(\acute{y}_{1i}^2 + \acute{y}_{2i}^2) = \tfrac{1}{(2tc_i-(1+\gamma^2))^2}[2tc_i^2\phi_i^2 - c_i(1+\gamma^2)\phi_i^2] \\ &= \tfrac{c_i\phi_i^2}{2tc_i-(1+\gamma^2)}.\end{aligned}$$

Consumer surplus is given by,

$$\begin{aligned}\acute{CS} &= \textstyle\int_0^{\acute{x}} \acute{u}_i = \int_0^{\acute{x}}(\phi_i + \acute{y}_{1i} + \gamma\acute{y}_{2i} - \acute{p}_i - tx)dx \\ &= (\phi_i + y_{1i} + \gamma\acute{y}_{2i} - \acute{p}_i - \tfrac{t\acute{x}}{2})\acute{x} \\ &= t\left(\tfrac{c_i\phi_i}{2tc_i-(1+\gamma^2)}\right)^2.\end{aligned}$$

Social welfare is the sum of producer surplus and consumer surplus, i.e.,

$$\acute{SW} = \acute{\pi}_i + \acute{CS} = \frac{c_i\phi_i^2}{2tc_i - (1+\gamma^2)} + t\left(\frac{c_i\phi_i}{2tc_i - (1+\gamma^2)}\right)^2$$
$$= \frac{c_i\phi_i^2}{2tc_i - (1+\gamma^2)}\left(1 + \frac{tc_i}{2tc_i - (1+\gamma^2)}\right).$$

□

We can see that an increase in γ benefits all—the consumer and producer surplus increases, which increases social welfare. But when γ increases, the accessibility cost t also increases. So, for a given t, there is an upper bound for γ. When consumers care more about altruistic CSR, the firm increases its altruistic CSR investment. Since the investment has a convex cost, it also increases the strategic CSR investment so that more CSR investment is made with less cost. Consumers also care about strategic CSR and are happy, which is why the demand increases. Now, firms increase the product's price, which gives them more revenue due to higher price and demand. Even though the firm's CSR spending has increased, the profit increases as it has captured the cost in its increased price and demand.

2.3.2 Strategic CSR Is Not Recognized

This section will study monopoly when strategic CSR is not recognized. It would be interesting to examine how consumer surplus is affected here compared to when strategic CSR is recognized. Intuitively, producer surplus here would be less as strategic CSR is not recognized here, directly benefiting the firm.

Consumer utility

$$u_i = \phi_i + \gamma a_i y_{2i} - p_i - tx. \tag{2.12}$$

Firm's utility

$$\pi_i = (p_i - g_i)d_i - c_i y_{2i}^2. \tag{2.13}$$

Proposition 2.3.3. *If there is no regulator and when* $t > \frac{1}{2}\left(\frac{\phi_i - g_i}{\pi} + \frac{(\gamma a_i)^2}{c_i}\right)$, *the equilibrium* *price, demand of the product and the CSR investments made by the firm i are given by:*

$$\grave{p}_i = \frac{1}{2tc_i - (\gamma a_i)^2} \left[tc_i\phi_i + \left(tc_i - (\gamma a_i)^2\right) g_i\right], \tag{2.14}$$

$$\grave{d}_i = \frac{2c_i(\phi_i - g_i)}{2tc_i - (\gamma a_i)^2}, \quad \grave{y}_{2i} = \frac{\gamma a_i(\phi_i - g_i)}{2tc_i - (\gamma a_i)^2}. \tag{2.15}$$

Proof. Consumers will buy the product iff $\grave{u}_i \geq 0$. To find the demand function, $u_i = 0$, gives

$$\grave{x} = \frac{1}{t}[\phi_i + \gamma a_i y_{2i} - p_i].$$

The demand,

$$d_i = 2\grave{x} = \frac{2}{t}[\phi_i + \gamma a_i y_{2i} - p_i].$$

Solving the below equations simultaneously,

$$\frac{d\pi_i}{dp_i} = 0, \quad \frac{d\pi_i}{dy_{2i}} = 0.$$

We have,

$$p_i = \frac{1}{2}[\phi_i + \gamma a_i y_{2i} + g_i], \quad y_{2i} = \frac{(p_i - g_i)\gamma a_i}{c_i}.$$

Solving the above two equations, we get the equilibrium outcomes, and after that, we update the demand.

Second-order conditions,

$$\frac{d^2\pi_i}{dp_i^2} = \frac{-4}{t} < 0.$$

Second order condition for y_{2i},

$$t > \frac{(\gamma a_i)^2}{2c_i}.$$

We get the equilibrium demand,

$$\grave{d}_i = \frac{2c_i(\phi_i - g_i)}{2tc_i - (\gamma a_i)^2}.$$

We need demand, $\grave{d}_i \in (0, 2\pi)$. Since $\grave{u}_i \geq 0$, we have $\grave{d}_i > 0$.
$\grave{d}_i < 2\pi$, implies, $\frac{2c_i(\phi_i - g_i)}{2tc_i - (\gamma a_i)^2} < 2\pi$.
Therefore,

$$t > \frac{1}{2}\left(\frac{\phi_i - g_i}{\pi} + \frac{(\gamma a_i)^2}{c_i}\right).$$

□

The expressions in Proposition 2.3.3 look similar to that of the case where strategic CSR is recognized. The denominator has changed, there is no $-a_i^2$ term. Therefore, the price, demand and altruistic CSR here are smaller than in the previous scenario.

Corollary 2.3.4. *If $a_i = 1$ and $g_i = 0$, there is no regulator and when $t > \frac{1}{2}\left(\frac{\phi_i}{\pi} + \frac{\gamma^2}{c_i}\right)$, the equilibrium price, the demand of the product; CSR investments made by the firm; Producer surplus, Consumer surplus and Social welfare; is given by:*

$$\grave{p}_i = \frac{tc_i\phi_i}{2tc_i - \gamma^2}, \quad \grave{d}_i = \frac{2c_i\phi_i}{2tc_i - \gamma^2}, \quad \grave{y}_{2i} = \frac{\gamma\phi_i}{2tc_i - \gamma^2}, \tag{2.16}$$

$$\grave{\pi}_i = \frac{c_i\phi_i^2}{2tc_i - \gamma^2}, \quad \grave{CS} = t\left(\frac{c_i\phi_i}{2tc_i - \gamma^2}\right)^2, \tag{2.17}$$

$$\grave{SW} = \frac{c_i\phi_i^2}{2tc_i - \gamma^2}\left(1 + \frac{tc_i}{2tc_i - \gamma^2}\right). \tag{2.18}$$

Proof. We get the price, demand, altruistic CSR investment and constraint for t directly from Proposition 2.3.3 by substituting values for a_i and g_i.
The profit of the firm is given by,

$$\begin{aligned}\grave{\pi}_i &= \grave{p}_i\grave{d}_i - c_i\grave{y}_{2i}^2 = \tfrac{1}{(2tc_i - \gamma^2)^2}[2tc_i^2\phi_i^2 - c_i\gamma^2\phi_i^2] \\ &= \tfrac{c_i\phi_i^2}{2tc_i - \gamma^2}.\end{aligned}$$

Consumer surplus is given by,

$$\begin{aligned}\grave{CS} &= \int_0^{\grave{x}} \grave{u}_i = \int_0^{\grave{x}} (\phi_i + \gamma \grave{y}_{2i} - \grave{p}_i - tx)dx \\ &= (\phi_i + \gamma \grave{y}_{2i} - \grave{p}_i - \tfrac{t\grave{x}}{2})\grave{x} \\ &= t\left(\tfrac{c_i\phi_i}{2tc_i-\gamma^2}\right)^2.\end{aligned}$$

Social welfare is the sum of producer surplus and consumer surplus, i.e.,

$$\begin{aligned}\grave{SW} = \grave{\pi}_i + \grave{CS} &= \frac{c_i\phi_i^2}{2tc_i-\gamma^2} + t\left(\frac{c_i\phi_i}{2tc_i-\gamma^2}\right)^2 \\ &= \frac{c_i\phi_i^2}{2tc_i-\gamma^2}\left(1+\frac{tc_i}{2tc_i-\gamma^2}\right).\end{aligned}$$

□

2.3.3 Comparisons: Strategic CSR is recognized v/s Not recognized

For feasible solutions in this subsection, we need $t > t_1 = \frac{1}{2}\left(\frac{\phi_i}{\pi} + \frac{1+\gamma^2}{c_i}\right)$.

Proposition 2.3.5 (Altruistic CSR investment)**.** *The altruistic CSR investment of a firm when strategic CSR is recognized is always greater than its altruistic CSR investment when strategic CSR is not recognized.*

$$\acute{y}_{2i} > \grave{y}_{2i}. \tag{2.19}$$

Proof.

$$\grave{y}_{2i} = \frac{\gamma a_i[\phi_i - g_i]}{2tc_i - (\gamma a_i)^2}, \quad \acute{y}_{2i} = \frac{\gamma a_i[\phi_i - g_i]}{2tc_i - (1+\gamma^2)a_i^2}.$$

Comparing $\grave{y}_{2i}$ and $\acute{y}_{2i}$, $2tc_i - (\gamma a_i)^2 - a_i^2 < 2tc_i - (\gamma a_i)^2$. Therefore, $\acute{y}_{2i} > \grave{y}_{2i}$. □

Recognizing strategic CSR is better in a monopoly when we compare the outcome of altruistic CSR investments! Due to the convex costs for CSR, the firm is better off investing in both CSRs rather than one. So, for a given cost of expenditure, a firm can have more CSR investment levels when strategic CSR is recognized v/s not recognized.

Corollary 2.3.6 (Total CSR investment)**.** *The total CSR investment made by a firm when strategic CSR is recognized is always greater than its total CSR investment when strategic CSR is not recognized.*

Proof. From the Proposition 2.3.5 we have, $\acute{y}_{2i} > \grave{y}_{2i}$. Total CSR investment when strategic CSR is recognized is $\acute{y}_{1i} + \acute{y}_{2i}$. Total CSR investment when strategic CSR is not recognized is $\grave{y}_{2i}$. Therefore, $\acute{y}_{1i} + \acute{y}_{2i} > \grave{y}_{2i}$. □

Proposition 2.3.7 (PS, CS, SW)**.** *The Producer Surplus, the Consumer Surplus, and the Social Welfare when strategic CSR is recognized is always greater than the respective when strategic CSR is not recognized.*

$$\acute{\pi}_i > \grave{\pi}_i, \quad \acute{CS} > \grave{CS}, \quad \acute{SW} > \grave{SW}. \tag{2.20}$$

Proof. Producer surplus,

$$\acute{\pi}_i = \frac{c_i \phi_i^2}{2tc_i - (1+\gamma^2)}, \quad \grave{\pi}_i = \frac{c_i \phi_i^2}{2tc_i - \gamma^2}.$$

As $2tc_i - (1+\gamma^2) < 2tc_i - \gamma^2$. We always have,

$$\acute{\pi}_i > \grave{\pi}_i. \tag{2.21}$$

Consumer Surplus,

$$\acute{CS} = t\left(\frac{c_i \phi_i}{2tc_i - (1+\gamma^2)}\right)^2, \quad \grave{CS} = t\left(\frac{c_h \phi_i}{2tc_i - \gamma^2}\right)^2.$$

As $2tc_i - (1+\gamma^2) < 2tc_i - \gamma^2$. We always have,

$$\acute{CS} > \grave{CS}. \tag{2.22}$$

$$\acute{SW} = \acute{\pi}_i + \acute{CS}, \quad \grave{SW} = \grave{\pi}_i + \grave{CS}.$$

From 2.21 and 2.22, we have, $\acute{SW} > \grave{SW}$.

□

The firm invests more in CSR at the same cost when strategic CSR is recognized v/s not recognized. Due to this, it can increase the product's price as there is an increased demand for the firm when strategic CSR is recognized compared to not recognized. The revenue is greater when strategic CSR is recognized v/s not recognized for the same CSR expenditure. Therefore, the firm's profit when CSR is recognized will be greater than when CSR is not recognized.

We know that the altruistic CSR when strategic CSR is recognized, is more than the altruistic CSR when strategic CSR is not recognized. Also, the firm has proportionately invested in strategic CSR with altruistic CSR when strategic CSR is recognized. So overall, consumers are very happy with CSR investment of the firm when strategic CSR is recognized v/s not recognized. Even though the product's price is higher when strategic CSR is recognized v/s not recognized, consumers are ok with it. Since, with a slightly higher price, the firm has invested much more in CSR when strategic CSR is recognized v/s not recognized.

The firm is more profitable when strategic CSR is recognized. Consumers are better off when strategic CSR is recognized. Hence, everyone is happier when strategic CSR is recognized v/s not recognized.

2.4 Duopoly

Now, we will see what happens in a duopoly when there is a high-type and a low-type firm where both are profit maximizers. Here, we could study how a competing firm influences the strategies of a firm, which we didn't see in a monopoly.

Symbol	Description
a_i	Benefit factor due to firm i's CSR investment.
c_i	Cost factor of firm i's CSR investment.
γ	The social concern of consumers
θ_i	The spillover of other firm's altruistic CSR that benefits firm i, $\theta_i \in [0, 1)$.
t	Accessibility cost of the product per unit distance for consumers.
d_i	Demand of consumers for buying a product from firm i.
π_i	Profit of firm i.
ϕ_i	Brand value of firm i.
g_i	Manufacturing cost of product of firm i.

Table 2.2: Symbols used

2.4.1 Strategic CSR Is Recognized

Consumers' utility is given by (2.1) and (2.2).

Firms' utility

$$\pi_h = (p_h - g_h)d_h - c_h(y_{1h}^2 + y_{2h}^2), \tag{2.23}$$

$$\pi_l = (p_l - g_l)d_l - c_l(y_{1l}^2 + y_{2l}^2). \tag{2.24}$$

We are taking the CSR costs to be separable, i.e., $y_{1h}^2 + y_{2h}^2$, as the costs correspond to the type of infrastructural investments that are separable, verifiable, and contractible. We don't assume a specific relationship between c_h and c_l such that either $c_h > c_l$ or $c_h \leq c_l$. This is because, for example, (Klapper et al. (2012),p. 9) observe (in their data set) that smaller firms (e.g., suppliers) are unlikely to have access to cheaper financing than bigger firms (e.g., buyers).

Lemma 2.4.1. *The equilibrium product prices of firm h and l when strategic CSR is allowed* *and there is no regulator is as follows:*

$$p_h(y_{1h}, y_{2h}, y_{1l}, y_{2l}) = \frac{1}{3}[\phi_h - \phi_l + a_h y_{1h} - a_l y_{1l} + \gamma[(1-\theta_l)a_h y_{2h} - (1-\theta_h)a_l y_{2l}] + g_h + g_l + 3t\pi], \tag{2.25}$$

$$p_l(y_{1h}, y_{2h}, y_{1l}, y_{2l}) = \frac{1}{3}[-(\phi_h - \phi_l) + a_l y_{1l} - a_h y_{1h} + \gamma[(1-\theta_h)a_l y_{2l} - (1-\theta_l)a_h y_{2h}] + g_h + g_l + 3t\pi]. \tag{2.26}$$

Proof. Let's find the demand for the product from Salop Circle.

Consider an indifferent consumer at x^*.

$$u_h(x^*) = u_l(x^*),$$

$$\phi_h + a_h y_{1h} + \gamma(a_h y_{2h} + \theta_h a_l y_{2l}) - p_h - tx^* = \phi_l + a_l y_{1l} + \gamma(a_l y_{2l} + \theta_l a_h y_{2h}) - p_l - t(\pi - x^*).$$

Demand functions,

$$d_h = 2x^* = \frac{1}{t}[\phi_h - \phi_l + a_h y_{1h} - a_l y_{1l} + \gamma[(1-\theta_l)a_h y_{2h} - (1-\theta_h)a_l y_{2l}] + p_l - p_h + t\pi], \tag{2.27}$$

$$d_l = 2\pi - 2x^* = \frac{1}{t}[-(\phi_h - \phi_l) + a_l y_{1l} - a_h y_{1h} + \gamma[(1-\theta_h)a_l y_{2l} - (1-\theta_l)a_h y_{2h}] + p_h - p_l + t\pi]. \tag{2.28}$$

Let us assume $d_h \in (0, 2\pi)$ and $u_h(x^*) > 0$. Later, we will express the feasible region in terms of t.

The backward induction technique is used to find this game's Subgame Perfect Nash Equilibrium(SPNE). The last stage of the game is the firm's deciding its product's price.

We have to find p_h and p_l that maximize π_h and π_l respectively.

$$\frac{d\pi_h}{dp_h} = d_h + (p_h - g_h)\frac{dd_h}{dp_h} = 0.$$

$$p_h = \frac{1}{2}[\phi_h - \phi_l + a_h y_{1h} - a_l y_{1l} + \gamma[(1-\theta_l)a_h y_{2h} - (1-\theta_h)a_l y_{2l}] + g_h + p_l + t\pi]. \tag{2.29}$$

$$\frac{d^2\pi_h}{dp_h^2} = -2/t < 0, \text{ as } t > 0.$$

Therefore, p_h maximizes π_h. Similarly,

$$p_l = \frac{1}{2}[-(\phi_h - \phi_l) + a_l y_{1l} - a_h y_{1h} + \gamma[(1-\theta_h)a_l y_{2l} - (1-\theta_l)a_h y_{2h}] + g_l + p_h + t\pi]. \quad (2.30)$$

Substituting p_l in p_h and vice-versa, we get, equilibrium prices $p_h(y_{1h}, y_{2h}, y_{1l}, y_{2l})$ and $p_l(y_{1h}, y_{2h}, y_{1l}, y_{2l})$.

□

Moving onto the next stage of backward induction, we get the equilibrium CSR investment efforts.

Proposition 2.4.2. *If there is no regulator and when* $t > \frac{1}{9}\left[\sum_{i\in\{h,l\}}(1+[\gamma(1-\theta_{-i})]^2)\frac{a_i^2}{c_i}\right]$ *, the equilibrium CSR investments made by firms h and l are given by:*

$$y_{1h}^* = \frac{1}{9ta_l\left[\frac{9tc_hc_l}{a_ha_l} - \sum_{i\in\{h,l\}}(1+[\gamma(1-\theta_i)]^2)\frac{c_ia_{-i}}{a_i}\right]}\left[9tc_l(\phi_h-\phi_l) - (18tc_l - a_l^2(1+[\gamma(1-\theta_h)]^2))\frac{g_h}{2} + (9tc_l + a_l^2(1+[\gamma(1-\theta_h)]^2))\frac{g_l}{2} + (9tc_l - 2a_l^2(1+[\gamma(1-\theta_h)]^2))3t\pi\right], \quad (2.31)$$

$$y_{1l}^* = \frac{1}{9ta_h\left[\frac{9tc_hc_l}{a_ha_l} - \sum_{i\in\{h,l\}}(1+[\gamma(1-\theta_i)]^2)\frac{c_ia_{-i}}{a_i}\right]}\left[-9tc_h(\phi_h-\phi_l) - (18tc_h - a_h^2(1+[\gamma(1-\theta_l)]^2))\frac{g_l}{2} + (9tc_h + a_h^2(1+[\gamma(1-\theta_l)]^2))\frac{g_h}{2} + (9tc_h - 2a_h^2(1+[\gamma(1-\theta_l)]^2))3t\pi\right], \quad (2.32)$$

$$y_{2h}^* = \gamma(1-\theta_l)y_{1h}^*, \quad y_{2l}^* = \gamma(1-\theta_h)y_{1l}^*. \quad (2.33)$$

Proof. We need $p_h - p_l$ which we get from Lemma 2.4.1,

$$p_h - p_l = \frac{2}{3}[\phi_h - \phi_l + a_h y_{1h} - a_l y_{1l} + \gamma[(1-\theta_l)a_h y_{2h} - (1-\theta_h)a_l y_{2l}]].$$

Updating the demand functions,

$$d_h = \frac{1}{3t}[\phi_h - \phi_l + a_h y_{1h} - a_l y_{1l} + \gamma[(1-\theta_l)a_h y_{2h} - (1-\theta_h)a_l y_{2l}] + 3t\pi], \tag{2.34}$$

$$d_l = \frac{1}{3t}[-(\phi_h - \phi_l) + a_l y_{1l} - a_h y_{1h} + \gamma[(1-\theta_h)a_l y_{2l} - (1-\theta_l)a_h y_{2h}] + 3t\pi]. \tag{2.35}$$

Moving onto the next stage of the backward induction, i.e., we have to find optimum y_{1h}, y_{1l}, y_{2h}, and y_{2l}.

We have to solve 4 equations simultaneously, i.e.,

$$\frac{d\pi_h}{dy_{1h}} = 0, \quad \frac{d\pi_l}{dy_{1l}} = 0, \tag{2.36}$$

$$\frac{d\pi_h}{dy_{2h}} = 0, \quad \frac{d\pi_l}{dy_{2l}} = 0. \tag{2.37}$$

Consider (2.36),

$$y_{1h} = \frac{1}{\frac{9tc_h}{a_h} - a_h}[\phi_h - \phi_l - a_l y_{1l} + \gamma[(1-\theta_l)a_h y_{2h} - (1-\theta_h)a_l y_{2l}] - g_h + \frac{g_l}{2} + 3t\pi]. \tag{2.38}$$

The second-order condition:

$$\frac{d^2\pi_h}{dy_{1h}^2} = \frac{2a_h^2}{9t} - 2c_h < 0, \quad \text{implies,} \quad t > \frac{a_h^2}{9c_h}.$$

Similarly,

$$y_{1l} = \frac{1}{\frac{9tc_l}{a_l} - a_l}[-(\phi_h - \phi_l) - a_h y_{1h} + \gamma[(1-\theta_h)a_l y_{2l} - (1-\theta_l)a_h y_{2h}] - g_l + \frac{g_h}{2} + 3t\pi]. \tag{2.39}$$

The second-order condition:

$$\frac{d^2\pi_l}{dy_{1l}^2} = \frac{2a_l^2}{9t} - 2c_l < 0 \quad \text{implies}, t > \frac{a_l^2}{9c_l}.$$

From (2.37),

$$y_{2h} = \frac{1}{(\frac{9tc_h}{\gamma(1-\theta_l)a_h} - \gamma(1-\theta_l)a_h)}[\phi_h - \phi_l + a_h y_{1h} - a_l y_{1l} - \gamma(1-\theta_h)a_l y_{2l} - g_h + \frac{g_l}{2} + 3t\pi]. \quad (2.40)$$

Second-order condition,

$$\frac{d^2\pi_h}{dy_{1h}^2} = \frac{2[\gamma(1-\theta_l)a_h]^2}{9t} - 2c_h < 0,$$

$$\text{gives}, \quad t > \frac{[\gamma(1-\theta_l)a_h]^2}{9c_h},$$

$$\text{implies}, \quad \frac{9tc_h}{\gamma(1-\theta_l)a_h} > \gamma(1-\theta_l)a_h.$$

Hence, the denominator of y_{2h} is positive.

$$y_{2l} = \frac{1}{(\frac{9tc_l}{\gamma(1-\theta_h)a_l} - \gamma(1-\theta_h)a_l)}[-(\phi_h - \phi_l) + a_l y_{1l} - a_h y_{1h} - \gamma(1-\theta_l)a_h y_{2h} - g_l + \frac{g_h}{2} + 3t\pi]. \quad (2.41)$$

Second-order condition,

$$\frac{d^2\pi_l}{dy_{1l}^2} = \frac{2[\gamma(1-\theta_h)a_l]^2}{9t} - 2c_l < 0, \quad \text{implies}, t > \frac{[\gamma(1-\theta_h)a_l]^2}{9c_l}.$$

Hence, the denominator of y_{2l} is positive.

Let $m = \frac{9tc_h}{a_h}$ and $n = \frac{9tc_l}{a_l}$.

Substituting 2.39 in 2.38,

$$y_{1h} = \frac{1}{mn - a_h n - a_l m}\Big[n\Big(\phi_h - \phi_l + \gamma[(1-\theta_l)a_h y_{2h} - (1-\theta_h)a_l y_{2l}]\Big) - (n - \frac{a_l}{2})g_h + (\frac{n + a_l}{2})g_l + (n - 2a_l)3t\pi\Big], \quad (2.42)$$

$$y_{1l} = \frac{1}{mn - a_h n - a_l m}\Big[m\Big(-(\phi_h - \phi_l) + \gamma[(1-\theta_h)a_l y_{2l} - (1-\theta_l)a_h y_{2h}]\Big) - (m - \frac{a_h}{2})g_l + (\frac{m + a_h)}{2})g_h + (m - 2a_h)3t\pi\Big]. \tag{2.43}$$

Let $r = mn - a_h n - a_l m = 9t\left[\frac{9tc_h c_l}{a_h a_l} - \frac{c_l a_h}{a_l} - \frac{c_h a_l}{a_h}\right]$

$$a_h y_{1h} - a_l y_{1l} = \frac{1}{r}\Big[(a_h n + a_l m)\Big(\phi_h - \phi_l + \gamma[(1-\theta_l)a_h y_{2h} - (1-\theta_h)a_l y_{2l}]\Big) - (a_h n + \frac{a_l m}{2})g_h + (a_l m + \frac{a_h n}{2})g_l + (a_h n - a_l m)3t\pi\Big].$$

We need $r > 0$, which gives , $t > \frac{a_h^2}{9c_h} + \frac{a_l^2}{9c_l}$.

$$\begin{aligned} v &= \frac{r}{\gamma^2(1-\theta_l)(1-\theta_h)} - [n(\frac{1-\theta_l}{1-\theta_h})a_h + m(\frac{1-\theta_h}{1-\theta_l})a_l] \\ &= \frac{9t}{\gamma^2(1-\theta_l)(1-\theta_h)}\Big[\frac{9tc_h c_l}{a_h a_l} - \sum_{i\in\{h,l\}}(1 + [\gamma(1-\theta_i)]^2)\frac{c_i a_{-i}}{a_i}\Big]. \end{aligned}$$

We need $v > 0$, which gives

$$t > \frac{1}{9}\left[\sum_{i\in\{h,l\}}(1 + [\gamma(1-\theta_{-i})]^2)\frac{a_i^2}{c_i}\right].$$

We have the altruistic CSR investments by both firms,

$$y_{2h}^* = \frac{1}{v}\left[\frac{n(\phi_h - \phi_l)}{\gamma(1-\theta_h)} + \frac{1}{2}\left(\gamma(1-\theta_h)a_l - \frac{2n - a_l}{\gamma(1-\theta_h)}\right)g_h + \frac{1}{2}\left(\gamma(1-\theta_h)a_l + \frac{n + a_l}{\gamma(1-\theta_h)}\right)g_l + \left(\frac{n - 2a_l}{\gamma(1-\theta_h)} - 2\gamma(1-\theta_h)a_l\right)3t\pi\right],$$

$$y_{2l}^* = \frac{1}{v}\left[\frac{m(\phi_l - \phi_h)}{\gamma(1-\theta_l)} + \frac{1}{2}\left(\gamma(1-\theta_l)a_h - \frac{2m - a_h}{\gamma(1-\theta_l)}\right) g_l + \frac{1}{2}\left(\gamma(1-\theta_l)a_h + \frac{m + a_h}{\gamma(1-\theta_l)}\right) g_h \right.$$
$$\left. + \left(\frac{m - 2a_h}{\gamma(1-\theta_l)} - 2\gamma(1-\theta_l)a_h\right) 3t\pi\right].$$

To calculate y_{1h}^* and y_{1l}^* we need,

$$\gamma[(1-\theta_l)a_h y_{2h}^* - (1-\theta_h)a_l y_{2l}^*] = \frac{1}{v}\left[\left(\left(\frac{1-\theta_l}{1-\theta_h}\right) a_h n + \left(\frac{1-\theta_h}{1-\theta_l}\right) a_l m\right)(\phi_h - \phi_l)\right.$$
$$-\left(\left(\frac{1-\theta_l}{1-\theta_h}\right) a_h(2n - a_l) + \left(\frac{1-\theta_h}{1-\theta_l}\right) a_l(m + a_h)\right)\frac{g_h}{2}$$
$$+\left(\left(\frac{1-\theta_l}{1-\theta_h}\right) a_h(n + a_l) + \left(\frac{1-\theta_h}{1-\theta_l}\right) a_l(2m - a_h)\right)\frac{g_l}{2}$$
$$+\left(\left(\frac{1-\theta_l}{1-\theta_h}\right) a_h(n - 2a_l) - \left(\frac{1-\theta_h}{1-\theta_l}\right) a_l(m - 2a_h)\right)3t\pi.$$

Using the above expression we get y_{1h}^* and y_{1l}^*.

□

The above proposition gives the CSR investments the high- and low-type firms make when competing with each other. The spillover factor θ_i where $i \in \{h, l\}$ significantly influences altruistic CSR investments made by firms. For instance, y_{2h}^* is $\gamma(1-\theta_l)y_{1h}^*$. So when θ_l tends to 1, i.e. almost all of the altruistic CSR investment made by h benefits l and the consumers who buy products from l, due to this y_{2h}^* tends to 0 even if y_{2h}^* fully benefits h.

We have the solutions for the last stage of the game, i.e., the equilibrium product prices by substituting equilibrium CSR outcomes from Proposition 2.4.2 in Lemma 2.4.1.

$$p_h^* = \frac{3tc_h}{a_h} y_{1h}^*, \quad p_l^* = \frac{3tc_l}{a_l} y_{1l}^*. \tag{2.44}$$

.

Let's see how Consumer Surplus for a duopoly is calculated, which can be used to understand how consumers are affected in the analysis part.

Consumer Surplus

The Consumer surplus for the duopoly, when strategic CSR is allowed, is,

Let $CS_i = u_i$.

$$CS = 2\left[\int_0^{x^*} CS_h + \int_{x^*}^{\pi} CS_l\right],$$

$$\begin{aligned} CS = 2\bigg[&\int_0^{x^*} (\phi_h + a_h y_{1h} + \gamma(a_h y_{2h} + \theta_h a_l y_{2l}) - p_h - tx)dx \\ &+ \int_{x^*}^{\pi} (\phi_l + a_l y_{1l} + \gamma(a_l y_{2l} + \theta_l a_h y_{2h}) - p_l - t(\pi - x))dx \bigg]. \end{aligned}$$

Implies,

$$\begin{aligned} CS = 2\bigg[&\left(\phi_h + a_h y_{1h} + \gamma(a_h y_{2h} + \theta_h a_l y_{2l}) - p_h - \frac{tx^*}{2}\right) x^* \\ &+ \left(\phi_l + a_l y_{1l} + \gamma(a_l y_{2l} + \theta_l a_h y_{2h}) - p_l - \frac{t(\pi - x^*)}{2}\right)(\pi - x^*) \bigg]. \end{aligned} \tag{2.45}$$

where x^* is the position of the indifferent consumer.

Corollary 2.4.3. *If $a_h = a_l = 1$, $\theta_h = \theta_l = \theta$ and $g_h = g_l = 0$, there is no regulator and when $t \in (max\{t_3, t_4, t_5, t_6\}, t_7)$ and $t_7 > max\{t_3, t_4, t_5, t_6\}$, the equilibrium price, the demand for the product, CSR investments made by the firm, and the Producer surplus are given by:*

CSR outcomes,

$$y_{1h}^* = \frac{9tc_l(\phi_h - \phi_l) + \left(9tc_l - 2(1 + [\gamma(1-\theta)]^2)\right)3t\pi}{9t\left(9tc_h c_l - (1 + [\gamma(1-\theta)]^2)(c_h + c_l)\right)}, \tag{2.46}$$

$$y^*_{1l} = \frac{-9tc_h(\phi_h - \phi_l) + \Big(9tc_h - 2(1 + [\gamma(1-\theta)]^2)\Big)3t\pi}{9t\left(9tc_hc_l - (1 + [\gamma(1-\theta)]^2)(c_h + c_l)\right)}, \tag{2.47}$$

$$y^*_{2h} = \gamma(1-\theta)y^*_{1h}, \quad y^*_{2l} = \gamma(1-\theta)y^*_{1l}. \tag{2.48}$$

Prices,

$$p^*_h = 3tc_hy^*_{1h}, \quad p^*_l = 3tc_ly^*_{1l}. \tag{2.49}$$

Demands,

$$d^*_h = 3c_hy^*_{1h}, \quad d^*_l = 3c_ly^*_{1l}. \tag{2.50}$$

Profits,

$$\pi^*_h = \left(9tc_h - (1 + [\gamma(1-\theta)]^2)\right)c_hy^{*2}_{1h}, \quad \pi^*_l = \left(9tc_l - (1 + [\gamma(1-\theta)]^2)\right)c_ly^{*2}_{1l}. \tag{2.51}$$

Proof. We get the CSR investments directly from Proposition 2.4.2 by substituting values for g_i, a_i and θ_i, $i \in \{h, l\}$.
We get equilibrium prices and demands by substituting these equilibrium CSR investments in 2.25, 2.26, 2.34 and 2.35.
Profits of firms are given by

$$\begin{aligned}\pi^*_h &= p^*_hd^*_h - c_h(y^{*2}_{1h} + y^{*2}_{2h}) = 3tc_hy^*_{1h} * 3c_hy^*_{1h} - c_h(1+\gamma^2)y^{*2}_{1h} \\ &= \left(9tc_h - (1 + [\gamma(1-\theta)]^2)\right)c_hy^{*2}_{1h}.\end{aligned}$$

$$\begin{aligned}\pi^*_l &= p^*_ld^*_l - c_l(y^{*2}_{1l} + y^{*2}_{2l}) = 3tc_ly_{1l} * 3c_ly_{1l} - c_l(1+\gamma^2)y^{*2}_{1l} \\ &= \left(9tc_l - (1 + [\gamma(1-\theta)]^2)\right)c_ly^{*2}_{1l}.\end{aligned}$$

Finding feasibility region,
We need $d^*_h \in (0, 2\pi)$

$d_h^* > 0$ gives,

$$t > \frac{-(\phi_h - \phi_l)}{3\pi} + \frac{2(1 + [\gamma(1-\theta)]^2)}{9c_l}.$$

$d_h^* < 2\pi$ gives,

$$t > \frac{\phi_h - \phi_l}{3\pi} + \frac{2(1 + [\gamma(1-\theta)]^2)}{9c_h}.$$

We need $u_h^*(x^*) > 0$ gives,

$$t > \frac{1}{18\pi c_h c_l}\Bigg[2\pi(1+\gamma^2(1-\theta))(c_h+c_l) + 3c_h c_l(\phi_h+\phi_l) - \sqrt{\begin{array}{l}(2\pi(1+\gamma^2(1-\theta))(c_h+c_l)+3c_h c_l(\phi_h+\phi_l))^2 \\ -8\pi c_h c_l\left[2\pi\left(1+\gamma^2(1-\theta)(2+[\gamma(1-\theta)]^2(1+\theta)\right)+3\sum_{i\in\{h,l\}} c_i[\gamma^2(1-\theta)(\phi_i-\theta\phi_{-i})+\phi_i]\right]\end{array}}\Bigg],$$

$$t < \frac{1}{18\pi c_h c_l}\Bigg[2\pi(1+\gamma^2(1-\theta))(c_h+c_l) + 3c_h c_l(\phi_h+\phi_l) + \sqrt{\begin{array}{l}(2\pi(1+\gamma^2(1-\theta))(c_h+c_l)+3c_h c_l(\phi_h+\phi_l))^2 \\ -8\pi c_h c_l\left[2\pi\left(1+\gamma^2(1-\theta)(2+[\gamma(1-\theta)]^2(1+\theta)\right)+3\sum_{i\in\{h,l\}} c_i[\gamma^2(1-\theta)(\phi_i-\theta\phi_{-i})+\phi_i]\right]\end{array}}\Bigg].$$

□

Altruistic CSR investments made by both firms are $\gamma(1-\theta)$ times its' strategic CSR investments. Considering the utility of a consumer who buys the product from a firm i: for 1 unit of strategic CSR investment made by the firm, it gets 1 unit of profit in return as the consumer gets 1 unit of benefit. For 1 unit of altruistic CSR investment made by firm i, consumers who buy a product from i get γ units as benefits. But, a consumer buying from another firm $(-i)$ also benefits from the altruistic CSR spent by firm i, which is $\gamma\theta$ unit of altruistic CSR in magnitude. For 1 unit of altruistic CSR investment made by i, it gets $\gamma(1-\theta)$ unit in return.

The CSR investment costs are convex, so investing only in 1 type of CSR, which gives more return per unit investment, is not a good strategy for i. For profit maximization, marginal cost should equal marginal return. Firm i gets 1 unit return in strategic CSR and

$\gamma(1-\theta)$ unit of return in altruistic CSR for 1 unit of investments made. When we take the derivative of the cost function w.r.t CSR investments, we get a linear equation of CSR investments, i.e., the marginal costs are some constant times CSR investments. We need marginal return equals marginal cost. Therefore, the investments made by the firm will be, for y^*_{1h} unit of strategic CSR invested by firm i, it will invest $\gamma(1-\theta)y^*_{1h}$ unit of altruistic CSR.

Let's see how a change in the model parameters affects the prices, demands and CSR investments of firms as given by Corollary 2.4.3.

First, consider the brand value difference ($\Delta\phi$). An increase in $\Delta\phi$ benefits h and is bad for l. Consumers care about the firm's brand value while buying their product and our model assumes that the brand value of h is always greater than that of l. So when the difference in brand value increases, the demand for product of h will increase and hence, the revenue of h will increase. With more revenue, h can invest more in CSR. Consumers care about firms' CSR investment, further increasing demand. With increased CSR investment, h can further increase the price. Cumulatively, this will increase the profit of h. For l, an increase in brand value difference decreases their demand among consumers, which will force them to reduce price. Due to decreased revenue, their CSR investment will decrease; hence, the price and demand will decrease again. Therefore, its profit decreases.

When γ increases, consumers care more about altruistic CSR. Hence, altruistic CSR investment of both h and l increases. Let's see how prices, demands and strategic CSR investments are affected when γ and θ change. The price and demand are some constant times strategic CSR investments. So, in the proposition below, we only consider strategic CSR.

Proposition 2.4.4. *1. If* $\Delta\phi \leq \frac{\Delta c}{c_h+c_l}3t\pi$,

- y^*_{1h} *decreases with* γ *and increases with* θ.
- y^*_{1l} *increases with* γ *and decreases with* θ.

2. If $\Delta\phi > \frac{\Delta c}{c_h+c_l}3t\pi$,

- y^*_{1h} *increases with* γ *and decreases with* θ.
- y^*_{1l} *decreases with* γ *and increases with* θ.

Proof.

$$\frac{dy_{1h}^*}{d\gamma} = \frac{2\gamma(1-\theta)^2 c_l[(c_h+c_l)\Delta\phi - \Delta c(3t\pi)]}{(9tc_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l))^2}, \quad \frac{dy_{1l}^*}{d\gamma} = -\frac{2\gamma(1-\theta)^2 c_h[(c_h+c_l)\Delta\phi - \Delta c(3t\pi)]}{(9tc_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l))^2},$$

$$\frac{dy_{1h}^*}{d\theta} = -\frac{2\gamma^2(1-\theta)c_l[(c_h+c_l)\Delta\phi - \Delta c(3t\pi)]}{(9tc_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l))^2}, \quad \frac{dy_{1l}^*}{d\theta} = \frac{2\gamma^2(1-\theta)c_h[(c_h+c_l)\Delta\phi - \Delta c(3t\pi)]}{(9tc_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l))^2}.$$

From above, we can see that $[(c_h + c_l)\Delta\phi - \Delta c(3t\pi)]$ is the sign determining factor of these derivatives. □

Let's consider the market region: $\Delta\phi \leq \frac{\Delta c}{c_h+c_l}3t\pi$, i.e., the brand value difference is relatively low compared to the cost difference and h is inefficient in spending CSR. This scenario favors l more than h. Hence, we got the result: when γ increases, the price, demand and strategic CSR investment of h decreases. For l, the opposite happens. As both firms are profit maximizers, the spillover factor θ negatively relates to firms' altruistic CSR investments. The above proposition shows that θ has the opposite behavior of γ. The spillover is an externality caused by the other firm. This highlights the need for a regulator when the spillovers are high, as it will tend profit-maximizing firms to invest less in altruistic CSR.

Consider the market region: $\Delta\phi > \frac{\Delta c}{c_h+c_l}3t\pi$. This situation can arise if: (i) h is efficient in spending CSR or (ii) h is inefficient in CSR spending and the brand value difference is relatively high compared to the cost difference in CSR expenditure(Δc). This scenario favors h more than l. Therefore, when γ increases, the price, demand and strategic CSR investment of h increases. For l, the opposite happens.

2.4.2 Strategic CSR Is Not Recognized

The regulator not recognizing strategic CSR may be a favorable situation for l. Firms can only invest in altruistic CSR as strategic CSR is not recognized. Intuitively, most of the time, h invests more in CSR as it has an inherent advantage of its higher brand value, which it will use for higher CSR investments. As altruistic CSR has this spillover component, h's

CSR investment can benefit both firms.

Consumer utility

$$u_h = \phi_h + \gamma(a_h y_{2h} + \theta_h a_l y_{2l}) - p_h - tx, \tag{2.52}$$

$$u_l = \phi_l + \gamma(a_l y_{2l} + \theta_l a_h y_{2h}) - p_l - t(\pi - x). \tag{2.53}$$

Firms' utility

$$\pi_h = (p_h - g_h)d_h - c_h y_{2h}^2, \tag{2.54}$$

$$\pi_l = (p_l - g_l)d_l - c_l y_{2l}^2. \tag{2.55}$$

Lemma 2.4.5. *The equilibrium product prices of firm h and l when strategic CSR is allowed and there is no regulator in a duopoly are as follows:*

$$p_h(y_{2h}, y_{2l}) = \frac{1}{3}[\phi_h - \phi_l + \gamma[(1-\theta_l)a_h y_{2h} - (1-\theta_h)a_l y_{2l}] + g_h + g_l + 3t\pi], \tag{2.56}$$

$$p_l(y_{2h}, y_{2l}) = \frac{1}{3}[-(\phi_h - \phi_l) + \gamma[(1-\theta_h)a_l y_{2l} - (1-\theta_l)a_h y_{2h}] + g_h + g_l + 3t\pi]. \tag{2.57}$$

Proof. Let's first find out the product demand from Salop Circle.

Consider an indifferent consumer at $\hat{x}$.

$$u_h(\hat{x}) = u_l(\hat{x}),$$

$$\phi_h + a_h y_{1h} + \gamma(a_h y_{2h} + \theta_h a_l y_{2l}) - p_h - t\hat{x} = \phi_l + a_l y_{1l} + \gamma(a_l y_{2l} + \theta_l a_h y_{2h}) - p_l - t(\pi - \hat{x}).$$

$$d_h = 2\hat{x} = \frac{1}{t}[\phi_h - \phi_l + \gamma[(1-\theta_l)a_h y_{2h} - (1-\theta_h)a_l y_{2l}] + p_l - p_h + t\pi], \tag{2.58}$$

$$d_l = 2\pi - 2\hat{x} = \frac{1}{t}[-(\phi_h - \phi_l) + \gamma[(1-\theta_h)a_l y_{2l} - (1-\theta_l)a_h y_{2h}] + p_h - p_l + t\pi]. \quad (2.59)$$

Let us assume $d_h \in (0, 2\pi)$ and $u_h(\hat{x}) > 0$. Later, we will express the feasible region in terms of t.

We have to find p_h and p_l that maximize π_h and π_l respectively.

$$\frac{d\pi_h}{dp_h} = d_h + (p_h - g_h)\frac{dd_h}{dp_h} = 0.$$

$$p_h = \frac{1}{2}[\phi_h - \phi_l + \gamma[(1-\theta_l)a_h y_{2h} - (1-\theta_h)a_l y_{2l}] + g_h + p_l + t\pi].$$

$$\frac{d^2\pi_h}{dp_h^2} = -2/t < 0, \text{ as } t > 0.$$

Therefore, p_h maximizes π_h. Similarly,

$$p_l = \frac{1}{2}[-(\phi_h - \phi_l) + \gamma[(1-\theta_h)a_l y_{2l} - (1-\theta_l)a_h y_{2h}] + g_l + p_h + t\pi].$$

Substituting p_l in p_h and vice-versa, we get, equilibrium prices $p_h(y_{2h}, y_{2l})$ and $p_l(y_{2h}, y_{2l})$.

□

Proposition 2.4.6. *If there is no regulator and when* $t > \frac{\gamma^2}{9}\left[\sum_{i\in\{h,l\}} \frac{[(1-\theta_{-i})a_i]^2}{c_i}\right]$, *the equilibrium CSR investments made by firms h and l are given by:*

$$\hat{y}_{2h} = \frac{\gamma(1-\theta_l)}{9ta_l\left[\frac{9tc_h c_l}{a_h a_l} - \sum_{i\in\{h,l\}}[\gamma(1-\theta_i)]^2\frac{c_i a_{-i}}{a_i}\right]}\Bigg[9tc_l(\phi_h - \phi_l) - (18c_l - a_l^2[\gamma(1-\theta_h)]^2)\frac{g_h}{2} + (9tc_l + a_l^2[\gamma(1-\theta_h)]^2)\frac{g_l}{2} + (9tc_l - 2a_l^2[\gamma(1-\theta_h)]^2)3t\pi\Bigg], \quad (2.60)$$

$$\hat{y}_{2l} = \frac{\gamma(1-\theta_h)}{9ta_h\left[\frac{9tc_hc_l}{a_ha_l} - \sum_{i\in\{h,l\}}[\gamma(1-\theta_i)]^2\frac{c_ia_{-i}}{a_i}\right]}\Bigg[-9tc_h(\phi_h-\phi_l) - (18c_h - a_h^2[\gamma(1-\theta_l)]^2)\frac{g_l}{2} + (9tc_h + a_h^2[\gamma(1-\theta_l)]^2)\frac{g_h}{2} + (9tc_h - 2a_h^2[\gamma(1-\theta_l)]^2)3t\pi\Bigg]. \tag{2.61}$$

Proof. Updating the demand functions by substituting the prices (2.56) and (2.57) in the demand functions (2.58) and (2.59) ,

$$d_h = \frac{1}{3t}[\phi_h - \phi_l + \gamma[(1-\theta_l)a_hy_{2h} - (1-\theta_h)a_ly_{2l}] + 3t\pi], \tag{2.62}$$

$$d_l = \frac{1}{3t}[-(\phi_h - \phi_l) + \gamma[(1-\theta_h)a_ly_{2l} - (1-\theta_l)a_hy_{2h}] + 3t\pi]. \tag{2.63}$$

Next stage of backward induction: Solving for CSR outcomes, we get $\hat{y}_{2h}$ and $\hat{y}_{2l}$.

Second-order condition for CSR outcomes,

$$t > \frac{\gamma^2}{9}\left[\sum_{i\in\{h,l\}}\frac{[(1-\theta_{-i})a_i]^2}{c_i}\right].$$

□

We have the solutions for the last stage of the game, i.e., the equilibrium product prices by substituting equilibrium CSR outcomes from Proposition 2.4.6 in Lemma 2.4.5.

$$\hat{p}_h = \frac{3tc_h}{\gamma(1-\theta_l)a_h}\hat{y}_{2h}, \quad \hat{p}_l = \frac{3tc_l}{\gamma(1-\theta_l)a_l}\hat{y}_{2l}. \tag{2.64}$$

The comparison of solutions when strategic CSR is recognized v/s not recognized is not as direct as the monopoly case, so we need careful analysis to interpret these.

Corollary 2.4.7. *If $a_h = a_l = 1$, $\theta_h = \theta_l = \theta$ and $g_h = g_l = 0$, there is no regulator and when$t \in (max\{t_8, t_9, t_{10}, t_{11}\}, t_{12})$ and $t_{12} > max\{t_8, t_9, t_{10}, t_{11}\}$, the equilibrium price, the demand for the product, CSR investments made by the firm, and the Producer surplus are given by:*

Let us consider 2 expressions,

$$\hat{y}_h = \frac{9tc_l(\phi_h - \phi_l) + (9tc_l - 2[\gamma(1-\theta)]^2)3t\pi}{9t(9tc_hc_l - [\gamma(1-\theta)]^2(c_h + c_l))}, \tag{2.65}$$

$$\hat{y}_l = \frac{-9tc_h(\phi_h - \phi_l) + (9tc_h - 2[\gamma(1-\theta)]^2)3t\pi}{9t(9tc_hc_l - [\gamma(1-\theta)]^2(c_h + c_l))}. \tag{2.66}$$

Prices,

$$\hat{p}_h = 3tc_h\hat{y}_h, \quad \hat{p}_l = 3tc_l\hat{y}_l. \tag{2.67}$$

Demands,

$$\hat{d}_h = 3c_h\hat{y}_h, \quad \hat{d}_l = 3c_l\hat{y}_l. \tag{2.68}$$

CSR,

$$\hat{y}_{2h} = \gamma(1-\theta)\hat{y}_h, \quad \hat{y}_{2l} = \gamma(1-\theta)\hat{y}_l. \tag{2.69}$$

Profits,

$$\hat{\pi}_h = \left(9tc_h - [\gamma(1-\theta)]^2\right)c_h\hat{y}_h^2, \quad \hat{\pi}_l = \left(9tc_l - [\gamma(1-\theta)]^2\right)c_l\hat{y}_l^2. \tag{2.70}$$

Proof. We get the CSR investments directly from Proposition 4 by substituting values for g_i, a_i and θ_i, $i \in \{h, l\}$.
We get equilibrium prices and demands by substituting equilibrium CSR investments in 2.56, 2.57, 2.62 and 2.63.
The profits of firms are,

$$\begin{aligned}\hat{\pi}_h &= (p_h - g_h)d_h - c_hy_{2h}^2 = 3tc_h\hat{y}_h * 3c_h\hat{y}_h - c_h(\gamma(1-\theta)\hat{y}_h^2) \\ &= \left(9tc_h - [\gamma(1-\theta)]^2\right)c_h\hat{y}_h^2.\end{aligned}$$

$$\hat{\pi}_l = (p_l - g_l)d_l - c_l y_{2l}^2 = 3tc_l\hat{y}_l * 3c_l\hat{y}_l - c_l(\gamma(1-\theta)\hat{y}_l^2)$$
$$= \left(9tc_l - [\gamma(1-\theta)]^2\right) c_l\hat{y}_l^2.$$

The feasibility region is given by,
We need $\hat{d}_h \in (0, 2\pi)$,

$\hat{d}_h < 2\pi$ gives,

$$t > \frac{\phi_h - \phi_l}{3\pi} + \frac{2[\gamma(1-\theta)]^2}{9c_h}.$$

$\hat{d}_h > 0$ gives,

$$t > \frac{-(\phi_h - \phi_l)}{3\pi} + \frac{2[\gamma(1-\theta)]^2}{9c_l}.$$

We need $\hat{u}_h(\hat{x}) > 0$, which implies,

$$t > \frac{1}{18\pi c_h c_l}\Bigg[2\pi\gamma^2(1-\theta)(c_h+c_l) + 3c_h c_l(\phi_h+\phi_l) - \sqrt{\begin{array}{l}(2\pi\gamma^2(1-\theta)(c_h+c_l) + 3c_h c_l(\phi_h+\phi_l))^2 \\ -\ 8\pi\gamma^2(1-\theta)c_h c_l\left[2\pi[\gamma(1-\theta)]^2(1+\theta) + 3[c_l(\phi_l - \theta\phi_h) + c_h(\phi_h - \theta\phi_l)]\right]\end{array}}\Bigg],$$

$$t < \frac{1}{18\pi c_h c_l}\Bigg[2\pi\gamma^2(1-\theta)(c_h+c_l) + 3c_h c_l(\phi_h+\phi_l) + \sqrt{\begin{array}{l}(2\pi\gamma^2(1-\theta)(c_h+c_l) + 3c_h c_l(\phi_h+\phi_l))^2 \\ -\ 8\pi\gamma^2(1-\theta)c_h c_l\left[2\pi[\gamma(1-\theta)]^2(1+\theta) + 3[c_l(\phi_l - \theta\phi_h) + c_h(\phi_h - \theta\phi_l)]\right]\end{array}}\Bigg].$$

□

Let's see how prices and demands change w.r.t γ and θ. The below proposition only gives price results. The same results hold for demands which are $\frac{price}{t}$.

Proposition 2.4.8. *1. If* $\Delta\phi \leq \frac{\Delta c}{c_h + c_l} 3t\pi$,

- *$\hat{p}_h$ decreases with γ and increases with θ.*
- *$\hat{p}_l$ increases with γ and decreases with θ.*

2. *If $\Delta\phi > \frac{\Delta c}{c_h+c_l}3t\pi$,*
   - *$\hat{p}_h$ increases with γ and decreases with θ.*
   - *$\hat{p}_l$ decreases with γ and increases with θ.*

Proof.

$$\frac{d\hat{p}_h}{d\gamma} = \frac{6t\gamma(1-\theta)^2 c_h c_l[(c_h+c_l)\Delta\phi - \Delta c(3t\pi)]}{\left(9tc_h c_l - [\gamma(1-\theta)]^2(c_h+c_l)\right)^2}, \quad \frac{d\hat{p}_l}{d\gamma} = -\frac{6t\gamma(1-\theta)^2 c_h c_l[(c_h+c_l)\Delta\phi - \Delta c(3t\pi)]}{\left(9tc_h c_l - [\gamma(1-\theta)]^2(c_h+c_l)\right)^2}.$$

$$\frac{d\hat{p}_h}{d\theta} = -\frac{6t\gamma^2(1-\theta) c_h c_l[(c_h+c_l)\Delta\phi - \Delta c(3t\pi)]}{\left(9tc_h c_l - [\gamma(1-\theta)]^2(c_h+c_l)\right)^2}, \quad \frac{d\hat{p}_l}{d\theta} = \frac{6t\gamma^2(1-\theta) c_h c_l[(c_h+c_l)\Delta\phi - \Delta c(3t\pi)]}{\left(9tc_h c_l - [\gamma(1-\theta)]^2(c_h+c_l)\right)^2}.$$

From above, we can see that $[(c_h+c_l)\Delta\phi - \Delta c(3t\pi)]$ is the sign determining factor of these derivatives. □

2.4.3 Comparisons

For feasible solutions in this subsection we need $t \in (max\{t_3, t_4, t_5, t_6\}, t_{12})$ and $t_{12} > max\{t_3, t_4, t_5, t_6\}$.

2.4.3.1 h v/s l

Let us compare the strategic CSR investments made by h and l when strategic CSR is recognized. This can also be used to understand the comparison of altruistic CSR as $y^*_{2i} = \gamma(1-\theta)y^*_{1i}$. For $i \in \{h, l\}$. Also, price and demand comparison is captured here as they can be written as constant factors times y^*_{1i}.

Proposition 2.4.9 (Strategic CSR investment: h v/s l)**.** *If*

$$\Delta\phi > \frac{\Delta c}{c_h+c_l}3t\pi, \quad \textit{then } y^*_{1h} > y^*_{1l}\ . \tag{2.71}$$

$$\Delta\phi \leq \frac{\Delta c}{c_h + c_l} 3t\pi, \quad \textit{then } y_{1h}^* \leq y_{1l}^* \ . \tag{2.72}$$

Proof. We have,

$$y_{1h}^* = \frac{9tc_l(\phi_h - \phi_l) + (9tc_l - 2(1 + [\gamma(1-\theta)]^2))\, 3t\pi}{9t\,(9tc_hc_l - (1 + [\gamma(1-\theta)]^2)(c_h + c_l))},$$

$$y_{1l}^* = \frac{-9tc_h(\phi_h - \phi_l) + (9tc_h - 2(1 + [\gamma(1-\theta)]^2))\, 3t\pi}{9t\,(9tc_hc_l - (1 + [\gamma(1-\theta)]^2)(c_h + c_l))}.$$

This implies,

$$y_{1h}^* - y_{1l}^* = \frac{(c_h + c_l)(\phi_h - \phi_l) - (c_h - c_l)3t\pi}{(9tc_hc_l - (1 + [\gamma(1-\theta)]^2)(c_h + c_l))}.$$

$$y_{1h}^* - y_{1l}^* > 0, \quad \text{hence,} \quad \phi_h - \phi_l > \frac{(c_h - c_l)}{c_h + c_l} 3t\pi.$$

Otherwise, we have the opposite result. □

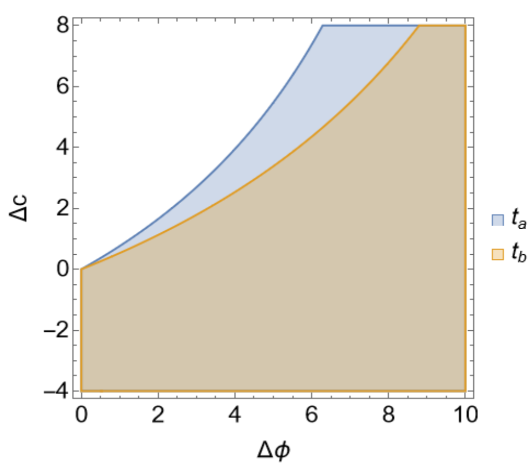


Figure 2.3: y_{1h}^* v/s y_{1l}^*

Parameter values: $\phi_l = 10$, $\phi_h \in (10, 20)$, $c_l = 5$,$c_h \in (1, 13)$, $\gamma = 1$, $\theta = 0.2$, $t_a = 1.5$, $t_b = 2.1$.

The shaded region is where the first variable is greater than the second vari-

able. The non-shaded region signifies the region where the first variable is lesser than or equal to the second. Here, the shaded region signifies $y^*_{1h} > y^*_{1l}$.

If the accessibility cost of the products (t) increases by keeping other parameters constant, the area covered by the region in the market where the strategic CSR of firm l is more than that of firm h increases.

When h is efficient in investing CSR, then always strategic CSR spending and altruistic CSR spending($y^*_{2i} = \gamma(1-\theta)y^*_{1i}$) of h is greater than l. When h becomes inefficient in spending CSR compared to l, it can go either way. l will spend more on CSR when it is highly efficient in spending CSR compared to h and the brand value difference is relatively low compared to the difference in their efficiency of spending CSR(Δc). Even if h is inefficient in spending on CSR, if it has a high brand value difference with l, it can use the revenue generated due to the brand value difference on CSR investment.

Producer Surplus

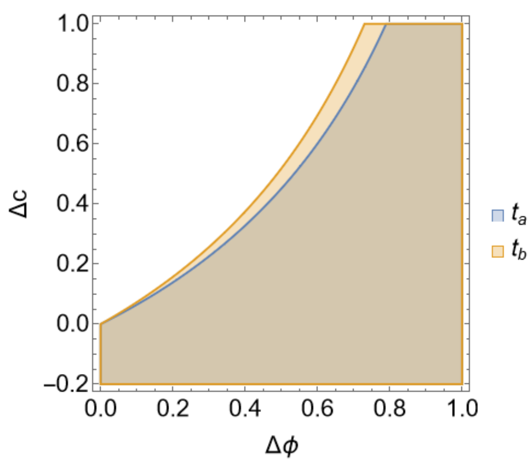


Figure 2.4: π^*_h v/s π^*_l

Parameter values: $\phi_l = 10$, $\phi_h \in (10, 11)$, $c_l = 1$,$c_h \in (0.8, 2)$, $\gamma = 1$, $\theta = 0.1$, $t_a = 0.6$, $t_b = 2$.

With higher brand value, h has an advantage over l. We can check which region favors l. Just above, we had seen that l will invest more in CSR v/s h in the region: $\Delta\phi \leq \frac{\Delta c}{c_h + c_l} 3t\pi$. But when we compare profits, l's dominance in this region reduces. In some parts of this region, l couldn't cover the expenses from the increased CSR spending in its revenue, which resulted in h's dominance in terms of profit. The increased CSR spending was due to the competition with h, which we couldn't see in the monopoly.

2.4.3.2 Strategic CSR allowed v/s Not allowed

Proposition 2.4.10 (Altruistic CSR investment of h, l)**.** *a If*

$$\Delta\phi > \frac{\Delta c}{c_h + c_l} 3t\pi, \quad \textit{then} \quad y^*_{2h} > \hat{y}_{2h}, y^*_{2l} < \hat{y}_{2l}. \tag{2.73}$$

$$\Delta\phi \leq \frac{\Delta c}{c_h + c_l} 3t\pi, \quad \textit{then} \quad y^*_{2h} \leq \hat{y}_{2h}, y^*_{2l} \geq \hat{y}_{2l}. \tag{2.74}$$

Proof.

$$y^*_{2h} = \frac{\gamma(1-\theta)}{9t\left(9tc_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l)\right)}\Big[9tc_l(\phi_h - \phi_l) + \big(9tc_l - 2(1+[\gamma(1-\theta)]^2)\big)\, 3t\pi\Big],$$

$$\hat{y}_{2h} = \frac{\gamma(1-\theta)}{9t\left(9tc_hc_l - [\gamma(1-\theta)]^2(c_h+c_l)\right)}\Big[9tc_l(\phi_h - \phi_l) + \big(9tc_l - 2[\gamma(1-\theta)]^2\big)\, 3t\pi\Big].$$

The difference in altruistic CSR investment when strategic CSR investment is recognized and not recognized,

$$y^*_{2h} - \hat{y}_{2h} = \frac{\gamma(1-\theta)c_l[(c_h+c_l)(\phi_h-\phi_l) - (c_h - c_l)3t\pi)]}{\Big(9tc_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l)\Big)\Big(9tc_hc_l - [\gamma(1-\theta)]^2(c_h+c_l)\Big)}.$$

The denominator of the above expression is greater than zero due to the constraint for t. We also know $\gamma(1-\theta)c_l > 0$ as $\gamma > 0$, $\theta \in [0, 1)$ and $c_l > 0$. $y^*_{2h} - \hat{y}_{2h} > 0$ implies,

$$\begin{aligned}(c_h + c_l)(\phi_h - \phi_l) - (c_h - c_l)3t\pi &> 0,\\ (c_h + c_l)(\phi_h - \phi_l) &> (c_h - c_l)3t\pi,\\ \phi_h - \phi_l &> \frac{(c_h - c_l)}{c_h + c_l}3t\pi.\end{aligned}$$

Otherwise, we have the opposite result.

For l,

$$y_{2l}^* = \frac{\gamma(1-\theta)}{9t\,(9tc_h c_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l))}\Big[-9tc_h(\phi_h-\phi_l)+\Big(9tc_h-2(1+[\gamma(1-\theta)]^2)\Big)3t\pi\Big],$$

$$\hat{y}_{2l} = \frac{\gamma(1-\theta)}{9t\,(9tc_h c_l - [\gamma(1-\theta)]^2(c_h+c_l))}\Big[-9tc_h(\phi_h-\phi_l)+\Big(9tc_h-2[\gamma(1-\theta)]^2\Big)3t\pi\Big],$$

$$y_{2l}^* - \hat{y}_{2l} = \frac{\gamma(1-\theta)c_h[-(c_h+c_l)(\phi_h-\phi_l)-(c_l-c_h)3t\pi)]}{\Big(9tc_h c_l-(1+[\gamma(1-\theta)]^2)(c_h+c_l)\Big)\Big(9tc_h c_l-[\gamma(1-\theta)]^2(c_h+c_l)\Big)}.$$

□

We can say that when strategic CSR is recognized, h and l have a severe competition where both can use both CSRs (strategic and altruistic) to increase the demand, hence price and cumulative revenue relative to the cost. When strategic CSR is not recognized, h and l compete moderately. From Proposition 2.4.9, when strategic CSR is recognized, we know that the altruistic CSR of h is more than l in the region $\Delta\phi > \frac{\Delta c}{c_h+c_l}3t\pi$. Therefore, here we can see that, for h, altruistic CSR, when strategic CSR is recognized, dominates the case that is not recognized. But, for l, which has a disadvantage in this region, the severity of downfall when strategic CSR is recognized is more than that of a non-recognized case. Therefore, here, strategic CSR not recognized dominates the recognized scenario. In $\Delta\phi \leq \frac{\Delta c}{c_h+c_l}3t\pi$ the opposite happens.

Let us compare the total altruistic CSR when strategic CSR is recognized v/s not recognized.

Proposition 2.4.11 (Total altruistic CSR investment)**.** *If*

$$c_h \leq c_l, \quad \textit{then} \sum_{i\in\{h,l\}} y_{2i}^* \geq \sum_{i\in\{h,l\}} \hat{y}_{2i}\ . \tag{2.75}$$

If $c_h > c_l$ *and*

$$\Delta\phi > \frac{\Delta c}{c_h + c_l} 3t\pi, \quad \textit{then} \sum_{i\in\{h,l\}} y^*_{2i} < \sum_{i\in\{h,l\}} \hat{y}_{2i} \ , \tag{2.76}$$

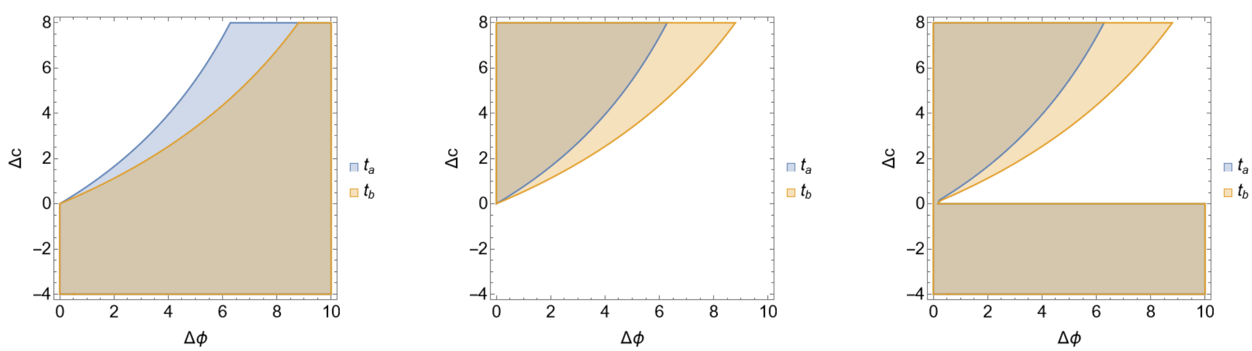
$$\Delta\phi \leq \frac{\Delta c}{c_h + c_l} 3t\pi, \quad \textit{then} \sum_{i\in\{h,l\}} y^*_{2i} \geq \sum_{i\in\{h,l\}} \hat{y}_{2i} \ . \tag{2.77}$$

Proof.

$$y^*_{2h} + y^*_{2l} - (\hat{y}_{2h} + \hat{y}_{2l}) = \frac{\gamma(1-\theta)(c_l - c_h)[(c_h + c_l)(\phi_h - \phi_l) - (c_h - c_l)3t\pi)]}{\Big(9tc_hc_l - (1 + [\gamma(1-\theta)]^2)(c_h + c_l)\Big)\Big(9tc_hc_l - [\gamma(1-\theta)]^2(c_h + c_l)\Big)}.$$

If $c_l \geq c_h$, look at the proof of h-type in the above proposition. If $c_h < c_l$, see the proof of l-type in the above proposition. □

Equation 2.75 implies that: If the High type's investment cost is lower than the low type's (i.e., h-type is efficient in investing), then the sum of altruistic CSR investment when strategic CSR is recognized is greater than the sum of altruistic CSR investment when strategic CSR is not recognized. If the High type is inefficient, then it can go either way (see 2.76 and 2.77).



(a) y^*_{2h} v/s $\hat{y}_{2h}$ (b) y^*_{2l} v/s $\hat{y}_{2l}$ (c) $y^*_{2h} + y^*_{2l}$ v/s $\hat{y}_{2h} + \hat{y}_{2l}$

Figure 2.5: Altruistic CSR comparison: Strategic CSR is recognized v/s Not recognized for different values of t.

Parameter values: $\phi_l = 10$, $\phi_h \in (10, 20)$, $c_l = 5$,$c_h \in (1, 13)$, $\gamma = 1$, $\theta = 0.2$, $t_a = 1.5$, $t_b = 2.1$.

From the above, we can infer that altruistic CSR acts more like a complement to strategic CSR than a substitute. Therefore, we can see allowing strategic CSR benefits in the increase of altruistic CSR investment by firms.

Remark 2.4.1 (Total CSR investment of h and l)**.**

1. *For h, the total CSR investment when strategic CSR is not recognized dominates the total CSR investment when strategic CSR is recognized, when the brand value difference is lower, h's CSR spending efficiency decreases and lower t.*

2. *For l, the total CSR investment when strategic CSR is not recognized dominates the total CSR investment when strategic CSR is recognized. When the brand value difference is higher, l's CSR spending efficiency decreases and lower t.*

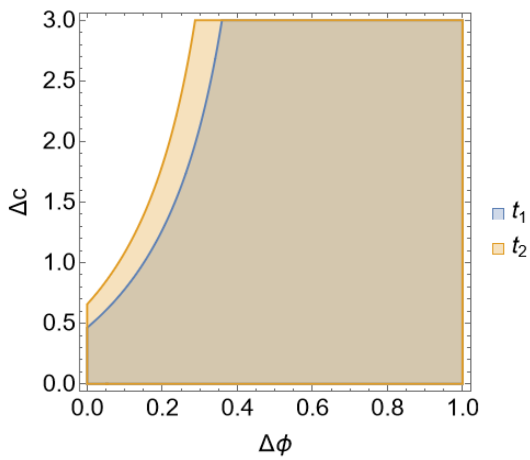


Figure 2.6: $y^*_{1h} + y^*_{2h}$ v/s $\hat{y}_{2h}$

Parameter values: $\phi_l = 5$, $\phi_h \in (5, 6)$, $c_l = 1$,$c_h \in (1, 4)$, $\gamma = 1$, $\theta = 0.1$, $t_a = 0.45$, $t_b = 0.46$.

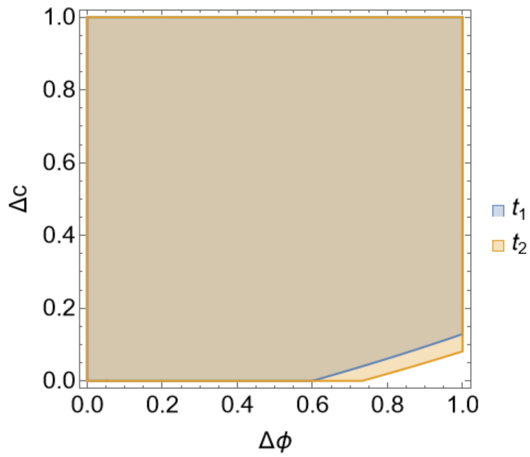


Figure 2.7: $y^*_{1l} + y^*_{2l}$ v/s $\hat{y}_{2l}$

Parameter values: $\phi_l = 5$, $\phi_h \in (5, 6)$, $c_l = 1$,$c_h \in (1, 2)$, $\gamma = 1$, $\theta = 0.1$, $t_a = 0.505$, $t_b = 0.525$.

The above remark and figures are surprising as we can see that there can be situations where the total CSR investment of firms h and l when only altruistic CSR is recognized can also be dominant in some market scenarios.

Let's see how the magnitude of price and demand changes for the firms when strategic CSR is recognized v/s not recognized w.r.t γ.

Proposition 2.4.12 (Magnitude of price and demand w.r.t γ)**.** *If $\Delta\phi \leq \frac{\Delta c}{c_h + c_l} 3t\pi$, when γ increases, the price and demand of h increases faster when strategic CSR is not recognized v/s recognized. For l, the price and demand increase faster when strategic CSR is recognized v/s not recognized.*
Otherwise, the opposite happens for h and l.

Proof.

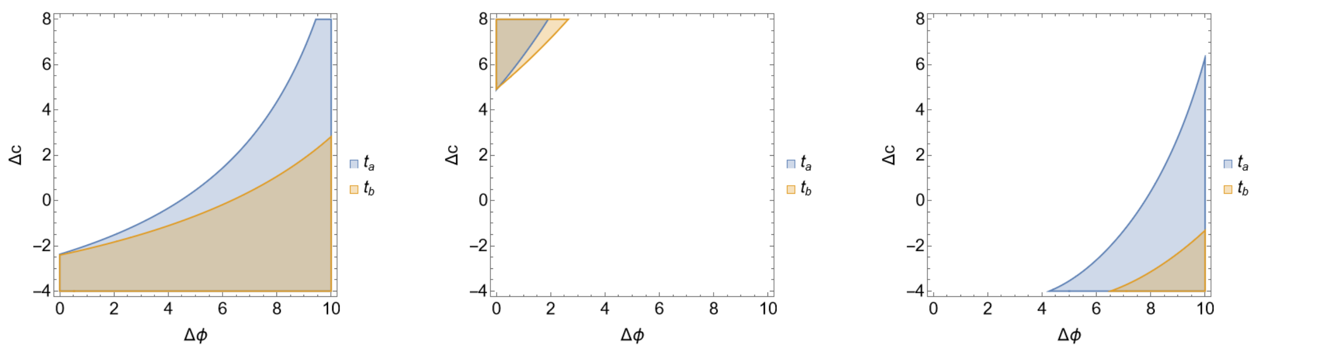
$$\begin{aligned} \frac{d(d^*_h - \hat{d}_h)}{d\gamma} =& \frac{dd^*_h}{d\gamma} - \frac{d\hat{d}_h}{d\gamma} \\ =& 6t\gamma(1-\theta)^2 c_h c_l [(c_h + c_l)\Delta\phi - \Delta c(3t\pi)] \\ & \left(\frac{1}{(9tc_h c_l - (1 + [\gamma(1-\theta)]^2)(c_h + c_l))^2} - \frac{1}{(9tc_h c_l - [\gamma(1-\theta)]^2 (c_h + c_l))^2} \right). \end{aligned} \tag{2.78}$$

We clearly have,
$(9tc_hc_l - (1 + [\gamma(1-\theta)]^2)(c_h + c_l))^2 < (9tc_hc_l - [\gamma(1-\theta)]^2(c_h + c_l))^2$.
Therefore, $\frac{1}{(9tc_hc_l-(1+[\gamma(1-\theta)]^2)(c_h+c_l))^2} > \frac{1}{(9tc_hc_l-[\gamma(1-\theta)]^2(c_h+c_l))^2}$.
Hence, $\left(\frac{1}{(9tc_hc_l-(1+[\gamma(1-\theta)]^2)(c_h+c_l))^2} - \frac{1}{(9tc_hc_l-[\gamma(1-\theta)]^2(c_h+c_l))^2}\right) > 0$.

When $\Delta\phi \leq \frac{\Delta c}{c_h+c_l}3t\pi$, $\frac{d(d_h-\hat{d}_h)}{d\gamma}$ is negative. In both scenarios, the demand for h decreases in this market region when γ increases. The demand for h when strategic CSR is recognized is decreasing at a faster rate than its demand when strategic CSR is recognized. A similar trend is observed for price. Here, $\frac{d(d_l^*-\hat{d}_l)}{d\gamma}$ is positive.

When $\Delta\phi > \frac{\Delta c}{c_h+c_l}3t\pi$, $\frac{d(d_h^*-\hat{d}_h)}{d\gamma}$ is positive. Here, the demand of h when strategic CSR is recognized increases faster than when strategic CSR is not recognized. Here, $\frac{d(d_l^*-\hat{d}_l)}{d\gamma}$ is negative □

Producer Surplus



(a) π_h^* v/s $\hat{\pi}_h$ (b) π_l^* v/s $\hat{\pi}_l$ (c) $\pi_h^* + \pi_l^*$ v/s $\hat{\pi}_h + \hat{\pi}_l$

Figure 2.8: Profit comparison: Strategic CSR recognized v/s Not recognized for different values of t.

Parameter values: $\phi_l = 10$, $\phi_h \in (10, 20)$, $c_l = 5$,$c_h \in (1, 13)$, $\gamma = 1$, $\theta = 0.2$, $t_a = 1.5$, $t_b = 2.1$.

Remark 2.4.2. *1. The market regions where strategic CSR is recognized dominates are disjoint for h and l. Hence, there is a common market region where strategic CSR is not recognized dominates for both h and l.*

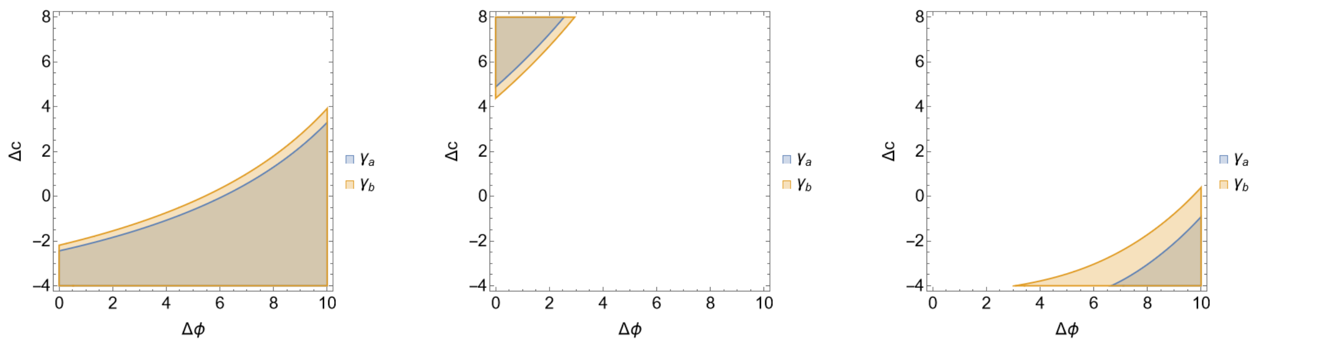
2. When strategic CSR is recognized, higher accessibility cost t, benefits l and is bad for h.

3. *If we compare the total Producer Surplus, the graph is similar to h's result but eats out the upper layer due to l's result.*

Consider accessibility cost t. Mathematically, if we see the expressions of profit of firm h and l, when t increases, the weight on brand value decreases(i.e., $3t\pi$ term will become more dominant), which will benefit firm l as it has an inherent disadvantage of low brand value. In the real world, when the accessibility cost per unit distance (t) increases, consumers who are far away from h and nearby to l, but used to buy from h, will prefer to buy from l as the disutility in reaching h will be greater than the brand value difference.

From the figures above, considering h and l separately, we can see that some regions in which strategic CSR recognized cases dominated in altruistic CSR comparison have gone in favor of strategic CSR not recognized in this profit comparison.

Consider h. The region of interest is $\Delta\phi > \frac{\Delta c}{c_h+c_l}3t\pi$. We have to understand why the top layer of the region has gone in favor of the case when strategic CSR is not recognized while comparing profits, even when h has been spending more altruistic CSR when strategic CSR is recognized v/s not recognized, as it indicates that the scenario should have been better for h. The top layer of this region indicates that the brand value difference is slightly greater than the cost difference of spending CSR. This region saw higher altruistic CSR investment when strategic CSR is recognized v/s not recognized. Therefore, h could incur more price and also have more demand when strategic CSR is recognized v/s not recognized. However, to increase the price and revenue, h also increased its investment in strategic CSR while competing with l. h had to incur the cost of both CSRs when strategic CSR is recognized. However, since the cost difference in CSR expenditure(Δc) is only slightly lower than the brand value difference, h couldn't exactly convert the increased spending of CSR into revenue. Even though the price and demand are higher when strategic CSR recognized v/s not recognized, the higher CSR investments in both types of CSRs when strategic CSR recognized v/s not recognized incurred h a greater cost.



(a) π_h^* v/s $\hat{\pi}_h$ (b) π_l^* v/s $\hat{\pi}_l$ (c) $\pi_h^* + \pi_l^*$ v/s $\hat{\pi}_h + \hat{\pi}_l$

Figure 2.9: Profit comparison: Strategic CSR recognized v/s not recognized for different values of γ.

Parameter values: $\phi_l = 10$, $\phi_h \in (10, 20)$, $c_l = 5$, $c_h \in (1, 13)$, $\theta = 0.1$, $t = 2$, $\gamma_a = 0.5$, $\gamma_b = 2$.

Remark 2.4.3. *When consumers care about altruistic CSR relative to strategic CSR increases, i.e., γ increases, both firms, which are intermediaries between consumers and the regulator, benefit when strategic CSR is recognized v/s not recognized.*

This remark is interesting because it feels counter-intuitive. When the weight given to altruistic CSR increases, it is better when strategic CSR is recognized rather than allowing only altruistic CSR.

Later in the consumer surplus, we will see that a strategic CSR-recognized scenario is always better for consumers than a non-recognized one. When γ increases, the incentives for altruistic CSR by allowing strategic CSR is higher compared to not allowing it. So, aiming to maximize social welfare, the regulator will be better off allowing strategic CSR when consumers' concern for altruistic CSR increases. **It is a win-win for all.**

Let's first consider h and see what happens to its profit when γ increases. Consider the region: $\Delta\phi \leq \frac{\Delta c}{c_h + c_l} 3t\pi$. The profit of h when strategic CSR is not allowed is always greater than its profit when strategic CSR is allowed($\pi_h^* < \hat{\pi}_h$). Since, $p_h^* \leq \hat{p}_h$ and $d_h^* \leq \hat{d}_h$. The total CSR investment can go either way in this region. If $y_{1h}^* + y_{2h}^* > \hat{y}_{2h}$, then $y_{1h}^{*2} + y_{2h}^{*2} > \hat{y}_{2h}^2$. Therefore, $p_h^* d_h^* - c_h(y_{1h}^{*2} + y_{2h}^{*2}) < \hat{p}_h \hat{d}_h - \hat{y}_{2h}^2$.In some parts, we can see that $y_{1h}^* + y_{2h}^* \leq \hat{y}_{2h}$, but here the revenue of h when strategic CSR is not recognized is higher that deficits its lose in CSR expenditure. This region always favors h in terms of profit when strategic CSR is not recognized v/s recognized. So here, we cannot see any effect on the profit of h for different values of γ.

Let's see what happens to h in the region: $\Delta\phi > \frac{\Delta c}{c_h+c_l}3t\pi$. Due to the competition with l, h has to invest in both CSRs when strategic CSR is recognized. In the top layer of this region, even though the product's demand is increased and there is an increase in the product's price, the revenue is not increased as par the cost when strategic CSR is recognized v/s not recognized. Since in this part of the region, the brand value difference is only slightly higher than the cost difference(Δc).

Mathematically, in the region: $\Delta\phi > \frac{\Delta c}{c_h+c_l}3t\pi$, we have,
$p_h^* > \hat{p}_h$, $d_h^* > \hat{d}_h$, $y_{2h}^* > \hat{y}_{2h}$.
Therefore, $p_h^* d_h^* > \hat{p}_h\hat{d}_h$.
We can also clearly say that $p_h^* d_h^* - c_h y_{2h}^{*2} > \hat{p}_h\hat{d}_h - c_h\hat{y}_{2h}^2$.

$$p_h^* d_h^* - c_h y_{2h}^{*2} = \frac{c_h}{9t}\left[c_h - \frac{[\gamma(1-\theta)]^2}{9t}\right]\left(\frac{9tc_l(\phi_h-\phi_l) + (9tc_l - 2(1+[\gamma(1-\theta)]^2))3t\pi}{9tc_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l)}\right)^2,$$

$$\hat{p}_h\hat{d}_h - c_h\hat{y}_{2h}^2 = \frac{c_h}{9t}\left[c_h - \frac{[\gamma(1-\theta)]^2}{9t}\right]\left(\frac{9tc_l(\phi_h-\phi_l) + (9tc_l - 2[\gamma(1-\theta)]^2)3t\pi}{9tc_hc_l - [\gamma(1-\theta)]^2(c_h+c_l)}\right)^2.$$

$p_h^* d_h^* - c_h y_{2h}^{*2} > \hat{p}_h\hat{d}_h - c_h\hat{y}_{2h}^2 \Leftrightarrow \Delta\phi > \frac{\Delta c}{c_h+c_l}3t\pi$.

Since π_h^* has an extra $-c_h y_{1h}^{*2}$ term, i.e., the cost of strategic CSR investment, this term eats into the region where the revenue of h when strategic CSR is allowed has been dominant over its profit when not allowed. In the top layer of the region where the brand value difference is slightly higher than the cost difference(Δc), h couldn't capitalize the increased CSR investment due to competition with l into the total profit. Therefore, this part is dominated by profit of h when strategic CSR is not recognized.

When γ increases, the price, demand and CSR investments of h when strategic CSR is recognized increases faster v/s not recognized as given by the derivatives w.r.t γ. Since the revenue is increasing faster when γ increases, firm h will be okay with spending on CSRs. With each unit of increase in CSR investment, spending in both CSR is a better strategy than spending only in one. Therefore, even if firm h has to spend more on CSR investments when strategic CSR is recognized, the revenue increase is significantly high when γ increases. Hence, this gives an advantage to the firm when strategic CSR is recognized.

Consumer Surplus

The consumer surplus is derived by substituting the equilibrium outcomes in the consumer surplus equation given in (2.45).

$$CS^* = \left(\phi_h + \left(1 + \gamma^2(1-\theta) - \frac{15tc_h}{4}\right) y^*_{1h} + \gamma^2\theta(1-\theta)y^*_{1l}\right) 3c_h y^*_{1h} + \left(\phi_l + \left(1 + \gamma^2(1-\theta) - \frac{15tc_l}{4}\right) y^*_{1l} + \gamma^2\theta(1-\theta)y^*_{1h}\right) 3c_l y^*_{1l}.$$

$$\hat{CS} = \left(\phi_h + \left(\gamma^2(1-\theta) - \frac{15tc_h}{4}\right) \hat{y}_h + \gamma^2\theta(1-\theta)\hat{y}_l\right) 3c_h\hat{y}_h + \left(\phi_l + \left(\gamma^2(1-\theta) - \frac{15tc_l}{4}\right) \hat{y}_l + \gamma^2\theta(1-\theta)\hat{y}_h\right) 3c_l\hat{y}_l.$$

We got the result below, which we had checked numerically for all possible parameter values.

Consumer surplus when strategic CSR is recognized is always greater than the consumer surplus when strategic CSR is not recognized ($CS^* > \hat{CS}$).

Consumers care about CSR investments. When we checked the total CSR investment made by firms, in most of the market scenarios, we have $y^*_{1i} + y^*_{2i} > \hat{y}_{2i}$, $i \in \{h, l\}$. Even if, in some parts, the price when strategic CSR is recognized is higher, the utility due to CSR investment will outperform this disutility due to the price. In the region where $y^*_{1i} + y^*_{2i} \leq \hat{y}_{2i}$, $i \in \{h, l\}$, we have $p^*_i < \hat{p}_i$. The disutility caused by the price is more than the utility due to CSR investments.

Social Welfare

We got the result below when we checked numerically for the social welfare comparison.

For social welfare, when strategic CSR is recognized always dominates when strategic

CSR is not recognized ($SW^* > S\hat{W}$). So, in the regions where strategic CSR is not recognized dominated along the producer surplus dimension, we can infer that consumer satisfaction when strategic CSR is recognized has a higher impact on its deficit in producer surplus.

2.5 Monopoly v/s Duopoly

For feasible solutions in this section we need, $t \in (max\{t_1, t_3, t_4, t_6\}, t_{12})$ and $t_{12} > max\{t_1, t_3, t_4, t_6\}$.

We first compare the altruistic CSR efforts between duopoly and monopoly when strategic CSR is recognized.

Proposition 2.5.1 (Altruistic CSR comparison when strategic CSR is recognized)**.** *1. For h-type,*

If

$$M_1\phi_h + c_l\phi_l < \left(3tc_l - \frac{2}{3}(1 + [\gamma(1-\theta)]^2)\right)\pi \quad \textit{then } y^*_{2h} > \acute{y}_{2h}, \tag{2.79}$$

$$M_1\phi_h + c_l\phi_l \geq \left(3tc_l - \frac{2}{3}(1 + [\gamma(1-\theta)]^2)\right)\pi \quad \textit{then } y^*_{2h} \leq \acute{y}_{2h}, \tag{2.80}$$

where $M_1 = \left(\frac{9tc_hc_l-(1+[\gamma(1-\theta)]^2)(c_h+c_l)}{(1-\theta)(2tc_h-(1+\gamma^2))} - c_l\right)$.

2. For l-type,

If

$$M_2\phi_l + c_h\phi_h < \left(3tc_h - \frac{2}{3}(1 + [\gamma(1-\theta)]^2)\right)\pi \quad \textit{then } y^*_{2l} > \acute{y}_{2l}, \tag{2.81}$$

$$M_2\phi_l + c_h\phi_h \geq \left(3tc_h - \frac{2}{3}(1 + [\gamma(1-\theta)]^2)\right)\pi \quad \textit{then } y^*_{2l} \leq \acute{y}_{2l}, \tag{2.82}$$

where $M_2 = \left(\frac{9tc_hc_l-(1+[\gamma(1-\theta)]^2)(c_h+c_l)}{(1-\theta)(2tc_l-(1+\gamma^2))} - c_h\right)$.

Proof.

$$y^*_{2h} = \frac{\gamma(1-\theta)}{9t\left(9tc_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l)\right)}\left[9tc_l(\phi_h - \phi_l) + \left(9tc_l - 2 - 2[\gamma(1-\theta)]^2\right)3t\pi\right],$$

$$\acute{y}_{2h} = \frac{\gamma\phi_h}{2tc_h - (1+\gamma^2)},$$

$$y^*_{2h} - \acute{y}_{2h} = \gamma \frac{\Big[-3\Big(9tc_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l) - (1-\theta)c_l(2tc_h - (1+\gamma^2))\Big)\phi_h + (1-\theta)(2tc_h - (1+\gamma^2))\Big((9tc_l - 2(1+[\gamma(1-\theta)]^2))\,\pi - 3c_l\phi_l\Big)\Big]}{3(2tc_h - (1+\gamma^2))\,(9tc_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l))}.$$

Simplifying $y^*_{2h} - \acute{y}_{2h} > 0$ and $y^*_{2h} - \acute{y}_{2h} \leq 0$, we get the results. Similarly, do for l. □

We now compare the altruistic CSR efforts between duopoly and monopoly when strategic CSR is not recognized.

Proposition 2.5.2 (Altruistic CSR comparison when strategic CSR is not recognized)**.** *1. For h-type,*

If

$$M_3\phi_h + c_l\phi_l < \left(3tc_l - \frac{2}{3}[\gamma(1-\theta)]^2\right)\pi \quad \textit{then } \hat{y}_{2h} > \grave{y}_{2h}, \tag{2.83}$$

$$M_3\phi_h + c_l\phi_l \geq \left(3tc_l - \frac{2}{3}[\gamma(1-\theta)]^2\right)\pi \quad \textit{then } \hat{y}_{2h} \leq \grave{y}_{2h}, \tag{2.84}$$

where $M_3 = \left(\frac{9tc_hc_l - [\gamma(1-\theta)]^2(c_h+c_l)}{(1-\theta)(2tc_h-\gamma^2)} - c_l\right)$.

2. For l-type,

If

$$M_4\phi_l + c_h\phi_h < \left(3tc_h - \frac{2}{3}[\gamma(1-\theta)]^2\right)\pi \quad \textit{then } \hat{y}_{2l} > \grave{y}_{2l}, \tag{2.85}$$

$$M_4\phi_l + c_h\phi_h \geq \left(3tc_h - \frac{2}{3}[\gamma(1-\theta)]^2\right)\pi \quad \textit{then } \hat{y}_{2l} \leq \grave{y}_{2l}, \tag{2.86}$$

where $M_4 = \left(\frac{9tc_hc_l - [\gamma(1-\theta)]^2(c_h+c_l)}{(1-\theta)(2tc_l-\gamma^2)} - c_h\right)$.

Proof.

$$\hat{y}_{2h} = \frac{\gamma(1-\theta)}{9t\,(9tc_hc_l - [\gamma(1-\theta)]^2(c_h+c_l))}\Big[9tc_l(\phi_h - \phi_l) + \Big(9tc_l - 2[\gamma(1-\theta)]^2\Big)3t\pi\Big],$$

$$\grave{y}_{2h} = \frac{\gamma\phi_h}{2tc_h - \gamma^2},$$

$$\hat{y}_{2h} - \grave{y}_{2h} = \gamma \frac{\begin{bmatrix} -3\Big(9tc_hc_l - [\gamma(1-\theta)]^2(c_h+c_l) - (1-\theta)c_l(2tc_h-\gamma^2)\Big)\phi_h \\ +(1-\theta)(2tc_h-\gamma^2)\Big((9tc_l - 2[\gamma(1-\theta)]^2)\pi - 3c_l\phi_l\Big) \end{bmatrix}}{3(2tc_h-\gamma^2)\left(9tc_hc_l - [\gamma(1-\theta)]^2(c_h+c_l)\right)}.$$

Simplifying $\hat{y}_{2h} - \grave{y}_{2h} > 0$ and $\hat{y}_{2h} - \grave{y}_{2h} \leq 0$, we get the results. Similarly, do for l.

□

If ϕ_h or ϕ_l increases by keeping other parameters constant, the market region where monopoly altruistic CSR is more than duopoly altruistic CSR increases. This trend can be seen in all four comparisons in the above Propositions.

When the brand value of the firm i is high in a monopoly, it can invest more in CSR to attract more consumers and then increase the product's price for a higher profit. However, if this firm is in a duopoly, despite the high brand value, it cannot increase the product's price as it did in the monopoly situation due to the price competition with the other firm. So, it will invest less in CSR than in the case of a monopoly when its brand value increases.

2.6 Conclusion

In this chapter, we started with Monopoly. Under Monopoly, the altruistic CSR investment, demand, price, profit, and CS [and hence SW] when strategic CSR is recognized are greater than the altruistic CSR investment, demand, price, profit, and CS [and hence SW] when strategic CSR is not recognized. Thus, a monopolist has an incentive to invest in strategic CSR. In Duopoly, our main motive was to check the altruistic CSR comparison. We saw that there are regions where altruistic CSR can be more when strategic CSR is recognized. When h is efficient, we always have that the total altruistic CSR investment when strategic CSR is recognized is more v/s not recognized. So, here, strategic and altruistic CSR act as complements. When l is efficient, it can go either way. Strategic CSR can be a substitute or complement to altruistic CSR. When consumers care about altruistic CSR increases, it is even better for the regulator to recognize strategic CSR as consumer surplus when strategic CSR is recognized is greater, we also have both firms benefitting here. There are regions where the total CSR investments of firms when only altruistic CSR is allowed can be greater. We got conditional results here except for CS and SW. Finally, we compared Monopoly v/s Duopoly, where we saw that when there is a higher brand value of firms, CSR investment of firms in monopoly dominates; otherwise, duopoly dominates.

Currently, in India, we have the CSR law, where there is a role of a regulator in deciding how much CSR investments the firms should do. Let's move on to the next chapter to understand how firms and consumers are affected.

Chapter 3

Strategic and Altruistic CSR efforts under Regulation

In this chapter, our main motivation is to study how society, i.e., firms and consumers, are affected by the government regulation on CSR spending and we have seen in the Introduction that a huge population of firms overspent CSR, so let's see why and look at the data to see some trend. We will compare the case of regulator vs. without regulator. We can check if there are any scenarios where the regulator favors firms and without the regulator favors consumers.

3.1 Model Building

The regulator sets a minimum threshold for the firms to spend on CSR. We have taken it as a certain fraction of the firm's revenue, i.e., if β_i is the factor($\beta_i \in (0,1)$), then the CSR expenditure threshold is $\beta_i p_i d_i$.

If a firm doesn't meet the required threshold, then the firm has to pay a penalty of a certain linear factor of the shortfall. Here, we have taken the linear factor to be k_i. If a firm invests more than the minimum threshold, it gets recognition from the regulator and the public, which increases its public image/ brand value. So, this over-investment benefits the firm, which is also captured by the same linear factor k_i.

Consumer utility

$$u_h = \phi_h + y_{1h} + \gamma(y_{2h} + \theta y_{2l}) - p_h - tx, \tag{3.1}$$

$$u_l = \phi_l + y_{1l} + \gamma(y_{2l} + \theta y_{2h}) - p_l - t(\pi - x). \tag{3.2}$$

Firms utility

$$\pi_h = p_h d_h - c_h(y_{1h}^2 + y_{2h}^2) - k_h(\beta_h p_h d_h - c_h(y_{1h}^2 + y_{2h}^2)), \tag{3.3}$$

$$\pi_l = p_l d_l - c_l(y_{1l}^2 + y_{2l}^2) - k_l(\beta_l p_l d_l - c_l(y_{1l}^2 + y_{2l}^2)). \tag{3.4}$$

(3.3) and (3.4) $\implies$

$$\pi_i = (1 - k_i\beta_i)p_i d_i - (1 - k_i)c_i(y_{1i}^2 + y_{2i}^2), \quad i \in \{h, l\}. \tag{3.5}$$

Proposition 3.1.1. *The equilibrium CSR investments made by firms; price and demand of the product; and profits of firms when strategic CSR is allowed with regulator in a duopoly are as follows:*

CSR investments,

$$\tilde{y}_{1h} = \frac{\left(\frac{1-k_l}{1-k_l\beta_l}\right) 9tc_l(\phi_h - \phi_l) + \left(\left(\frac{1-k_l}{1-k_l\beta_l}\right) 9tc_l - 2(1 + [\gamma(1-\theta)]^2)\right) 3t\pi}{9t\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right) 9tc_h c_l - (1 + [\gamma(1-\theta)]^2)\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right) c_h + \left(\frac{1-k_l}{1-k_l\beta_l}\right) c_l\right)\right)}, \tag{3.6}$$

$$\tilde{y}_{1l} = \frac{-\left(\frac{1-k_h}{1-k_h\beta_h}\right) 9tc_h(\phi_h - \phi_l) + \left(\left(\frac{1-k_h}{1-k_h\beta_h}\right) 9tc_h - 2(1 + [\gamma(1-\theta)]^2)\right) 3t\pi}{9t\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right) 9tc_h c_l - (1 + [\gamma(1-\theta)]^2)\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right) c_h + \left(\frac{1-k_l}{1-k_l\beta_l}\right) c_l\right)\right)}, \tag{3.7}$$

$$\tilde{y}_{2h} = \gamma(1-\theta)\tilde{y}_{1h}, \quad \tilde{y}_{2l} = \gamma(1-\theta)\tilde{y}_{1l}. \tag{3.8}$$

Prices,

$$\tilde{p}_h = \left(\frac{1-k_h}{1-k_h\beta_h}\right)3tc_h\tilde{y}_{1h}, \quad \tilde{p}_l = \left(\frac{1-k_l}{1-k_l\beta_l}\right)3tc_l\tilde{y}_{1l}. \tag{3.9}$$

Demands,

$$\tilde{d}_h = \left(\frac{1-k_h}{1-k_h\beta_h}\right)3c_h\tilde{y}_{1h}, \quad \tilde{d}_l = \left(\frac{1-k_l}{1-k_l\beta_l}\right)3c_l\tilde{y}_{1l}. \tag{3.10}$$

Profits,

$$\tilde{\pi}_h = \left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)9tc_h - (1+[\gamma(1-\theta)]^2)\right)(1-k_h)c_h\tilde{y}_{1h}^2. \tag{3.11}$$

$$\tilde{\pi}_l = \left(\left(\frac{1-k_l}{1-k_l\beta_l}\right)9tc_l - (1+[\gamma(1-\theta)]^2)\right)(1-k_l)c_l\tilde{y}_{1l}^2. \tag{3.12}$$

Proof. Let's first find the demand of h and l from Salop Circle.
Consider an indifferent consumer at $\tilde{x}$.

$$u_h(\tilde{x}) = u_l(\tilde{x}),$$

$$\phi_h + y_{1h} + \gamma(y_{2h} + \theta y_{2l}) - p_h - t\hat{x} = \phi_l + y_{1l} + \gamma(y_{2l} + \theta y_{2h}) - p_l - t(\pi - \tilde{x}).$$

$$\tilde{x} = \frac{1}{2t}[\phi_h - \phi_l + y_{1h} - y_{1l} + \gamma(1-\theta)(y_{2h} - y_{2l}) + p_l - p_h + t\pi].$$

Demand functions,

$$d_h = 2\tilde{x} = \frac{1}{t}[\phi_h - \phi_l + y_{1h} - y_{1l} + \gamma(1-\theta)(y_{2h} - y_{2l}) + p_l - p_h + t\pi],$$

$$d_l = 2\pi - 2\tilde{x} = \frac{1}{t}[-(\phi_h - \phi_l) + y_{1l} - y_{1h} + \gamma(1-\theta)(y_{2l} - y_{2h}) + p_h - p_l + t\pi].$$

The next step of backward induction is finding the equilibrium prices.

Maximizing π_h w.r.t p_h,

$$\frac{d\pi_h}{dp_h} = (1 - k_h\beta_h)\left(d_h + p_h\frac{dd_h}{dp_h}\right) = 0 \quad \text{gives,} \quad d_h + p_h\frac{dd_h}{dp_h} = 0.$$

$$p_h = \frac{1}{2}[\phi_h - \phi_l + y_{1h} - y_{1l} + \gamma(1-\theta)(y_{2h} - y_{2l}) + p_l + t\pi].$$

The second-order condition,

$$\frac{d^2\pi_h}{dp_h^2} = -2(1 - k_h\beta_h)/t < 0, \text{ as } t > 0 \text{ and } k_h\beta_h \text{ should be less than 1.}$$

Similarly, maximizing π_l w.r.t p_l, we get,

$$p_l = \frac{1}{2}[-(\phi_h - \phi_l) + y_{1l} - y_{1h} + \gamma(1-\theta)(y_{2l} - y_{2h}) + p_h + t\pi].$$

The second-order condition gives $k_l\beta_l < 1$.

Substituting p_l in p_h, we get equilibrium price $p_h(y_{1h}, y_{2h}, y_{1l}, y_{2l})$,

$$p_h(y_{1h}, y_{2h}, y_{1l}, y_{2l}) = \frac{1}{3}[\phi_h - \phi_l + y_{1h} - y_{1l} + \gamma(1-\theta)(y_{2h} - y_{2l}) + 3t\pi],$$

$$p_l(y_{1h}, y_{2h}, y_{1l}, y_{2l}) = \frac{1}{3}[-(\phi_h - \phi_l) + y_{1l} - y_{1h} + \gamma(1-\theta)(y_{2l} - y_{2h}) + 3t\pi].$$

Updating the demand functions,

$$d_h = \frac{1}{3t}[\phi_h - \phi_l + y_{1h} - y_{1l} + \gamma(1-\theta)(y_{2h} - y_{2l}) + 3t\pi] = \frac{p_h(y_{1h}, y_{2h}, y_{1l}, y_{2l})}{t},$$

$$d_l = \frac{1}{3t}[-(\phi_h - \phi_l) + y_{1l} - y_{1h} + \gamma(1-\theta)(y_{2l} - y_{2h}) + 3t\pi] = \frac{p_l(y_{1h}, y_{2h}, y_{1l}, y_{2l})}{t}.$$

Moving onto the next stage of the backward induction, i.e., we have to find optimum y_{1h}, y_{1l}, y_{2h}, and y_{2l}.

We have to solve 4 equations simultaneously, i.e.,

$$\frac{d\pi_h}{dy_{1h}} = 0, \quad \frac{d\pi_l}{dy_{1l}} = 0, \tag{3.13}$$

$$\frac{d\pi_h}{dy_{2h}} = 0, \quad \frac{d\pi_l}{dy_{2l}} = 0. \tag{3.14}$$

Consider (3.13),

$$y_{1h} = \frac{1}{\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right) 9tc_h - 1\right)}[\phi_h - \phi_l - y_{1l} + \gamma(1-\theta)(y_{2h} - y_{2l}) + 3t\pi],$$

$$y_{1l} = \frac{1}{\left(\left(\frac{1-k_l}{1-k_l\beta_l}\right) 9tc_l - 1\right)}[-(\phi_h - \phi_l) - y_{1h} + \gamma(1-\theta)(y_{2l} - y_{2h}) + 3t\pi].$$

The second-order condition for y_{1h} and y_{1l} is given by,

$$\frac{d^2\tilde{\pi}_i}{dy_{1i}^2} = \frac{2(1-k_i\beta_i)}{9t} - 2(1-k_i)c_i < 0,$$

$$\text{implies,} \qquad t > \frac{1}{9c_i}\left(\frac{1-k_i\beta_i}{1-k_i}\right).$$

Therefore, $k_i < 1$.

Consider (3.14),

$$y_{2h} = \frac{\gamma(1-\theta)}{\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right) 9tc_h - (\gamma(1-\theta))^2\right)}[\phi_h - \phi_l + y_{1h} - y_{1l} - \gamma(1-\theta)y_{2l} + 3t\pi],$$

$$y_{2l} = \frac{\gamma(1-\theta)}{\left(\left(\frac{1-k_l}{1-k_l\beta_l}\right) 9tc_l - (\gamma(1-\theta))^2\right)}[-(\phi_h - \phi_l) - y_{1h} - y_{1l} - \gamma(1-\theta)y_{2h} + 3t\pi].$$

The second-order condition for y_{2h} and y_{2l} is given by,

$$\frac{d^2\pi_i}{dy_{2i}^2} = \frac{2[\gamma(1-\theta_{-i})]^2(1-k_i\beta_i)}{9t} - 2(1-k_i)c_i < 0.$$

Hence, $$t > (\frac{1-k_i\beta_i}{1-k_i})\frac{[\gamma(1-\theta_{-i})]^2}{9c_i}.$$

Let $m = \left(\frac{1-k_h}{1-k_h\beta_h}\right)9tc_h$ and $n = \left(\frac{1-k_l}{1-k_l\beta_l}\right)9tc_l$.
Substituting y_{1l} in y_{1h},

$$y_{1h} = \frac{1}{mn-m-n}[n(\phi_h - \phi_l + \gamma(1-\theta)(y_{2h}-y_{2l})) + (n-2)3t\pi], \tag{3.15}$$

$$y_{1l} = \frac{1}{mn-m-n}[-m(\phi_h - \phi_l + \gamma(1-\theta)(y_{2h}-y_{2l})) + (m-2)3t\pi]. \tag{3.16}$$

$$y_{1h} - y_{1l} = \frac{1}{mn-m-n}[(n+m)(\phi_h - \phi_l + \gamma(1-\theta)(y_{2h}-y_{2l})) + (n-m)3t\pi]. \tag{3.17}$$

Substituting $y_{1h} - y_{1l}$ in y_{2h} and y_{2l}. Then solving for y_{2h} by substituting y_{2l} in it.

$$\tilde{y}_{2h} = \frac{\gamma(1-\theta)\left(\left(\frac{1-k_l}{1-k_l\beta_l}\right)9tc_l(\phi_h-\phi_l) + \left(\left(\frac{1-k_l}{1-k_l\beta_l}\right)9tc_l - 2(1+[\gamma(1-\theta)]^2)\right)3t\pi\right)}{9t\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right)9tc_hc_l - (1+[\gamma(1-\theta)]^2)\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h + \left(\frac{1-k_l}{1-k_l\beta_l}\right)c_l\right)\right)},$$

$$\tilde{y}_{2l} = \frac{\gamma(1-\theta)\left(-\left(\frac{1-k_h}{1-k_h\beta_h}\right)9tc_h(\phi_h-\phi_l) + \left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)9tc_h - 2(1+[\gamma(1-\theta)]^2)\right)3t\pi\right)}{9t\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right)9tc_hc_l - (1+[\gamma(1-\theta)]^2)\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h + \left(\frac{1-k_l}{1-k_l\beta_l}\right)c_l\right)\right)}.$$

Substituting $\tilde{y}_{2h}$ and $\tilde{y}_{2l}$ in 3.15 and 3.16, we get $\tilde{y}_{1h}$ and $\tilde{y}_{2h}$ respectively.

We need the denominators of equilibrium CSR outcomes to be positive. Denominator>0 implies,

$$t > \frac{1}{9}\left(1 + [\gamma(1-\theta)]^2\right)\left(\frac{(1-k_h\beta_h)}{(1-k_h)c_h} + \frac{(1-k_l\beta_l)}{(1-k_l)c_l}\right).$$

We can update the prices and demands by substituting equilibrium CSR outcomes.

We need $\tilde{d}_h \in (0, 2\pi)$,

$\tilde{d}_h > 0$ gives,

$$t > \frac{-(\phi_h - \phi_l)}{3\pi} + \frac{2(1-k_l\beta_l)(1+[\gamma(1-\theta)]^2)}{9(1-k_l)c_l}.$$

$\tilde{d}_h < 2\pi$ gives,

$$t > \frac{\phi_h - \phi_l}{3\pi} + \frac{2(1-k_h\beta_h)(1+[\gamma(1-\theta)]^2)}{9(1-k_h)c_h}.$$

$\tilde{u}_h(\tilde{x}) > 0$ gives,

$$t > \frac{(1-k_h\beta_h)(1-k_l\beta_l)}{18\pi(1-k_h)(1-k_l)c_hc_l}\Bigg[2\pi(1+\gamma^2(1-\theta))\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h + \left(\frac{1-k_h}{1-k_h\beta_h}\right)c_l\right) + 3\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_hc_l(\phi_h+\phi_l) - \sqrt{\begin{array}{l}\left(2\pi(1+\gamma^2(1-\theta))\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h + \left(\frac{1-k_l}{1-k_l\beta_l}\right)c_l\right) + 3\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right)c_hc_l(\phi_h+\phi_l)\right)^2 \\ -\,8\pi\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right)c_hc_l \\ \qquad \left[2\pi\left(1+\gamma^2(1-\theta)(2+[\gamma(1-\theta)]^2(1+\theta)\right) + 3\sum_{i\in\{h,l\}}\left(\frac{1-k_i}{1-k_i\beta_i}\right)c_i[\gamma^2(1-\theta)(\phi_i-\theta\phi_{-i})+\phi_i]\right]\end{array}}\Bigg],$$

$$t < \frac{(1-k_h\beta_h)(1-k_l\beta_l)}{18\pi(1-k_h)(1-k_l)c_hc_l}\Bigg[2\pi(1+\gamma^2(1-\theta))\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h + \left(\frac{1-k_h}{1-k_h\beta_h}\right)c_l\right) + 3\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_hc_l(\phi_h+\phi_l) + \sqrt{\begin{array}{l}\left(2\pi(1+\gamma^2(1-\theta))\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h + \left(\frac{1-k_l}{1-k_l\beta_l}\right)c_l\right) + 3\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right)c_hc_l(\phi_h+\phi_l)\right)^2 \\ -\,8\pi\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right)c_hc_l \\ \qquad \left[2\pi\left(1+\gamma^2(1-\theta)(2+[\gamma(1-\theta)]^2(1+\theta)\right) + 3\sum_{i\in\{h,l\}}\left(\frac{1-k_i}{1-k_i\beta_i}\right)c_i[\gamma^2(1-\theta)(\phi_i-\theta\phi_{-i})+\phi_i]\right]\end{array}}\Bigg].$$

□

3.2 With regulator v/s Without regulator

In this section, we are taking $k_h = k_l = k$ and $\beta_h = \beta_l = \beta$.

For feasible solutions in this section we need $t \in (max\{t_6, t_{13}, t_{14}, t_{15}, t_{16}\}, min\{t_7, t_{17}\})$ and $min\{t_7, t_{17}\} > max\{t_6, t_{13}, t_{14}, t_{15}, t_{16}\}$.

Proposition 3.2.1 (Strategic CSR comparison: h and l)**.** *Let*

$$B_h = -3t + \frac{2(1+[\gamma(1-\theta)]^2)}{27\left(\frac{1-k}{1-k\beta}\right)tc_hc_l^2}\left[9t\left(1+\frac{1-k}{1-k\beta}\right)c_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l)\right], \quad (3.18)$$

and

$$B_l = 3t - \frac{2(1+[\gamma(1-\theta)]^2)}{27\left(\frac{1-k}{1-k\beta}\right)tc_h^2c_l}\left[9t\left(1+\frac{1-k}{1-k\beta}\right)c_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l)\right]. \quad (3.19)$$

1. *If*
$$\frac{\Delta\phi}{\pi} > B_h \quad \text{then } \tilde{y}_{1h} > y_{1h}^*. \quad (3.20)$$
Otherwise, $\tilde{y}_{1h} \leq y_{1h}^*$.

2. *If*
$$\frac{\Delta\phi}{\pi} < B_l \quad \text{then } \tilde{y}_{1l} > y_{1l}^*. \quad (3.21)$$
Otherwise, $\tilde{y}_{1l} \leq y_{1l}^*$.

3. *We always have* $B_h < B_l$.

Proof. 1.

$$y_{1h}^* = \frac{9tc_l(\phi_h-\phi_l) + (9tc_l - 2(1+[\gamma(1-\theta)]^2))\,3t\pi}{9t\,(9tc_hc_l - (1+[\gamma(1-\theta)]^2)(c_h+c_l))},$$

$$\tilde{y}_{1h} = \frac{9t\left(\frac{1-k}{1-k\beta}\right)c_l(\phi_h-\phi_l) + \left(9t\left(\frac{1-k}{1-k\beta}\right)c_l - 2(1+[\gamma(1-\theta)]^2)\right)3t\pi}{9t\left(\frac{1-k}{1-k\beta}\right)\left(9t\left(\frac{1-k}{1-k\beta}\right)c_hc_l - (1+[\gamma(1-\theta)]^2)\,(c_h+c_l)\right)}.$$

Do $\tilde{y}_{1h} - y_{1h}^* > 0$, write in terms of $\frac{\Delta\phi}{\pi}$.

2.

$$y_{1l}^* = \frac{-9tc_h(\phi_h - \phi_l) + (9tc_h - 2(1 + [\gamma(1-\theta)]^2))\, 3t\pi}{9t\,(9tc_hc_l - (1 + [\gamma(1-\theta)]^2)(c_h + c_l))},$$

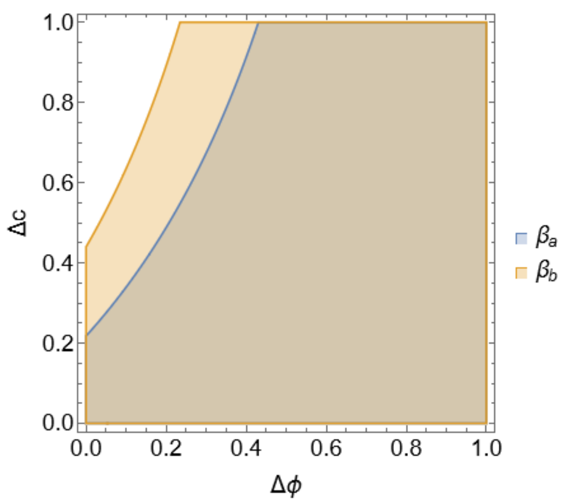
$$\tilde{y}_{1l} = \frac{-9t\left(\frac{1-k}{1-k\beta}\right)c_h(\phi_h - \phi_l) + \left(9t\left(\frac{1-k}{1-k\beta}\right)c_h - 2(1 + [\gamma(1-\theta)]^2)\right)3t\pi}{9t\left(\frac{1-k}{1-k\beta}\right)\left(9t\left(\frac{1-k}{1-k\beta}\right)c_hc_l - (1 + [\gamma(1-\theta)]^2)\,(c_h + c_l)\right)}.$$

Do $\tilde{y}_{1l} - y_{1l}^* > 0$, write in terms of $\frac{\Delta\phi}{\pi}$.

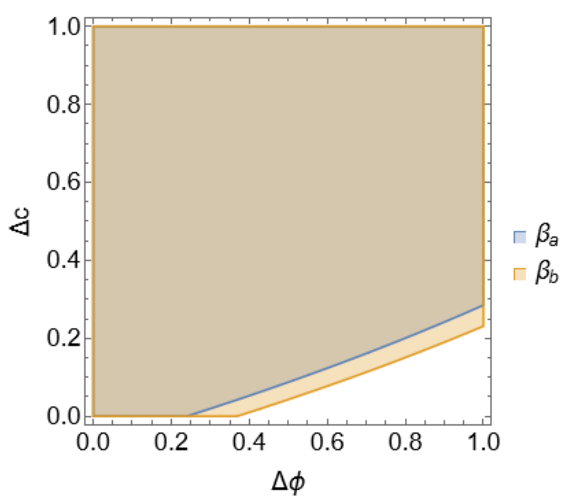
3. $B_h \geq B_l$, implies,
$t \in \left[\frac{1}{9}\,(1 + [\gamma(1-\theta)]^2)\left(\frac{1}{c_h} + \frac{1}{c_l}\right), \frac{1}{9}\left(\frac{1-k\beta}{1-\beta}\right)(1 + [\gamma(1-\theta)]^2)\left(\frac{1}{c_h} + \frac{1}{c_l}\right)\right]$.
If we see t_{13}(A.13) in Bounds of t, we need $t > \frac{1}{9}\left(\frac{1-k\beta}{1-\beta}\right)(1 + [\gamma(1-\theta)]^2)\left(\frac{1}{c_h} + \frac{1}{c_l}\right)$.
Therefore, $B_h < B_l$.

□

Similar result holds for altruistic CSR comparison of $i \in \{h, l\}$, i.e., y_{2i}^* v/s $\tilde{y}_{2i}$. As, $y_{2i}^* = \gamma(1-\theta)y_{1i}^*$ and $\tilde{y}_{2i} = \gamma(1-\theta)\tilde{y}_{1i}$.



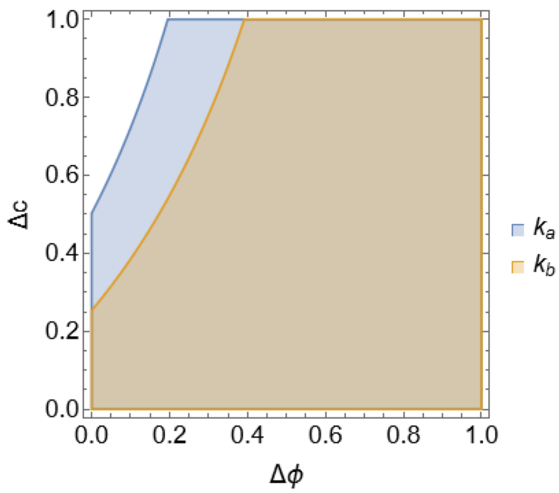
(a) $\tilde{y}_{1h}$ v/s y_{1h}^*



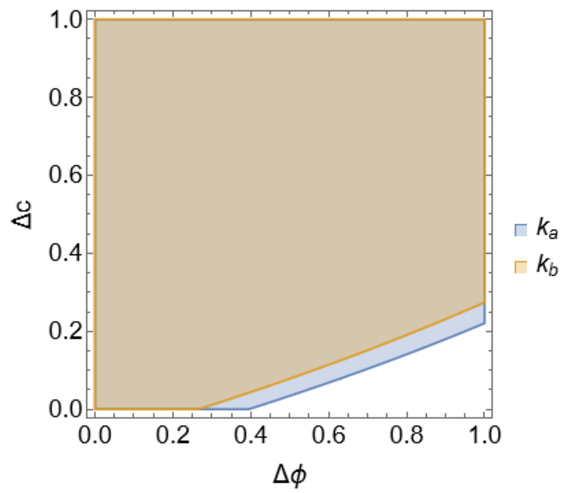
(b) $\tilde{y}_{1l}$ v/s y_{1l}^*

Figure 3.1: Strategic CSR comparison: With regulator v/s Without regulator of h and l for different values of β.

Parameter values: $\phi_l = 5$, $c_l = 1$, $\gamma = 0.8$, $\theta = 0.1$, $t = 0.5$, $k = 0.2$, $\beta_a = 0.001$, $\beta_b = 0.5$.



(a) $\tilde{y}_{1h}$ v/s y^*_{1h}



(b) $\tilde{y}_{1l}$ v/s y^*_{1l}

Figure 3.2: Strategic CSR comparison: With regulator v/s Without regulator of h and l for different values of k.

Parameter values: $\phi_l = 5$, $c_l = 1$, $\gamma = 0.8$, $\theta = 0.1$, $t = 0.5$, $\beta = 0.1$, $k_a = 0.1$, $k_b = 0.2$.

We have a proposition regarding $y_{1h} + y_{1l}$ v/s $\tilde{y}^*_{1h} + \tilde{y}^*_{1l}$. The same proposition will hold for total altruistic CSR, i.e.,$y^*_{2h} + y^*_{2l}$ v/s $\tilde{y}_{2h} + \tilde{y}_{2l}$. As we have, $y^*_{2h} + y^*_{2l} = \gamma(1-\theta)[y^*_{1h} + y^*_{1l}]$ and $\tilde{y}_{2h} + \tilde{y}_{2l} = \gamma(1-\theta)[\tilde{y}_{1h} + \tilde{y}_{1l}]$.
Hence the total CSR comparison, $y^*_{1h} + y^*_{1l} + y^*_{2h} + y^*_{2l}$ v/s $\tilde{y}_{1h} + \tilde{y}_{1l} + \tilde{y}_{2h} + \tilde{y}_{2l}$, will also have the same result.

Proposition 3.2.2 (Total strategic CSR comparison: $h + l$)**.** *Let*

$$B = \frac{1}{c_h - c_l}\left(3t(c_h + c_l) - \frac{4(1 + [\gamma(1-\theta)]^2)}{27\left(\frac{1-k}{1-k\beta}\right)tc_hc_l}\left[9t\left(1 + \frac{1-k}{1-k\beta}\right)c_hc_l - (1 + [\gamma(1-\theta)]^2)(c_h + c_l)\right]\right). \tag{3.22}$$

We have $B_h < B_l$ from the previous Proposition.

1. *For $\frac{\Delta\phi}{\pi} \in (B_h, B_l)$, we always have $\tilde{y}_{1h} + \tilde{y}_{1l} > y^*_{1h} + y^*_{1l}$.*

2. (a) *When $c_h < c_l$,*
   If $\frac{\Delta\phi}{\pi} > B$, we have $\tilde{y}_{1h} + \tilde{y}_{1l} > y^*_{1h} + y^*_{1l}$. Otherwise, $\tilde{y}_{1h} + \tilde{y}_{1l} \le y^*_{1h} + y^*_{1l}$. Here $(B_h, B_l) \subset (B, \infty)$.

(b) *When* $c_h > c_l$,
If $\frac{\Delta\phi}{\pi} < B$, *we have* $\tilde{y}_{1h} + \tilde{y}_{1l} > y^*_{1h} + y^*_{1l}$. *Otherwise,* $\tilde{y}_{1h} + \tilde{y}_{1l} \leq y^*_{1h} + y^*_{1l}$. *Here* $(B_h, B_l) \subset (-\infty, B)$.

(c) *When* $c_h = c_l$, *we always have* $\tilde{y}_{1h} + \tilde{y}_{1l} > y^*_{1h} + y^*_{1l}$.

Proof. Earlier we had seen that when $\frac{\Delta\phi}{\pi} \in (B_h, B_l)$, $\tilde{y}_{1h} > y^*_{1h}$ and $\tilde{y}_{1l} > y^*_{1l}$. Therefore, $\tilde{y}_{1h} + \tilde{y}_{1l} > y^*_{1h} + y^*_{1l}$.

$$y^*_{1h} + y^*_{1l} = \frac{9t(c_l - c_h)(\phi_h - \phi_l) + [9t(c_l + c_h) - 4(1 + [\gamma(1-\theta)]^2)]\, 3t\pi}{9t\,(9tc_hc_l - (1 + [\gamma(1-\theta)]^2)(c_h + c_l))}, \tag{3.23}$$

$$\tilde{y}_{1h} + \tilde{y}_{1l} = \frac{9t\left(\frac{1-k}{1-k\beta}\right)(c_l - c_h)(\phi_h - \phi_l) + \left(9t\left(\frac{1-k}{1-k\beta}\right)(c_l + c_h) - 4(1 + [\gamma(1-\theta)]^2)\right) 3t\pi}{9t\left(\frac{1-k}{1-k\beta}\right)\left(9t\left(\frac{1-k}{1-k\beta}\right)c_hc_l - (1 + [\gamma(1-\theta)]^2)\,(c_h + c_l)\right)}. \tag{3.24}$$

Now, when we evaluate $\tilde{y}_{1h} + \tilde{y}_{1l} > y^*_{1h} + y^*_{1l}$. We get the inequalities based on whether h or l is efficient in investing.
If h is efficient in investing($c_h < c_l$), we get $\frac{\Delta\phi}{\pi} > B$.
Now checking whether $B < B_h$. If $B \geq B_h$, we have,
$t \in \left[\frac{1}{9}(1 + [\gamma(1-\theta)]^2)\left(\frac{1}{c_h} + \frac{1}{c_l}\right), \frac{1}{9}\left(\frac{1-k\beta}{1-\beta}\right)(1 + [\gamma(1-\theta)]^2)\left(\frac{1}{c_h} + \frac{1}{c_l}\right)\right]$.
We need $t > \frac{1}{9}\left(\frac{1-k\beta}{1-\beta}\right)(1 + [\gamma(1-\theta)]^2)\left(\frac{1}{c_h} + \frac{1}{c_l}\right)$.
Therefore, $B < B_h$. This gives, $(B_h, B_l) \subset (B, \infty)$.

If l is efficient in investing($c_h > c_l$), we get $\frac{\Delta\phi}{\pi} < B$.
Now checking whether $B > B_l$. If $B \leq B_l$, we have,
$t \in \left[\frac{1}{9}(1 + [\gamma(1-\theta)]^2)\left(\frac{1}{c_h} + \frac{1}{c_l}\right), \frac{1}{9}\left(\frac{1-k\beta}{1-\beta}\right)(1 + [\gamma(1-\theta)]^2)\left(\frac{1}{c_h} + \frac{1}{c_l}\right)\right]$.
We need $t > \frac{1}{9}\left(\frac{1-k\beta}{1-\beta}\right)(1 + [\gamma(1-\theta)]^2)\left(\frac{1}{c_h} + \frac{1}{c_l}\right)$.
Therefore, $B > B_l$. This gives, $(B_h, B_l) \subset (-\infty, B)$.

When $c_h = c_l = c$,
(3.23) gives,

$$y_{1h}^* + y_{1l}^* = \frac{[18tc - 4(1+[\gamma(1-\theta)]^2)]\,\pi}{3c\,(9tc - 2(1+[\gamma(1-\theta)]^2))}.$$

(3.24) gives,

$$\tilde{y}_{1h} + \tilde{y}_{1l} = \frac{\left(18t\left(\frac{1-k}{1-k\beta}\right)c - 4(1+[\gamma(1-\theta)]^2)\right)\pi}{3c\left(\frac{1-k}{1-k\beta}\right)\left(9t\left(\frac{1-k}{1-k\beta}\right)c - 2(1+[\gamma(1-\theta)]^2)\right)}. \tag{3.25}$$

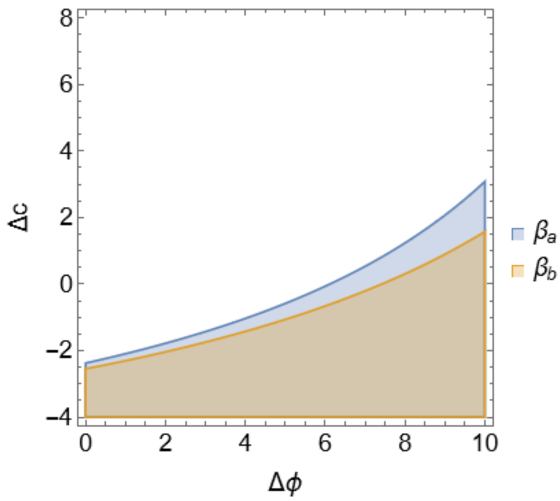
We can see that the above equations don't have $\Delta\phi$ dependence.

Claim: We always have $\tilde{y}_{1h} + \tilde{y}_{1l} > y_{1h}^* + y_{1l}^*$, when $c_h = c_l = c$.
Consider (3.25),
Taking derivative of $\tilde{y}_{1h} + \tilde{y}_{1l}$ w.r.t k.

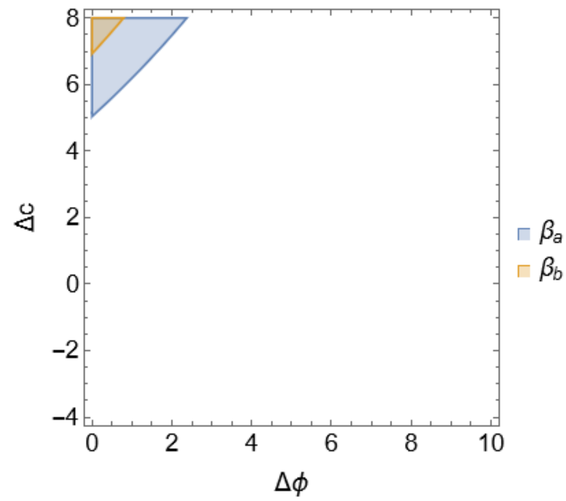
$$\frac{d}{dk}(\tilde{y}_{1h} + \tilde{y}_{1l}) = \frac{2\pi(1-\beta)}{3c(1-k)^2} > 0.$$

Hence, $\tilde{y}_{1h} + \tilde{y}_{1l}$ is increasing in k. Therefore, we have the minimum value of $\tilde{y}_{1h} + \tilde{y}_{1l}$, when $k = 0$. Without regulator case is a special case of with regulator when $k = 0$. This gives, $\tilde{y}_{1h} + \tilde{y}_{1l} > y_{1h}^* + y_{1l}^*$. □

Producer Surplus (PS)



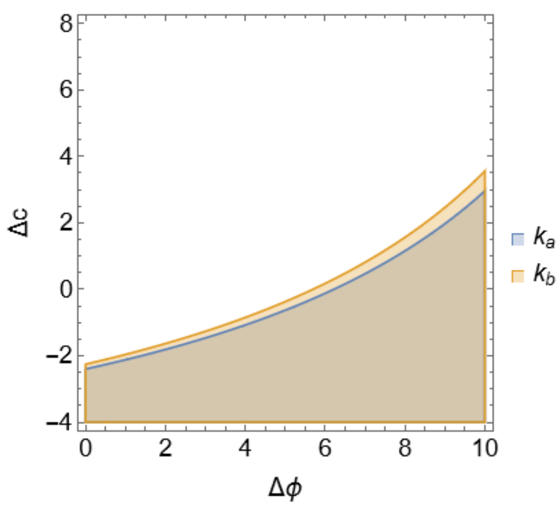
(a) $\tilde{\pi}_h$ v/s π_h^*



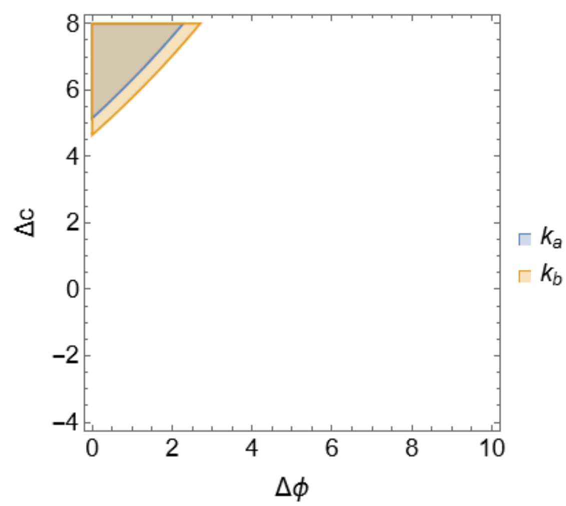
(b) $\tilde{\pi}_l$ v/s π_l^*

Figure 3.3: Producer surplus comparison: With regulator v/s Without regulator of h and l for different values of β.

Parameter values: $\phi_l = 15$, $c_l = 5$, $\gamma = 1$, $\theta = 0.1$, $t = 2$, $k = 0.2$, $\beta_a = 0.001$, $\beta_b = 0.005$.



(a) $\tilde{\pi}_h$ v/s π_h^*



(b) $\tilde{\pi}_l$ v/s π_l^*

Figure 3.4: Producer surplus comparison: With regulator v/s Without regulator of h and l for different values of k.

Parameter values: $\phi_l = 15$, $c_l = 5$, $\gamma = 1$, $\theta = 0.1$, $t = 2$, $\beta = 0.001$, $k_a = 0.05$, $k_b = 0.6$.

From the above figures, we have the following remarks.

Remark 3.2.1. *1. PS of h and l have negative relation with β.*

2. PS of h and l have positive relation with k.

3. The region dominated by PS with regulator for h and l are disjoint. Hence, there is a common region where PS without regulator of h and l dominate.

Recommended CSR v/s Actual CSR expenditure

The recommended CSR expenditure of firm i is $\beta\tilde{p}_i\tilde{d}_i$.

The total CSR spent by firm i is $c_i(\tilde{y}_{1i}^2 + \tilde{y}_{2i}^2)$.

$\frac{d}{dk}(\beta\tilde{p}_i\tilde{d}_i - c_i(\tilde{y}_{1i}^2 + \tilde{y}_{2i}^2)) < 0$ implies, firm i tend to overspend when k increases.

Let's see when firms tend to overspend/underspend w.r.t k.

Proposition 3.2.3. *Let*

$$M = \frac{1}{(\beta(1-k)c_i - (1-\beta)c_{-i})}\left[3t\left[\beta(1-k)c_i + (1-\beta)c_{-i}\right] - \frac{2(1-k\beta)^3(1+[\gamma(1-\theta)]^2)}{27(1-k)^2 tc_i c_{-i}}\left[18\left(\frac{1-k}{1-k\beta}\right)tc_i c_{-i} - (1+[\gamma(1-\theta)]^2)\left(c_i + c_{-i}\right)\right]\right). \tag{3.26}$$

1. *When $c_i < \frac{(1-\beta)}{\beta(1-k)}c_{-i}$,*

$$\text{If} \quad \frac{(\phi_i - \phi_{-i})}{\pi} > M \quad \text{then } \frac{d}{dk}(\beta\tilde{p}_i\tilde{d}_i - c_i(\tilde{y}_{1i}^2 + \tilde{y}_{2i}^2)) < 0. \tag{3.27}$$

   Otherwise, $\frac{d}{dk}(\beta\tilde{p}_i\tilde{d}_i - c_i(\tilde{y}_{1i}^2 + \tilde{y}_{2i}^2)) > 0$.

2. *When $c_i > \frac{(1-\beta)}{\beta(1-k)}c_{-i}$,*

$$\text{If} \quad \frac{(\phi_i - \phi_{-i})}{\pi} < M \quad \text{then } \frac{d}{dk}(\beta\tilde{p}_i\tilde{d}_i - c_i(\tilde{y}_{1i}^2 + \tilde{y}_{2i}^2)) < 0. \tag{3.28}$$

   Otherwise, $\frac{d}{dk}(\beta\tilde{p}_i\tilde{d}_i - c_i(\tilde{y}_{1i}^2 + \tilde{y}_{2i}^2)) > 0$.

3. *When $c_i = \frac{(1-\beta)}{\beta(1-k)}c_{-i}$, we always have $\frac{d}{dk}(\beta\tilde{p}_i\tilde{d}_i - c_i(\tilde{y}_{1i}^2 + \tilde{y}_{2i}^2)) < 0$.*

Proof.

$$\beta\tilde{p}_i\tilde{d}_i = \beta\left(\frac{1-k}{1-k\beta}\right)^2 9tc_i^2\tilde{y}_{1i}^2,$$

$$c_i(\tilde{y}_{1i}^2+\tilde{y}_{2i}^2) = (1+[\gamma(1-\theta)]^2)c_i\tilde{y}_{1i}^2.$$

Let's take the derivative of (Recommended CSR-Actual CSR expenditure) of firm i w.r.t k.

$$\frac{d}{dk}(\beta\tilde{p}_i\tilde{d}_i - c_i(\tilde{y}_{1i}^2+\tilde{y}_{2i}^2)) = \frac{d}{dk}\left(\left[\beta\left(\frac{1-k}{1-k\beta}\right)^2 9tc_i - (1+[\gamma(1-\theta)]^2)\right]c_i\tilde{y}_{1i}^2\right)$$

$$= \chi\Big[\pi(1-k\beta)^2(1+[\gamma(1-\theta)]^2)\left(36t(1-k)c_ic_{-i} - 2(1-k\beta)(1+[\gamma(1-\theta)]^2)(c_i+c_{-i})\right)$$
$$- 27(1-k)^2tc_ic_{-i}[(1-k\beta c_i(3t\pi-(\phi_i-\phi_{-i}))+(1-\beta)c_{-i}(3t\pi+(\phi_i-\phi_{-i})]\Big],$$

where

$$\chi = \frac{6tc_i(1-\beta)(1-k\beta)(1+[\gamma(1-\theta)]^2)\left(9t\left(\frac{1-k}{1-k\beta}\right)c_{-i}(3t\pi+(\phi_i-\phi_{-i}))-6t\pi(1+[\gamma(1-\theta)]^2)\right)}{(1-k)^3(1-k\beta)^3\left(9t\left(\frac{1-k}{1-k\beta}\right)c_ic_{-i}-(1+[\gamma(1-\theta)]^2)(c_i+c_{-i})\right)^3}.$$

We have $\chi > 0$ due to various feasibility conditions discussed above. Therefore, the sign of the derivative depends on the second term in the bracket. We express it in terms of $\frac{(\phi_i-\phi_{-i})}{\pi}$. □

Remark 3.2.2 (CS, SW)**.**

1. *CS with regulator is greater than CS without regulator ($\tilde{CS} > CS^*$).*
2. *There are some regions where SW with regulator is greater than SW without regulator and in the complement regions, we have vice-versa.*

When we checked numerically for all possible parameters, we got the above remark for consumer surplus. From Proposition 3.2.2, the total CSR with regulator dominates the total CSR investment without regulator in most cases. In the region where without regulator dominates in total CSR comparison, the disutility due to high product price causes its consumer surplus to go down.

In producer surplus we got a conditional result where with regulator case and without regulator case can be dominant. Therefore, this influences social welfare.

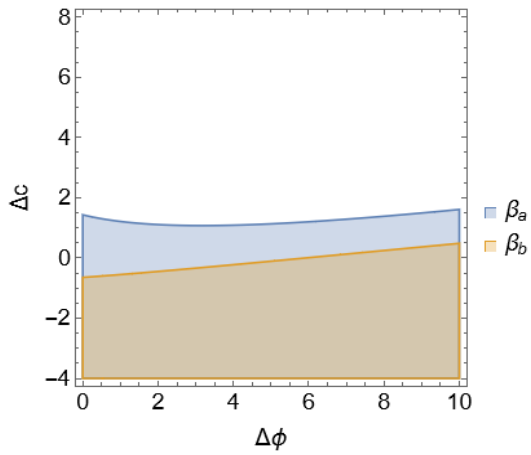


Figure 3.5: $\tilde{SW}$ v/s SW^*

Parameter values: $\phi_l = 15$, $c_l = 5$, $\gamma = 1$, $\theta = 0.1$, $t = 2$, $k = 0.2$, $\beta_a = 0.005$, $\beta_b = 0.006$.

3.3 k-value: Linking the model with real data

The data from Indian industries show that firms may spend more under CSR than the regulator prescribes. A question is what incentives exist for firms to invest more under CSR than prescribed. In this regard, we determine the perceived value of CSR spending for firms.

In previous section we saw that the recommended CSR is $\beta_i \left(\frac{1-k_i}{1-k_i\beta_i}\right)^2 9tc_i^2\tilde{y}_{1i}^2$ and CSR spending by firm is $(1 + [\gamma(1-\theta)]^2)c_i\tilde{y}_{1i}^2$. In this section, the CSR investment $\tilde{y}_{1i}$ is not dependent on k_i. If firms are overspending, we need Actual CSR spent > Recommended CSR,

$$(1 + [\gamma(1-\theta)]^2)c_i\tilde{y}_{1i}^2 > \beta_i \left(\frac{1-k_i}{1-k_i\beta_i}\right)^2 9tc_i^2\tilde{y}_{1i}^2,$$

this implies,

$$(1 + [\gamma(1-\theta)]^2) > \beta_i \left(\frac{1-k_i}{1-k_i\beta_i}\right)^2 9tc_i.$$

Considering RHS of the above inequality, i.e., $\beta_i \left(\frac{1-k_i}{1-k_i\beta_i}\right)^2 9tc_i$.

$$\frac{d}{dk_i}\left(\beta_i \left(\frac{1-k_i}{1-k_i\beta_i}\right)^2 9tc_i\right) = -\frac{(1-k_i)(1-\beta_i)18tc_i\beta_i}{(1-k_i\beta_i)^3} < 0. \tag{3.29}$$

$\therefore$ The RHS of the inequality(3.3) is decreasing in k_i. Firms tend to overspend when k_i increases.

Therefore, **the parameter k_i signifies firm i's perceived value of CSR spending**. The higher the k_i, the higher the perceived value, suggesting the higher CSR spending. The question is whether we can impute firms' perceived valuations from the existing dataset.

Finding k_i

Let $\check{\pi}_i$ be the average profit before tax of the past 3 years of firm i from real data, i.e., revenue in our model.

Let $\check{Y}_i$ be the total amount spent by firm i for CSR.

Let us assume our model is true. Then let's consider the equilibrium solutions given in Proposition 3.1.1.

In our model, $\check{Y}_i = c_i(\tilde{y}_{1i}^2 + \tilde{y}_{2i}^2) = c_i(\tilde{y}_{1i}^2 + [\gamma(1-\theta)\tilde{y}_{1i}]^2) = (1 + [\gamma(1-\theta)]^2)c_i\tilde{y}_{1i}^2$.

$\check{\pi}_i = \tilde{p}_i\tilde{d}_i = \left(\frac{1-k_i}{1-k_i\beta_i}\right)^2 9tc_i^2\tilde{y}_{1i}^2 = \left(\frac{1-k_i}{1-k_i\beta_i}\right)^2 \frac{9tc_i\check{Y}_i}{(1+[\gamma(1-\theta)]^2)}$.

$\therefore$ We have to solve the following equation for finding the value of k_i taking $\check{\pi}_i$ and $\check{y}_i$ from real data.

$$\left(\frac{1-k_i\beta_i}{1-k_i}\right)^2 = \frac{9tc_i\check{Y}_i}{(1+[\gamma(1-\theta)]^2)\check{\pi}_i}.$$

$k_i \in (0,1)$ and $k_i\beta_i < 1 \implies \left(\frac{1-k_i\beta_i}{1-k_i}\right) > 0$.

Therefore, $\left(\frac{1-k_i\beta_i}{1-k_i}\right) = \sqrt{\frac{9tc_i\check{Y}_i}{(1+[\gamma(1-\theta)]^2)\check{\pi}_i}}$. This gives,

$$k_i = \frac{\sqrt{\frac{9tc_i\check{Y}_i}{(1+[\gamma(1-\theta)]^2)\check{\pi}_i}} - 1}{\sqrt{\frac{9tc_i\check{Y}_i}{(1+[\gamma(1-\theta)]^2)\check{\pi}_i}} - \beta_i}. \tag{3.30}$$

Let's see what we can infer from the expression of k_i given in (3.30).

Proposition 3.3.1. *1. k_i is increasing in $\frac{\check{Y}_i}{\check{\pi}_i}$.*

2. k_i of any Overspent firm is greater than k_i of any Underspent firm for $\beta_i = \beta$, $\forall i$.

Proof. 1. We have,

$$k_i = \frac{C\sqrt{x_i} - 1}{C\sqrt{x_i} - \beta_i}, \quad \text{where} \quad C = \frac{9tc_i}{(1 + [\gamma(1-\theta)]^2)} \quad \text{and} \quad x_i = \frac{\check{Y}_i}{\check{\pi}_i}.$$

$$\frac{dk_i}{dx_i} = \frac{1 - \beta_i}{2\sqrt{x_i}(C\sqrt{x_i} - \beta_i)^2} > 0.$$

$\therefore$ k_i is increasing in x_i.

2. We can write the total CSR spent by firm i as,

$$\check{Y}_i = \beta\check{\pi}_i \pm z_i, \quad \text{where } z_i \text{ is the overspent/underspent amount.}$$

$$\frac{\check{Y}_i}{\check{\pi}_i} = \beta \pm \frac{z_i}{\check{\pi}_i}, \quad \text{where} \quad z_i < \beta\check{\pi}_i.$$

Let's consider overspending firms,

$$k_i = \frac{C\sqrt{\beta + \frac{z_i}{\check{\pi}_i}} - 1}{C\sqrt{\beta + \frac{z_i}{\check{\pi}_i}} - \beta} > \frac{C\sqrt{\beta} - 1}{C\sqrt{\beta} - \beta} = \bar{k}.$$

Let's consider underspending firms,

$$k_i = \frac{C\sqrt{\beta - \frac{z_i}{\check{\pi}_i}} - 1}{C\sqrt{\beta - \frac{z_i}{\check{\pi}_i}} - \beta} < \frac{C\sqrt{\beta} - 1}{C\sqrt{\beta} - \beta} = \bar{k}.$$

□

From Proposition 3.3.1, k_i essentially measures how much firms spend on CSR relative to their profit. It is a good measure to understand a firm's perceived value of CSR.

From the above proposition, we learned the k-value of overspent and underspent firms. But we don't know how different profit-level or market-cap firms behave on CSR spending in India. We would like to know the pattern of CSR perception among firms classified along these lines. So, let's study the data which we have collected.

3.3.1 Methodology: Data Collection

- To find the k-value, we needed the average profit before tax of the past 3 years(PROFIT) and the total CSR spent by the firms.
- The data was taken from the CMIE ProwessIQ database.
- The data contains the PROFITs, recommended CSR expenditure and the total CSR spent by each firm for the previous 5 financial years, i.e., from FY2018-19 to FY2022-23.
- Then, the average of these 3 indicators was calculated for each firm over these 5 years. We did this because some firms carry forward the recommended CSR to future years as the law gives this provision. Within 3 years, they have to use this CSR fund. So, taking the average for 5 years will nullify this effect. There are many reasons to carry forward CSR; the main reason is that the firm is uncertain about where to spend it.
- Our dataset has 6431 firms, of which 3244 (50.4%)are CSR Overspent firms and 3187 (49.6%) are CSR Underspent firms. Here, 'Underspent' firm is a term used to refer to underspent and exactly spent firms. 355 out of 3187 underspent firms spent exactly the recommended CSR. In underspent firms, we do not include CSR zero-spent firms that comply with CSR law and are recommended to do CSR activities.
- We classified firms into 3 categories according to their profit level. They are classified as low: 0-25 Crore; mid: 25-750 Crore; high: 750 Crore and Above.
- Overspent firms have 95 high-profit firms, 1433 mid-profit firms and 1716 low-profit firms.
- Underspent firms had 108 high-profit firms, 1573 mid-profit firms, and 1506 low-profit firms.

- We also took the market capitalization of the firms from CMIE ProwessIQ. We got information on 1652 firms from these 6431 firms. Out of 1652 firms, 856 were overspent firms and 796 were underspent firms.
- We classified firms with a market cap of 5000 Crores and above as high-cap firms and below 5000 Crores as low-cap firms.
- Among Overspent firms, 267 were high-cap firms and 589 were low-cap firms.
- Among Underspent firms, 269 were high-cap firms and 527 were low-cap firms.

3.3.2 Calculating the k-value

Consider (3.30),

- Here, we are taking same value for $c_i, \forall i$,i.e., the cost of spending CSR for firm i. If we are going onto more specifications to calculate different c_i, we can approximate it as the weighted average cost of capital (WACC), i.e., the weighted average of cost of debt and cost of equity (Liu and Wang 2022). Liu and Wang (2022) gives a method to calculate WACC.
- We took the constant($\sqrt{\frac{9tc_i}{(1+[\gamma(1-\theta)]^2)}}$) value to be 100 so that all firms in our dataset have $k_i \in (0,1)$. Also, 100 is a feasible constant considering the values for the parameters in the expression.
- $\beta_i = 0.02$, $\forall i$.
- The average PROFIT, the average CSR expenditure, β_i and the above constant were substituted in the equation (3.30) to get k_i.

3.4 Data Analysis

The current CSR regulation says that a firm has to spend a certain percentage of its profits on CSR. So, studying firms with different profit levels will give us an idea of how big/ small firms are affected by this regulation. Is it better for certain groups of firms? Since

CSR spending is a relative value of profit, big firms have to spend a lot in absolute terms compared to small firms competing with it. So, do they care that much and overspend? Let's see in this part.

3.4.1 Profit levels

- For overspent and underspent firms separately, we did a t-test: high v/s mid, high v/s low and mid v/s low.
- For high, mid and low–profit firms separately, we did a t-test: Overspent v/s Underspent.
- If $P(T \leq t)$ two-tail is less than 0.05, we would say a significant difference exists between the mean k_is of the groups we compare.

* HMD in the table is the Hypothesized Mean Difference.

	High	Mid
Mean	0.923248	0.935853
Variance	0.007398	8.15E-05
Observations	95	1433
H M D	0	
df	94	
t Stat	-1.42787	
$P(T \leq t)$ one-tail	0.078322	
t Critical one-tail	1.661226	
$P(T \leq t)$ two-tail	0.156645	
t Critical two-tail	1.985523	

	High	Low
Mean	0.923248	0.940718
Variance	0.007398	0.000129
Observations	95	1716
H M D	0	
df	94	
t Stat	-1.97868	
$P(T \leq t)$ one-tail	0.025389	
t Critical one-tail	1.661226	
$P(T \leq t)$ two-tail	0.050779	
t Critical two-tail	1.985523	

	Mid	Low
Mean	0.935853	0.940718
Variance	8.15E-05	0.000129
Observations	1433	1716
H M D	0	
df	3139	
t Stat	-13.3736	
$P(T \leq t)$ one-tail	5.16E-40	
t Critical one-tail	1.645339	
$P(T \leq t)$ two-tail	1.03E-39	
t Critical two-tail	1.96072	

Table 3.1: Overspent firms

	High	Mid
Mean	0.92104	0.913184
Variance	0.000227	0.001505
Observations	108	1573
H M D	0	
df	224	
t Stat	4.492827	
$P(T \leq t)$ one-tail	5.63E-06	
t Critical one-tail	1.651685	
$P(T \leq t)$ two-tail	1.13E-05	
t Critical two-tail	1.970611	

	High	Low
Mean	0.92104	0.918403
Variance	0.000227	0.000947
Observations	108	1506
H M D	0	
df	180	
t Stat	1.596011	
$P(T \leq t)$ one-tail	0.05612	
t Critical one-tail	1.653363	
$P(T \leq t)$ two-tail	0.112241	
t Critical two-tail	1.973231	

	Mid	Low
Mean	0.913184	0.918403
Variance	0.001505	0.000947
Observations	1573	1506
H M D	0	
df	2975	
t Stat	-4.14473	
$P(T \leq t)$ one-tail	1.75E-05	
t Critical one-tail	1.645366	
$P(T \leq t)$ two-tail	3.5E-05	
t Critical two-tail	1.960762	

Table 3.2: Underspent firms

High	Over	Under
Mean	0.923248	0.92104
Variance	0.007398	0.000227
Observations	95	108
H M D	0	
df	99	
t Stat	0.246915	
$P(T \leq t)$ one-tail	0.402743	
t Critical one-tail	1.660391	
$P(T \leq t)$ two-tail	0.805486	
t Critical two-tail	1.984217	

Mid	Over	Under
Mean	0.935853	0.913184
Variance	8.15E-05	0.001505
Observations	1433	1573
H M D	0	
df	1758	
t Stat	22.51584	
$P(T \leq t)$ one-tail	3.8E-99	
t Critical one-tail	1.645721	
$P(T \leq t)$ two-tail	7.5E-99	
t Critical two-tail	1.961314	

Low	Over	Under
Mean	0.940718	0.000947
Variance	0.000129	0.000947
Observations	1716	1506
H M D	0	
df	1864	
t Stat	26.58857	
$P(T \leq t)$ one-tail	1.2E-132	
t Critical one-tail	1.645672	
$P(T \leq t)$ two-tail	2.5E-132	
t Critical two-tail	1.961237	

Table 3.3: Overspent v/s Underspent (Profits)

	High v/s Mid	High v/s Low	Mid v/s Low
Overspent	×	✓	✓
Underspent	✓	×	✓

Table 3.4: Mean difference significance comparison for different profit groups.

	Overspent v/s Underspent
High	×
Mid	✓
Low	✓

Table 3.5: Mean difference significance comparison: Overspent v/s Underspent firms.

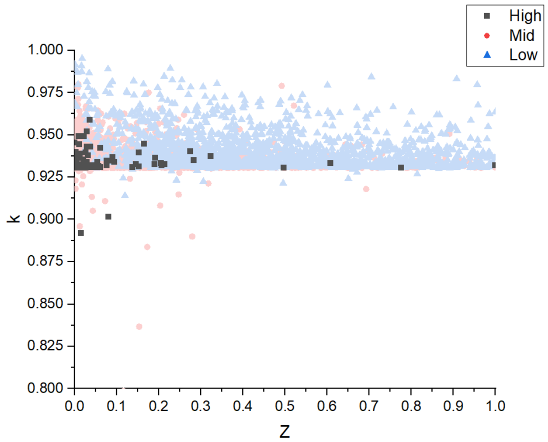
Observations and Inferences from the t-test:

- From the Table 3.4, for overspent firms, high v/s mid doesn't have any significant mean difference and for underspent firms, high v/s low doesn't have any significant mean difference. For high-profit firms in Table 3.5, overspent v/s underspent doesn't have any significant mean difference. We can see that for high-profit firms, their CSR perception, whether overspending or underspending, is nearly the same.
- Consider Table 3.1, low firms have a relatively high k-value, signifying it has a higher perceived notion of CSR among these 3 groups. Even if there wasn't any CSR regulation, firms competed on CSR spending as consumers cared about CSR. So, while competing with a high-profit firm, a low-profit firm has to spend a lot relative to its profit on CSR compared to a high/mid-profit firm.
- Ordering overspent firms according to k-value is $k_{low} > k_{mid} > k_{high}$. For underspent firms, $k_{high} > k_{low} > k_{mid}$.

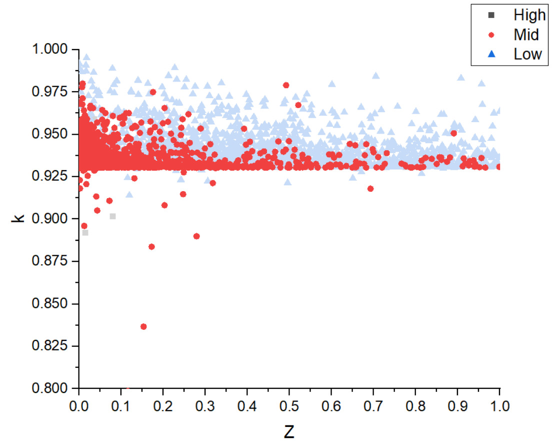
Now, moving on to plots. To compare high, mid and low-profit firms, we normalize the profits. We use Z_i to denote normalized profits. We will take the smallest and largest firms from respective profit groups to calculate Z_i.

$$Z_i = \frac{\breve{\pi}_i - \breve{\pi}_{smallest}}{\breve{\pi}_{largest} - \breve{\pi}_{smallest}}. \tag{3.31}$$

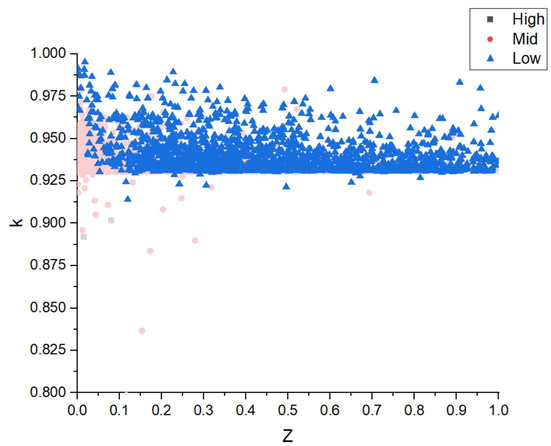
Therefore, $Z_{smallest} = 0$, $Z_{largest} = 1$ and $Z_i \in [0, 1]$.



(a) High

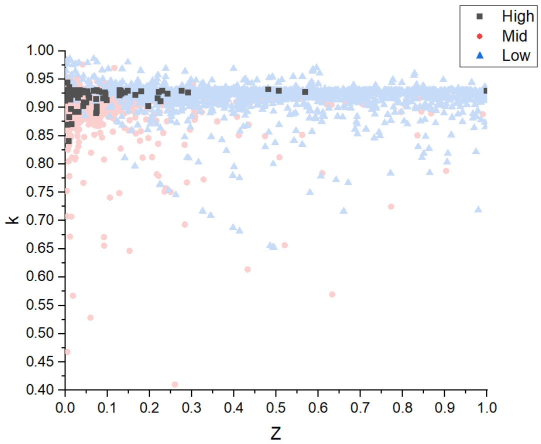


(b) Mid

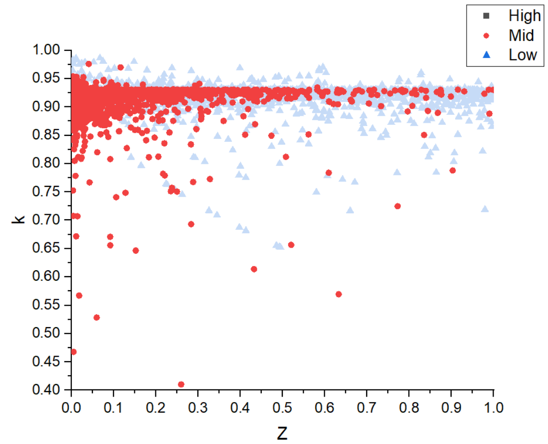


(c) Low

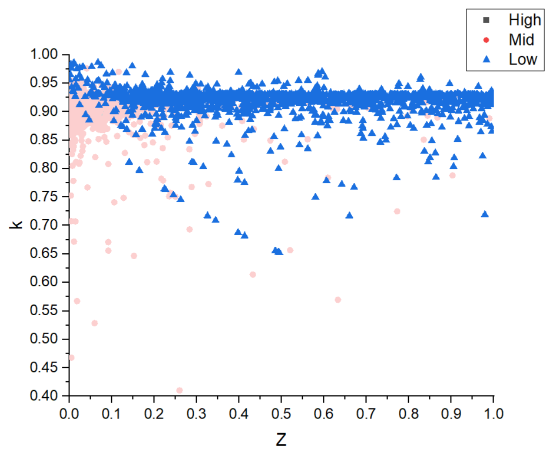
Figure 3.6: Overspent firms



(a) High



(b) Mid



(c) Low

Figure 3.7: Underspent firms

Observations and Inferences from Plots:

- We could see the k v/s Z trend for overspent firms to be like a **rectangular hyperbola** curve.
- For underspent firms, the trend seems to reflect overspent firms along a line.
- The reflecting line is the $\bar{k}$ discussed in the proof of Proposition 3.3.1.
- Both overspent and underspent high-type firms are near the line, $k_i = \bar{k}$.
- We could see that some overspent firms have lesser values than $\bar{k}$ and underspent firms have higher values than $\bar{k}$. We classified firms as overspent and underspent by

comparing the recommended CSR and CSR spent obtained from CMIE ProwessIQ. So, in the data we could see that for some firms, the recommended CSR is not 2 percent of average net profit before tax of the past 3 years. Maybe they were considering any extra profit characteristics while calculating the recommended CSR, which is unavailable to us. So, that's why we get some disparity in the plot, which we don't expect due to the Proposition 3.3.1.

From the above Tables and plots, we have the following remark.

Remark 3.4.1. *Low firms generally have a high perceived notion of CSR. In overspent firms, we saw it is completely dominating other groups. In underspent firms, low firms are just behind high firms, but their mean k-value is not significantly different.*

3.4.2 Market Capitalization

Our model considered two types of firms: h and l. They have different brand values ϕ_h and ϕ_l where $\phi_h > \phi_l$. To get an idea of their perceived notion of CSR from the real data, we thought of grouping firms into h and l according to their market capitalization, which is mentioned in the data collection section. First, let's see how firms in different market-cap groups are distributed as overspent and underspent. The table below shows that overspent and underspent firms have roughly the same distribution.

Market cap	Overspent	Underspent
1 Lakh- Above	39	26
10k-1 Lakh	152	163
5k-10k	76	80
2.5k-5k	89	91
1k-2.5k	143	131
100-1k	281	227
0-100	76	78
Total	856	796

Table 3.6: Distribution of firms according to their market capitalization in Crores in our dataset.

Two sample t-test assuming unequal variances:

Overspent	High	Low
Mean	0.935712	0.938663
Variance	7.89E-05	0.000119
Observations	267	589
H M D	0	
df	623	
t Stat	-4.18306	
$P(T \leq t)$ one-tail	1.64E-05	
t Critical one-tail	1.647303	
$P(T \leq t)$ two-tail	3.29E-05	
t Critical two-tail	1.963779	

Underspent	High	Low
Mean	0.919733	0.918974
Variance	0.00075	0.000855
Observations	269	573
H M D	0	
df	572	
t Stat	0.361413	
$P(T \leq t)$ one-tail	0.358962	
t Critical one-tail	1.647522	
$P(T \leq t)$ two-tail	0.717924	
t Critical two-tail	1.96412	

Table 3.7: High v/s Low

Overspent	Very High	Very Low
Mean	0.936618	0.943734
Variance	9.19E-05	0.000226
Observations	39	76
H M D	0	
df	108	
t Stat	-3.08228	
$P(T \leq t)$ one-tail	0.001304	
t Critical one-tail	1.659085	
$P(T \leq t)$ two-tail	0.002607	
t Critical two-tail	1.982173	

Underspent	Very High	Very Low
Mean	0.923611	0.911599
Variance	0.000115	0.001624
Observations	26	78
H M D	0	
df	99	
t Stat	2.391287	
$P(T \leq t)$ one-tail	0.009339	
t Critical one-tail	1.660391	
$P(T \leq t)$ two-tail	0.018679	
t Critical two-tail	1.984217	

Table 3.8: Very High v/s Very Low

High	Overspent	Underspent
Mean	0.935712	0.919733
Variance	7.89E-05	0.00075
Observations	267	269
H M D	0	
df	324	
t Stat	9.097989	
$P(T \leq t)$ one-tail	4.78E-18	
t Critical one-tail	1.64957	
$P(T \leq t)$ two-tail	9.55E-18	
t Critical two-tail	1.967313	

Low	Overspent	Underspent
Mean	0.938663	0.918974
Variance	0.000119	0.000855
Observations	589	527
H M D	0	
df	656	
t Stat	14.57527	
$P(T \leq t)$ one-tail	3.42E-42	
t Critical one-tail	1.64718	
$P(T \leq t)$ two-tail	6.84E-42	
t Critical two-tail	1.963587	

Table 3.9: Overspent v/s Underspent (Market cap)

Very High firms have a market cap of 1 lakh crore and above. Very low firms are the firms having a market cap of 100 crore and below.

	High v/s Low
Overspent	✓
Underspent	×

	Very High v/s Very Low
Overspent	✓
Underspent	✓

	Overspent v/s Underspent
High	✓
Low	✓

Table 3.10: Mean difference significance comparison: Market capitalization

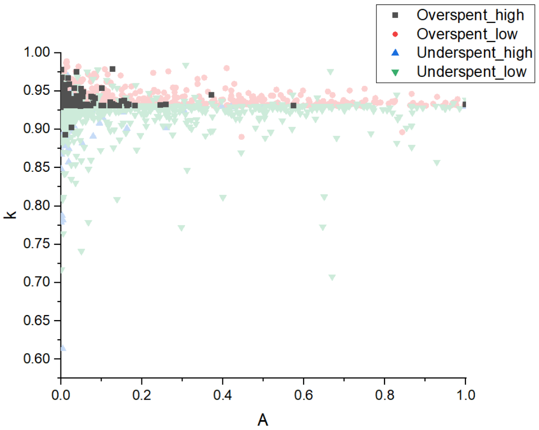
Observations and Inferences from the t-test:

- For overspent firms, $k_{low} > k_{high}$ and $k_{very-low} > k_{very-high}$.
- For underspent firms, $k_{low} < k_{high}$ and $k_{very-low} < k_{very-high}$.
- For underspent firms, k_{high} and k_{low} don't have a significant difference. That's why we compared very-high with very-low to check if there is any significant difference and yes, there is.
- High firms behave moderately in the perception of CSR. Low firms are more dynamic; some have high perceptions and overspend. Others have relatively lower perceptions compared to high firms when they underspend.

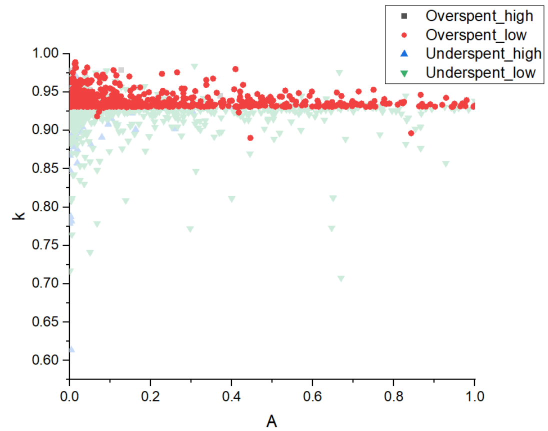
Let μ_i denote the market cap of firm i. Let's normalize market-cap using A_i.

$$A_i = \frac{\mu_i - \mu_{smallest}}{\mu_{largest} - \mu_{smallest}}. \tag{3.32}$$

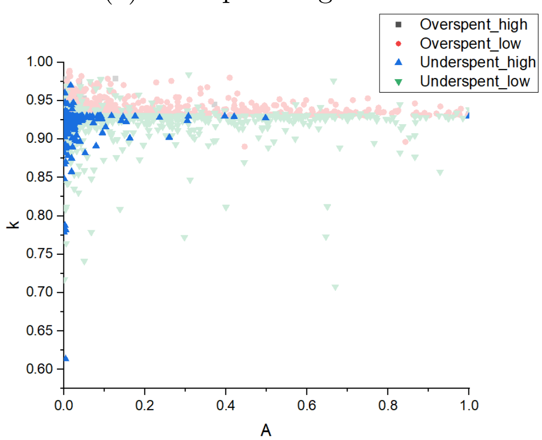
Therefore, $A_{smallest} = 0$, $A_{largest} = 1$ and $A_i \in [0, 1]$.



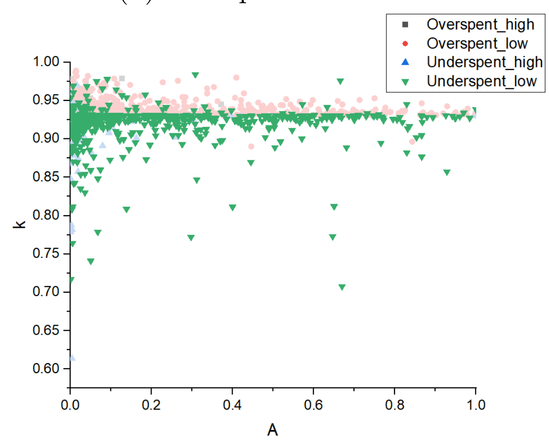
(a) Overspent High firms



(b) Overspent Low firms



(c) Underspent High firms



(d) Underspent Low firms

Figure 3.8: Firms' distribution according to their k-value (Market cap).

The above plots are similar to the trend seen in the above subsection where we dealt with different profit firms.

3.5 Conclusion

In this chapter, we extended our primary model by introducing β_i, which is the factor to which profit is multiplied to get the CSR threshold expenditure and k_i, which is the linear penalty/reward. We considered these as an exogenous parameter. When we compared with regulator v/s without regulator, we saw that there can be market scenarios where CSR investment without regulator dominates with regulator. We got a stronger and weaker result for a region where the total CSR investment with regulator dominates without regulator. The consumer surplus with regulator always dominated without regulator, but Social welfare could go either way as we got conditional results for producer surplus.

We could consider k_i as a firm's CSR perception as an increase in k_i means a firm's tendency to overspend increases. Assuming our model to be true, we got an expression to find the k-value from real data. We studied 6431 firms, classified as high, mid and low-profit firms, and 1652 of these 6431 firms, classified according to their market cap. As expected, overspent firms had greater k-value than underspent firms. There wasn't a significant difference between high profit overspent and underspent firms. Low firms generally had a higher perception of CSR compared to high and mid-profit firms. From plots of normalized profits v/s k and normalized market cap v/s k, we could see a rectangular hyperbola trend for overspent firms and a reflection of it along a line $k_i = \bar{k}$ for underspent firms.

Appendix A

Bounds for t

A.1 Without regulator

A.1.1 Monopoly: Strategic CSR Is Recognized

$$t > t_1 = \frac{1}{2}\left(\frac{\phi_i}{\pi} + \frac{1+\gamma^2}{c_i}\right), \quad i \in \{h, l\}. \tag{A.1}$$

A.1.2 Monopoly: Strategic CSR Is Not Recognized

$$t > t_2 = \frac{1}{2}\left(\frac{\phi_i}{\pi} + \frac{\gamma^2}{c_i}\right) \quad i \in \{h, l\}. \tag{A.2}$$

A.1.3 Duopoly: Strategic CSR Is Recognized

$$t > t_3 = \frac{1}{9}\left(1 + [\gamma(1-\theta)]^2\right)\left(\frac{1}{c_h} + \frac{1}{c_l}\right). \tag{A.3}$$

$$t > t_4 = \frac{-(\phi_h - \phi_l)}{3\pi} + \frac{2(1 + [\gamma(1-\theta)]^2)}{9c_l}. \tag{A.4}$$

$$t > t_5 = \frac{\phi_h - \phi_l}{3\pi} + \frac{2(1 + [\gamma(1-\theta)]^2)}{9c_h}. \tag{A.5}$$

$$t > t_6 = \frac{1}{18\pi c_h c_l}\Bigg[2\pi(1+\gamma^2(1-\theta))(c_h+c_l) + 3c_h c_l(\phi_h+\phi_l) - \sqrt{\begin{aligned}&(2\pi(1+\gamma^2(1-\theta))(c_h+c_l) + 3c_h c_l(\phi_h+\phi_l))^2 \\ &- 8\pi c_h c_l\left[2\pi\left(1+\gamma^2(1-\theta)(2+[\gamma(1-\theta)]^2(1+\theta)\right) + 3\sum_{i\in\{h,l\}} c_i[\gamma^2(1-\theta)(\phi_i - \theta\phi_{-i}) + \phi_i]\right]\end{aligned}}\Bigg]. \tag{A.6}$$

$$t < t_7 = \frac{1}{18\pi c_h c_l}\Bigg[2\pi(1+\gamma^2(1-\theta))(c_h+c_l) + 3c_h c_l(\phi_h+\phi_l) + \sqrt{\begin{aligned}&(2\pi(1+\gamma^2(1-\theta))(c_h+c_l) + 3c_h c_l(\phi_h+\phi_l))^2 \\ &- 8\pi c_h c_l\left[2\pi\left(1+\gamma^2(1-\theta)(2+[\gamma(1-\theta)]^2(1+\theta)\right) + 3\sum_{i\in\{h,l\}} c_i[\gamma^2(1-\theta)(\phi_i - \theta\phi_{-i}) + \phi_i]\right]\end{aligned}}\Bigg]. \tag{A.7}$$

A.1.4 Duopoly: Strategic CSR Is Not Recognized

$$t > t_8 = \frac{1}{9}\left([\gamma(1-\theta)]^2\right)\left(\frac{1}{c_h} + \frac{1}{c_l}\right). \tag{A.8}$$

$$t = t_9 > \frac{-(\phi_h - \phi_l)}{3\pi} + \frac{2[\gamma(1-\theta)]^2}{9c_l}. \tag{A.9}$$

$$t = t_{10} > \frac{\phi_h - \phi_l}{3\pi} + \frac{2[\gamma(1-\theta)]^2}{9c_h}. \tag{A.10}$$

$$t > t_{11} = \frac{1}{18\pi c_h c_l}\Bigg[2\pi\gamma^2(1-\theta)(c_h+c_l)+3c_hc_l(\phi_h+\phi_l) - \sqrt{\begin{array}{l}(2\pi\gamma^2(1-\theta)(c_h+c_l)+3c_hc_l(\phi_h+\phi_l))^2 \\ -8\pi\gamma^2(1-\theta)c_hc_l\left[2\pi[\gamma(1-\theta)]^2(1+\theta)+3[c_l(\phi_l-\theta\phi_h)+c_h(\phi_h-\theta\phi_l)]\right]\end{array}}\Bigg]. \quad \text{(A.11)}$$

$$t < t_{12} = \frac{1}{18\pi c_h c_l}\Bigg[2\pi\gamma^2(1-\theta)(c_h+c_l)+3c_hc_l(\phi_h+\phi_l) + \sqrt{\begin{array}{l}(2\pi\gamma^2(1-\theta)(c_h+c_l)+3c_hc_l(\phi_h+\phi_l))^2 \\ -8\pi\gamma^2(1-\theta)c_hc_l\left[2\pi[\gamma(1-\theta)]^2(1+\theta)+3[c_l(\phi_l-\theta\phi_h)+c_h(\phi_h-\theta\phi_l)]\right]\end{array}}\Bigg]. \quad \text{(A.12)}$$

For monopoly comparison, we need $t > t_1$ as $t_1 > t_2$.
For duopoly comparison, we need $t \in (max\{t_3, t_4, t_5, t_6\}, t_{12})$ and $t_{12} > (max\{t_3, t_4, t_5, t_6\}$. As for upper bound, we have $t_{12} < t_7$ and for lower bound, we have $t_3 > t_8$, $t_4 > t_9$, $t_5 > t_{10}$, $t_6 > t_{11}$.
For monopoly v/s duopoly, we need $t \in (max\{t_1, t_3, t_4, t_6\}, t_{12})$ and $t_{12} > max\{t_1, t_3, t_4, t_6\}$. As additionally, we have $t_1 > t_2$ and $t_1 > t_5$.

A.2 With regulator

A.2.1 Duopoly: Strategic CSR Is Recognized

$$t > t_{13} = \frac{1}{9}\left(1+[\gamma(1-\theta)]^2\right)\left(\frac{(1-k_h\beta_h)}{(1-k_h)c_h}+\frac{(1-k_l\beta_l)}{(1-k_l)c_l}\right). \quad \text{(A.13)}$$

$$t > t_{14} = \frac{-(\phi_h-\phi_l)}{3\pi}+\frac{2(1-k_l\beta_l)(1+[\gamma(1-\theta)]^2)}{9(1-k_l)c_l}. \quad \text{(A.14)}$$

$$t > t_{15} = \frac{\phi_h - \phi_l}{3\pi} + \frac{2(1 - k_h\beta_h)(1 + [\gamma(1-\theta)]^2)}{9(1-k_h)c_h}. \tag{A.15}$$

$$t > t_{16} = \frac{(1-k_h\beta_h)(1-k_l\beta_l)}{18\pi(1-k_h)(1-k_l)c_h c_l}\Bigg[2\pi(1+\gamma^2(1-\theta))\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h + \left(\frac{1-k_h}{1-k_h\beta_h}\right)c_l\right) + 3\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h c_l(\phi_h+\phi_l) - \sqrt{\begin{aligned}&\left(2\pi(1+\gamma^2(1-\theta))\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h + \left(\frac{1-k_l}{1-k_l\beta_l}\right)c_l\right) + 3\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right)c_h c_l(\phi_h+\phi_l)\right)^2 \\ &- 8\pi\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right)c_h c_l \\ &\quad\left[2\pi\left(1+\gamma^2(1-\theta)(2+[\gamma(1-\theta)]^2(1+\theta)\right) + 3\sum_{i\in\{h,l\}}\left(\frac{1-k_i}{1-k_i\beta_i}\right)c_i[\gamma^2(1-\theta)(\phi_i - \theta\phi_{-i}) + \phi_i]\right]\end{aligned}}\;\Bigg]. \tag{A.16}$$

$$t < t_{17} = \frac{(1-k_h\beta_h)(1-k_l\beta_l)}{18\pi(1-k_h)(1-k_l)c_h c_l}\Bigg[2\pi(1+\gamma^2(1-\theta))\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h + \left(\frac{1-k_h}{1-k_h\beta_h}\right)c_l\right) + 3\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h c_l(\phi_h+\phi_l) + \sqrt{\begin{aligned}&\left(2\pi(1+\gamma^2(1-\theta))\left(\left(\frac{1-k_h}{1-k_h\beta_h}\right)c_h + \left(\frac{1-k_l}{1-k_l\beta_l}\right)c_l\right) + 3\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right)c_h c_l(\phi_h+\phi_l)\right)^2 \\ &- 8\pi\left(\frac{1-k_h}{1-k_h\beta_h}\right)\left(\frac{1-k_l}{1-k_l\beta_l}\right)c_h c_l \\ &\quad\left[2\pi\left(1+\gamma^2(1-\theta)(2+[\gamma(1-\theta)]^2(1+\theta)\right) + 3\sum_{i\in\{h,l\}}\left(\frac{1-k_i}{1-k_i\beta_i}\right)c_i[\gamma^2(1-\theta)(\phi_i - \theta\phi_{-i}) + \phi_i]\right]\end{aligned}}\;\Bigg]. \tag{A.17}$$

For comparison with regulator v/s without regulator, we are taking $k_h = k_l = k$ and $\beta_h = \beta_l = \beta$.

As $\beta < 1$, $k\beta < k$. This gives $1 - k\beta > 1 - k$. Therefore, $\left(\frac{1-k\beta}{1-k}\right) > 1$.

Hence, when we see bounds of duopoly when strategic CSR is recognized for with regulator and without regulator, we have $t_{13} > t_3$, $t_{14} > t_4$ and $t_{15} > t_5$.

For with regulator v/s without regulator, we need $t \in (max\{t_6, t_{13}, t_{14}, t_{15}, t_{16}\}, min\{t_7, t_{17}\})$ and $min\{t_7, t_{17}\} > max\{t_6, t_{13}, t_{14}, t_{15}, t_{16}\}$.

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