

Loop Corrections to Cosmological Correlators

विद्या वाचस् की

उपाधि की अपेक्षाओं की आंशिक पूर्ति में प्रस् शोध प्रबंध

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द्वारा / By

सुप्रथा भौमिक / Supritha Bhowmick

पंजीकरण सं. / Registration No. 20202013

शोध प्रबंध पर्यवेक्षक / Thesis Supervisor:

प्रो. दीप्तिमय घोष / Prof. Diptimoy Ghosh



भारतीय विज्ञान शिक्षा एवं अनुसंधान संस् पुणे

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Dedicated to my Pa and Ma.

Certificate

Certified that the work incorporated in the thesis entitled “Loop Corrections to Cosmological Correlators” submitted by Supritha Bhowmick was carried out by the candidate, under my supervision. The work presented here or any part of it has not been included in any other thesis submitted previously for the award of any degree or diploma from any other University or institution.



Prof. Diptimoy Ghosh

Date : May 11, 2026

Declaration

Name of Student : Supritha Bhowmick

Reg. No. : 20202013

Thesis Supervisor : Diptimoy Ghosh

Department : Physics

Date of joining program : August 01, 2020

Date of Pre-Synopsis Seminar : April 28, 2026

Title of Thesis : Loop Corrections to Cosmological Correlators

I declare that this written submission represents my idea in my own words and where others' ideas have been included; I have adequately cited and referenced the original sources. I declare that I have acknowledged collaborative work and discussions wherever such work has been included. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

The work reported in this thesis is the original work done by me under the guidance of Prof. Diptimoy Ghosh.



Supritha Bhowmick

Date : May 11, 2026

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Abstract

Inflationary correlation functions are fundamental to probing the physics of the primordial universe. These correlations are the theoretical predictions, that can be mapped to present-day cosmological observations, such as the Cosmic Microwave Background (CMB) and Large-Scale Structure, to extract information about the inflationary era. While the tree-level structure of these correlators has been extensively analyzed, loop corrections in inflation remain technically challenging, and fully explicit computations are still lacking. These loop effects, although expected to be highly suppressed and practically too small to be observable, offer invaluable “theoretical data” that shed light on the behavior of inflationary dynamics at the 1-loop level. In this thesis, we work on a controlled set of 1-loop diagrams and explicitly compute them to investigate their analytic structure.

One of the first loop computations in cosmology is that of the scalar power spectrum at 1-loop, performed by Weinberg [1]. This correction featured a worrisome and unphysical $\log(k/\mu)$, where k is the comoving scale and μ is the physical renormalization scale. It was later pointed out in [2] that due to dimensional dependence of the modes in de-Sitter, the implementation of dimensional regularization is subtle, and once full $d(=3+\delta)$ -dimensional modes are used, the unphysical logarithms cancel. Motivated by this, we perform the first explicit computation of the scalar 3-point correlation function at 1-loop with interactions from the Effective Field Theory (EFT) of Inflation in [3], with the goal of exploring the structure of loop corrections beyond power spectrum. Interestingly, we found that these correlators indeed feature unphysical logarithms despite implementing dimensional regularization correctly, and the cancellation in the unrenormalized power spectrum was a happy coincidence. We show that for loops in higher point correlators these unphysical logarithms are removed only once the correlator is renormalized, a feature

which was previously unrecognized.

It is important to understand how general physical principles, such as unitarity, locality and causality, manifest as analytic properties of cosmological observables in the complex plane of kinematics. In the flat space S-matrix program, a key ingredient to achieving the same was the knowledge of analytic properties of scattering amplitudes. However for cosmological correlators, the analytic structure beyond tree level was previously unexplored. To address this gap, we performed the first systematic study of the singularity structure of 1-loop (2-vertex loops) correlation functions of massless scalars in [4]. Interestingly, we find poles and branch cuts only along negative real axis in the complex plane of external energies. We also formulate diagrammatic rules to obtain singularity structure (higher order poles and logarithms) of any n -point function at 1-loop, based on the energy injection into specific subgraphs. Our analysis clarified the origin of total energy branch points in off-shell loop correlators as a feature of the time dependent cosmological background.

Also, almost all results on loop correlations only apply to simple polynomial or derivative interactions. The effect of non-local interaction terms on the singularity structure of loop corrections was largely unexplored. This is an important gap, since realistic inflationary effective actions naturally contain non-local interactions, for example, when using the ADM formalism to enforce constraints. To address this gap, we investigate the effects of such interactions on the analytic structure of loops in [5], which poses considerable computational challenges due to complicated angular dependencies in the loop integrals. We find that the singularities arising still follow the diagrammatic rules, providing the first systematic investigation into non-local interactions in inflationary loops. We also establish for the first time the flat space limit of 1-loop correlators, where this correspondence was previously only known at tree level.

List of Publications

- * D. Bardhan, S. Bhowmick, D. Ghosh, A. Guha, D. Sachdeva, *Bounds on boosted dark matter from direct detection: Role of energy dependent cross sections*, Phys. Rev. D 107, 015010, *arXiv:2208.09405 [hep-ph]*.
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- S. Bhowmick, D. Ghosh, F. Ullah *Bispectrum at 1-loop in the Effective Field Theory of Inflation*, JHEP 10 (2024) 057, *arXiv:2405.10374 [hep-th]*.
- S. Bhowmick, M.H.G. Lee, D. Ghosh, F. Ullah, *Singularities in Cosmological Loop Correlators*, JHEP 02 (2026) 116, *arXiv: 2503.21880 [hep-th]*.
- E. Ansari, S. Bhowmick, D. Ghosh, *Singularities in Cosmological Loop Correlators II : Non Local Interactions and Flat Space limits*, JHEP 05 (2026) 114, *arXiv: 2512.11040 [hep-th]*.

Entries marked with * are not included in this thesis.

Chapter 1

Introduction

1.1 Overview

The primordial universe is believed to have undergone a phase of accelerated expansion, known as Inflation [6, 7, 8, 9]. Quantum fluctuations in the field driving this era are imprinted in cosmological observables [10, 11]. The correlation functions of these fluctuations evaluated at the end of inflation constitute the primary theoretical predictions of inflationary models. By mapping these cosmological correlations onto observables, such as large scale structures or the Cosmic Microwave Background (CMB), one can probe the underlying inflationary theory (Fig. 1.1), in close analogy with how flat space scattering amplitudes are compared with collider data and detector signal to test quantum field theories in flat space. As an example, the amplitude of the inflationary power spectrum $\langle \zeta_{\vec{k}} \zeta_{-\vec{k}} \rangle$ has been inferred from the statistics of the CMB photon temperature fluctuations $\langle \delta T \delta T \rangle$ [12, 13]. While primordial non-Gaussianities such as $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle$ have not yet been detected, bounds exist for various momentum configurations $(\vec{k}_1, \vec{k}_2, \vec{k}_3)$ [14] (Fig. 1.2), and there is growing effort for their measurement because they encode information about

inflationary interactions.

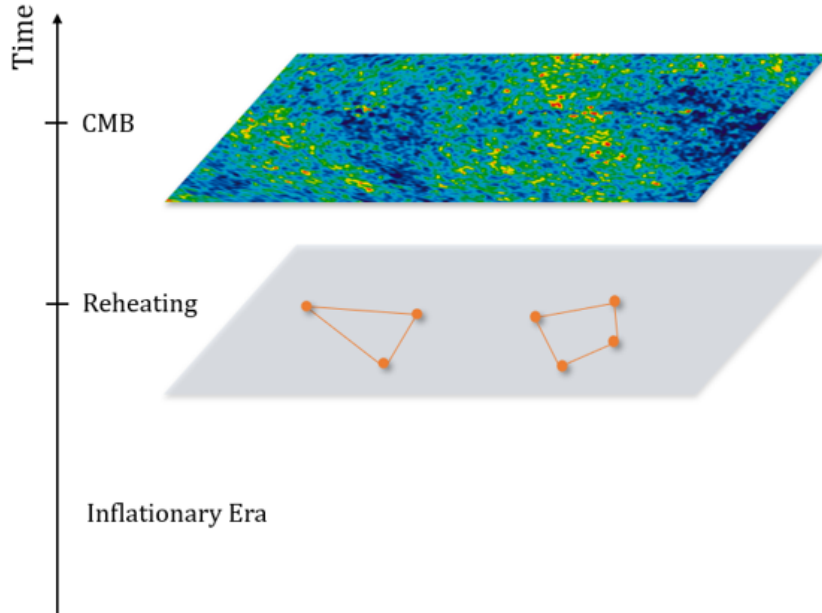


Figure 1.1: The statistics of the fluctuations (e.g. 3-pt, 4-pt and so on) in the fields towards the end of inflation can be mapped to current day cosmological observations, allowing us to study the underlying inflationary dynamics.

Tree level inflationary correlations have been studied in great detail, with a lot of formalism developed to study their analytic structure (see for e.g. [15, 16, 17]). These efforts include exploiting symmetries (see for e.g. [18, 19, 20, 21]), investigating effects of heavy or spinning spectator fields during inflation (see for e.g. [22, 23, 24]), characterizing the role of the initial state (see for e.g. [25, 26, 27, 28, 29, 30]), and identifying constraints imposed by general physical principles such as unitarity, causality and locality (see [31, 32, 33, 34, 35, 36]).

Beyond tree level, however, inflationary loop computations are notoriously tricky, and explicit calculations are still lacking. The reason why these loop corrections remained uncharted territory for a long time is because loop effects are expected to be highly suppressed and observationally inaccessible. Weinberg [1] was the first to emphasize that loop effects nevertheless constitute invaluable “theoretical data”.

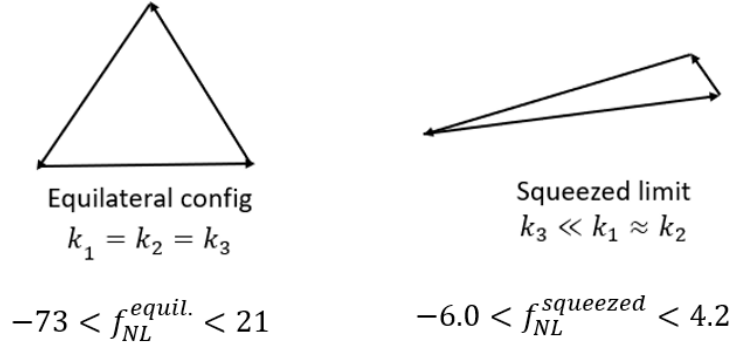


Figure 1.2: Momentum conservation imposes the momenta of the three legs ($\vec{k}_{i=1,2,3}$) in the bispectrum to form a triangle. In the figure above, the bounds on the non-Gaussianity parameter $f_{NL} \sim \frac{\langle \zeta \zeta \zeta \rangle}{\langle \zeta \zeta \rangle \langle \zeta \zeta \rangle}$ corresponding to two such configurations are shown. The equilateral configuration corresponds to all three modes of the same order, whereas the squeezed limit corresponds to one of the modes being much larger than the other two.

Since then, there has been a lot of interest towards improving the theoretical understanding of inflation at loops. Various subtleties of implementing regularization in time-dependent backgrounds have been identified [2, 37, 38], disregarding which would lead to *unphysical* features in cosmological correlators. Loops have also been studied in the context of large non-gaussianities [39, 40, 41, 42, 43, 44, 45, 46], IR-divergences in de-Sitter [47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60], and time independence of correlators (e.g. [37, 38]), particularly in inflationary models involving transient features in the potential, such as an intermediate Ultra-Slow Roll phase in the context of Primordial Black Hole (PBH) production [61, 62, 63, 64, 65, 66, 67]. Notably, the question of whether loop-level power spectra at CMB scales are affected by short modes associated with primordial black hole production remains under active debate.

A further motivation for studying loop correlators parallels the goals of the flat-space S-matrix program: namely understanding how general physical principles, such as unitarity, causality and locality, manifest as analytic properties of observ-

ables. In flat space, a key ingredient to achieving this was the knowledge of the analytic structure of the S-matrix in the complex plane of kinematics. This has led to the understanding of, for e.g. how S-matrix develops poles and branch cuts whenever intermediate particles go on-shell and multi-particle thresholds are met respectively [68, 69, 70, 71, 72, 73]. However, an analogous systematic understanding of the singularity structure of cosmological correlators beyond tree level was missing.

Importantly, inflationary effective actions naturally involve non-local spatial inverse laplacian operators, arising due to solutions of constrained degrees of freedom being plugged back in the action [15, 74, 75]. The effect of such non-local operators on analytic structure of correlators has not been studied beyond tree level. Computations involving such interactions are technically challenging, since the inverse laplacian operators produce non-trivial vertex factors. This leaves an important gap in understanding structure of inflationary correlators at loop level.

Finally, cosmological correlators computed in the Bunch Davies vacuum have been shown to demonstrate a remarkable correspondence with flat space scattering amplitude at tree level [76, 77, 78, 18, 79, 20, 33]. This can be understood as a consequence of the modes being sub-Hubble and essentially experiencing flat space conditions in the far past, and is obtained in the limit of the total energy (i.e. the sum of the energies of the external legs) of a correlator being taken to 0. However this correspondence was only known at tree level, and the flat space limit of the full loop correlator was previously unknown.

1.2 Inflation

This section follows the lecture notes [80, 81, 82]

1.2.1 Motivation

Our universe, observed to be spatially homogeneous and isotropic on large scales ($\gtrsim 100$ Mpc), is described by the FRW metric,

$$ds^2 = a^2(\tau) \left(-d\tau^2 + \frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega^2 \right), \quad (1.1)$$

where τ is the conformal time, κ is the spatial curvature, and physical lengthscales grow linearly with the scale factor $a(\tau)$.

Inflation is a hypothesized era when the universe underwent accelerated expansion, with $\ddot{a} > 0$, and the growth of the physical lengthscales was exponential $a(t) = e^{Ht}$, where $H = \frac{1}{a} \frac{da}{dt}$ is the Hubble and t is cosmic time. This epoch in the early universe was proposed to explain a few observational inconsistencies with the standard model of cosmology.

As an example, consider the Cosmic Microwave Background (CMB). At the redshift of $z \sim 1100$, the universe cooled down enough for electrons to be captured, forming light atoms in the early universe. As the universe became neutral, the primordial photons could finally stream freely instead of being scattering around, forming the CMB we observe today. Now, the comoving separation χ_{sep} between an anti-podal pair of CMB photons is related to the comoving particle horizon $\chi_{\text{p.h.}}$ at the surface of last scattering as,

$$\frac{\chi_{\text{sep}}}{\chi_{\text{p.h.}}} \sim \frac{aH}{H_0} \sim \left(\frac{1}{a} \right)^{\frac{3w+1}{2}}, \quad (1.2)$$

where $\sim$ denotes a $\mathcal{O}(1)$ factor, H_0 is the current Hubble rate, and a, H are the scale factor and Hubble rate at the surface of last scattering respectively. $\chi_{\text{p.h.}}$ is the comoving distance that light has traveled from the very beginning ($a = 0$). w is given

by the equation of state ($P = w\rho$) of the dominant energy density in the universe, e.g. during radiation domination (RD) and matter domination (MD), $w = \frac{1}{3}, 0$ respectively, which implies the RHS of Eqn. (1.2) is much larger than 1. In other words, at the surface of last scattering, the distance between the CMB photons was larger than their horizon size. Infact, it can be shown that CMB photons that are separated by $\gtrsim 2^\circ$ have never been in causal contact with each other. However, the CMB sky, mapped by experiments (such as COBE, WMAP and Planck), has been observed to have an extremely uniform background at a temperature of $T \sim 2.7$ K, with tiny fluctuations ($\frac{\delta T}{T} \sim 10^{-5}$). Such uniform temperature of the CMB photons is then a puzzle, since patches in the CMB sky are causally disconnected, and is known as the Horizon problem.

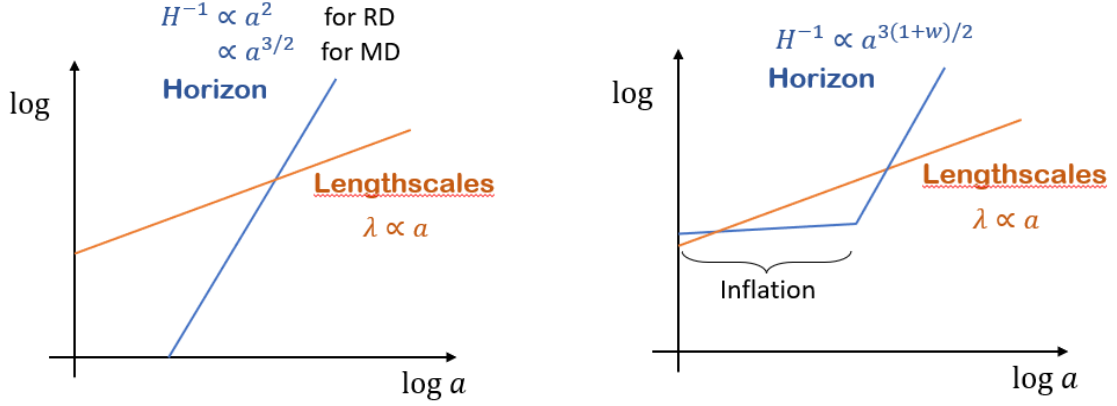
At the heart of this puzzle lies the problem that the Hubble radius (H^{-1}) always grows faster than the lengthscales λ (Fig. 1.3a) throughout the evolution of the universe, hence lengthscales never enter the Hubble horizon in the past. This is resolved by an era in the early universe, where lengthscales grow faster than the Hubble radius (Fig. 1.3b). During this inflationary epoch, causally connected patches are stretched out to large scales beyond horizon size, effectively resolving the Horizon problem.

For the modes to enter Hubble radius in the past, the lengthscales must grow faster than the horizon. Hence the inflationary candidate must satisfy,

$$\frac{d}{dt} \frac{\lambda}{H^{-1}} = \ddot{a}(t) > 0 \implies \rho + 3P < 0, \quad (1.3)$$

where the second equality follows from the Friedmann equation,

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3P). \quad (1.4)$$



(a) The horizon ($\sim H^{-1}$) grows as a^2 during RD and $a^{3/2}$ during MD. This growth is faster than lengthscales, which grow as $\lambda \propto a$. Hence lengthscales never enter the horizon in the past.

(b) In the inflationary epoch, lengthscales enter the Hubble radius in the past, establishing causal contact.

Figure 1.3: The growth of physical lengthscales λ and Hubble radius $\sim H^{-1}$, during RD, MD (Fig. 1.3a) and Inflationary era (Fig. 1.3b).

In other words, the dominant energy density during inflation must follow an equation of state $P = w\rho$ with $w < -\frac{1}{3}$.

1.2.2 Single field slow roll inflation

One of the simplest candidates of inflation, satisfying Eqn. (1.3), involves a single massless scalar field ϕ in a potential $V(\phi)$, minimally coupled to gravity, given by the action,

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]. \quad (1.5)$$

The scalar field is split into a homogeneous time-dependent background field $\bar{\phi}(t)$ and inhomogeneous fluctuations $\varphi(\vec{x}, t)$ as follows,

$$\phi(\vec{x}, t) = \bar{\phi}(t) + \varphi(\vec{x}, t). \quad (1.6)$$

The equation of motion satisfied by the background is given by,

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + \frac{d}{d\bar{\phi}}V(\bar{\phi}) = 0. \quad (1.7)$$

Since the pressure and energy density of this field is given by,

$$P = \frac{1}{2}\dot{\bar{\phi}}^2 - V(\bar{\phi}), \quad \rho = \frac{1}{2}\dot{\bar{\phi}}^2 + V(\bar{\phi}), \quad (1.8)$$

this field acts as an inflaton with $P = -\rho$ (i.e., $w = -1$) if the potential energy of the field $V(\bar{\phi})$ dominates the kinetic energy $\frac{1}{2}\dot{\bar{\phi}}^2$.

This slow roll of the field down a flat potential (Fig. 1.4) is parameterized by the Hubble slow roll parameters,

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{\dot{\bar{\phi}}^2}{2M_{\text{pl}}^2 H^2}, \quad \eta = \epsilon - \frac{\dot{\epsilon}}{2H\epsilon} = -\frac{\ddot{\bar{\phi}}}{H\dot{\bar{\phi}}}. \quad (1.9)$$

Inflation continues till slow roll condition $\epsilon \ll 1$ holds true, and ends when the field is no longer slowly rolling.

The second Friedmann equation gives the Hubble rate $H \propto a^{-\frac{3(1+w)}{2}}$, where we ignore the spatial curvature contribution κ/a^2 , which is quickly driven to zero during inflation. Setting $w \approx -1$, we find the $H \approx \text{constant}$, which gives a scale factor of $a(t) = e^{Ht}$ upto slow roll corrections. Hence the inflationary FRW metric

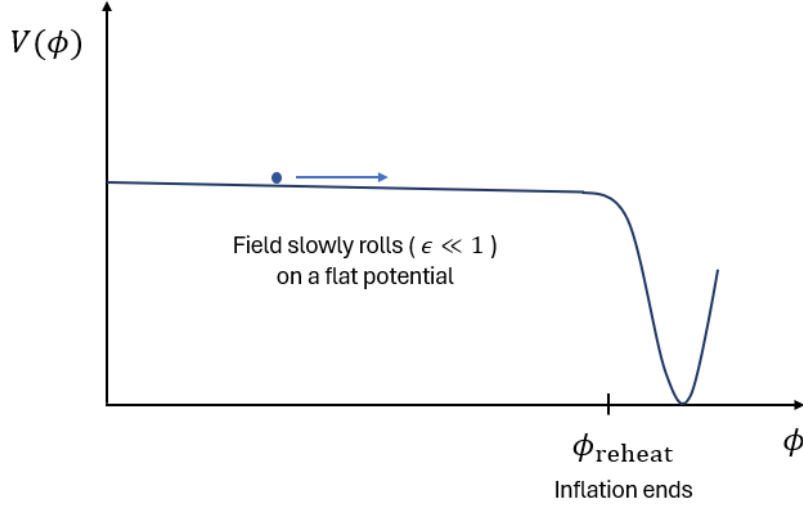


Figure 1.4: The background scalar field slowly rolls down the flat potential, giving an inflationary background.

is given by the *quasi* de-Sitter metric in flat slicing,

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2. \quad (1.10)$$

1.2.3 QFT in curved spacetime

The homogeneous background field in Eqn. (1.6) leads to an inflationary background spacetime, given by Eqn. (1.10). On the other hand, studying the dynamics of the scalar field fluctuations is more complicated, since $\varphi(\vec{x}, t)$ would induce metric fluctuations due to the latter being coupled to energy-momentum tensor perturbations via Einstein's equations. However it is useful to first study the dynamics of the scalar field fluctuations in *rigid de-Sitter*, which approximates upto slow roll corrections the dynamics of the scalar degree of freedom in the theory, as will be discussed in Sec. 1.3.

Massless scalar in rigid de-Sitter

The action of a free massless scalar in de-Sitter background is given by,

$$S_2 = \int d^3x dt a^3 \frac{1}{2} [\dot{\varphi}^2 - \frac{1}{a^2} \partial_i \varphi \partial^i \varphi] . \quad (1.11)$$

Quantization by promoting the field φ to an operator gives,

$$\varphi_{\vec{k}} = a_{\vec{k}} f_k + a_{-\vec{k}}^\dagger f_k^* , \quad (1.12)$$

where the creation and annihilation operators $a_{\vec{k}}, a_{\vec{k}}^\dagger$ must satisfy the following for the equal time commutation relation $[\varphi(\vec{x}, t), \pi(\vec{y}, t)] = i\delta(\vec{x} - \vec{y})$ to hold,

$$[a_{\vec{k}}, a_{\vec{k}'}] = 0, \quad [a_{\vec{k}}, a_{\vec{k}'}^\dagger] = \delta(\vec{k} - \vec{k}') , \quad (1.13)$$

while f_k solves the equation of motion (EOM),

$$\ddot{f}_k + 3H\dot{f}_k + \frac{k^2}{a^2} f_k = 0 , \quad (1.14)$$

$$\implies (af_k)'' + \left(k^2 - \frac{2}{\tau^2}\right)(af_k) = 0 , \quad (1.15)$$

and satisfies the normalization condition,

$$\dot{f}_k f_k^* - f_k \dot{f}_k^* = \frac{i}{a^2} . \quad (1.16)$$

Derivatives with respect to t and τ are denoted with dot and prime respectively. When modes are sub-Hubble, i.e. $\lambda = \frac{a}{k} \ll H^{-1}$, they oscillate with a time dependent frequency $\omega(t) = k/a$. This is apparent from Eqn. (1.14), where we can ignore the second term in the sub-Hubble limit. When modes are super-Hubble, i.e.

$\lambda \gg H^{-1}$, the second term in Eqn. (1.15) can be dropped, which implies that once they exit the horizon, these modes freeze with an amplitude $f_k \sim H^{-1}$.

The most general solution to the second order linear differential equation Eqn. (1.15) is given by the following linear combination,

$$f_k(\tau) = \alpha_k(1 + ik\tau)e^{-ik\tau} + \beta_k(1 - ik\tau)e^{ik\tau}, \quad (1.17)$$

where the k -dependent coefficients α_k and β_k satisfy,

$$|\alpha_k|^2 - |\beta_k|^2 = \frac{H^2}{2k^3}, \quad (1.18)$$

due to Eqn. (1.16). Different choices of α_k corresponds to different bases of mode function g_k , related to f_k as, $g_k = C_1 f_k^* + C_2 f_k$, for some C_1, C_2 . Since both bases correspond to the same quantized field,

$$\varphi_{\vec{k}} = b_{\vec{k}} g_k + b_{-\vec{k}}^* g_k^* = f_k(C_2 b_{\vec{k}} + C_1^* b_{-\vec{k}}^\dagger) + f_k^*(C_1 b_{\vec{k}} + C_2^* b_{-\vec{k}}^\dagger) = a_{\vec{k}} f_k + a_{-\vec{k}}^\dagger f_k^*. \quad (1.19)$$

Thus the choice of α_k directly corresponds to the choice of the creation/annihilation operator and hence the choice of vacuum. The vacuum of one basis is an excited state of the other, since the occupation number of b -particles (i.e., excited by $b_{\vec{k}}^\dagger$) in the vacuum $|0\rangle_{\hat{a}}$ (i.e., the vacuum annihilated by $a_{\vec{k}}$) is not zero,

$${}_{\hat{a}}\langle 0|b_{\vec{k}}^\dagger b_{\vec{k}}|0\rangle_{\hat{a}} = {}_{\hat{a}}\langle 0|C_2 a_{-\vec{k}} C_2^* a_{-\vec{k}}^\dagger|0\rangle_a = |C_2|^2 \delta^3(0). \quad (1.20)$$

One is faced with a similar choice when studying a massless scalar in flat space, where the EOM is again a second order linear differential equation, and the most

general solution for the modes is again a linear combination,

$$f_{k,\text{flat}}(t) = \alpha_k e^{ikt} + \beta_k e^{-ikt}. \quad (1.21)$$

However, in flat space a unique choice of vacuum exists by demanding that the vacuum does not evolve into an excited state in the free theory, i.e., that the vacuum be an eigenstate of the Hamiltonian,

$$H = \frac{1}{2} \int_{\vec{k}} \left(\dot{\phi}_{\vec{k}} \dot{\phi}_{-\vec{k}} + k^2 \phi_{\vec{k}} \phi_{-\vec{k}} \right) = \frac{1}{2} \int_{\vec{k}} \left[a_{\vec{k}} a_{-\vec{k}} F_k + a_{\vec{k}}^\dagger a_{-\vec{k}}^\dagger F_k^* + 2a_{\vec{k}}^\dagger a_{\vec{k}} E_k + \delta^3(0) E_k \right], \quad (1.22)$$

where $F_k = \dot{f}_k \dot{f}_k + k^2 f_k f_k$ and $E_k = \dot{f}_k f_k^* + k^2 f_k f_k^*$. Clearly, unless the mode satisfies the (non-linear) equation $F_k = 0$, the vacuum does not remain invariant under time evolution. This is satisfied by $f_k = e^{ikt}$ and $f_k = e^{-ikt}$, but not by a linear combination of both solutions, hence allowing us to select a time-translationally invariant vacuum as either $\alpha_k = 0$ or $\beta_k = 0$.

A unique vacuum in de-Sitter cannot be chosen by similar arguments, since the analogous condition for the massless scalar in de-Sitter turns out to be the time dependent equation $F_k(t) = \dot{f}_k^2 + \frac{k^2}{a^2(t)} f_k^2 = 0$. The time dependent background results in the instantaneous vacuum to evolve into superposition of Hamiltonian eigenstates, despite the theory being free. Demanding invariance under de-Sitter isometries corresponds to a family of vacua with k -independent coefficients α_k and β_k , also known as α -vacua [83, 84]. However, in the following chapters of this thesis, we work with a choice of the vacuum known as the Bunch Davies vacuum. This choice follows naturally from the fact that modes in the infinite past were sub-Hubble, and effectively experienced flat spacetime. This can be seen from Eqn. (1.15), where the third term $(-\frac{2}{\tau^2})$ can be dropped from the EOM in the sub-Hubble limit, and

the resulting EOM is that of a massless scalar in Minkowski. Hence matching the de-Sitter mode function and its time derivative to Minkowski modes in the infinite past sets $\beta_k = 0$, and we get the Bunch Davies mode function,

$$f_k^{\text{BD}}(\tau) = \frac{H}{\sqrt{2k^3}}(1 + ik\tau)e^{-ik\tau}. \quad (1.23)$$

Using Eqn. (1.12) with the Bunch Davies vacuum, we find the following 2-pt correlation function for the massless scalar field in rigid de-Sitter at late time $\tau \rightarrow 0$,

$$\lim_{\tau \rightarrow 0} \langle 0 | \varphi_{\vec{k}_1} \varphi_{\vec{k}_2} | 0 \rangle = \lim_{\tau \rightarrow 0} f_{k_1} f_{k_2}^* \langle 0 | a_{\vec{k}_1} a_{-\vec{k}_2}^\dagger | 0 \rangle = \frac{H^2}{2k^3} \delta^3(\vec{k}_1 + \vec{k}_2), \quad (1.24)$$

where we have 3-momentum conservation as a consequence of spatially homogeneous background. The $\frac{1}{k^3}$ dependence in the above arises due to dilatation invariance of the de-Sitter background, i.e. $\langle \varphi(\vec{x}) \varphi(\vec{x}') \rangle = \langle \varphi(\lambda \vec{x}) \varphi(\lambda \vec{x}') \rangle$.

1.3 Massless scalar in dynamic de-Sitter : Cosmological perturbation theory

This section follows the lecture notes [85, 86]

Fluctuations in the scalar field $\varphi(\vec{x}, t)$ result in fluctuations of energy-momentum tensor $\delta T^{\mu\nu}(\vec{x}, t)$, which induces inhomogeneous metric fluctuations $h_{\mu\nu}(\vec{x}, t)$ on top of the de Sitter background $\bar{g}_{\mu\nu}(t)$ as,

$$g_{\mu\nu}(\vec{x}, t) = \bar{g}_{\mu\nu}(t) + h_{\mu\nu}(\vec{x}, t). \quad (1.25)$$

We are interested in the evolution of the fluctuations in this *dynamic* de Sitter spacetime.

1.3.1 SVT Decomposition

To study fluctuations in dynamic de Sitter, we first count the degrees of freedom (DOF) in the theory. Exploiting the spatial translational and rotational invariance of the background, we can decompose the components of metric fluctuations into objects that are scalars, vectors and tensors under spatial rotation. Importantly, the EOM of the (rotation-)scalars, (rotation-)vectors and (rotation-)tensors are decoupled, hence they evolve independent of each other. This SVT (scalar-vector-tensor) decomposition follows from Hodge decomposition (for h_{i0}) and Helmholtz theorem (for h_{ij}), and is given by,

$$\begin{aligned} h_{ij} &= a^2(t) \left(\delta_{ij} A + \partial_{ij} B + \partial_{(i} C_{j)} + D_{ij} \right), \\ h_{i0} &= a \left(\partial_i F + G_i \right), \quad h_{00} = E, \end{aligned} \tag{1.26}$$

where A, B, E, F are rotation scalars, contributing in total 4 DOF. C_i and G_i are transverse rotation vectors, i.e. $\partial_i C^i = \partial_i G^i = 0$, hence contributing 2 DOF each. D_{ij} is a transverse ($\partial_i D^{ij} = 0$), traceless ($D_i^i = 0$) rotation tensor, hence contributing 2 DOF. Adding up, we get a total of 10 DOF, which matches with that of the symmetric 4×4 tensor $h_{\mu\nu}$. Together with the scalar field fluctuation, which is a rotation scalar, we have 5 scalar, 4 vector and 2 tensor degrees of freedom.

1.3.2 Identifying non-dynamical DOF in the ADM formalism

However, not all of these degrees of freedom are dynamical. To see this, consider the Einstein's equations $G_{\mu\nu} = -8\pi G T_{\mu\nu}$. Since these are 10 of 2nd order differential equations, the initial conditions are provided in terms of $g_{\mu\nu}(\vec{x}, t_0)$ and $\dot{g}_{\mu\nu}(\vec{x}, t_0)$ at

some initial time t_0 to specify unique evolution. Also, demanding a diffeomorphism invariant theory of GR requires the Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$ to be satisfied. Expanding the time and spatial components, we find Bianchi identity to take the form,

$$\dot{G}^{t\beta} = -\partial_k G^{k\beta} - \Gamma_{\alpha\gamma}^\alpha G^{\beta\gamma} - \Gamma_{\alpha\gamma}^\beta G^{\alpha\gamma}. \quad (1.27)$$

Since the Christoffel connection has terms with atmost one time derivative on the metric, the equation above implies there is no $\ddot{g}$ term in the EOM for $G^{t\beta} = 0$, hence 4 of Einstein's equations are non-dynamical constraints.

To identify the dynamical degrees of freedom, ADM (Arnowitt–Deser–Misner) formalism is considered. The full spacetime, equipped with coordinates x^α , is sliced into constant-time hypersurfaces Σ_t , equipped with coordinates y^a , such that $x^\alpha = x^\alpha(t, y^a)$ (Fig. 1.5). The ADM parameterized metric, given below, reflects this slicing,

$$ds^2 = -N^2 dt^2 + h_{ab}(\mathcal{N}^a dt + dy^a)(\mathcal{N}^b dt + dy^b), \quad (1.28)$$

where h_{ab} is the spatial part of the metric obtained from restriction of the full spacetime metric onto Σ_t , and $N, \mathcal{N}^a$ are the lapse function and the shift vector respectively. Geometrically they correspond to the perpendicular projection length and the parallel projection of the time evolution curve $t^\alpha (= \frac{\partial x^\alpha}{\partial t})$ onto Σ_t respectively, i.e.,

$$\frac{\partial x^\alpha}{\partial t} = N n^\alpha + \mathcal{N}^a \frac{\partial x^\alpha}{\partial y^a}, \quad (1.29)$$

where the time evolution curve connects the same spatial points across different

equal time hypersurfaces, and n^μ is the perpendicular vector to Σ_t , as shown in Fig. 1.5.

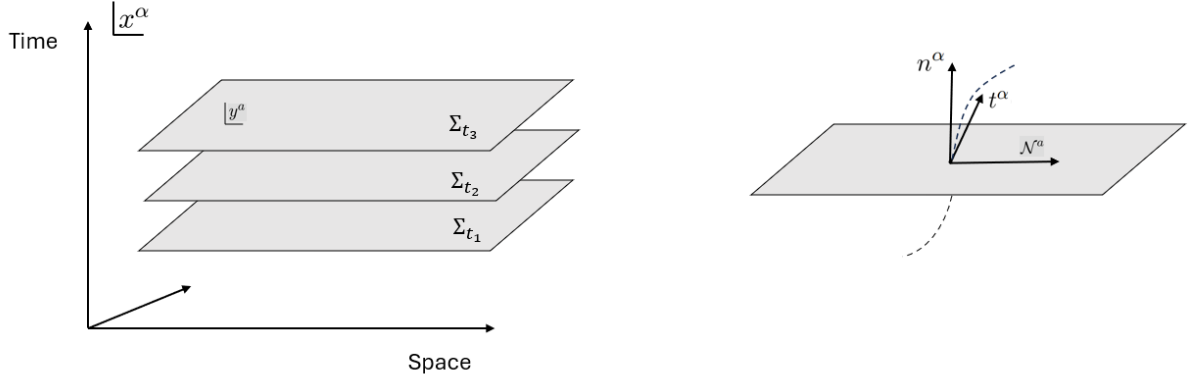


Figure 1.5: In the ADM parameterization of the metric, the full spacetime is foliated into spatial slices at constant time Σ_t . The time evolution vector $t^\alpha = \frac{\partial x^\alpha}{\partial t}$ at a spatial slice has a parallel projection of $\mathcal{N}^a$, and the length of the perpendicular (n^α) projection is N , called the shift vector and the lapse function respectively.

Importantly, this foliation results in the Gauss-Codazzi decomposition of the Ricci scalar R of the full spacetime,

$$R = R^{(3)} - (K^2 - K^{\alpha\beta} K_{\alpha\beta}) - 2\nabla_\alpha (n^\beta \nabla_\beta n^\alpha - n^\alpha \nabla_\beta n^\beta), \quad (1.30)$$

where the last term is a total derivative term, corresponding to the Gibbons–Hawking–York boundary term in the Einstein–Hilbert action. The first term $R^{(3)}$ is the 3–Ricci scalar on the hypersurface, entailing information about the geometry intrinsic to the foliations. The second term carries information about $K_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_{(i}\mathcal{N}_{j)})$, the extrinsic curvature, which captures how the hypersurfaces are embedded in the full spacetime. Eqn. (1.30) leads to the following ADM action,

$$S_{\text{ADM}} = \frac{M_{\text{pl}}^2}{2} \int dt d^3x \sqrt{h} N \left[R^{(3)} - (K^2 - K^{ab} K_{ab}) \right]. \quad (1.31)$$

Clearly the extrinsic curvature involves no time derivatives of $N, \mathcal{N}^i$. Neither does $R^{(3)}$ since, as previously stated, it is the Ricci scalar intrinsic to Σ_t , and hence it is constructed from spatial derivatives of the metric, without relying on the notion of time derivative. Thus this parameterization of the metric establishes the EOM corresponding to the lapse and the shift as non-dynamical, i.e. $\frac{\partial \mathcal{L}}{\partial N} = \frac{\partial \mathcal{L}}{\partial \mathcal{N}^i} = 0$. This implies $N, \mathcal{N}^i$ are constraints, and from Eqn. (1.28), the non-dynamical metric components are $g_{00} = \mathcal{N}^\alpha \mathcal{N}_\alpha - N^2$ and $g_{0i} = \mathcal{N}_i$. Solving these equations order by order in perturbation allows us to express the constraints in terms of the dynamical degrees of freedom, which we will return to in a moment.

1.3.3 Gauge choice

Once the non-dynamical degrees of freedom have been identified, we get the metric component $g_{ij} = h_{ij}$ as dynamical. Thus from Eqn. (1.26) we see that there are 6 dynamical degrees of freedom (A, B, C_i, D_{ij}) . However since the theory is diffeomorphism invariant, we can choose a suitable gauge to remove 4 of these. To see this, we must first consider how the perturbations $(\delta A_{\mu\nu}(\vec{x}, t))$ and background $(\bar{A}_{\mu\nu}(\vec{x}, t))$ transform under a small diffeomorphism $x^\mu \rightarrow x'^\mu = x^\mu + \epsilon^\mu(x)$ to leave the full object $A_{\mu\nu}(\vec{x}, t) = \bar{A}_{\mu\nu}(\vec{x}, t) + \delta A_{\mu\nu}(\vec{x}, t)$ covariant. In practice, the background is kept fixed, and all changes to maintain covariance of the full tensor is attributed to the perturbations. This results in the following change in perturbations at $\mathcal{O}(\epsilon)$,

$$\begin{aligned} \Delta\varphi &= -\epsilon^0 \dot{\bar{\phi}}, & \Delta A &= 2H\epsilon_0, & \Delta B &= -\frac{2}{a^2}\epsilon^S, \\ \Delta C_i &= -\frac{1}{a^2}\epsilon_i^V, & \Delta D_{ij} &= 0, \end{aligned} \tag{1.32}$$

where $\epsilon^\mu = \{\epsilon^0, \partial^i \epsilon^S + \epsilon_V^i\}$, with $\partial_i \epsilon_V^i = 0$. With suitable choice of gauge, one is left with a scalar and a tensor degree of freedom. A few convenient gauge choices are

discussed below,

1. Unitary gauge : Time diffeomorphisms are fixed to set scalar field perturbation to zero, i.e. $\varphi = 0$, with all degrees of freedom in the spatial part of the metric.
2. Comoving gauge : In this gauge, we have $\varphi = B = C_i = 0$ and $A = 2\zeta$, where ζ is the gauge invariant curvature perturbation, given by $\zeta = \frac{A}{2} - \frac{H}{\dot{\phi}}\varphi$. In this gauge, the spatial part of the metric is given by,

$$g_{ij} = a^2(t) \left[(1 + 2\zeta) \delta_{ij} + D_{ij} \right]. \quad (1.33)$$

3. Spatially flat gauge : In this gauge, we have $A = B = C_i = 0$, with the scalar degree of freedom in the scalar field perturbation $\varphi(\vec{x}, t)$, and the spatial part of the metric as,

$$g_{ij} = a^2(t) \left[\delta_{ij} + D_{ij} \right]. \quad (1.34)$$

1.4 Action in the ADM formalism

1.4.1 Power Spectrum of ζ

Finally, we go back to the case of a massless scalar in dynamic de-Sitter, whose action is given by

$$S = \frac{1}{2} \int \sqrt{g} \left(M_{pl}^2 R - (\nabla \phi)^2 - 2V \right). \quad (1.35)$$

We now use the ADM parameterized action Eqn. (1.28) and the Gauss-Codazzi decomposition of the Ricci scalar in Eqn. (1.30). The constraints are solved for and plugged back into the action. To determine the quadratic and cubic order

action, it is sufficient to solve for the constraints upto first order [15], while for the quartic action, we need the constraints solved upto second order [74, 75]. Concretely, consider the constraint EOM in the comoving gauge (Eqn. (1.33)),

$$\begin{aligned} \nabla_i [N^{-1}(E_j^i - \delta_j^i E)] &= 0, \\ R^{(3)} - \frac{2V}{M_{pl}^2} - N^{-2}(E_{ij}E^{ij} - E^2 + \frac{\dot{\phi}^2}{M_{pl}^2}) &= 0. \end{aligned} \quad (1.36)$$

where $E_{ij} = NK_{ij}$ involves spatial derivatives $\nabla_{(i}\mathcal{N}_{j)}$. Since the constraint equations in Eqn. (1.36) are second order in spatial derivatives, the solutions will involve non-local inverse spatial laplacian operators (∂^{-2}) , given by (to first order in ζ),

$$N = 1 + \frac{\dot{\zeta}}{H}, \quad \mathcal{N}_i = \partial_i \left(-e^{-2H} \frac{\zeta}{H} + \frac{\dot{\phi}^2}{2H^2} \partial^{-2} \dot{\zeta} \right). \quad (1.37)$$

Plugging this back in the action and keeping terms leading in slow roll, we get the following quadratic action in ζ ,

$$S_2 = \frac{1}{2} \int d^3x dt \, \epsilon M_{pl}^2 \left[a^3 \dot{\zeta}^2 - a(\partial\zeta)^2 \right], \quad (1.38)$$

and we can identify this action with that of the massless scalar in rigid de-Sitter from Eqn. (1.11), by using $\varphi = \sqrt{\epsilon} M_{pl} \zeta$. Hence under slow roll approximations, sub-Hubble modes of ζ oscillate, and they freeze once they grow to super-Hubble scales. From Eqn. (1.24), we also obtain the following late time 2-pt correlation function for ζ ,

$$\lim_{\tau \rightarrow 0} \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle = \frac{1}{2k^3} \frac{H_*^2}{\epsilon_* M_{pl}^2} \left(\frac{k}{k_*} \right)^{n_s} \delta^3(\vec{k}_1 + \vec{k}_2), \quad (1.39)$$

where the 2 pt function is computed with H_*, ϵ_* , i.e., the values of the Hubble and

the slow roll parameter at the time of horizon crossing t_* of the pivot scale k_* , where $H(t_*) = \frac{k_*}{a(t_*)}$. The spectral tilt is given by,

$$n_s - 1 = \frac{d}{d \log k} \log \left(\frac{H^2}{\epsilon M_{\text{pl}}^2} \right) = 2\eta - 4\epsilon, \quad (1.40)$$

where ϵ and η are the slow roll parameters, defined in Eqn. (1.9). The factor of k/k_* raised to the spectral tilt n_s in Eqn. (1.39) is the additional momentum dependence due to the fact that H, ϵ are not exactly constant throughout inflation, and different modes cross the horizon at different times.

The current Planck measurement of the scalar field amplitude A_s , defined as,

$$\frac{k^3}{2\pi^2} \langle \zeta_{\vec{k}} \zeta_{-\vec{k}} \rangle = A_s \left(\frac{k}{k_*} \right)^{n_s}, \quad (1.41)$$

and the spectral tilt n_s are as follows [12, 13],

$$A_s = 2.098 \pm 0.023 \times 10^{-9}, \quad n_s + 1 = 0.965 \pm 0.004. \quad (1.42)$$

1.4.2 Non-local interactions from constraints

The solution to constraint equations in Eqn. (1.37) in the comoving gauge involves second order spatial derivatives. When these solutions are plugged back into the action, they naturally give rise to non-local interaction terms involving spatial inverse-laplacian operators. This is also observed in other gauge choices, for e.g., plugging back the solutions of these constraints upto first (second) order in φ in the spatially flat gauge of Eqn. (1.34), gives the following quadratic and cubic (quartic) action

[15, 74, 75],

$$S_2 = \frac{1}{2} \int dt d^3x [a^3 \dot{\varphi}^2 - a(\partial\varphi)^2] , \quad (1.43)$$

$$S_3 = \int dt d^3x a^3 \left(-\frac{\dot{\phi}}{4\dot{\rho}} \varphi \dot{\varphi}^2 - a^{-2} \frac{\dot{\phi}}{4\dot{\rho}} \varphi (\partial\varphi)^2 - \dot{\varphi} \partial_i \varphi \partial_i \partial^{-2} \left[\frac{\dot{\phi}^2}{2\dot{\rho}^2} \frac{d}{dt} \left(-\frac{\dot{\rho}}{\dot{\phi}} \varphi \right) \right] \right) , \quad (1.44)$$

$$S_4 = \frac{1}{4} \int dt d^3x a^3 \left[\partial^{-2} \partial_i (\dot{\varphi} \partial_i \varphi) \partial^{-2} \partial_j (\dot{\varphi} \partial_j \varphi) - \frac{1}{\dot{\rho}} \partial^{-2} \partial_j (\dot{\varphi} \partial_j \varphi) (\dot{\varphi}^2 + e^{-2\rho} \partial_i \varphi \partial_i \varphi) + 4 \dot{\varphi} \partial_i \varphi \partial^{-2} (\dot{\varphi} \partial_i \varphi) \right] , \quad (1.45)$$

where we have set $M_{pl}^2 = 1$. We see that the cubic and quartic interaction terms involve inverse laplacian operators ∂^{-2} acting on φ , due to the fact that the constraint equations are second order in spatial derivatives. Such non local interaction terms produce complicated vertex factors in the in-in computations, as we shall see in Chapter 4.

1.5 In-In Formalism

This section follows [87, 88]

1.5.1 In-In correlation functions

In cosmology, we are interested in studying the correlation of field fluctuations at late times, given by,

$$\lim_{\tau \rightarrow 0} \langle \mathcal{O}_I(\tau) \rangle = \lim_{\tau \rightarrow 0} {}_I \langle \psi(\tau) | \mathcal{O}_I(\tau) | \psi(\tau) \rangle_I , \quad (1.46)$$

computed at late time $\tau \rightarrow 0$, and the subscript I indicates this is evaluated in the interaction picture, where both the field and the state evolves. In other words, Eqn. (1.46) is the expectation value of some operator $\mathcal{O}$ on the late-time state $|\psi\rangle$ of the universe, provided $|\psi\rangle$ has evolved from the vacuum state $|\Omega\rangle$ at some early time τ_0 , where we identify all pictures, see Fig. 1.6. This is done because the vacuum evolves into superposition of Hamiltonian eigenstates in de Sitter, due to the time dependence of the background. Modes are effectively free only in the far past, where they effectively experience a flat spacetime in the sub-Hubble limit. However once they grow and couple to gravity, it is unphysical to expect this coupling to switch off, hence there is no “out” vacuum to project onto. As a result, both the bra and the ket states are evolved from the *in*-vacuum in the far past, and is thus called the “in-in” formalism. This is different from the “in-out” formalism used to compute scattering amplitudes in flat space, where the in-state and the out-state are asymptotically free in the far past and future respectively.

Evolving the vacuum state to $|\psi\rangle$, we get,

$$\lim_{\tau \rightarrow 0} \langle \mathcal{O}_I(\tau) \rangle = {}_I \langle \Omega(\tau_0) | U_I^\dagger(0, \tau_0) \mathcal{O}_I(0) U_I(0, \tau_0) | \Omega(\tau_0) \rangle_I = \langle \Omega | U_I^\dagger(0, \tau_0) \mathcal{O}_I(0) U_I(0, \tau_0) | \Omega \rangle, \quad (1.47)$$

where $|\Omega(\tau_0)\rangle_I = |\Omega\rangle$ is the Heisenberg picture ket.

For any general state $|\tilde{\psi}\rangle$ having non-zero overlap with the interacting vacuum, we use the $i\epsilon$ prescription of flat-space QFT to get,

$$\lim_{T \rightarrow \infty(1-i\epsilon)} e^{-iHT} |\tilde{\psi}\rangle = e^{-iE_0 T} |\Omega\rangle \langle \Omega | \tilde{\psi} \rangle, \quad (1.48)$$

which is obtained by inserting a complete basis of Hamiltonian eigenstates, and in the limit of $T \rightarrow \infty(1-i\epsilon)$, the contribution from excited states die faster than

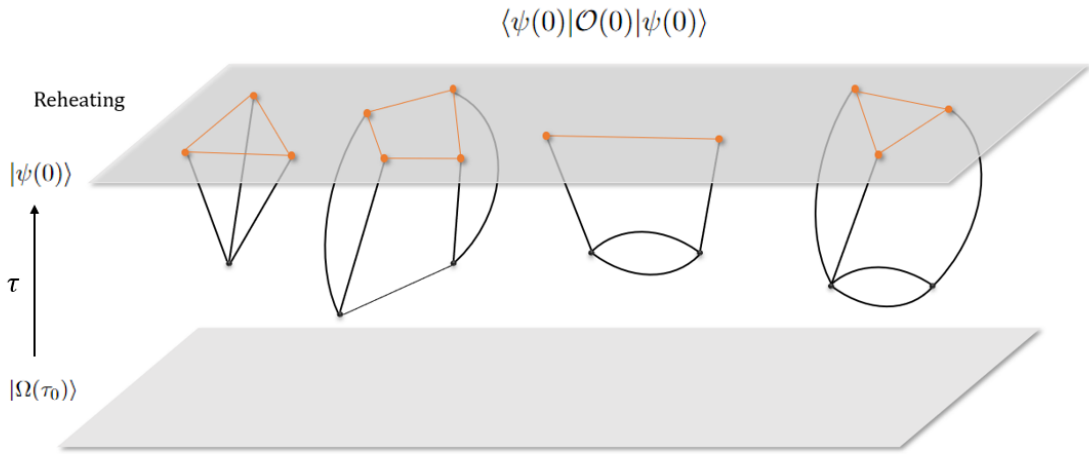


Figure 1.6: The cosmological correlation functions at late time, obtained from evolution of bulk interactions in the past. From left to right, the diagrammatic contributions above correspond to tree level 3-pt and 4-pt from cubic and quartic interactions, as well as 1-loop corrections to the power spectrum and the bispectrum respectively. Here we show contributions coming only from time-ordered part of the in-in correlator. There are, however, additional contributions from the anti-time ordered part of the correlator, which is diagrammatically represented as vertices lying above the $\tau = 0$ surface.

the vacuum contribution in Eqn. (1.48). In general, we may use the free vacuum $|\tilde{\psi}\rangle = |0\rangle$ in the projector equation above, to relate the interacting vacuum to an object with which we can compute the in-in expression perturbatively. This also works in cosmological setup as long as the time τ_0 is early enough such that the relevant modes are deep inside the horizon and, because of the equivalence principle, are effectively in flat space. Therefore, for these early times, the interaction picture operator has the usual flat space form,

$$U_I(\tau, \tau') = e^{iH_0(\tau-\tau_0)} e^{-iH(\tau-\tau')} e^{-iH_0(\tau'-\tau_0)}, \quad (1.49)$$

and using the projector, we have,

$$\begin{aligned} \lim_{T \rightarrow \infty(1-i\epsilon)} U_I(\tau_0, -T)|0\rangle &= e^{-iE_0(T+\tau_0)} |\Omega\rangle \langle \Omega|0\rangle, \\ \lim_{T \rightarrow \infty(1+i\epsilon)} \langle 0|U_I^\dagger(\tau_0, -T) &= e^{iE_0(T+\tau_0)} \langle \Omega| \langle 0|\Omega\rangle. \end{aligned} \quad (1.50)$$

Using this in Eqn. (1.48), we get the in-in correlation function at τ as,

$$\begin{aligned} \lim_{\tau \rightarrow 0} \langle \mathcal{O}(\tau) \rangle &= \langle 0|U_I^\dagger(0, -\infty^+) \mathcal{O}(0) U_I(0, -\infty^-) |0\rangle \\ &= \langle 0|\bar{T} e^{i \int_{-\infty^+}^\tau d\tau' H_{\text{int}}(\tau')} \mathcal{O}(\tau) T e^{-i \int_{-\infty^-}^\tau d\tau' H_{\text{int}}(\tau')} |0\rangle, \end{aligned} \quad (1.51)$$

where the superscript $\pm$ on ∞ defines the integration contour in the far past as $\tau' \rightarrow -\infty(1 \pm i\epsilon)$.

1.5.2 Feynman Rules to compute in-in correlation functions

Equivalently, the in-in correlation can be obtained by following a simple set of Feynman rules, developed in [89, 90]. For a given diagram, such as the ones in

Fig. 1.6, one can have vertices lying above or below the $\tau = 0$ surface, depending on whether the interaction is from the (anti-)time ordered part of the correlator. The Feynman rules state that,

1. Every vertex lying below or above the $\tau = 0$ surface, contributes iV or $-iV$ respectively, alongwith spatial momentum conserved at every vertex. This vertex factor V depends on interactions in the theory and is determined by the action of spatial derivatives acting on the mode functions.
2. Every exchange between interaction vertices at τ_1 and τ_2 , obtained from the time ordered part of in-in correlator, gives a Feynman propagator,

$$G_k(\tau_1, \tau_2) = f_k^*(\tau_1)f_k(\tau_2)\Theta(\tau_1 - \tau_2) + (\tau_1 \longleftrightarrow \tau_2) , \quad (1.52)$$

where k is the momenta flowing through the exchange line. If $\tau_{1,2}$ are interaction vertices coming from anti-time ordered part, the propagator is $G_k^*(\tau_1, \tau_2)$.

3. Every bulk to boundary exchange, or bulk to bulk exchange between vertices lying on opposite sides of the $\tau = 0$ surface, gives a Wightman propagator,

$$W_k(\tau_1, \tau_2) = f_k^*(\tau_1)f_k(\tau_2) , \quad (1.53)$$

where τ_1 is the uppermost vertex, i.e. $\tau_1 > \tau_2$.

4. f_k in Eqns. (1.52) and (1.53) is the mode function, or time derivatives acted on them, depending on H_{int} .
5. Finally, all bulk points and undetermined momenta must be integrated over, leading to time and loop momenta integrals.

As previously stated, a given n -pt function will have contributions from diagrams with all possible combinations of vertices lying above or below $\tau = 0$ surface. However, not all diagrams need to be computed, since half of the cases are mere complex conjugates of the other half. Hence it suffices to only compute half the cases and take the Real part of the answer to obtain the full correlator.

1.5.3 Examples of tree-level in-in computations

As a concrete example, let us consider the computation of the contact 3-pt function of massless scalars ϕ , using cubic $g_3\phi^3$ interaction. Notice since this derivative interaction pulls enough factors of τ in the integrand, the correlators are late-time finite, and hence we compute them at external time $\tau_0 = 0$. This correlator has the diagrammatic contributions in Fig. 1.7, where the contact interaction can lie above or below $\tau = 0$ surface. Using Feynman rules, the time ordered and anti-time

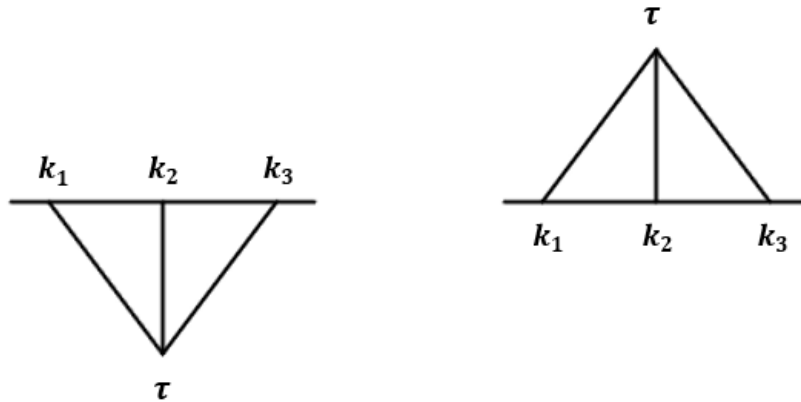


Figure 1.7: The contact contributions to the bispectrum (3-pt. function) from cubic interaction at τ . k_i denote external momenta.

ordered correlators, stripped of momentum conserving delta function $\delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$,

are given by,

$$\begin{aligned}\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{\text{ord.}} &= ig_3 \int_{-\infty}^0 d\tau a(\tau) W_{k_1}(0, \tau) W_{k_2}(0, \tau) W_{k_3}(0, \tau) \\ &= ig_3 f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}^*(0) \int_{-\infty}^0 d\tau \frac{1}{-H\tau} f'_{k_1}(\tau) f'_{k_2}(\tau) f'_{k_3}(\tau),\end{aligned}\quad (1.54)$$

$$\begin{aligned}\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{\text{anti-ord.}} &= -ig_3 \int_{-\infty}^0 d\tau a(\tau) W_{k_1}(\tau, 0) W_{k_2}(\tau, 0) W_{k_3}(\tau, 0) \\ &= -ig_3 f_{k_1}(0) f_{k_2}(0) f_{k_3}(0) \int_{-\infty}^0 d\tau \frac{1}{-H\tau} f'^*_{k_1}(\tau) f'^*_{k_2}(\tau) f'^*_{k_3}(\tau),\end{aligned}\quad (1.55)$$

respectively. The factor of $a(\tau)$ arises due to imposing dilatation invariance interaction. The Wightman propagators are constructed as $W_{k_1}(0, \tau) = f_{k_1}^*(0) f'_{k_1}(\tau)$, where ' denotes derivative with respect to conformal time τ . Clearly, the anti-time ordered contribution in Eqn. (1.55) is obtained from complex conjugating the time ordered contribution in Eqn. (1.54). Using Bunch Davies massless mode functions of Eqn. (1.23), the full correlator is obtained as,

$$\begin{aligned}\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle &= \langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{\text{ord.}} + \langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{\text{anti-ord.}} \\ &= 2 \operatorname{Re} \left[ig_3 f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}^*(0) \int_{-\infty}^0 d\tau \frac{1}{-H\tau} f'_{k_1}(\tau) f'_{k_2}(\tau) f'_{k_3}(\tau) \right] \\ &= -2g_3 \frac{H^5}{4k_1 k_2 k_3} \frac{1}{k_T^3},\end{aligned}\quad (1.56)$$

where we have stripped a momentum conserving delta function $\delta^3(\vec{k}_T)$. We find the correlator to feature a third order pole at total energy $k_T = k_1 + k_2 + k_3$.

Let us also consider the computation of exchange diagram of a 4-pt function. The contributions are given in Fig. 1.8, and we once again find that it is sufficient to

compute only half the number of total cases, since the two diagrams in the second row are complex conjugate of the two diagrams in the first row.

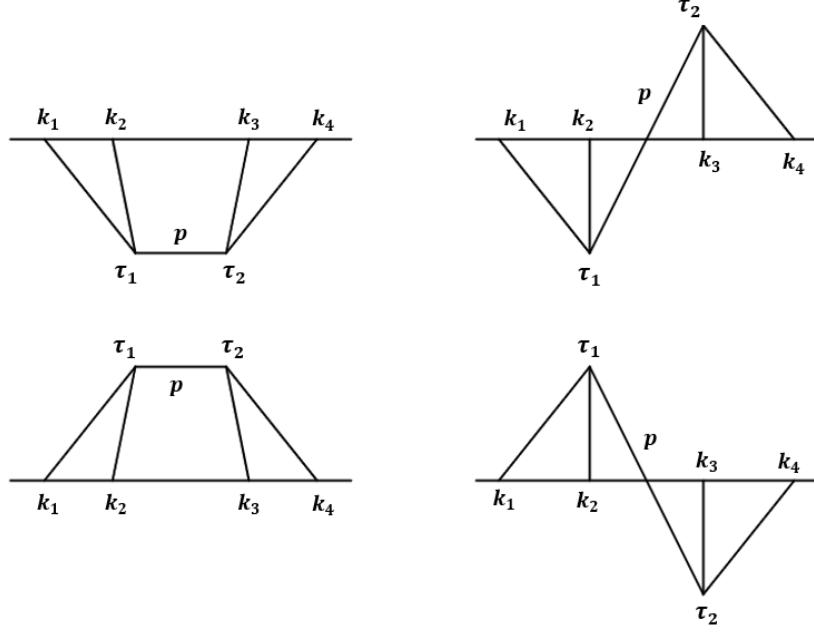


Figure 1.8: The exchange contributions to the trispectrum (4-pt. function) from cubic interaction vertices at $\tau_{1,2}$. k_i, p denote external and internal momentum respectively.

The diagrams in the first row are given by,

$$\begin{aligned}
\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle_{\text{ord.}} &= -g_3^2 \int_{-\infty}^0 d\tau_1 d\tau_2 a(\tau_1) a(\tau_2) W_{k_1}(0, \tau_1) W_{k_2}(0, \tau_1) W_{k_3}(0, \tau_2) \\
&\quad \times W_{k_4}(0, \tau_2) G_p(\tau_1, \tau_2) \\
&= -\frac{g_3^2}{H^2} f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}^*(0) f_{k_4}^*(0) \int_{-\infty}^0 d\tau_1 d\tau_2 \frac{1}{\tau_1 \tau_2} f'_{k_1}(\tau_1) f'_{k_2}(\tau_1) \\
&\quad \times f'_{k_3}(\tau_2) f'_{k_4}(\tau_2) \left[f_p'^*(\tau_1) f_p'(\tau_2) \Theta(\tau_1 - \tau_2) + (\tau_1 \longleftrightarrow \tau_2) \right], \quad (1.57)
\end{aligned}$$

$$\begin{aligned}
\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle_{\text{unord.}} &= g_3^2 \int_{-\infty}^0 d\tau_1 d\tau_2 a(\tau_1) a(\tau_2) W_{k_1}(0, \tau_1) W_{k_2}(0, \tau_1) W_p(\tau_2, \tau_1) \\
&\times W_{k_3}(\tau_2, 0) W_{k_4}(\tau_2, 0) \\
&= \frac{g_3^2}{H^2} f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}(0) f_{k_4}(0) \int_{-\infty}^0 d\tau_1 d\tau_2 \frac{1}{\tau_1 \tau_2} f'_{k_1}(\tau_1) f'_{k_2}(\tau_1) \\
&\times f'_{k_3}(\tau_2) f'_{k_4}(\tau_2) f'_p(\tau_2) f'_p(\tau_1), \tag{1.58}
\end{aligned}$$

where $\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle_{\text{ord.}}$ is obtained using both cubic interactions coming from time-ordered part, whereas $\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle_{\text{unord.}}$ is obtained using one cubic interaction each from the time-ordered and anti-time ordered part of the in-in correlator.

Evaluating the integrals, we find the full correlator with all four contributions given in Fig. 1.8 to be,

$$\begin{aligned}
\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle &= 2g_3^2 \frac{H^8}{8k_1 k_2 k_3 k_4 k_T^5} \frac{p}{k_T^5} \left[\frac{1}{(k_{34} + p)^3} \left\{ -2k_1 p - 2k_2 p + 10(k_{34} + p)^2 \right. \right. \\
&- 5(-k_{12} + p)(k_{34} + p) + k_{12}^2 + p^2 \Big\} + \frac{1}{(k_{12} + p)^3} \left\{ 5k_2(k_{34} + 3p) \right. \\
&\left. \left. + k_{34}(k_{34} + 3p) + 5k_1(4k_2 + k_{34} + 3p) + 10k_1^2 + 10k_2^2 + 6p^2 \right\} \right], \tag{1.59}
\end{aligned}$$

where $k_{34} = k_3 + k_4$ and so on. We find the correlator has a total energy pole $1/k_T^5$, as well as poles $1/(k_1 + k_2 + p)^3$ and $1/(k_3 + k_4 + p)^3$ at energy entering vertices τ_1 and τ_2 respectively.

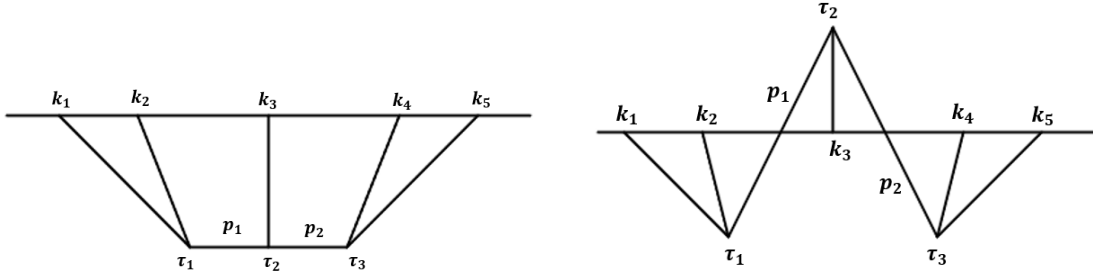
1.6 The analytic structure of cosmological correlation functions

There is extensive literature on the structure of cosmological correlation functions at tree level (see e.g. [15, 18, 19, 20, 21, 31, 32, 91, 30, 92, 93, 94, 22, 78, 95] and

references within). In this section, we discuss a few of these results.

1.6.1 Singularity structure of tree level correlation functions

The singularity structure of correlation functions has been studied in detail at tree level [96, 20, 78, 97, 79, 33]. Correlation functions involving massless fields in particular exhibit poles whenever a vertex or a subgraph becomes energy conserving. A subgraph is defined as a set of (*consecutive*) vertices, where every vertex in the set must be connected to atleast one other vertex in the set by a Feynman propagator.



(a) The 5-pt function with all interaction vertices lying below $\tau = 0$ surface. (b) The 5-pt function with interaction vertices lying above and below $\tau = 0$ surface.

Figure 1.9: Diagrams contributing to the 5-pt function at tree level with cubic interactions. The interaction vertices are labeled by $\tau_{1,2,3}$. k_i and p_j denote external and internal momenta respectively.

As an example, consider a double-exchange contribution to the 5-pt function with cubic interactions in flat space in Fig. 1.9a. The bulk to bulk exchange propagators $G_{p_1}(\tau_1, \tau_2)$ and $G_{p_2}(\tau_1, \tau_2)$ contribute Heaviside step functions leading to nested time integrals, which in turn result in energy mixing in the arguments of the exponentials in the integrand, arising from the mode functions. Consider one such time ordering $\Theta(\tau_1 - \tau_2)\Theta(\tau_2 - \tau_3)$, given by the nested time integrals,

$$\int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 e^{i(k_1+k_2-p_1)\tau_1} e^{i(k_3+p_1-p_2)\tau_2} e^{i(k_4+k_5+p_2)\tau_3} . \quad (1.60)$$

The integral structure implies that upon evaluation of the τ_3 integral, a simple pole at $k_4 + k_5 + p_2$ would be produced. This is the pole produced when τ_3 vertex becomes energy conserving. Similarly evaluating τ_2 and τ_1 integrals produce simple poles at $k_3 + k_4 + k_5 + p_1$ and $k_1 + k_2 + k_3 + k_4 + k_5$, which is the energy entering the subgraph consisting of vertices τ_2, τ_3 and the full graph respectively. Similarly, considering the other time orderings yield nested time integrals, that produce poles at energies entering all possible subgraphs, giving *partial energy poles*. The same argument follows for de Sitter correlators, however unlike flat space correlators, the partial and total energy poles produced are higher order, since mode functions and measure contribute additional factors of τ in the integrand.

However, at 5-pt we also have contributions from diagrams such as Fig. 1.9b. In this diagram, the only subgraphs are trivial, i.e. they either consist of only one vertex or the full graph. This is because all internal lines are Wightman propagators, which do not produce nested time integrals. The integral structure of this contribution in flat space is given by,

$$\int_{-\infty}^0 d\tau_1 \int_{-\infty}^0 d\tau_2 \int_{-\infty}^0 d\tau_3 e^{i(k_1+k_2+p_1)\tau_1} e^{-i(p_1+k_3+p_2)\tau_2} e^{i(p_2+k_4+k_5)\tau_3}. \quad (1.61)$$

Thus the evaluation of the time integrals produce simple poles at the energies entering the vertices τ_1, τ_2, τ_3 , i.e. at $k_1 + k_2 + p_1$, $k_3 + p_1 + p_2$ and $p_2 + k_4 + k_5$ respectively. Once again, the same argument follows for de Sitter correlators, and we get higher order poles instead of simple poles whenever the vertices become energy conserving.

However, this systematic understanding only existed at tree level correlators, and hence for loop integrands. For loop correlators, the analogous structure of poles and branch cuts was previously unexplored.

1.6.2 From Wavefunction to Correlations : Total energy branch cuts

Below, we briefly review the wavefunction of the universe formalism, the map between wavefunction coefficients and cosmological correlators, and the Cosmological KLN theorem.

Wavefunction of the Universe

Consider a field $\hat{\Phi}(\vec{x}, \tau)$ and its eigenstates $|\Phi(\vec{x}, \tau)\rangle$ corresponding to the field configuration eigenvalue of $\Phi(\vec{x}, \tau)$. The wavefunction of the universe [15, 98, 99, 100] in the ground state gives us the propagation amplitude from the zero field eigenstate in infinite past to some configuration $\phi(\vec{x})$ late time τ_0 ,

$$\Psi[\phi] = \int_{\Phi(\vec{x}, \tau=-\infty^-)=0}^{\Phi(\vec{x}, \tau_0)=\phi(\vec{x})} \mathcal{D}\Phi e^{iS[\Phi]}. \quad (1.62)$$

The wavefunction can be expanded as follows for small fluctuations,

$$\Psi[\phi] = \exp \left[-\frac{1}{2} \int d^3k_1 d^3k_2 \psi_2(\vec{x}_1) \phi(\vec{k}_1) \phi(\vec{k}_2) + \sum_{n=3}^{\infty} \frac{1}{n!} \int d^3k_1 \dots d^3k_n \psi_n(\vec{k}) \phi(\vec{k}_1) \dots \phi(\vec{k}_n) \right], \quad (1.63)$$

where ψ_n are the wavefunction coefficients. These can be computed diagrammatically using Feynman rules, similar to in-in correlation functions. However there are a few crucial differences [101, 102, 103, 104] in the Feynman rules, such as different bulk-bulk propagator,

$$K_k(\tau) = \frac{f_k(\tau)}{f_k(\tau_0)}, \quad (1.64)$$

and different bulk-boundary propagator,

$$G_k(\tau_1, \tau_2) = iP_k \left(\Theta[\tau_1 - \tau_2] K_k^*(\tau_1) K_k(\tau_2) + \Theta[\tau_2 - \tau_1] K_k^*(\tau_2) K_k(\tau_1) - K_k(\tau_1) K_k(\tau_2) \right), \quad (1.65)$$

due to different boundary conditions in the past. Here P_k is the momentum conserving delta function stripped 2-pt function, i.e. $\langle \hat{\phi}_{\vec{k}_1} \hat{\phi}_{\vec{k}_2} \rangle = P_k \delta^3(\vec{k}_1 + \vec{k}_2)$. Also, due to the difference in the integration domain, unlike correlators the diagrams of wavefunction coefficients do not involve vertices on both sides of spatial slice at late time τ_0 .

The Cosmological KLN theorem

The correlators of late time field fluctuations are obtained from wavefunction coefficients via the Born rule,

$$\langle \hat{\phi}_{\vec{k}_1} \dots \hat{\phi}_{\vec{k}_n} \rangle = \frac{\int \mathcal{D}\phi \phi_{\vec{k}_1} \dots \phi_{\vec{k}_n} |\Psi[\phi]|^2}{\int \mathcal{D}\phi |\Psi[\phi]|^2}. \quad (1.66)$$

To compute tree level correlators using the above map, it suffices to compute the tree level wavefunction coefficients using saddle point approximation $\Psi[\phi] \simeq \exp[iS[\Phi_d]]$. However, at loop level, correlators are obtained from wavefunction coefficients at loops, as well as momentum integrals over tree level wavefunction coefficients. The latter are often referred to as *classical-loops* [60].

The various wavefunction contributions from the map in Eqn. (1.66) can often lead to cancellations of poles, leading to a cleaner analytic structure of the correlators. The simplicity of the analytic structure of correlation functions over the corresponding wavefunction counterparts has been discussed in great detail [105, 106]. One such discussion is regarding the absence of total energy branch cut in cor-

relations of massless scalar field fluctuations, and is known as the Cosmological KLN theorem [34], since it is reminiscent of the KLN (Kinoshita–Lee–Nauenberg) theorem in flat space, involving cancellation of IR divergences in the inclusive scattering cross-section.

The cosmological KLN theorem is argued from the various wavefunction contributions to the correlator at loop level, alongwith the Cosmological Tree theorem. The latter is obtained from the bulk to bulk propagator for the wavefunction coefficient being expressed in terms of the retarded propagator $G_k^{\text{ret.}}$ as,

$$G_k^{\text{ret.}}(\tau_1, \tau_2) = 2P_k \text{Im}[K_k(\tau_1)K_k^*(\tau_2)]\Theta[\tau_1 - \tau_2] = G_k(\tau_1, \tau_2) - 2P_k K_k(\tau_1) \text{Im}[K_k(\tau_2)]. \quad (1.67)$$

When applied to diagram involving loops, the cosmological tree theorem provides a way to cut open loops and express the wavefunction coefficient as a sum of terms involving momentum integral over tree level wavefunction coefficients, with one or more bulk-boundary propagators being replaced with $\text{Im}(K_k)$. In mapping the wavefunction to the correlator, such contributions combine with the classical loops, yielding terms where none of the partial energy poles produced from time integrals ever mix the loop momenta and total energy. The cosmological KLN theorem follows from the observation that since the time integral never produces a threshold in total energy, hence the correlator at loop level can only produce poles in total energy, never a branch cut.

We discuss in Chapter 3 of this thesis, that while the arguments above hold for flat space, correlators in de-Sitter indeed feature total energy branch cuts. We point out the loophole in the arguments of the cosmological KLN theorem, where important contributions due to the time dependence of the background are missed. These contributions produce total energy branch cuts from time integrals, rather

than momentum integrals, in de-Sitter. These total energy branch cuts are a feature of time-dependent backgrounds.

1.6.3 Flat space limit of correlators

It is well known that for cosmological correlators at tree level, the leading singular terms in the limit of the total energy taken to zero recovers the flat space scattering amplitude corresponding to the same process, while the leading partial energy is given by the product of a lower-point correlator and a lower-point scattering amplitude [20, 33, 76, 77, 78, 79, 96, 97]. Such a correspondence is expected, since in the far past, the Bunch Davies modes were deep inside the Hubble radius and hence effectively experienced flat space conditions. However, contribution from early times in the time integral of the in-in correlation is exponentially suppressed due to the $i\epsilon$ prescription. This suppression goes away in the limit of energy argument in the exponential (either total energy ω_T or partial energy ω_i) goes to zero, hence probing the early times and recovering the flat space answer.

In particular, for a tree level n -pt diagram D_n with V vertices, I internal lines and a total of N spatial and time derivatives acting on all interaction fields, the leading singular term in the limit of total energy $\omega_T \rightarrow 0$ is given by [33],

$$\lim_{\omega_T \rightarrow 0} D_n = (H^{\alpha+n}) (-1)^{n+\sum S_V} i^n \frac{(\alpha!) (-1)^\alpha}{(i\omega_T)^{\alpha+1}} \mathcal{A}_n \left(\prod_{i \in \text{ext.}} \frac{1}{2\omega_i^2} \right), \quad (1.68)$$

where $\alpha = -4V + 2I + N + n$, $\mathcal{A}_n$ is the flat space n -pt scattering amplitude, $\sum S_V$ denotes the total number of time derivatives acting on all fields contracting in the diagram, and the product $\prod_i$ runs over all external legs. This is the *flat space limit* for cosmological correlators. Notice this is unrelated to the flat space limit in the sense of $H \rightarrow 0$, and the $\lim$ in Eqn. (1.68) is to be understood as extracting the

leading singular term when $\omega_T \rightarrow 0$.

This relation holds for tree level, and hence for loop integrands. However, such a flat space limit was not known at the level of loop correlation functions.

1.7 Loops in Cosmological Correlators

Loop corrections to inflationary correlators were not studied for a long time in the literature, since they are expected to be extremely small due to suppression by factors of slow roll and H/M_{pl} , and hence are practically unobservable. As an example, consider the power spectrum $\langle \zeta_k^2 \rangle$ at tree versus at 1-loop. The amplitude of the tree level power spectrum $H^2/(\epsilon M_{pl}^2)$ has been measured from the CMB temperature fluctuations to be $\sim 10^{-10}$, and since $\epsilon \ll 1$ during inflation, we have H/M_{pl} during inflation to be much smaller than 10^{-5} . Now, the cubic action $S_3(\zeta) \sim \epsilon^2 M_{pl}^2 \int \mathcal{O}(\zeta^3)$. A rough back of the envelope calculation gives us the 1-loop power spectrum to be $\langle \zeta_k^2 \rangle_{1\text{-loop}} \sim \frac{H^4}{M_{pl}^4} \mathcal{O}(\frac{1}{k^3})$, with two interaction insertions contributing $\sim \epsilon^4 M_{pl}^4$ and 8 ζ modes, each contributing $\sim H/(\sqrt{\epsilon} M_{pl})$ (see Fig. 1.10). This implies that not only is the power spectrum at 1-loop slow roll ϵ suppressed, it also suffers a $(H/M_{pl})^2$ suppression, and hence it is *atleast* 10 orders of magnitude smaller than the tree level result. This is an incredibly small effect and it is practically inaccessible from experiments.

However, Weinberg first stressed that despite the loop effects being extremely tiny, one should still study these corrections to explore and better understand the inflationary theory. With this motivation, in [1] he computes the 1-loop correction to the power spectrum in Fig. 1.10 and obtains,

$$\langle \zeta_k^2 \rangle_{1\text{-loop}} \sim \frac{H^4}{M_{pl}^4} \frac{1}{k^3} \left(c + \log \frac{k}{\mu} \right), \quad (1.69)$$

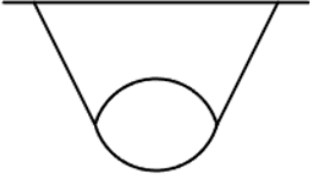
<p>2-pt at tree</p> $\langle \zeta_k^2 \rangle_{\text{tree}} \sim \frac{H^2}{\epsilon M_{pl}^2} \frac{1}{k^3}$ <p>From CMB : $\frac{H^2}{\epsilon M_{pl}^2} \sim 10^{-10}$</p>	<p>2-pt at 1-loop</p>  $\langle \zeta^2 \rangle_{1\text{-loop}} \sim \frac{H^4}{M_{pl}^4} \mathcal{O}\left(\frac{1}{k^3}\right)$
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Figure 1.10: The leading contribution to the power spectrum at 1-loop, coming from two cubic interaction insertions in the bulk, is $\sim \epsilon \frac{H^2}{M_{pl}^2}$ suppressed with respect to the tree level result.

where c is a scheme-dependent constant and μ is the renormalization scale. Subsequent studies on loops in inflation also found a similar $\log(k/\mu)$ in their computations [107, 108, 109, 110, 111]. However, it was pointed out in [2] that this logarithm implies a mistake was made in the computation, since the argument of this logarithm is the ratio of a comoving scale k and a physical scale μ . Apart from the fact that this logarithm breaks dilatation invariance, there is another problem with such a 1-loop correction. Importantly, one also obtains logarithms in flat space loop computations of the type $\log(s/\mu^2)$, where s is the Mandelstam invariant, and the renormalization scale is chosen close to the external kinematics to avoid the problem of large logarithms. However the same cannot be done with the logarithm obtained in Weinberg's answer, since a physical scale μ cannot be compared against a comoving scale k . One can convert the comoving k to a physical scale $k_{\text{phys}} = k/a$, but this implies that the power spectrum is sensitive to the scale factor. However, the scale factor is, by definition, unobservable, corresponding only to a rescaling of coordinates.

As was shown in [2], the subtlety lies in recognizing that unlike in flat space, modes in de Sitter have non-trivial dependence on dimension due to the time de-

pendence of the background. Hence, when implementing dimensional regularization to regulate loop divergences, using 3-dimensional modes and measure will lead to *unphysical* logarithms of k/μ , which cancel from $\mathcal{O}(\delta)$ corrections of modes and measure, and get corrected to $\log(H/\mu)$ once $d = 3 + \delta$ dimensional modes and measure are used.

The previous observation naturally leads to the investigation of the analytic structure of higher point functions at loops. In particular, momentum conservation imposes both legs of the power spectrum to have the same scale k , but does not limit the external legs of the bispectrum to have the same scale, and one can in principle observe a richer structure involving logarithms of ratios of comoving scales. It is also interesting to investigate whether, similar to the power spectrum, *unphysical* logarithms cancel out in loop corrections to the bispectrum and higher point functions, once dimensional regularization is implemented properly.

As discussed in Sec. 1.1, loop correlators are also essential to understand how unitarity, locality and causality constrain the structure of correlators. However this requires a systematic understanding of the singularity structure of cosmological correlators at loop level, which was missing. There has been some progress on understanding loop corrections to wavefunction coefficients [101, 102, 103, 104], however the analytic structure of wavefunction coefficients can be significantly different from that of correlators due to different choice of boundary conditions [105, 106, 112, 113].

1.8 Notation and Convention

In this section, we set the notations and conventions followed in this thesis.

We work in the inflationary background, well approximated by the flat de Sitter

metric,

$$ds^2 = \frac{-d\tau^2 + d\vec{x}^2}{(H\tau)^2}, \quad (1.70)$$

where H is the Hubble parameter. Derivatives with respect to the cosmic time t and conformal time τ are denoted by a dot ($\dot{}$) and a prime ($\prime$) respectively, unless stated otherwise. The external momenta are denoted by $\{\vec{k}_i\}$ and exchange momenta by $\{\vec{p}_i\}$ with $\vec{p}_1$ & $\vec{p}_2$ reserved to label the loop arms. For *on-shell* propagators, the modulus of momenta are referred to as energies and denoted by momentum label without the vector sign e.g., $k_i = |\vec{k}_i|$. However, we will often treat the *energies* of external legs as independent variables (this is the *off-shell* limit of the correlator) and in this case the energies are denoted by $\{\omega_i\}$. The energy entering the loop site is denoted by s , given by $s = |\vec{s}| = |\vec{p}_1 + \vec{p}_2|$. Often, we will denote sum of off-shell energies in the following manner : $\omega_1 + \omega_2 + \omega_5 = \omega_{125}$ and so on. Whenever we set $M_{pl}^2 = 1$, this convention is stated explicitly.

The correlation functions are computed via the in-in formalism. To be precise, we employ in-in Feynman rules to compute the diagrams. In this work, the notation we follow for the diagrams are as follows : vertices drawn at $\tau < 0$ come from the time ordered part of the in-in correlator, and vertices drawn at $\tau > 0$ come from anti-time ordered part. These vertices are also called “right” (or ‘+’) and “left” (or ‘−’) vertices respectively ([1, 89]). Of course a n -point correlator will have contributions from diagrams with all possible combinations of left and right vertices. However not all of these diagram need to be computed since half of these combinations are obtained from complex conjugation of the other half.

Given a mode function $f_k(\tau)$, the propagators are defined as follow:

$$G_{++}(k; \tau_1, \tau_2) = f_k^*(\tau_1)f_k(\tau_2)\Theta(\tau_1 - \tau_2) + f_k(\tau_1)f_k^*(\tau_2)\Theta(\tau_2 - \tau_1), \quad (1.71)$$

$$G_{+-}(k; \tau_1, \tau_2) = f_k(\tau_1)f_k^*(\tau_2), \quad (1.72)$$

$$G_{--}(k; \tau_1, \tau_2) = G_{++}^*(k; \tau_1, \tau_2), \quad (1.73)$$

$$G_{-+}(k; \tau_1, \tau_2) = G_{+-}^*(k; \tau_1, \tau_2), \quad (1.74)$$

where $+$ ($-$) in the subscripts denotes whether τ_1, τ_2 lie below (above) the $\tau = 0$ surface.

Dimensional regularization Throughout this thesis we will employ dimensional regularization (dim reg) to tame the divergences in loop integrals. In flat space this amounts to computing the loop integral in d -dimensions, then setting $d = 3 + \delta$. In de Sitter this is more subtle: mode functions in de-Sitter depend on the dimension. Therefore, when we compute correlators with dimensional regularization, we need to use the appropriate d -dimensional mode functions as well, as they may include crucial contributions to the structure of the correlator. A famous example would be the $\log(k)$ term mentioned in the introduction: using the incorrect mode function would lead us to conclude that there is secular divergence in the one loop correction of the power spectrum [1], and this is tamed by properly regulating the mode functions as well [2].

For correlators in flat space, since there is no dimensional dependence of modes, simple 3-dimensional flat space modes, given by,

$$f_k(\tau) = \frac{1}{\sqrt{2k}} \exp(ik\tau), \quad (1.75)$$

are employed in loop computations.

In de Sitter, the mode function in d -dimensions for a scalar field with mass m is given by,

$$f_k(\tau) = H^{-\frac{1}{2}}(-H\tau)^{d/2} H_{i\nu}^{(2)}(-k\tau), \quad (1.76)$$

where $\nu = \sqrt{\frac{m^2}{H^2} - \frac{d^2}{4}}$. However it is difficult to compute correlators in arbitrary d -dimension: in general we would need to compute time integrals and loop momentum integrals with Hankel functions, and these don't usually give nice analytic expressions.

Fortunately for our purposes there are two major simplifications. The first is that we are mainly interested in massless fields ($m^2 = 0$) and conformally coupled fields ($m^2 = 2H^2$) in $d = 3$. For $d = 3$ these Hankel functions reduces to simple plane waves multiplying some polynomials. The second simplification comes from modifying the mass slightly when we perform dimensional regularization: on top of setting $d = 3 + \delta$, we also modify the mass as $m^2 \rightarrow m^2 + H^2 \left(\frac{3}{2}\delta + \frac{\delta^2}{4} \right)$ [36, 45]¹. Under this scheme the mode function simplifies into the following:

$$f_k(\tau) = (-H\tau)^{\delta/2} \frac{H}{\sqrt{2k^3}} (1 - ik\tau) e^{ik\tau} \quad (\text{for massless scalar}), \quad (1.77)$$

$$f_k(\tau) = (-H\tau)^{\delta/2} \frac{(-H\tau)}{\sqrt{2k}} e^{ik\tau} \quad (\text{for conformally coupled scalar}). \quad (1.78)$$

In practice we would frequently need to expand in δ to obtain analytic expressions, and this generally produces logarithmic terms. For example, for the massless scalar we have:

$$f_k(\tau) = \left(1 + \frac{\delta}{2} \log(-H\tau) \right) \frac{H}{\sqrt{2k^3}} (1 - ik\tau) e^{ik\tau} + \mathcal{O}(\delta^2). \quad (1.79)$$

¹This is different from the scheme used in [2] where the mass is not modified. While the scheme of modifying the mass simplifies the calculations further, one should be careful when employing this modified scheme on gauge fields or gravitons, since introducing a mass may break gauge invariance.

Since we are mainly concerned with one loop corrections, it suffices to expand the mode function to $\mathcal{O}(\delta)$, but in principle one would need the higher order terms for higher loop corrections as well.

Similarly, the time integral measure is also dependent on the dimension d , and receives correction in dimensional regularization,

$$a^\delta(\tau) = (-H\tau)^{-\delta} = (1 - \delta \log(-H\tau) + \mathcal{O}(\delta^2)). \quad (1.80)$$

Off-shell correlators In flat space the mode functions are simply plane waves, and as a result the correlators always depends on the total energy entering each vertex (or subgraphs of these vertices), regardless of the number of external legs attached to it. For example, if in a contact diagram there are N_a external legs attached to a vertex A , then the correlator only depends on the external energy $|\mathbf{k}_i|$ in the combination:

$$\Omega_A = \sum_{i=1}^{N_a} \sqrt{|\mathbf{k}_i|^2 + m_i^2}. \quad (1.81)$$

For de Sitter, if we look at fields with general mass, the correlator will be some complicated function of the energies of individual external legs $k_i = |\mathbf{k}_i|$, rather than depending on simply their sum. However, since we are mainly working with massless and conformally coupled scalars, the mode functions are plane waves multiplied by polynomials of conformal time and energies. This implies that the time integral at each vertex takes the following form:

$$\int_{-\infty}^{\tau'} d\tau \text{Poly}(k, \tau) e^{i\Omega\tau}, \quad (1.82)$$

where $\Omega = \sum_i k_i$ is the energy entering a vertex if τ is the first vertex to be integrated over, and the energy entering a subgraph otherwise. $\text{Poly}(k, \tau)$ is simply a polyno-

mial in energies and conformal time. It is not hard to see that after time integration, the denominators will depend only on the external energies in the combination Ω .

It is useful to define the *off-shell correlator* as an analytic extension of the physical correlator, where the energy of the internal legs remains *on-shell*, i.e. $p_i = \vec{p}_i$, but the energy of the external legs ω_i is treated as independent from the momentum $\mathbf{k}_i$. Physical answers are obtained when we put the external energy *on-shell*, i.e. take $\omega_i = |\vec{k}_i|$.

Analytic continuation of the off-shell energy has been fruitful in understanding the structure of wavefunctions and correlators [114, 34, 115]. In particular, it is possible to keep track of the analytic structure in terms of ω , and in the case of the wavefunction this has led us to rules which determine the possible poles and branch points. It will be helpful for us to work with these off-shell energies here as well. As a result we will work with the off-shell external energies ω in our dS examples, since it contains interesting features about the analyticity of the correlators.

Cubic and Quartic order action in the ADM formalism The metric in the spatially flat gauge is given by,

$$ds^2 = -N^2 dt^2 + e^{2\rho(t)}(dx^i + N^i dt)(dx^j + N^j dt), \quad (1.83)$$

where $\dot{\rho}(t)$ is the Hubble parameter (H), considered to be time independent here to leading order in the slow-roll parameters. The lapse function (N) and the shift vector (N_i) are the constraints, evaluated to second order, to get the action till fourth order in perturbation². Since the constraint equations are of second order in spatial derivatives, the constraint solutions contain inverse laplacian, giving non-

²higher order terms in the constraints multiply terms that vanish since they satisfy equation of motion; see Appendix of [116]

local terms in cubic and quartic actions. The background and fluctuations of the scalar field are denoted by,

$$\phi(x, t) = \bar{\phi}(t) + \varphi(x, t). \quad (1.84)$$

Focusing on the scalar sector, the quadratic and cubic action at leading order in slow roll in the spatially flat gauge are given by [15],

$$\begin{aligned} S_2 &= \frac{1}{2} \int dt d^3x \left[e^{3\rho} \dot{\varphi}^2 - e^\rho (\partial\varphi)^2 \right], \\ S_3 &= \int dt d^3x e^{3\rho} \left(-\frac{\dot{\phi}}{4\dot{\rho}} \varphi \dot{\varphi}^2 - e^{-2\rho} \frac{\dot{\phi}}{4\dot{\rho}} \varphi (\partial\varphi)^2 - \dot{\varphi} \partial_i \beta \partial_i \varphi \right), \end{aligned} \quad (1.85)$$

where

$$\partial^2 \beta = \left[\frac{\dot{\phi}^2}{2\dot{\rho}^2} \frac{d}{dt} \left(-\frac{\dot{\rho}}{\dot{\phi}} \varphi \right) \right]. \quad (1.86)$$

The quartic action, to leading order in slow-roll, computed in [74, 75] is given by,

$$\begin{aligned} S_4 &= \frac{1}{4} \int dt d^3x e^{3\rho} \left[\partial^{-2} \partial_i (\dot{\varphi} \partial_i \varphi) \partial^{-2} \partial_j (\dot{\varphi} \partial_j \varphi) - \frac{1}{\dot{\rho}} \partial^{-2} \partial_j (\dot{\varphi} \partial_j \varphi) (\dot{\varphi}^2 + e^{-2\rho} \partial_i \varphi \partial_i \varphi) \right. \\ &\quad \left. + 4 \dot{\varphi} \partial_i \varphi \partial^{-2} (\dot{\varphi} \partial_i \varphi) \right]. \end{aligned} \quad (1.87)$$

Note that in Eqn. (1.85) and (1.87), we set $M_{pl}^2 = 1$. Since the interactions contain temporal derivative of fields, one should be careful when evaluating the interaction Hamiltonian, since Legendre transforming the Lagrangian will now generate higher order operators. However, since we work in leading order in slow-roll, the quartic terms generated from the cubic interaction can be neglected (see Appendix A.8).

1.9 Organisation of this thesis

This thesis is organized as follows. In Chapter 2 we perform the first explicit computation of scalar bispectrum at 1-loop, with interactions from the Effective Field Theory of Inflation (EFToI), with the goal of exploring loop corrections beyond power spectrum. We find that the final results feature analytic structures of the form $\log(k_i/k_T)$ and $\log(H/\mu)$, where H is the Hubble parameter and μ is the renormalisation scale. We find that unlike the 1-loop correction to the power-spectrum computed in [2] the unrenormalised answers always produce unphysical logarithms of co-moving momenta, which are canceled only after renormalisation.

In Chapter 3, we perform the first systematic study of the singularity structure of inflationary correlations at 1-loop. We derive diagrammatic rules to extract the singularities of any two-site 1-loop diagram, by analyzing the general structure of the time and momentum integrals of the correlators. Using these rules, the poles and branch cuts produced can be predicted by simply identifying the energies flowing through certain subgraphs, without having to perform complicated integrals. An interesting feature of de-Sitter correlators at 1-loop is the presence of an *off-shell* total energy branch point, which is present in dimensional regularization as well as cutoff regularization. We probe the source of this branch cut in detail, while revisiting the cosmological KLN theorem [34] in this context. Finally, we show that the branch cuts produced in a renormalised correlator always repackage themselves in a dilatation invariant form to produce logarithms of ratios of comoving scales.

In Chapter 4, we extend the analysis of Chapter 3 and compute the 1-loop bispectrum during slow-roll inflation, including the non-local, inverse-Laplacian operators present at cubic and quartic order in the ADM action (in the spatially flat gauge). These non-local vertices introduce non-trivial angular dependencies that significantly complicate the evaluation of loop integrals. We find that the diagram-

matic rules, discussed in Chapter 3, identifying poles and branch cuts in correlators without explicit integration, remain valid even in the presence of such non-local interactions. Finally, we derive the flat-space limit of the 1-loop inflationary correlator and verify it with explicit examples, thereby establishing a direct correspondence between its leading total-energy singularities ($\omega_T \rightarrow 0$) and the flat-space scattering amplitude.

In Chapter 5, we conclude by summarizing the main results of this thesis and discussing open questions that remain for future investigation.

Chapter 2

Bispectrum at 1-loop in the Effective Field Theory of Inflation

This thesis builds on a sequence of works on cosmological loop correlators. In particular, results from the author's publication [3], in collaboration with Diptimoy Ghosh and Farman Ullah, are reviewed and summarized in this Chapter, to establish notion and provide context. Portions of this material have previously appeared in the PhD thesis of Farman Ullah. Their inclusion here is intended solely to ensure a self-contained presentation; the novel contributions of this thesis arise in the subsequent chapters 3 and 4.

As emphasized in Sec. 1.1 and 1.7, it is of utmost importance to study cosmological correlation functions at loop level to understand the underlying inflationary theory. Loop corrections to cosmological correlation functions were first studied by Weinberg [1], where he computed the 1-loop correction to the 2-point correlation function. However, it was pointed out in [2] that there were mistakes in the computation, since the loop correction computed in [1] and subsequent related studies [107, 108, 109, 110, 111] have a $\log(k/\mu)$ dependence. This logarithm is unphysical,

since its argument involves the ratio of the co-moving momentum k and the physical renormalization scale μ , and hence is signature of some mistake in the computation. In [2] it was shown that once the mode functions are properly continued to d dimensions in dimensional regularization, the unphysical logarithms cancel and the log dependence instead is $\log(H/\mu)$.

It is essential to move beyond the power spectrum at 1-loop in order to resolve interaction-driven features encoded in the shapes of cosmological correlators. Importantly, the analytic structure of higher-point correlators at loop level remains largely uncharted, necessitating a systematic theoretical analysis. In this chapter, we examine one-loop corrections to the scalar bispectrum in the decoupling limit of the Effective Field Theory of Inflation (EFToI). To focus solely on the analytic properties of these correlators, we carry out the computation in a tuned setup (see Sec. 2.1) that minimizes the number of contributing Feynman diagrams.

Our main results are summarized as follows,

- The correlation function is not a rational function unlike the parity odd correlators computed in [45].
- Unlike [2], the unphysical logs do not cancel despite using correct d dimensional mode functions. They are removed only after renormalizing the results.
- Our final renormalized result for the correlation function contains logarithms of ratios of comoving scale, of the form $\log \frac{k_i}{k_T}$ and $\log \frac{H}{\mu}$, where $k_T = k_1 + k_2 + k_3$.

The rest of this chapter is organised as follows. In Sec. 2.1 we discuss the decoupling limit of EFToI and consider the tuned action used for our computations. In Sec. 2.2 we give the computational scheme of the loop computations. In Sec. 2.3

we discuss how renormalisation results in the cancellation of unphysical logarithms. In Sec. 2.4 we perform a cross check of our results with cutoff regularisation.

2.1 Decoupling limit of EFToI

The EFToI [117, 118, 119] Lagrangian is constructed in the unitary gauge by fixing time diffeomorphisms, using operators that are invariant under spatial diffeomorphisms. Using the Stueckelberg trick, time diffeomorphism invariance can be restored by performing a time diffeomorphism $t \rightarrow \tilde{t} = t + \xi^0(x)$, $\vec{x} \rightarrow \tilde{\vec{x}} = \vec{x}$ and then promoting the transformation parameter ξ^0 to a field π . This field $\pi(\vec{x}, t)$ now corresponds to the Goldstone mode for broken time diffeomorphisms, and must transform as $\pi(x) \rightarrow \pi(x) - \xi^0(x)$ under time diffeomorphisms. The action for $\pi(x)$ is given by,

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{pl}^2 R - M_{pl}^2 \left(3H^2 + \dot{H} \right) + M_{pl}^2 \dot{H} \tilde{g}^{00} + \frac{M_2^4}{2!} (\tilde{g}^{00} + 1)^2 + \frac{M_3^4}{3!} (\tilde{g}^{00} + 1)^3 + \frac{M_4^4}{4!} (\tilde{g}^{00} + 1)^4 + \dots \right], \quad (2.1)$$

where we neglect time dependence in the couplings, ‘...’ denotes terms which are higher order in derivatives and/or field fluctuations, and we also have,

$$\tilde{g}^{00} = \frac{\partial(t + \pi)}{\partial x^\mu} \frac{\partial(t + \pi)}{\partial x^\nu} g^{\mu\nu} = (1 + \dot{\pi})^2 g^{00} + 2(1 + \dot{\pi}) \partial_i \pi g^{0i} + g^{ij} \partial_i \pi \partial_j \pi. \quad (2.2)$$

The action simplifies drastically in the limit where the mixing energy E_{mix} between metric and scalar fluctuations is considered to be much smaller than the energy scale of the modes. This is known as the decoupling limit of EFToI. We further tune the coupling $M_2 = 0$, since our goal is to explore the analytic structure arising in 1-loop corrections of the bispectrum, rather than obtaining its full analytic structure. The

tuned action of π in the decoupling limit simplifies to,

$$\begin{aligned}
S &= \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{pl}^2 R - M_{pl}^2 \left(3H^2 + \dot{H} \right) + M_{pl}^2 \dot{H} \left(-(1 + \dot{\pi})^2 + \frac{1}{a^2} (\partial_i \pi)^2 + 1 \right) + \right. \\
&\quad \left. + \frac{1}{3!} M_3^4 \left(-(1 + \dot{\pi})^2 + \frac{1}{a^2} (\partial_i \pi)^2 + 1 \right)^3 + \frac{1}{4!} M_4^4 \left(-(1 + \dot{\pi})^2 + \frac{1}{a^2} (\partial_i \pi)^2 + 1 \right)^4 \right] \\
&= \int d^4x \sqrt{-g} \left[-M_{pl}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial \pi)^2}{a^2} \right) + g_3 \dot{\pi}^3 + g_4 \dot{\pi}^4 - \frac{3g_3}{2} \dot{\pi}^2 \left(\frac{\partial \pi}{a} \right)^2 \right], \quad (2.3)
\end{aligned}$$

where we have,

$$g_3 = -\frac{4}{3} M_3^4, \quad g_4 = -2M_3^4 + \frac{2}{3} M_4^4. \quad (2.4)$$

Note that lifting the $M_2 = 0$ restriction produces the following changes,

- The speed of sound is modified to $1/c_s^2 = 1 + 2M_2^2/(\epsilon H^2 M_{pl}^2)$. This produces a c_s dependent normalization factor in the mode functions, with the comoving momenta appropriately scaled as $k_i \rightarrow c_s k_i$.
- It produces additional cubic and quartic interaction vertices, such as $\dot{\pi}(\partial\pi)^2$ and $(\partial\pi)^4$.

One can ask how the inclusion of these interactions will affect our final results, and whether they can lead to additional singularities which are not captured by the contributions computed with interactions from Eqn. (2.3). However, we shall see in Chapter 3 that there is a systematic way to obtain the singularity structure of 1-loop correlations, computed with interactions involving temporal and spatial derivatives acted on all the fields. While the exact correlator is dependent on the interactions participating in the bispectrum, the location of the poles and branch cuts of the loop correlator depend only on the energy flow through specific subgraphs. Thus

the interactions in Eqn. (2.3) are a representative set of interactions for the purposes of probing analytic structure of loop corrections to the bispectrum. The additional contributions from the untuned ($M_2 \neq 0$) action will not produce additional poles and branch cuts in the correlator.

2.2 Explicit computation of Bispectrum at 1-loop

In this section, we compute the bispectrum at 1-loop with cubic and quartic interactions from the EFToI in Eqn. (2.3), using the in-in formalism,

$$\langle \pi^3(0) \rangle = \frac{\langle 0 | \bar{T} e^{i \int_{-\infty(1+i\epsilon)}^0 H_I d\tau} \pi^3(0) T e^{-i \int_0^{-\infty(1-i\epsilon)} H_I d\tau} | 0 \rangle}{\langle 0 | \bar{T} e^{i \int_{-\infty(1+i\epsilon)}^0 H_I d\tau} T e^{-i \int_0^{-\infty(1-i\epsilon)} H_I d\tau} | 0 \rangle}. \quad (2.5)$$

The time integrals are performed with the usual $i\epsilon$ prescription, with $\tau \rightarrow \tau(1 \pm i\epsilon)$, to project the free vacuum onto the interacting vacuum.

At a particular order in the EFT expansion, the 1-loop corrections can have contributions either from one insertion of a cubic and a quartic operator each (Fig. 4.3), which we refer to as Diagram I, or from diagrams with three cubic insertions, such as the ones in Fig. 2.2. In Sec. 2.2.2 and 2.2.3, we provide computational details of Diagram I and II respectively. All other contributions from a cubic and a quartic interaction are different time orderings of Fig. 4.3, obtained by complex conjugation of the same. Note that among the diagrams involving three cubic interaction insertions, we only inspect Fig. 2.2a, leaving an explicit computation of all other diagrams for the future. Finally, there are contributions from diagrams where the loop closes on itself, including tadpole diagrams, resulting in a scaleless integral, such as the ones in Fig. 2.3. Such contributions are scaleless, and vanish in dim-reg, as shown in Sec. 2.2.4.

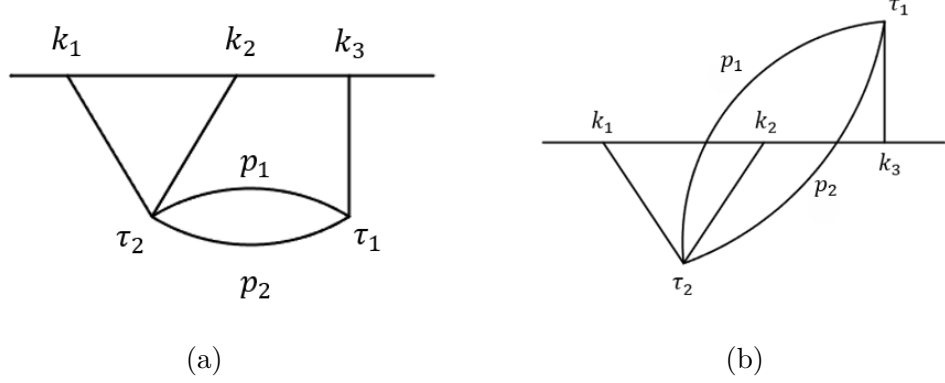


Figure 2.1: Diagram I : Loop contribution to the bispectrum from one insertion of the cubic and a quartic operator. k_1 , k_2 and k_3 denote external momenta. p_1 and p_2 denote loop momenta. τ_1 and τ_2 are points in the bulk. Time ordered diagram is shown in the left panel and the diagram with no specific time ordering is drawn in the right panel.

2.2.1 Calculational scheme

We will now sketch the computational scheme for the loop diagrams. We use the well-known Feynman rules [89]. The $d = 3 + \delta$ dimensional mode function and the measure are given by,

$$f_k(\tau) = (-H\tau)^{\delta/2} \frac{H}{\sqrt{-\dot{H}M_{pl}^2}} \frac{1}{\sqrt{2k^3}} (1 - ik\tau) e^{ik\tau} = \left(1 + \frac{\delta}{2} \log(-H\tau)\right) f_k^{3d}(\tau) + \mathcal{O}(\delta^2),$$

$$a(\tau)^\delta = 1 - \delta \log(-H\tau) + \mathcal{O}(\delta^2). \quad (2.6)$$

In our computations, the external mode functions (at $\tau = 0$) and the external momenta conserving delta function ($\delta^d(\sum_i \vec{k}_i)$) are taken to be in 3 dimensions, since the counter terms cancel the $\mathcal{O}(\delta)$ contribution from these terms. The integrals then

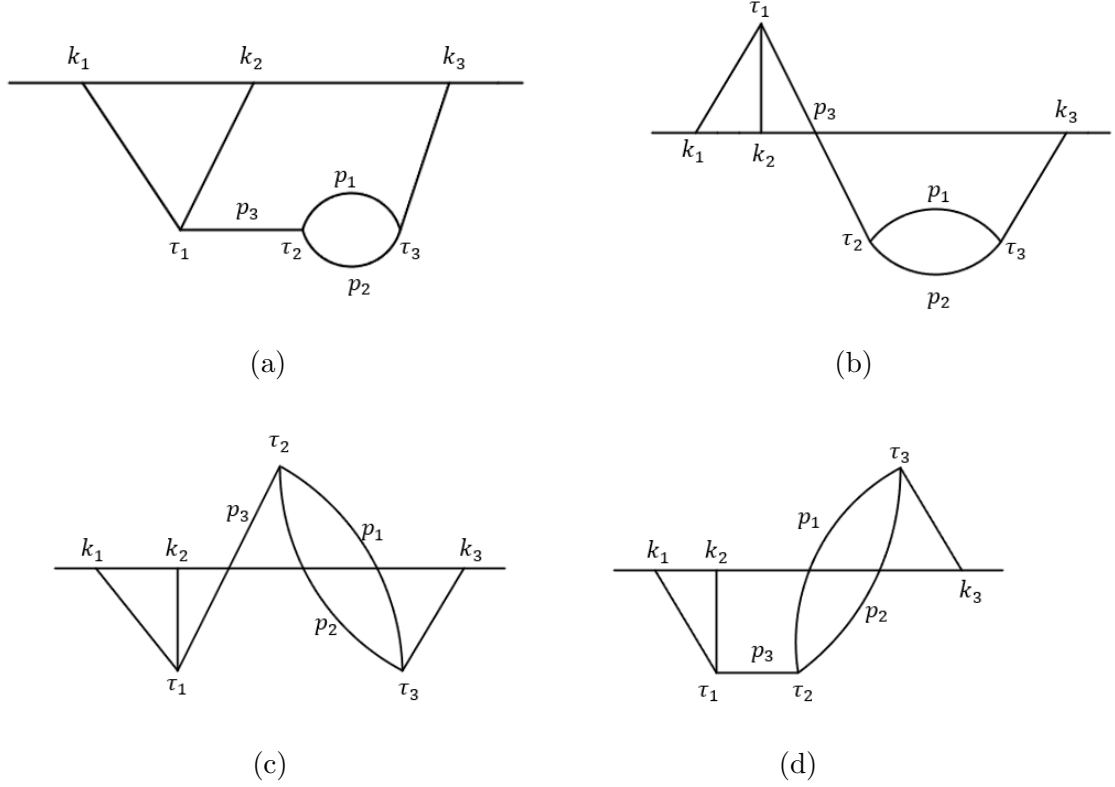


Figure 2.2: Diagram II : Loop contribution to the bispectrum from three insertions of the cubic operator. k_1 , k_2 and k_3 denote external momenta. p_1 , p_2 are loop momenta and $p_3 = k_3$ due to momentum conservation. τ_1 , τ_2 and τ_3 are points in the bulk.

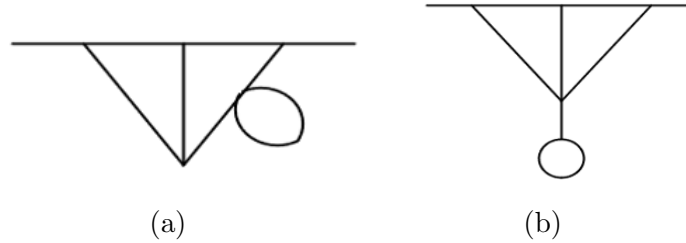


Figure 2.3: Diagrams with scaleless loops (left panel) and tadpoles (right panel). Contributions from scaleless loops vanish in dimensional regularization. Since all the interactions have derivatives, the tadpole diagram integrand either vanishes, by virtue of the tadpole leg $k = 0$, or is finite and its contribution vanishes from scaleless loop arguments (see Sec. 2.2.4)

take the following form,

$$\begin{aligned}
D(k_1, k_2, k_3) &= \delta^3\left(\sum_i \vec{k}_i\right) \mu^{-\frac{3\delta}{2}} \int d^d \vec{p}_1 d^d \vec{p}_2 P(p_1, p_2, \{k_i\}, \{\tau_i\}) \delta^{(d)}(\vec{p}_1 + \vec{p}_2 - \vec{k}_3) \\
&\quad \times \prod_i [(-H\tau_i)^{(n_i)\delta} d\tau_i] , \\
D(k_1, k_2, k_3) &= \delta^3\left(\sum_i \vec{k}_i\right) \mu^{-\frac{3\delta}{2}} \int d^d \vec{p}_1 d^d \vec{p}_2 P(p_1, p_2, \{k_i\}, \{\tau_i\}) \delta^{(d)}(\vec{p}_1 + \vec{p}_2 - \vec{k}_3) \\
&\quad \times \left[1 + \sum_i n_i \delta \log(-H\tau_i)\right] \prod_i d\tau_i , \quad (2.7)
\end{aligned}$$

where μ is the renormalisation scale. Note from Eq. (2.6) that $n_i = -1, 1/2$ for contributions from each vertex $(a(\tau)^\delta)$ and mode function respectively. Next we may split the entire correlator as $D = D_1 + D_2$, with,

$$\begin{aligned}
D_1(k_1, k_2, k_3) &= \delta^3\left(\sum_i \vec{k}_i\right) \mu^{-\frac{3\delta}{2}} \int d^d \vec{p}_1 d^d \vec{p}_2 P(p_1, p_2, \{k_i\}, \{\tau_i\}) \delta^{(d)}(\vec{p}_1 + \vec{p}_2 - \vec{k}_3) \prod_i d\tau_i , \\
D_2(k_1, k_2, k_3) &= \delta^3\left(\sum_i \vec{k}_i\right) \mu^{-\frac{3\delta}{2}} \int d^d \vec{p}_1 d^d \vec{p}_2 P(p_1, p_2, \{k_i\}, \{\tau_i\}) \sum_i n_i \delta \log(-H\tau_i) \\
&\quad \times \delta^{(d)}(\vec{p}_1 + \vec{p}_2 - \vec{k}_3) \prod_i d\tau_i , \quad (2.8)
\end{aligned}$$

where D_1 is the computation of the correlator with 3-dimensional mode functions and scale factors, and D_2 includes the $\mathcal{O}(\delta)$ contribution from them. Computing the time integrals in D_1 yields,

$$D_1(k_1, k_2, k_3) = \delta^3\left(\sum_i \vec{k}_i\right) \mu^{-\frac{3\delta}{2}} \int d^d \vec{p}_1 d^d \vec{p}_2 Q(p_1, p_2, \{k_i\}) \delta^{(d)}(\vec{p}_1 + \vec{p}_2 - \vec{k}_3) . \quad (2.9)$$

Next, the momentum integrals are evaluated using the identity in Appendix A.1, to

obtain,

$$D_1(k_1, k_2, k_3) = \delta^3 \left(\sum_i \vec{k}_i \right) \mu^{-\frac{3\delta}{2}} \frac{S_{d-2}}{2} \int_{k_3}^{\infty} dp_+ \int_{-k_3}^{k_3} dp_- \frac{p_1^{d-2} p_2}{k_3} Q(p_1, p_2, \{k_i\}). \quad (2.10)$$

The computation is further simplified by factoring out powers of k_3 and turning the integral dimensionless as follows,

$$D_1(k_1, k_2, k_3) = \delta^3 \left(\sum_i \vec{k}_i \right) \mu^{-\frac{3\delta}{2}} \frac{S_{d-2} k_3^{\delta+N}}{2} \int_1^{\infty} dq_+ \int_{-1}^1 dq_- q_1^{d-2} q_2 Q(q_1, q_2, \{x_i\}), \quad (2.11)$$

where $q_i = \frac{p_i}{k_3}$ and $x_i = \frac{k_i}{k_3}$ are scaled internal and external momenta respectively. A straightforward evaluation gives the following,

$$D_1(k_1, k_2, k_3) = \delta^3 \left(\sum_i \vec{k}_i \right) \mu^{-\frac{3\delta}{2}} \frac{S_{d-2} k_3^{\delta+N}}{2} R(d, \{x_i\}). \quad (2.12)$$

Finally, we expand the result about $\delta = 0$. If the function $R(3+\delta, \{x_i\})$ in Eqn. (2.12) is divergent in the limit $\delta \rightarrow 0$, then $R(d, \{x_i\})$ will have the form,

$$R(3+\delta, \{x_i\}) = \frac{F_0(\{x_i\})}{\delta} + F_1(\{x_i\}) + \mathcal{O}(\delta),$$

where F_0 is the coefficient of the $\mathcal{O}(1/\delta)$ divergent term, and F_1 is a function involving polynomial or logarithmic dependence on the scaled momenta x_i . This gives,

$$D_1(k_1, k_2, k_3) = \delta^3 \left(\sum_i \vec{k}_i \right) \frac{S_{d-2} k_3^N}{2} \left\{ \frac{F_0(\{x_i\})}{\delta} + F_0(\{x_i\}) \left(\log k_3 - \frac{3}{2} \log \mu \right) + F_1(\{x_i\}) \right\}. \quad (2.13)$$

The result of integrals involved in D_2 are directly obtained from the equation above, since the computation of D_2 is exactly the same as that of D_1 albeit with an additional factor of $\log(H\tau)$. The evaluation of time integrals with this extra factor is equivalent, upto non-log finite terms, to the logs being pulled outside the integrals as $\log(H/k_T)$, as shown in Appendix A.3. Therefore we get the following for D_2 ,

$$D_2(k_1, k_2, k_3) = \delta \left[\int d^d \vec{p}_1 d^d \vec{p}_2 P(p_1, p_2, \{k_i\}, \{\tau_i\}) \delta^{(d)}(\vec{p}_1 + \vec{p}_2 - \vec{k}_3) \prod_i d\tau_i \right] \\ \times \sum_i n_i \log(H/k_T) \delta^3\left(\sum_i \vec{k}_i\right) ,$$

$$D_2(k_1, k_2, k_3) = \delta D_1 \sum_i n_i \log(H/k_T) . \quad (2.14)$$

Combining D_1 and D_2 gives the full correlator as,

$$D = D_1 \left(1 + \delta \sum_i n_i \log(H/k_T) \right) . \quad (2.15)$$

Therefore, the final expression for the correlator is given by,

$$D = \frac{S_{d-2} k_m^N}{2} \left[\frac{F_0(\{x_i\})}{\delta} + F_0(\{x_i\}) \left(\log k_m - \frac{3}{2} \log \mu + \sum_i n_i \log(H/k_T) \right) \right. \\ \left. + F_1(\{x_i\}) \right] \delta^3\left(\sum_i \vec{k}_i\right) . \quad (2.16)$$

If D_1 does not have a divergence then $D = D_1$.

For the loop correction to the power spectrum in [2], $\sum_i n_i = 1$. It is clear from the equation above, that unless the number of internal mode functions (i.e. $f_k(\tau)$ at $\tau \neq 0$) and scale factors is such that $\sum_i n_i = 1$, the unphysical logs in D_1 will not cancel with contributions from D_2 .

2.2.2 Loop contribution from Diagram I

Here, we follow the procedure outlined in Sec. 2.2.1 to compute the 1-loop contribution in Fig. 2.1. We consider the diagram where k_3 contracts with the cubic vertex τ_1 . There will be 2 more contributions coming from contractions of k_1 and k_2 with the cubic vertex, obtained by substituting $k_3 \longleftrightarrow k_1$ and $k_3 \longleftrightarrow k_2$ respectively in the final expressions.

We first consider the time-ordered diagram Fig. 2.1a, with the quartic interaction as $\dot{\pi}^4$. This gives,

$$\begin{aligned}
D = & -72 \, g_3 g_4 \, \delta^3 \left(\sum_i \vec{k}_i \right) f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}^*(0) \int_{-\infty}^0 d\tau_1 d\tau_2 \, \mu^{-\frac{3\delta}{2}} f'_{k_1}(\tau_2) f'_{k_2}(\tau_2) f'_{k_3}(\tau_1) \\
& \times a^{d-2}(\tau_1) a^{d-3}(\tau_2) \int d^d \vec{p}_1 d^d \vec{p}_2 \left\{ f'_{p_1}(\tau_2) f'_{p_1}^*(\tau_1) f'_{p_2}(\tau_2) f'_{p_2}^*(\tau_1) \Theta(\tau_1 - \tau_2) \right. \\
& \left. + f'_{p_1}(\tau_1) f'_{p_1}^*(\tau_2) f'_{p_2}(\tau_1) f'_{p_2}^*(\tau_2) \Theta(\tau_2 - \tau_1) \right\} \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3), \tag{2.17}
\end{aligned}$$

As outlined in Sec. 2.2.1, we first compute $D_1(k_1, k_2, k_3)$, i.e. the correlator computed with 3-dimensional contribution to the modes and measure, to obtain,

$$\begin{aligned}
D_1 = & 72 \, g_3 g_4 \, \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{k_3(k_1 + k_2)(k_3 - 2k_1 - 2k_2)}{k_1 k_2 k_T^7} \left(\frac{1}{\delta} + \frac{1}{2} \log(k_T k_3) \right. \\
& \left. - \frac{3}{2} \log \mu \right) + \text{NLf}, \tag{2.18}
\end{aligned}$$

where NLf (Non-logarithmic functions) are simple rational functions of external momenta. Next, we compute D_2 . As discussed in Section. 2.2.1, and explicitly computed in Appendix A.2), D_2 is obtained from D_1 by bringing the $\log(H\tau)$ term out of the time integrals as $\log(H/k_T)$, along with some NLf of k_i . We note that there are 3 internal mode functions and 1 scale factor at τ_1 , 4 internal mode functions

and 1 scale factor at τ_2 , which gives,

$$D_2 = \delta \left\{ \left(\frac{3}{2} - 1 \right) + \left(\frac{4}{2} - 1 \right) \right\} \log \left(\frac{H}{k_T} \right) D_1 + \text{NLf}.$$

Adding this $\mathcal{O}(\delta)$ correction to D_1 , alongwith contribution from the other contractions involving contraction of k_1 and k_2 with the cubic vertex, gives :

$$D = 72 \, g_3 g_4 \, \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{k_3(k_1 + k_2)(k_3 - 2k_1 - 2k_2)}{k_1 k_2 k_T^7} \left\{ \frac{1}{\delta} + \frac{1}{2} \log \frac{k_3}{k_T} \right. \\ \left. - \frac{1}{2} \log k_T + \frac{3}{2} \log \frac{H}{\mu} \right\} + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} + \text{NLf}. \quad (2.19)$$

Next, we compute the unordered diagram in Fig. 2.1b for quartic interaction $\dot{\pi}^4$. This gives the following finite expression,

$$D = -72 \, g_3 g_4 \, \delta^3 \left(\sum_i \vec{k}_i \right) f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}(0) \int_{-\infty}^0 d\tau_1 d\tau_2 \, \mu^{-\frac{3\delta}{2}} f'_{k_1}(\tau_2) f'_{k_2}(\tau_2) f'_{k_3}^*(\tau_1) \\ \times a^{d-2}(\tau_1) a^{d-3}(\tau_2) \int d^d \vec{p}_1 d^d \vec{p}_2 \, f'_{p_1}(\tau_2) f'_{p_1}^*(\tau_1) f'_{p_2}(\tau_2) f'_{p_2}^*(\tau_1) \, \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3), \quad (2.20)$$

and the final answer, given as follows,

$$D = D_1 = 72 \, g_3 g_4 \, \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{k_3(k_1 + k_2)(k_T + k_1 + k_2)}{2k_1 k_2 (k_3 - k_1 - k_2)^7} \log \left(\frac{k_T}{2k_3} \right) \\ + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} + \text{NLf}, \quad (2.21)$$

involves neither a $1/\delta$ divergence nor any unphysical logarithms. The apparent

folded pole at $k_3 - k_1 - k_2$ is actually spurious, and the correlator is finite when series expanded about $k_3 = k_1 + k_2$, with crucial cancellations coming from NLf terms, given by,

$$\begin{aligned}
\text{NLf} = & 72 \, g_3 g_4 \, \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{1}{8 k_1 k_2 k_3} \frac{1}{120 (k_3 - k_{12})^7 k_T^4} \left(-9 k_3^8 + 388 k_{12} k_3^7 \right. \\
& + 1380 k_{12}^2 k_3^6 + 2180 k_{12}^3 k_3^5 - 190 k_{12}^4 k_3^4 - 2196 k_{12}^5 k_3^3 - 1388 k_{12}^6 k_3^2 - 180 k_{12}^7 k_3 + 15 k_{12}^8 \\
& - 480 k_{12} k_3^7 \log(2) - 2880 k_{12}^2 k_3^6 \log(2) - 6720 k_{12}^3 k_3^5 \log(2) - 7680 k_{12}^4 k_3^4 \log(2) \\
& \left. - 4320 k_{12}^5 k_3^3 \log(2) - 960 k_{12}^6 k_3^2 \log(2) \right) + \dots
\end{aligned} \tag{2.22}$$

where the dots represent finite terms in NLf that do not have folded singularity.

Next, we go back to the contribution in Fig. 2.1a computed with quartic interaction $\dot{\pi}^2(\partial\pi)^2$. There are three types of contractions. First, when both spatial derivatives act on external k_i modes (i.e. f_{k_i}), we have,

$$\begin{aligned}
D = & 18 \, g_3^2 \, \delta^3 \left(\sum_i \vec{k}_i \right) f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}^*(0) \{ \vec{k}_1 \cdot \vec{k}_2 \} \int_{-\infty}^0 d\tau_1 d\tau_2 \, \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2) \\
& f_{k_1}(\tau_2) f_{k_2}(\tau_2) f_{k_3}'(\tau_1) \int d^d \vec{p}_1 d^d \vec{p}_2 \{ f_{p_1}'(\tau_2) f_{p_1}^*(\tau_1) f_{p_2}'(\tau_2) f_{p_2}^*(\tau_1) \Theta(\tau_1 - \tau_2) \\
& + f_{p_1}'(\tau_1) f_{p_1}^*(\tau_2) f_{p_2}'(\tau_1) f_{p_2}^*(\tau_2) \Theta(\tau_2 - \tau_1) \} \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3),
\end{aligned} \tag{2.23}$$

where $\vec{k}_1 \cdot \vec{k}_2 = -\vec{k}_1 \cdot (\vec{k}_1 + \vec{k}_3) = \frac{k_3^2 - k_1^2 - k_2^2}{2}$.

Second, when the spatial derivatives act on a loop mode (i.e. f_{p_j}) and on an

external k_i mode, we have,

$$\begin{aligned}
D = & 72 \, g_3^2 \, \delta^3 \left(\sum_i \vec{k}_i \right) f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}^*(0) \int_{-\infty}^0 d\tau_1 d\tau_2 \, \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2) \\
& \left\{ f_{k_1}(\tau_2) f'_{k_2}(\tau_2) f'_{k_3}(\tau_1) \vec{k}_1 + f_{k_2}(\tau_2) f'_{k_1}(\tau_2) f'_{k_3}(\tau_1) \vec{k}_2 \right\} \cdot \int d^d \vec{p}_1 d^d \vec{p}_2 \, \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3) \\
& \left\{ f'_{p_1}^*(\tau_1) f'_{p_2}^*(\tau_1) \Theta(\tau_1 - \tau_2) \left(f'_{p_1}(\tau_2) f_{p_2}(\tau_2) \vec{p}_2 + f_{p_1}(\tau_2) f'_{p_2}(\tau_2) \vec{p}_1 \right) \right. \\
& \left. + f'_{p_1}(\tau_1) f'_{p_2}(\tau_1) \Theta(\tau_2 - \tau_1) \left(f_{p_1}^*(\tau_2) f_{p_2}^*(\tau_2) \vec{p}_2 + f_{p_1}^*(\tau_2) f'_{p_2}^*(\tau_2) \vec{p}_1 \right) \right\}, \quad (2.24)
\end{aligned}$$

with

$$\vec{k}_1 \cdot \vec{p}_1 = k_1 p_1 [\cos \theta_1 \cos \theta_k + \sin \theta_1 \sin \theta_k \cos \theta_2], \quad (2.25)$$

where $\cos \theta_k$ is angle between k_1 and k_3 , given by $\cos \theta_k = \frac{k_2^2 - k_1^2 - k_3^2}{2k_1 k_3}$. θ_1, θ_2 are polar angles of $\vec{p}_1$, with $\cos \theta_1 = \frac{k_3^2 + p_1^2 - p_2^2}{2k_3 p_1}$. However we only need to keep the first term in Eq. (2.25) when evaluating the momentum integrals using the identity in Appendix A.1, since the integrand is odd over $\theta_2 \in (0, \pi)$ due to the $\cos \theta_2$ term. All the dot products may be expressed in terms of $\vec{k}_1 \cdot \vec{p}_1$ using the momentum conserving delta functions.

Finally, when both spatial derivatives act on loop modes, we have,

$$\begin{aligned}
D = & 18 \, g_3^2 \, \delta^3 \left(\sum_i \vec{k}_i \right) f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}^*(0) \int_{-\infty}^0 d\tau_1 d\tau_2 \, \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2) \\
& f'_{k_1}(\tau_2) f'_{k_2}(\tau_2) f'_{k_3}(\tau_1) \int d^d \vec{p}_1 d^d \vec{p}_2 \left\{ f_{p_1}(\tau_2) f'_{p_1}^*(\tau_1) f_{p_2}(\tau_2) f'_{p_2}^*(\tau_1) \Theta(\tau_1 - \tau_2) \right. \\
& \left. + f'_{p_1}(\tau_1) f_{p_1}^*(\tau_2) f'_{p_2}(\tau_1) f_{p_2}^*(\tau_2) \Theta(\tau_2 - \tau_1) \right\} \{ \vec{p}_1 \cdot \vec{p}_2 \} \, \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3), \quad (2.26)
\end{aligned}$$

with $\vec{p}_1 \cdot \vec{p}_2 = \vec{p}_1 \cdot (\vec{k}_3 - \vec{p}_1) = p_1 k_3 \cos \theta_1 - p_1^2$.

The $\mathcal{O}(\delta)$ contribution from the mode functions to this computation is given by $D_2 = \frac{3\delta}{2} \log\left(\frac{H}{k_T}\right) D_1$, obtained due to contribution from 3 internal mode functions and 1 scale factor at τ_1 , as well as 4 mode functions and 1 scale factor at τ_2 . We explicitly show this in Appendix A.3. Computing the integrals above, the 1-loop correlator alongwith contribution from D_2 is given by,

$$\begin{aligned}
D = & -\frac{3}{40} g_3^2 \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{k_3}{k_1^3 k_2^3} \frac{\vec{k}_1 \cdot \vec{k}_2}{k_T^7} (280 k_2 k_1^3 + 55 k_3 k_1^3 + 480 k_2^2 k_1^2 \\
& + 21 k_3^2 k_1^2 + 105 k_2 k_3 k_1^2 + 280 k_2^3 k_1 + 7 k_3^3 k_1 + 42 k_2 k_3^2 k_1 + 105 k_2^2 k_3 k_1 + 40 k_2^4 + 40 k_1^4 \\
& + k_3^4 + 7 k_2 k_3^3 + 21 k_2^2 k_3^2 + 55 k_2^3 k_3) \left(\frac{1}{\delta} + \frac{1}{2} \log\left(\frac{k_3}{k_T}\right) - \frac{1}{2} \log k_T + \frac{3}{2} \log\left(\frac{H}{\mu}\right) \right) \\
& - \frac{3}{40} g_3^2 \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{1}{k_1^3 k_2^3 k_3} \frac{1}{k_T^7} (20 k_1^8 + 140 k_2 k_1^7 + 140 k_3 k_1^7 + 200 k_2^2 k_1^6 \\
& - 152 k_3^2 k_1^6 + 840 k_2 k_3 k_1^6 - 140 k_2^3 k_1^5 - 264 k_3^3 k_1^5 - 1904 k_2 k_3^2 k_1^5 + 560 k_2^2 k_3 k_1^5 \\
& - 440 k_2^4 k_1^4 + 260 k_3^4 k_1^4 + 56 k_2 k_3^3 k_1^4 - 248 k_2^2 k_3^2 k_1^4 - 1540 k_2^3 k_3 k_1^4 - 140 k_2^5 k_1^3 \\
& + 124 k_3^5 k_1^3 + 1764 k_2 k_3^4 k_1^3 - 2592 k_2^2 k_3^3 k_1^3 + 3008 k_2^3 k_3^2 k_1^3 - 1540 k_2^4 k_3 k_1^3 + 200 k_2^6 k_1^2 \\
& - 128 k_3^6 k_1^2 - 896 k_2 k_3^5 k_1^2 + 3008 k_2^2 k_3^4 k_1^2 - 2592 k_2^3 k_3^3 k_1^2 - 248 k_2^4 k_3^2 k_1^2 + 560 k_2^5 k_3 k_1^2 \\
& + 140 k_2^7 k_1 - 896 k_2^2 k_3^5 k_1 + 1764 k_2^3 k_3^4 k_1 + 56 k_2^4 k_3^3 k_1 - 1904 k_2^5 k_3^2 k_1 + 840 k_2^6 k_3 k_1 + 20 k_2^8 \\
& - 128 k_2^2 k_3^6 + 124 k_2^3 k_3^5 + 260 k_2^4 k_3^4 - 264 k_2^5 k_3^3 - 152 k_2^6 k_3^2 + 140 k_2^7 k_3) \\
& \left(\frac{1}{\delta} + \frac{1}{2} \log\left(\frac{k_3}{k_T}\right) - \frac{1}{2} \log k_T + \frac{3}{2} \log\left(\frac{H}{\mu}\right) \right) + \frac{9}{4} g_3^2 \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \\
& \frac{1}{k_1 k_2 k_3 k_T^7} (k_1^4 + 4 k_2 k_1^3 + 7 k_3 k_1^3 + 6 k_2^2 k_1^2 + 7 k_3^2 k_1^2 + 21 k_2 k_3 k_1^2 + 4 k_2^3 k_1 - 23 k_3^3 k_1 \\
& + 14 k_2 k_3^2 k_1 + 21 k_2^2 k_3 k_1 + k_2^4 - 48 k_3^4 - 23 k_2 k_3^3 + 7 k_2^2 k_3^2 + 7 k_2^3 k_3) \\
& \left(\frac{1}{\delta} + \frac{1}{2} \log\left(\frac{k_3}{k_T}\right) - \frac{1}{2} \log k_T + \frac{3}{2} \log\left(\frac{H}{\mu}\right) \right) + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} + \text{NLf}.
\end{aligned} \tag{2.27}$$

We compute one last contribution, i.e. the diagram in Fig. 2.1b with quartic interaction $\dot{\pi}^2(\partial\pi)^2$. We can again have three types of contractions. First, when both spatial derivatives act on the external k mode functions, we get,

$$D = 18 g_3^2 \delta^3 \left(\sum_i \vec{k}_i \right) f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}(0) \{ \vec{k}_1 \cdot \vec{k}_2 \} \int_{-\infty}^0 d\tau_1 d\tau_2 \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2) f_{k_1}(\tau_2) f_{k_2}(\tau_2) f_{k_3}'^*(\tau_1) \int d^d \vec{p}_1 d^d \vec{p}_2 f_{p_1}'(\tau_2) f_{p_1}'^*(\tau_1) f_{p_2}'(\tau_2) f_{p_2}'^*(\tau_1) \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3). \quad (2.28)$$

Secon, when the two spatial derivatives act on an external k mode and a loop mode, we get,

$$D = 72 g_3^2 \delta^3 \left(\sum_i \vec{k}_i \right) f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}(0) \int_{-\infty}^0 d\tau_1 d\tau_2 \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2) \left\{ f_{k_1}(\tau_2) f_{k_2}'(\tau_2) f_{k_3}'^*(\tau_1) \vec{k}_1 + f_{k_2}(\tau_2) f_{k_1}'(\tau_2) f_{k_3}'^*(\tau_1) \vec{k}_2 \right\} \cdot \int d^d \vec{p}_1 d^d \vec{p}_2 f_{p_1}'^*(\tau_1) f_{p_2}'^*(\tau_1) (f_{p_1}'(\tau_2) f_{p_2}(\tau_2) \vec{p}_2 + f_{p_1}(\tau_2) f_{p_2}'(\tau_2) \vec{p}_1) \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3). \quad (2.29)$$

Lastly, when both spatial derivatives act on loop modes, we have,

$$D = 18 g_3^2 \delta^3 \left(\sum_i \vec{k}_i \right) f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}(0) \int_{-\infty}^0 d\tau_1 d\tau_2 \mu^{-\frac{3\delta}{2}} a^{d-2}(\tau_1) a^{d-3}(\tau_2) f_{k_1}'(\tau_2) f_{k_2}'(\tau_2) f_{k_3}'^*(\tau_1) \int d^d \vec{p}_1 d^d \vec{p}_2 f_{p_1}(\tau_2) f_{p_1}'^*(\tau_1) f_{p_2}(\tau_2) f_{p_2}'^*(\tau_1) \{ \vec{p}_1 \cdot \vec{p}_2 \} \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3). \quad (2.30)$$

Once again this diagram does not produce any divergences or unphysical logarithms, and is given by,

$$\begin{aligned}
D = D_1 = & \frac{3}{20} g_3^2 \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{\{\vec{k}_1 \cdot \vec{k}_2\}}{k_1^3 k_2^3} \frac{k_3}{(k_1 + k_2 - k_3)^7} \log \left(\frac{2k_3}{k_T} \right) (40k_1^4 \\
& + 280k_2k_1^3 - 55k_3k_1^3 + 480k_2^2k_1^2 + 21k_3^2k_1^2 - 105k_2k_3k_1^2 + 280k_2^3k_1 - 7k_3^3k_1 + 42k_2k_3^2k_1 \\
& - 105k_2^2k_3k_1 + 40k_2^4 + k_3^4 - 7k_2k_3^3 + 21k_2^2k_3^2 - 55k_2^3k_3) \\
& + \frac{3}{80} g_3^2 \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{1}{k_1^3 k_2^3 k_3} \frac{1}{(k_1 + k_2 - k_3)^7} \log \left(\frac{2k_3}{k_T} \right) (20k_1^8 + 140k_2k_1^7 \\
& + 200k_2^2k_1^6 - 152k_3^2k_1^6 - 840k_2k_3k_1^6 - 140k_2^3k_1^5 + 264k_3^3k_1^5 - 1904k_2k_3^2k_1^5 - 560k_2^2k_3k_1^5 \\
& - 440k_2^4k_1^4 + 260k_3^4k_1^4 - 56k_2k_3^3k_1^4 - 248k_2^2k_3^2k_1^4 + 1540k_2^3k_3k_1^4 - 140k_2^5k_1^3 - 124k_3^5k_1^3 \\
& + 1764k_2k_3^4k_1^3 + 2592k_2^2k_3^3k_1^3 + 3008k_2^3k_3^2k_1^3 + 1540k_2^4k_3k_1^3 + 200k_2^6k_1^2 - 128k_3^6k_1^2 \\
& + 896k_2k_3^5k_1^2 + 3008k_2^2k_3^4k_1^2 + 2592k_2^3k_3^3k_1^2 - 248k_2^4k_3^2k_1^2 - 560k_2^5k_3k_1^2 + 140k_2^7k_1 \\
& + 896k_2^2k_3^5k_1 + 1764k_2^3k_3^4k_1 - 56k_2^4k_3^3k_1 - 1904k_2^5k_3^2k_1 - 840k_2^6k_3k_1 + 20k_2^8 - 128k_2^2k_3^6 \\
& - 124k_2^3k_3^5 + 260k_2^4k_3^4 + 264k_2^5k_3^3 - 152k_2^6k_3^2 - 140k_2^7k_3 - 140k_3k_1^7) \\
& + \frac{9}{8} g_3^2 \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{1}{k_1 k_2 k_3} \frac{1}{(k_1 + k_2 - k_3)^7} \log \left(\frac{k_3}{k_T} \right) (-k_1^4 - 4k_2k_1^3 + 7k_3k_1^3 \\
& - 7k_3^2k_1^2 + 21k_2k_3k_1^2 - 4k_2^3k_1 - 23k_3^3k_1 - 14k_2k_3^2k_1 + 21k_2^2k_3k_1 - k_2^4 + 48k_3^4 - 23k_2k_3^3 \\
& - 6k_2^2k_1^2 - 7k_2^2k_3^2 + 7k_2^3k_3) + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} + \text{NLf}, \tag{2.31}
\end{aligned}$$

and the apparent folded poles are canceled from NLf terms.

Eq. (2.19) and Eq. (2.27) depend on the logarithm of a dimensionful quantity, i.e., $\log k_T$. This is due to the fact that factors of $\mu^{-\frac{3\delta}{2}}$ keep mass dimension of couplings unchanged in d -dimensions, however the correlator's mass dimension changes and depends on δ . Importantly, factors of μ^δ are scheme dependent and only affect the constant finite part of the answer. Hence, we can insert them at will. Inserting a factor of $\mu^{-\delta}$, such that the mass dimension of the correlator is continued

to d -dimensions, we see that the unrenormalized answer will then feature a $\log \frac{k_T}{\mu}$. This is precisely the *unphysical* logarithm that arose in Weinberg's computation of the power spectrum at 1-loop in [118], and canceled due to $\mathcal{O}(\delta)$ contribution coming from modes and measure in [2]. However we clearly see from Eqns. (2.19) and (2.27), that for the bispectrum at 1-loop, such unphysical logarithms persist despite adding the $\mathcal{O}(\delta)$ contributions from modes and measure.

2.2.3 Loop contribution from Diagram II

In this section, we will compute one of the contribution to the bispectrum at 1-loop, in Fig. 2.2a, coming from three cubic interaction insertions, and comment on the analytic structure of the other three-cubic-vertex contributions. The correlator is given by,

$$\begin{aligned}
D = & -648i \, g_3^3 \, \delta^3 \left(\sum_i \vec{k}_i \right) f_{k_1}^*(0) f_{k_2}^*(0) f_{k_3}^*(0) \int \int \int_{-\infty}^0 d\tau_1 d\tau_2 d\tau_3 \, \mu^{-\frac{3\delta}{2}} f'_{k_1}(\tau_1) f'_{k_2}(\tau_1) f'_{k_3}(\tau_3) \\
& \{a(\tau_1) a(\tau_2) a(\tau_3)\}^{d-2} \int d^d \vec{p}_1 d^d \vec{p}_2 \, \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3) \\
& \{ f'_{k_3}(\tau_1) f'_{k_3}^*(\tau_2) f'_{p_1}(\tau_2) f'_{p_1}^*(\tau_3) f'_{p_2}(\tau_2) f'_{p_2}^*(\tau_3) \Theta(\tau_3 - \tau_2) \Theta(\tau_2 - \tau_1) \\
& + f'_{k_3}(\tau_1) f'_{k_3}^*(\tau_2) f'_{p_1}^*(\tau_2) f'_{p_1}(\tau_3) f'_{p_2}^*(\tau_2) f'_{p_2}(\tau_3) \Theta(\tau_2 - \tau_1) \Theta(\tau_2 - \tau_3) \\
& + f'_{k_3}^*(\tau_1) f'_{k_3}(\tau_2) f'_{p_1}(\tau_2) f'_{p_1}^*(\tau_3) f'_{p_2}(\tau_2) f'_{p_2}^*(\tau_3) \Theta(\tau_1 - \tau_2) \Theta(\tau_3 - \tau_2) \\
& + f'_{k_3}^*(\tau_1) f'_{k_3}(\tau_2) f'_{p_1}^*(\tau_2) f'_{p_1}(\tau_3) f'_{p_2}^*(\tau_2) f'_{p_2}(\tau_3) \Theta(\tau_1 - \tau_2) \Theta(\tau_2 - \tau_3) \} , \\
\end{aligned} \tag{2.32}$$

To compute D_2 , we note that the diagram features 3 internal mode functions and 1 scale factor at each of the three time vertices τ_1 , τ_2 and τ_3 . Following arguments of Sec. 2.2.1, we have $D_2 = \frac{3\delta}{2} \log \left(\frac{H}{k_T} \right) D_1$. The final correlator $D = D_1 + D_2$ turns

out to be,

$$\begin{aligned}
D = & -648 \, g_3^3 \, \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^6} \frac{1}{8k_1 k_2 k_3} \frac{1}{15k_T^8} \left(k_1^5 + 5k_2 k_1^4 + 11k_3 k_1^4 + 10k_2^2 k_1^3 \right. \\
& + 58k_3^2 k_1^3 + 44k_2 k_3 k_1^3 + 10k_2^3 k_1^2 + 198k_3^3 k_1^2 + 174k_2 k_3^2 k_1^2 + 66k_2^2 k_3 k_1^2 + 5k_2^4 k_1 - 99k_3^4 k_1 \\
& + 396k_2 k_3^3 k_1 + 174k_2^2 k_3^2 k_1 + 44k_2^3 k_3 k_1 + k_2^5 + 3k_3^5 - 99k_2 k_3^4 + 198k_2^2 k_3^3 \\
& + 58k_2^3 k_3^2 + 11k_2^4 k_3 \left. \right) \left\{ \frac{1}{\delta} + \log \left(\frac{k_3}{k_T} \right) - \frac{1}{2} \log k_T + \frac{3}{2} \log \left(\frac{H}{\mu} \right) \right\} \\
& + 648 \, g_3^3 \, \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^6} \frac{1}{8k_1 k_2} \frac{2k_3^3}{15k_T^8 (k_3 - k_1 - k_2)^3} \log \left(\frac{k_T}{k_3} \right) (150k_1^4 \\
& + 600k_2 k_1^3 - 195k_3 k_1^3 + 900k_2^2 k_1^2 + 133k_3^2 k_1^2 - 585k_2 k_3 k_1^2 + 600k_2^3 k_1 - 25k_3^3 k_1 \\
& + 266k_2 k_3^2 k_1 - 585k_2^2 k_3 k_1 + 150k_2^4 + k_3^4 - 25k_2 k_3^3 + 133k_2^2 k_3^2 - 195k_2^3 k_3) \\
& + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} + \text{NLf}, \tag{2.33}
\end{aligned}$$

and the apparent folded pole at $k_3 = k_1 + k_2$ cancels from the following NLf contribution,

$$\begin{aligned}
\text{NLf} = & 648 \, g_3^3 \, \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^6} \frac{1}{64k_1 k_2 k_3 k_T^8} \frac{1}{(450(k_3 - k_{12})^3)} (3k_3^8 (80 \log(2) - 2593) \\
& + 96k_{12} k_3^7 (229 - 145 \log(2)) + 24k_{12}^2 k_3^6 (2380 \log(2) - 4173) + 8k_{12}^3 k_3^5 (29681 - 13380 \log(2)) \\
& + 10k_{12}^4 k_3^4 (5520 \log(2) - 19729) + 16k_{12}^5 k_3^3 (2293 - 840 \log(2)) + 8k_{12}^6 k_3^2 (943 - 840 \log(2)) \\
& + 8k_{12}^7 k_3 (173 - 240 \log(2)) + k_{12}^8 (173 - 240 \log(2))) + \dots \tag{2.34}
\end{aligned}$$

We once again find that the unphysical logarithms $-\frac{1}{2} \log k_T$ in Eq. (2.33) persist despite using d -dimensional modes and measure, and it cancels only after renormalization, as discussed later in Sec 2.3.

Let us now consider the three-cubic-vertex contributions in Fig. 2.2 with vertices

from both, the ordered as well as unordered part of the in-in correlator. The simplest case is the contribution of Fig. 2.2c, which is UV finite, features a total energy pole, and has no branch cuts. The diagram in Fig. 2.2d is also UV finite, given by,

$$\begin{aligned}
D \sim \delta^3 \left(\sum_i \vec{k}_i \right) \frac{g_3^3 \log \left(\frac{k_T}{k_3} \right)}{k_1 k_2 (k_{12} - k_3)^8 k_T^3 (-\dot{H} M_{pl}^2)^6} \frac{H^9}{(-\dot{H} M_{pl}^2)^6} \left\{ k_3^3 \left(150 k_1^4 + 15 (40 k_2 + 13 k_3) k_1^3 \right. \right. \\
+ \left(900 k_2^2 + 585 k_3 k_2 + 133 k_3^2 \right) k_1^2 + (600 k_2^3 + 585 k_3 k_2^2 + 266 k_3^2 k_2 + 25 k_3^3) k_1 \\
\left. \left. + 150 k_2^4 + k_3^4 + 25 k_2 k_3^3 + 133 k_2^2 k_3^2 + 195 k_2^3 k_3 \right) \right\} + \text{NLf}, \quad (2.35)
\end{aligned}$$

and involves no unphysical logarithm. The apparent folded pole at $k_1 + k_2 = k_3$ is spurious and the correlator is finite in this folded limit due to crucial cancellations from NLf terms.

Computing the diagram in Fig. 2.2b, we find D_1 and D_2 as,

$$\begin{aligned}
D_1 \sim \frac{g_3^3}{k_1 k_2 k_3 k_T^3} \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{(-\dot{H} M_{pl}^2)^6} \left(\frac{1}{\delta} + \log k_3 + \frac{3}{2} \log \left(\frac{H}{\mu} \right) \right) + \text{NLf}, \\
D_{2,\tau_1} = -\frac{1}{2} \delta D_1 \log k_T, \quad D_{2,\tau_2} = D_{2,\tau_3} = -\frac{1}{2} \delta D_1 \log k_3. \quad (2.36)
\end{aligned}$$

where D_{2,τ_i} is obtained from $\mathcal{O}(\delta)$ contribution of modes and measure at vertex τ_i . Clearly, the contributions of the unrenormalized correlator add up to cancel $\log k_3$, leaving behind an unphysical $-\frac{1}{2} \log k_T$, which is absorbed by the counter term contributions as discussed later in Sec 2.3. This gives a renormalized contribution that does not involve any branch cuts. Unlike ordered diagrams which only produce $\log k_T$ in D_2 , notice that now D_2 also produces a $\log k_3$ at vertices τ_2, τ_3 . The reason can be intuitively understood by analyzing the two-fold effect of nested integrals. First, they allow for mixing of energies in the argument of the exponential coming from the mode functions. Second, the nested time integral produces logarithm of the

vertex in the integral limit, as well as exponential integral functions. For example, consider the $\Theta(\tau_2 - \tau_3)$ piece of the time integrals for Fig. 2.2b,

$$I = \delta \int_{-\infty}^0 d\tau_1 \int_{-\infty}^0 d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 e^{ia\tau_1} e^{ib\tau_2} e^{ic\tau_3} (\tau_1 \tau_2 \tau_3)^2 \log(-\tau_3), \quad (2.37)$$

where $a = -k_T$, $b = k_3 - p_+$ and $c = p_+ + k_3$. Evaluation of τ_3 integral gives,

$$I = \delta \int_{-\infty}^0 d\tau_1 \int_{-\infty}^0 d\tau_2 (\tau_1 \tau_2)^2 e^{ia\tau_1} \left[e^{i(b+c)\tau_2} f_1(k_i, p_j) + \text{Ei}[i(b+c)\tau_2] f_2(\tau_2, k_i, p_j) + e^{i(b+c)\tau_2} \log(-\tau_2) f_3(\tau_2, k_i, p_j) \right], \quad (2.38)$$

where f_1, f_2, f_3 are polynomial functions in k_i, p_j and τ_2 . Evaluating the rest of the time integrals over the first two terms gives rise to finite non-log functions, as well as additional logarithms of the form $\log\left(\frac{k_3 + p_+}{k_3}\right)$. The additional logarithm contribution, however, is finite as $p_+ \rightarrow 0$ and thus vanishes as $\delta \rightarrow 0$. Importantly, f_3 is the result of the τ_3 integral Eqn. (2.37) performed without the logarithm. Had the remaining integral been a nested one, i.e. $\int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2$, the result would have featured a $\log k_T$ by virtue of the two-fold effect discussed above, as well as shown in Appendix. A.3. However, the evaluation of the (non-nested) τ_2 integral in Eqn. (2.38) instead produces a logarithm at $b + c = 2k_3$, leading to Eqn. (2.36).

To summarize, we find that the unordered contributions in Fig. 2.2c and Fig. 2.2d are UV finite, and they feature no branch cuts and branch cuts of the form $\log \frac{k_3}{k_T}$ respectively. The unordered contribution in Fig. 2.2b is UV divergent, and we find unphysical logarithms $-\frac{1}{2} \log k_T$ in the unrenormalized contribution, which cancels only after renormalization, as discussed in Sec. 2.3.

2.2.4 Contribution from scaleless loops

In this section, we show that the scaleless integral contributions from diagrams in Fig. 2.3a vanish [45]. In all these diagrams the loop closes on itself, therefore there is no injection of external kinematics. The bulk-to-bulk propagator for the loop is given by,

$$G_p(\tau, \tau) = \frac{H^2}{2p^3} (1 + p^2 \tau^2) . \quad (2.39)$$

There are no exponentials with energy p , which implies that after time integration we only get scale-less integrals of p of the form,

$$\int p^\alpha d^3 p , \quad (2.40)$$

with some k dependent coefficients. Substituting $p = \lambda p'$ in the above integral in d dimensions, the limits of the integral $\int_{-\infty}^{\infty}$ allows us to bring factors of λ out the loop integral as,

$$\lambda^{d+\alpha} \int p'^\alpha d^d p' , \quad (2.41)$$

which implies,

$$\int p^\alpha d^d p = \lambda^{d+\alpha} \int p'^\alpha d^d p' . \quad (2.42)$$

Since $d + \alpha \neq 0$, the only possibility is that the integral is zero. Therefore, all 1-loop diagrams where the loop closes on itself are zero. However, one must be careful with tadpole diagrams such as the one in Fig. 2.3b. The tadpole leg has

zero momentum due to translational invariance, which may cause the integrand to diverge. However, this does not happen in our case since the interaction we use have derivatives acting on the fields, for which the integrand is either zero or finite. The former case evaluates to zero due to the scaleless loop argument.

To see this, note that the propagator arises due to contraction of the tadpole leg coming from the cubic ($\dot{\pi}^3$) vertex with a mode function coming from either the cubic or a quartic ($\dot{\pi}^4$, $\dot{\pi}^2(\partial\pi)^2$) vertex. When mode functions with time derivatives contract to give the propagator, the integrand is proportional to the tadpole leg momentum, k and hence vanishes for $k = 0$. When modes with a time and a spatial derivative contract, the propagator is non-vanishing but finite for $k = 0$, hence the integrand is finite and the tadpole contribution vanishes by the scaleless arguments given in this section.

2.3 Renormalisation: canceling the unphysical logarithms

Effective Field theories can be renormalised by including a finite number of higher dimensional counter-term operators, at a particular order in the EFT expansion. For example, a loop diagram constructed out of two insertions of a dimension six operator is of order $\left(\frac{E}{\Lambda}\right)^4$ and therefore, requires an eight-dimensional counter term operators to absorb loop divergences.

In this chapter, we used interactions from the following EFT,

$$S = \int d^4x \sqrt{-g} \left[\left(\dot{\pi}_c^2 - \frac{(\partial\pi_c)^2}{a^2} \right) + \frac{\tilde{g}_3}{\Lambda^2} \dot{\pi}_c^3 + \frac{\tilde{g}_4}{\Lambda^4} \dot{\pi}_c^4 - \frac{3\tilde{g}_3}{2\Lambda^4} \dot{\pi}_c^2 \left(\frac{\partial\pi_c}{a} \right)^2 \right], \quad (2.43)$$

where Eqn. (2.3) has been rewritten in terms of dimensionless couplings $\{\tilde{g}_i\}$, canon-

ically normalized $\pi_c = \sqrt{-\dot{H}} M_{pl} \pi$, and inverse powers of some UV cutoff scale Λ , assuming all operators are generated by integrating out the same scale in the UV [120]. It is now clear that our loop results are of order $\sim (\frac{H}{\Lambda})^6$, therefore we need ten-dimensional cubic operators as counter terms. In this section, we will not explicitly renormalize the theory by writing down all such possible counter terms. Instead, we will show that, provided the counter-terms, giving the same kinematical structures as that of terms multiplying the $1/\delta$ divergence in the loop, exist, then they *necessarily* cancel the unphysical logarithms. To see this, consider the structure of counter-term operators required to renormalize the bispectrum at 1-loop,

$$\mathcal{O}^{\text{CT}} \sim \frac{1}{\delta} \int \mu^{-\delta/2} a^{-3+\delta} \partial^7 (\pi^3) , \quad (2.44)$$

where ∂^7 are seven space/time derivatives. Such an operator will produce a tree level contribution to the bispectrum, and as mentioned before, we will consider the external modes in this tree level contribution to be in 3-dimensions, and the internal modes to be in d - dimensions. The three internal mode functions plus the scale factors bring the following δ dependence in the in-in integrals,

$$\frac{1}{\delta} \int d\tau \mu^{-\delta/2} (-H\tau)^{\delta/2} \dots , \quad (2.45)$$

where the dots represent the remaining terms involving factors of external kinematics. Expanding about $\delta = 0$ yields,

$$\int d\tau \left(\frac{1}{\delta} - \frac{1}{2} \log \mu + \frac{1}{2} \log (-H\tau) \right) \dots . \quad (2.46)$$

Once again the $\log (-H\tau)$ can be pulled out of the time integrals as $\log \left(\frac{H}{k_T} \right)$ upto non-log finite terms. This gives a common factor of $\left(\frac{1}{\delta} + \frac{1}{2} \log \left(\frac{H}{\mu} \right) - \frac{1}{2} \log k_T \right)$ from

all the counter term contributions to the tree level bispectrum. We clearly see that the coefficient of the unphysical logarithm $\log k_T$ and the $1/\delta$ divergent piece have a relative factor of $-1/2$, which is exactly the case for the unrenormalised answer of each diagram as well. This remarkable observation allows us to conclude that if the counter-term is set appropriately to absorb the loop divergence, then it must necessarily cancel the unphysical logarithm.

Concretely, the diagrams computed in Sec. 2.2, which have the following structure,

$$D(k_1, k_2, k_3) = \frac{S_{d-2} k_3^N}{2} F_0(\{x_i\}) \times \left(\frac{1}{\delta} + \frac{1}{2} \log \left(\frac{k_3}{k_T} \right) - \frac{1}{2} \log k_T + \frac{3}{2} \log \left(\frac{H}{\mu} \right) + F_1(\{x_i\}) \right) + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} , \quad (2.47)$$

are renormalized by an appropriate linear combination of dimension-10 cubic counter-term operators, whose tree level contributions have the form,

$$D^{\text{CT}} = - \left[\frac{S_{d-2} k_3^N}{2} F_0(\{x_i\}) \times \left(\frac{1}{\delta} + \frac{1}{2} \log \left(\frac{H}{\mu} \right) - \frac{1}{2} \log k_T \right) + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} \right] . \quad (2.48)$$

To see this without explicitly renormalizing the full bispectrum, consider the contact contribution of the dimension-10 cubic counter term : $\frac{C_1}{\delta} \frac{g_3 g_2}{(-\dot{H} M_{pl}^2)^2} \ddot{\pi} \ddot{\pi} \ddot{\pi}$, where C_1 is a dimensionless number, which is determined together with the coefficients of the other counter terms by imposing cancellation of $1/\delta$ divergence of the unrenormalized correlator. The time integrals computed with $\mathcal{O}(\delta^0)$ and $\mathcal{O}(\delta)$ contributions

from modes and measure, denoted by D_1^{CT} and D_2^{CT} respectively, are given by,

$$\begin{aligned}
D_1^{\text{CT}} &\sim C_1 g_3 g_4 \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{(-\dot{H} M_{pl}^2)^5} \frac{1}{k_2 k_3 k_T^7} \left(k_1^3 - 5(k_2 + k_3) k_1^2 + (-3k_2^2 \right. \\
&\quad \left. + 34k_3 k_2 - 3k_3^2) k_1 + (k_2 + k_3) (3k_2^2 - 14k_3 k_2 + 3k_3^2) \right) \left\{ \frac{1}{\delta} - \frac{1}{2} \log \mu \right\}, \\
D_2^{\text{CT}} &= \frac{1}{2} \delta D_1^{\text{CT}} \log \left(\frac{H}{k_T} \right) + \text{NLf}, \tag{2.49}
\end{aligned}$$

and we see that these two add up to give an unphysical logarithm of $-\frac{1}{2} \log k_T$ in $D^{\text{CT}} = D_1^{\text{CT}} + D_2^{\text{CT}}$, as in Eqn. (2.48). As mentioned previously, we do not explicitly renormalize the bispectrum in this work, which involves determination of C_1 along with coefficients of all other allowed dimension 10 cubic counter term operators (such as $(\partial^2 \dot{\pi})^2 \dot{\pi}$, $\partial^2 \dot{\pi} \partial^2 \pi \ddot{\pi}$, and others). However we stress that, when considering a different counter term contribution, only the k_i dependent polynomial in D_1^{CT} changes, while the relation of D_2^{CT} with D_1^{CT} as given by Eqn. (2.49) remains unchanged. This is because evaluation of time integrals with an additional factor of $\log \tau$ is equivalent, up to finite terms, to pulling the factor out of the integrals by substituting $\tau \rightarrow 1/k_T$, as discussed throughout this chapter and shown in Appendix. A.3.

Adding Eq. (2.47) and Eq. (2.48) we get,

$$\begin{aligned}
D^{\text{renorm}} &= \delta^3 \left(\sum_i \vec{k}_i \right) \frac{S_{d-2} k_3^N}{2} \left(F_0(\{x_i\}) \log \left(\frac{k_3}{k_T} \right) + F_0(\{x_i\}) \log \left(\frac{H}{\mu} \right) + F_1(\{x_i\}) \right) \\
&\quad + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\}. \tag{2.50}
\end{aligned}$$

Hence, this results in the cancellation of unphysical logarithms in all the diagrams computed, leaving us with logarithms of the kind $\log(k_i/k_T)$ and $\log(H/\mu)$.

We emphasize that the crucial takeaway from this section is that the unphysical logarithms, of the form $\log \frac{k}{\mu}$, obtained in the loop computations of the previous

sections, are canceled only after renormalisation.

It is important to note that the unrenormalized power spectrum at 1-loop, computed in [2], did not feature any unphysical logarithm, since the $\mathcal{O}(\delta)$ contribution from modes (6 internal modes) and measure (2 factors of a^δ) was just right to cancel the unphysical logs. Schematically, the power spectrum at 1-loop is given by,

$$\mu^{-\delta/2} \int d\tau d^{3+\delta} p_1 d^{3+\delta} p_2 (-H\tau)^\delta \dots, \quad (2.51)$$

Again the logarithm can be factored out at scale k which is the only scale in the problem and the integral evaluates to the form given in Eq. (2.16) with $\sum_i n_i = 1$, and we get,

$$\begin{aligned} D &= \frac{\mu^{-\delta/2} S_{d-2} k^N}{2} \left(\frac{F_0}{\delta} + F_0 \log k + F_0 \log(H/k) + \text{finite} \right) \\ &= \frac{S_{d-2} k^N}{2} \left(\frac{F_0}{\delta} + F_0 \log \frac{H}{\mu} + \text{finite} \right). \end{aligned} \quad (2.52)$$

So, in this case, the cancellation of unphysical logarithms did not require renormalization. However, this was a happy coincidence, and for general one-loop diagrams beyond the power spectrum, the unphysical logarithms will cancel only after considering contribution from counter terms.

2.4 Cutoff Regularisation: a cross check

In this chapter, we work with the modes expanded about $\delta = 0$, since working with the full, unexpanded mode functions results in complicated hypergeometric functions once the time integrals are performed, and hence the loop integrals become intractable. Now, the expansion in δ for $(-H\tau)^\delta = e^{\delta \log(-H\tau)}$ converges for all values of δ . However, we then commute the summation operator across the integrals, to

sum over each of the integrals, whereas we should indeed be integrating over the full (sum) series. This commutation is subtle, and is allowed only if the series converges. However, for the loop integrals to converge, δ must be analytically continued to some point in the complex plane, and arguing the convergence of the sum of integrals is not straightforward at this value of δ . For these reasons, we compute one of the diagrams previously computed in Sec. 2.2 in cutoff regularization, as a cross check of our results, to demonstrate that our implementation of dimensional regularisation reliably computes the logarithmic running of the correlator, despite the subtlety mentioned above.

We focus on the time ordered contribution to Diagram I computed with one insertion each of the cubic and quartic interactions $\dot{\pi}^3$ and $\dot{\pi}^4$ respectively. Although there is another contribution when one considers the quartic interaction $\dot{\pi}^2(\partial\pi)^2$ instead, it cannot alter the log structure we compute below since it has a different coupling constant. Moreover, since the mode functions used in cutoff regularisation 3-dimensional, the tree level contributions from the counter terms cannot produce log k terms. Thus the unrenormalised correlator computed in cutoff regularization should not feature any unphysical logarithms, which we find to be indeed true in the following.

In cutoff regularisation, the momentum integrals are regulated with a time dependent cutoff $\Lambda a(\tau_f)$, where Λ is the physical cutoff and τ_f is the time being integrated first, e.g. for the $\Theta(\tau_1 - \tau_2)$ piece, $\tau_f = \tau_2$. Due to this time dependence, the loop integrals now must be integrated first. We use,

$$\int d^3\vec{p}_1 d^3\vec{p}_2 Q(p_1, p_2, k_i) \delta^3(\vec{p}_1 + \vec{p}_2 + \vec{k}_3) = 2\pi \int_0^{\Lambda a(\tau_f)} dp_1 \int_{|\vec{p}_1 - \vec{k}_3|}^{\vec{p}_1 + \vec{k}_3} dp_2 \frac{p_1 p_2}{k_3} Q(p_1, p_2, k_i). \quad (2.53)$$

Evaluating the $\tau_1 > \tau_2$ piece of the loop integrals gives the following :

$$\begin{aligned}
D = & -36 \, g_3 g_4 \, \delta^3 \left(\sum_i \vec{k}_i \right) \int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1 \left(1 + \frac{H}{\Lambda}\right)} d\tau_2 \left\{ \frac{1}{16k_3 (\tau_1 - \tau_2)^6} \tau_1^2 \tau_2^4 \left(e^{i(k_1 + k_2 + k_3)\tau_2} \right. \right. \\
& \left. \left(-k_3^2 (\tau_1 - \tau_2)^2 + 5ik_3 (\tau_1 - \tau_2) + 8 \right) + e^{i(k_1 + k_2 + 3k_3)\tau_2 - 2ik_3\tau_1} (8k_3^4 (\tau_1 - \tau_2)^4 \right. \\
& \left. - 20ik_3^3 (\tau_1 - \tau_2)^3 + k_3^2 (\tau_1 - \tau_2)^2 (-27 + 8e^{2ik_3(\tau_1 - \tau_2)}) - ik_3 (\tau_1 - \tau_2) \left(-21 \right. \right. \\
& \left. \left. + 16e^{2ik_3(\tau_1 - \tau_2)} \right) - 16e^{2ik_3(\tau_1 - \tau_2)} + 8 \right) \right) + \dots \\
& + \frac{1}{240k_3 (\tau_1 - \tau_2)^6} \tau_1^2 \tau_2^4 \left(-2ik_3^5 (\tau_1 - \tau_2)^5 e^{2ik_3(\tau_1 - \tau_2)} - 10k_3^4 (\tau_1 - \tau_2)^4 \left(12 + e^{2ik_3(\tau_1 - \tau_2)} \right) \right. \\
& \left. + 20ik_3^3 (\tau_1 - \tau_2)^3 \left(15 + 2e^{2ik_3(\tau_1 - \tau_2)} \right) - 15k_3^2 (\tau_1 - \tau_2)^2 \left(-27 + e^{2ik_3(\tau_1 - \tau_2)} \right) + 15ik_3 \right. \\
& \left. (\tau_1 - \tau_2) \left(-21 + 5e^{2ik_3(\tau_1 - \tau_2)} \right) + 120 \left(-1 + e^{2ik_3(\tau_1 - \tau_2)} \right) \right) e^{i(k_1 + k_2)\tau_2 - ik_3(2\tau_1 - 3\tau_2)} \left. \right\}, \tag{2.54}
\end{aligned}$$

where we regulate the $\frac{1}{\tau_1 - \tau_2}$ divergence in the time integrals with the same cutoff

Λ . The first 4 lines correspond to $q_1 > k_3$, the last 3 lines correspond to $q_1 < k_3$.

Similarly, evaluating the $\tau_2 > \tau_1$ piece gives,

$$\begin{aligned}
D = & -36 \, g_3 g_4 \, \delta^3 \left(\sum_i \vec{k}_i \right) \int_{-\infty}^0 d\tau_2 \int_{-\infty}^{\tau_2 \left(1 + \frac{H}{\Lambda}\right)} d\tau_1 \left\{ \frac{1}{16k_3 (\tau_1 - \tau_2)^6} \tau_1^2 \tau_2^4 e^{i(2k_3\tau_1 + (k_{12} - 3k_3)\tau_2)} \right. \\
& \left(8k_3^4 (\tau_1 - \tau_2)^4 e^{2ik_3\tau_1} + 20ik_3^3 (\tau_1 - \tau_2)^3 e^{2ik_3\tau_1} - k_3^2 (\tau_1 - \tau_2)^2 \left(27e^{2ik_3\tau_1} - 7e^{2ik_3\tau_2} \right) \right. \\
& \left. - ik_3 (\tau_1 - \tau_2) \left(21e^{2ik_3\tau_1} - 11e^{2ik_3\tau_2} \right) + 8 \left(e^{2ik_3\tau_1} - e^{2ik_3\tau_2} \right) \right) + \dots \\
& + \frac{1}{4k_3 (\tau_1 - \tau_2)^3} \tau_1^2 \tau_2^4 e^{i(k_3(2\tau_1 - \tau_2) + (k_1 + k_2)\tau_2)} \left(\frac{1}{5} ik_3^5 (\tau_1 - \tau_2)^2 e^{i(k_3(2\tau_1 - \tau_2) + (k_1 + k_2)\tau_2)} \right. \\
& + \frac{1}{2} k_3^4 (\tau_1 - \tau_2) (1 - ik_3 (\tau_1 - \tau_2)) e^{i(k_3(2\tau_1 - \tau_2) + (k_1 + k_2)\tau_2)} + \frac{1}{3} ik_3^3 (k_3^2 (\tau_1 - \tau_2)^2 \\
& + 2ik_3 (\tau_1 - \tau_2) - 2) - \frac{1}{4 (\tau_1 - \tau_2)^3} e^{i(2k_3\tau_1 + (k_1 + k_2 - k_3)\tau_2)} \left(k_3^2 (\tau_1 - \tau_2)^2 + 5ik_3 (\tau_1 - \tau_2) \right. \\
& \left. - 8 \right) + \frac{1}{4 (\tau_1 - \tau_2)^3} e^{i(4k_3\tau_1 + (k_1 + k_2 - 3k_3)\tau_2)} \left(-8k_3^4 (\tau_1 - \tau_2)^4 - 20ik_3^3 (\tau_1 - \tau_2)^3 \right. \\
& \left. \left. + 27k_3^2 (\tau_1 - \tau_2)^2 + 21ik_3 (\tau_1 - \tau_2) - 8 \right) \right) \left. \right\}, \tag{2.55}
\end{aligned}$$

where the first 3 lines correspond to $q_1 > k_3$ and the last 5 lines correspond to $q_1 < k_3$. In Eqns. (2.54) and (2.55) the dots represent contributions that are proportional to the term $(\text{Exp}\{\frac{\Lambda}{H}(\frac{\tau_s}{\tau_f} - 1)\})$, where the Θ function imposes $\tau_s > \tau_f$. These terms do not produce any logarithms, as was noted in [2], and hence we ignore them.

Evaluation of the time integrals gives the following correlator,

$$D = 72 \, g_3 g_4 \, \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{k_3(k_1 + k_2)(k_3 - 2k_1 - 2k_2)}{k_1 k_2 k_T^7} \left\{ \log \frac{H}{\Lambda} \right. \\ \left. + \frac{1}{2} \log \frac{k_3}{k_T} \right\} + \{k_3 \longleftrightarrow k_1\} + \{k_3 \longleftrightarrow k_2\} + \text{power law div in } \Lambda + \text{NLf}. \quad (2.56)$$

Upon renormalisation, the $\log \frac{H}{\Lambda}$ is expected to turn into $\longrightarrow \log \frac{H}{\mu}$. We find that the logarithm running, as well as the structure of the coefficients multiplying them, matches with the renormalized correlator computed in dimensional regularization, which confirms that the procedure of expanding the mode functions in δ reliably computes the log running in the correlator.

Chapter 3

Singularities in Cosmological Loop Correlators

The material presented in this chapter is based on the author's publication [4].

There is an important lesson in Chapter 2: properly understanding loop effects is essential for understanding the predictions as well as the robustness of inflationary theories. In many cases secular growth are a genuine feature of the theory and has to be properly resummed (for instance, through stochastic inflation)[47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60]; loop effects can also appreciably affect predictions on non-Gaussianities [39, 40, 41, 42, 43, 44, 45, 46], or lead to interesting predictions such as production of primordial black holes [61, 62, 63, 64, 65, 66, 67].

Aside from secular divergences, there are many other reasons to care about loop effects in dS and inflation. One of the important lessons from the S-matrix community is that physical constraints are often manifest as analyticity properties in the observable: unitarity, locality and causality all leave imprints in the complex plane of the kinematics of the amplitude [68, 69, 70, 71, 72, 73]. Since loop effects may introduce branch cuts (through transcendental functions such as polylogarithms),

understanding loop corrections can lead us to a better understanding of the analytic structure of inflationary correlators, and may bring us a step closer towards bootstrapping cosmological observables.

While in-in correlators are the observable of choice for many cosmologists, the analytic properties of the wavefunction is better understood [79, 121, 114, 101]. In cases where IR divergences are absent (for instance the effective field theory of inflation [117]), the wavefunction or correlator calculations in dS or inflation can be related to those in flat space by taking derivatives [122, 45]. We know that the singularities of the flat space wavefunction are given by the *energy conservation condition*: poles and branch cuts occur when energies entering a subgraph of a Feynman diagram vanishes [96, 20, 78, 97, 79, 33]. In addition, in-in correlators can be computed from wavefunction coefficients, so their singularities should be related to each other. This give us an easy way to study the analyticity properties of in-in correlators in some phenomenologically relevant inflationary theories. An effort in this direction was made in [34].

In this chapter we will re-examine the analytic structure of in-in correlators carefully, by studying the features of the time and momentum integrals for the in-in correlators and regularizing the integrals properly. This chapter is organized as follow: we compute a few examples of correlation functions at 1-loop with 2 sites in Sec. 3.1. Motivated by the trend in the results, we deduce diagrammatic rules to extract the singularity structure of loop corrections in Sec. 3.2.1. These rules may be used to bypass explicit computations and obtain all possible poles and branch cuts simply based on the energy injection into the loop sites and various subgraphs. This is similar to how Feynman rules can be followed to write down integral expressions for a given diagram without evaluating the full in-in expression. In Sec. 3.2.2 we consider the structure of a general n -point function at 1-loop with 2-sites and study the

expected log structure arising once the loop integrals are performed, demonstrating how the previously mentioned diagrammatic rules follow. In Sec. 3.3 we revisit the cosmological KLN theorem [34], which forbids the presence of total energy branch points in off-shell cosmological correlators of massless fields. We discuss a loophole which allows us to bypass the cosmological KLN theorem: namely, de Sitter regularization schemes (including dimensional regularization with d -dimensional mode function as well as time dependent cutoff regularization) can give rise to terms which generates total energy branch points. We also show that all branch cuts produced will always repackage themselves in a renormalised correlator to produce logarithms of ratios of comoving scales, thereby not producing any scale-breaking terms.

The main results of this chapter are as follows,

- The singularity structure of any *off-shell* correlation function at 1-loop with 2-sites can be extracted without explicit computation, using diagrammatic rules (see Sec. 3.2.1). To proceed, one must identify certain subgraphs in the diagram, which we call the “*loop*”-, the “*left*”- and the “*right*”- subgraphs, defined in Sec. 3.2.1.
 - If the energy flowing into the *loop*-subgraph through the left and right loop site is $\mathcal{S}_L$ and $\mathcal{S}_R$ respectively, then the loop correction will feature a logarithm of $\mathcal{S}_L + s$ and $\mathcal{S}_R + s$.
 - If the energy flowing through the *left*- (*right*-) subgraph through all vertices except the right (left) loop site is $\mathcal{S}_{L_i}$ ($\mathcal{S}_{R_j}$), then the loop correction will feature a logarithm of $\mathcal{S}_{L_i} + s$ ($\mathcal{S}_{R_j} + s$).

These results hold for correlation functions in flat space as well as de-Sitter. In de Sitter, for the case involving only massless scalars, the rules hold for

interactions with derivative(s) acting on each field insertion. For correlators in flat space and de Sitter correlators where the fields running in the loop are conformally coupled scalars, our rules work with polynomial as well as derivative interactions.

- Additionally, correlation functions in de-Sitter feature logarithm of total energy from time integrals due to $\mathcal{O}(\delta)$ contributions from modes and measure.
- In a renormalised correlator, all branch cuts will combine to produce logarithms of ratios of comoving scales, such that final answer is scale invariant.

3.1 Explicit Computations

In an effort to better understand the analytic structure of correlators at the loop level, we explicitly compute some correlations in flat space and de-Sitter. We will use dimensional regularisation to regulate loop divergences. Later on we discuss that our results are not just artifacts of the regularisation scheme, and hold true even in cutoff regularisation (See Sec. 3.3.1). In this section, we only compute correlator contributions with vertices on the same side of $\tau = 0$. Later on in Sec. 3.2.3, we consider all possible contributions where vertices can be $\tau > 0$ or $\tau < 0$, and show that the results of our work follow through.

For flat space computations, we will consider a theory with polynomial quartic ($g_4\phi^4$) and cubic ($g_3\phi^3$) interactions of a massless scalar ϕ . For correlators in de Sitter, we will work with interactions involving time derivatives acting on massless scalars and polynomials of conformal scalars (σ), such as $\dot{\phi}^3$, $\dot{\phi}\sigma^2$ and so on. Also, we will choose to drop overall factors of normalization ($\prod_{ij} \omega_i p_j$) coming from modes as well as numerical constants when stating the results of computations.

3.1.1 Correlators in Flat Space

Bispectrum at 1-loop with cubic and quartic interaction: Consider the $(++)$ contribution to the bispectrum in the diagram of Fig. 3.1.

Let us denote the magnitude of the momentum exchanged between the two sites

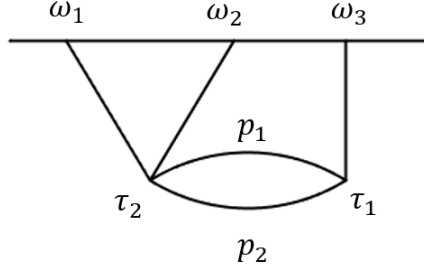


Figure 3.1: Bispectrum at 1-loop with interactions $g_3\phi^3$ and $g_4\phi^4$.

as $s = |\vec{p}_1 + \vec{p}_2|$, and the energy flowing through the loop sites as follows,

$$E_L = \omega_1 + \omega_2 + p_1 + p_2, \quad E_R = \omega_3 + p_1 + p_2. \quad (3.1)$$

Evaluation of the time integrals leads to partial and total energy poles,

$$\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle^{1\text{-loop}} \sim \frac{g_3 g_4}{\omega_T} \int d^d \vec{p}_1 d^d \vec{p}_2 \frac{1}{p_1 p_2} \frac{1}{E_L E_R} (\omega_T + 2p_+). \quad (3.2)$$

where $\omega_T = \omega_1 + \omega_2 + \omega_3$ and $p_1 + p_2 = p_+$. Both the partial energies E_L, E_R depend on loop momenta, and these simple poles turn into branch cuts once the loop integrals are carried out, resulting in the following,

$$\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle^{1\text{-loop}} \sim \frac{g_3 g_4}{\omega_T} \left(\frac{1}{\delta} + \frac{1}{2} \log \left(\frac{(\omega_3 + s)(\omega_{12} + s)}{\mu^2} \right) \right) + \text{NLF}, \quad (3.3)$$

where μ is the renormalization scale and Nlf denotes Non-Logarithmic finite terms.

Trispectrum at 1-loop with quartic interactions : Consider the $(++)$ contribution to the trispectrum in the diagram of Fig. 3.2.

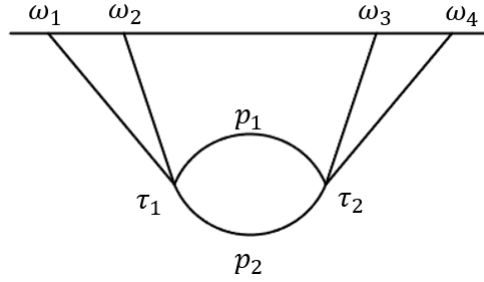


Figure 3.2: Trispectrum at 1-loop with interaction $g_4\phi^4$.

Once again let us denote the magnitude of the momentum exchanged between the two sites as $s = |\vec{p}_1 + \vec{p}_2| = |\vec{k}_1 + \vec{k}_2| = |\vec{k}_3 + \vec{k}_4|$, and the energies flowing through the vertices as,

$$E_L = \omega_1 + \omega_2 + p_1 + p_2, \quad E_R = \omega_3 + \omega_4 + p_1 + p_2. \quad (3.4)$$

Evaluation of time integrals gives partial and total energy poles as follows,

$$\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle^{1\text{-loop}} \sim \frac{g_4^2}{\omega_T} \int d^d \vec{p}_1 d^d \vec{p}_2 \frac{1}{p_1 p_2} \frac{1}{E_L E_R} (\omega_T + 2p_+) . \quad (3.5)$$

Upon performing the loop integrals both partial energy poles once again evaluate to

branch cuts as follows,

$$\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle^{1\text{-loop}} \sim \frac{g_4^2}{\omega_T} \left(\frac{1}{\delta} + \frac{1}{2} \log \left(\frac{(\omega_{12} + s)(\omega_{34} + s)}{\mu^2} \right) \right) + \text{NLf}, \quad (3.6)$$

where $\omega_{12} = \omega_1 + \omega_2$ and $\omega_{34} = \omega_3 + \omega_4$.

Note that in both diagrams of Fig. 3.1 and 3.2, there are no non-trivial subgraphs and only involve the loop. Let us now consider diagrams which have non-trivial subgraphs.

Bispectrum at 1-loop with three cubic interactions : Consider the $(++ +)$ contribution to the bispectrum in the diagram of Fig. 3.3. Notice there are three subgraphs to this diagram, one of which is trivial. Two of the subgraphs involve the loop, including the trivial one, and are shaded in blue and red. The magnitude of the momentum flowing between loop sites is $s = |\vec{p}_1 + \vec{p}_2|$. The energy flowing through the vertices τ_1, τ_2 and τ_3 are E_1, E_2 and E_3 respectively. The partial energies flowing through subgraphs of this diagram are given by,

$$\begin{aligned} E_1 &= \omega_1 + \omega_2 + p_3, & E_2 &= p_3 + p_1 + p_2, & E_3 &= p_1 + p_2 + \omega_3, \\ E_L &= \omega_1 + \omega_2 + p_1 + p_2, & E_R &= \omega_3 + p_3. \end{aligned} \quad (3.7)$$

Evaluation of the time integrals leads to poles in total energy and partial energies as follows,

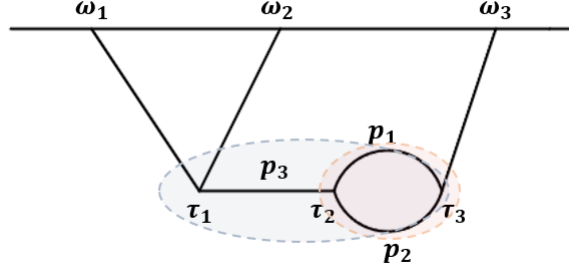


Figure 3.3: Bispectrum at 1-loop correction with interaction $g_3\phi^3$. The blue-shaded subgraph is trivial. The red-shaded part denotes the subgraph of this diagram only involving the loop.

$$\begin{aligned}
\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle^{1\text{-loop}} &\sim \frac{g_3^3}{p_3 E_1 E_R \omega_T} \int d^d \vec{p}_1 d^d \vec{p}_2 \frac{1}{p_1 p_2} \frac{1}{E_2 E_3 E_L} \left[2p_3^2 (\omega_T + 2p_+) \right. \\
&+ (\omega_{12} + p_+) (\omega_3 + 2p_+) \omega_T + p_3 \left(\omega_1 (2\omega_2 + 3\omega_3 + 5p_+) + 5\omega_3 p_1 + p_2 (5\omega_3 + 8p_1) \right. \\
&\left. \left. + \omega_2 (3\omega_3 + 5p_+) + \omega_1^2 + \omega_2^2 + 2\omega_3^2 + 4p_1^2 + 4p_2^2 \right) \right]. \quad (3.8)
\end{aligned}$$

The poles in total energy ω_T and partial energies E_1, E_R are independent of loop momenta and hence are pulled out of the integrals. After performing the loop integrals the partial energy poles in E_2, E_3 and E_L once again evaluate to logarithms as follows,

$$\begin{aligned}
\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle^{1\text{-loop}} &\sim \frac{g_3^3}{p_3 E_1 E_R \omega_T} (\omega_T + 2p_3) \left[\frac{1}{\delta} + \frac{1}{2} \log \frac{(\omega_3 + s)(p_3 + s)}{\mu^2} \right] \\
&+ \frac{g_3^3}{\omega_T E_1 E_R} \frac{\omega_3 + p_3}{\omega_{12} - p_3} \log \left(\frac{\omega_{12} + s}{p_3 + s} \right) + \text{NLF}, \quad (3.9)
\end{aligned}$$

where in the second line there is no folded pole at $\omega_{12} = p_3$.

Of course the *on-shell correlator* can be found from Eqn. (3.9) by putting the

external legs onshell, i.e. $\omega_i = k_i$ and using $s = p_3 = |\vec{k}_3| = \omega_3$, which simplifies the previous expression to,

$$\begin{aligned} \langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle^{1\text{-loop}} &\sim \frac{g_3^3}{k_3 E_1 E_R k_T} \left((k_T + 2k_3) \left[\frac{1}{\delta} + \frac{1}{2} \log \frac{k_3 E_R}{\mu^2} \right] \right. \\ &\quad \left. + \frac{2k_3^2}{k_{12} - k_3} \log \left(\frac{k_T}{E_R} \right) \right) + \text{NLF}. \end{aligned} \quad (3.10)$$

Notice that the *off-shell* correlator does not have a total energy branch point ($\log \omega_T$), and it is only after imposing external legs onshell that we find a total energy logarithm ($\log k_T$). This result goes by the name of Cosmological KLN Theorem [34]. However, we will see that de Sitter examples will feature total energy logarithms in the *off-shell* correlators. We discuss this in Sec. 3.3.

5-point function at 1-loop with cubic and quartic interactions : Finally let us consider the $(+ + + +)$ contribution to the 5-point function in the diagram of Fig. 3.4. Notice there are six subgraphs for this diagram, one of which is trivial. Four of these involve the loop, including the trivial one, and the non-trivial ones are shaded in red, blue and green in Fig. 3.4. The red-shaded subgraph involves only the loop.

Here the partial energies entering vertices and subgraphs of the diagram are,

$$\begin{aligned} E_1 &= \omega_1 + \omega_2 + p_3, & E_2 &= \omega_3 + p_3 + p_1 + p_2, & E_3 &= p_1 + p_2 + p_4, \\ E_4 &= p_4 + \omega_4 + \omega_5, & E_{L_1} &= \omega_1 + \omega_2 + \omega_3 + p_4, & E_L &= \omega_1 + \omega_2 + \omega_3 + p_1 + p_2, \\ E_M &= p_3 + \omega_3 + p_4, & E_R &= p_1 + p_2 + \omega_4 + \omega_5, & E_{R_1} &= p_3 + \omega_3 + \omega_4 + \omega_5. \end{aligned} \quad (3.11)$$

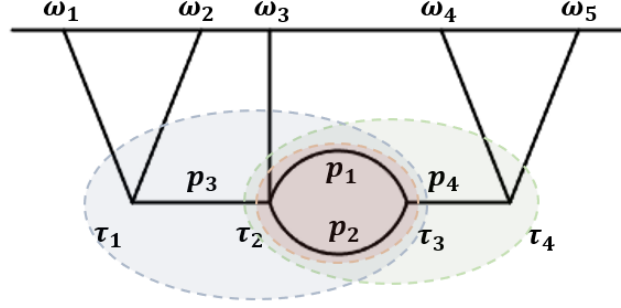


Figure 3.4: 5-point function at 1-loop with interactions $g_3\phi^3$ and $g_4\phi^4$. The red, green and blue shaded parts denote non-trivial subgraphs of this diagram involving the loop. The red shaded subgraph involves only the loop.

The time integrals produce poles in total energy and partial energies. Once again the partial energies depending on loop momenta (i.e. E_2, E_3, E_L and E_R) turn into logarithms once loop integrals are evaluated, giving us the following,

$$\begin{aligned}
 \langle \phi_{\vec{k}_1} \dots \phi_{\vec{k}_5} \rangle^{1\text{-loop}} \sim g_4 g_3^3 & \left\{ \frac{2}{\delta} \frac{1}{\omega_T E_1 p_4} \left(\frac{-1}{E_{L_1}} - \frac{2E_1 + E_M}{E_{L_1} E_M} - 2 \frac{E_1}{E_M E_{R_1}} - 2 \frac{E_1 + E_{R_1}}{E_{R_1} E_4} \right) \right. \\
 & + 4 \frac{E_1 + E_{R_1}}{\omega_T E_1 E_{R_1} E_4} \frac{\log[(\omega_{45} + s)/\mu]}{\omega_{45} - p_4} - 2 \frac{E_1 + E_M}{p_4 E_1 E_{L_1} E_M} \frac{\log[(p_4 + s)/\mu]}{\omega_{45} - p_4} \\
 & \left. - 2 \frac{E_M + E_4}{p_4 E_{R_1} E_4 E_M} \frac{\log[(\omega_3 + p_3 + s)/\mu]}{\omega_{12} - p_3} + 4p_3 \frac{E_{L_1} + E_4}{\omega_T p_4 E_1 E_{L_1} E_4} \frac{\log[(\omega_{123} + s)/\mu]}{\omega_{12} - p_3} \right\} + \text{NLf},
 \end{aligned} \tag{3.12}$$

where the correlator above is finite in the limit $\omega_{12} \rightarrow p_3$ and $\omega_{45} \rightarrow p_4$, and hence there are no folded poles.

3.1.2 Correlators in de-Sitter

When the internal and external fields are either massless or conformally coupled scalars, we can write the correlator as a derivative of flat space expressions [122, 45], with one important caveat: since the de-Sitter mode functions are d -dimensional, they contribute additional logarithmic terms. We will expand on this point in Sec-

tion 3.3.1.

Bispectrum at 1-loop with cubic interactions: As a simple example let us consider the 1-loop corrections to the 3-pt function in Fig. 3.3 with the vertices ordered as $(+++)$, along with the contributions $(-++)$, $(-- +)$, $(+ - +)$. The left vertex is mediated by a $\lambda_1 \dot{\phi}^3$ interaction while the middle and right vertex is mediated by the interaction $\lambda_2 \dot{\phi} \sigma^2$. Hence the internal line with momentum p_3 as well as the external lines are all massless scalars, and conformally coupled scalars run in the loop.

The correlator is given by:

$$\begin{aligned}
\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{dS}^{1\text{-loop}} &= \sum_{a_L, M, R = \pm} \frac{i^3 a_L a_R a_M \lambda_1 \lambda_2^2}{8 k_1^3 k_2^3 k_3^3} \\
&\times \int_{-\infty}^0 \frac{d\tau_L}{\tau_L^{4+\delta}} \frac{d\tau_M}{\tau_M^{4+\delta}} \frac{d\tau_R}{\tau_R^{4+\delta}} \int \frac{d^{3+\delta} p}{(2\pi)^3} k_1^2 k_2^2 \tau_L^{4+\delta} k_3^2 \tau_R^{2+\frac{\delta}{2}} e^{ia_L \omega_L \tau_L} e^{ia_R \omega_R \tau_R} \\
&\times \tau_L \tau_M \partial_{\tau_L} \partial_{\tau_M} G_{a_L a_M}^\phi(p_3; \tau_L, \tau_M) G_{a_M a_R}^\sigma(p_1; \tau_M, \tau_R) G_{a_M a_R}^\sigma(p_2; \tau_M, \tau_R).
\end{aligned} \tag{3.13}$$

where G is the propagator defined in Eqn. (4.5), $\omega_L = \omega_{12}$ and $\omega_R = \omega_3$.

From the time integral representation (3.13), it is straightforward to see that the off-shell correlator is given by:

$$\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{dS}^{1\text{-loop}}(\omega_L, \omega_R) = - \left(1 - \frac{3}{2} \delta \log\left(\frac{\omega_T}{H}\right) \right) \partial_{\omega_L}^2 \left(\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{Flat}^{1\text{-loop}}(\omega_L, \omega_R) \right). \tag{3.14}$$

Here $\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{Flat}^{1\text{-loop}}(\omega_L, \omega_R)$ refers to the same diagram computed in flat space.

In this example, the *unrenormalized* correlator gets an additional logarithmic correction. These logarithmic terms come from expanding the mode function as

well as the integration measure in the dim reg parameter δ (see (1.79) and (1.80)). There are 3 time integrals (each contributing $-\delta \log(-H\tau)$) and 9 mode function (3 from external mode functions and 6 from internal propagators, each contributing $\frac{\delta}{2} \log(-H\tau)$), and so upon time integration this gives a contribution of $-\frac{3\delta}{2} \log(\omega_T)$. This will be expanded further in section 3.2.2.

A straightforward calculation leads us to the following expression:

$$\begin{aligned} \langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{dS}^{1\text{-loop}} &\equiv \lambda_1 \lambda_2^2 \left[\frac{p_3^2 (\omega_R + 3\omega_L) + p_3 (3\omega_R \omega_L + \omega_R^2 + 3\omega_L^2) + p_3^3 + \omega_T^3}{E_1^3 E_R \omega_T^3} \right. \\ &\times \left(\frac{1}{\delta} - \frac{3}{2} \log\left(\frac{\omega_T}{H}\right) \right) + \frac{\omega_R (\omega_L^3 + 3p_3^2 \omega_L)}{E_R E_1^3 (p_3 - \omega_R) (p_3 - \omega_L)^3} \log(p_3 + s) \\ &- \frac{\omega_R p_3 (p_3^4 (k_3^2 + 3\omega_L^2) - p_3^2 (9\omega_L^4 - k_3^4) - 9k_3^2 \omega_L^4 + 3k_3^4 \omega_L^2 + 10\omega_L^6)}{E_1^3 \omega_T^3 (\omega_L - \omega_R)^3 (p_3 - \omega_L)^3} \log(\omega_L + s) \\ &\left. - \frac{p_3 \omega_L (3\omega_R^2 + \omega_L^2)}{\omega_T^3 E_R (p_3 - \omega_R) (\omega_R - \omega_L)^3} \log(\omega_R + s) \right]. \end{aligned} \quad (3.15)$$

Just like the flat space example, the folded singularities are spurious.

We would like to renormalize this divergent loop contribution. The contribution from the counterterm comes in the form of a 3-point exchange diagram (where the loop in Fig. 3.3 collapses into a vertex). In this diagram the left vertex is mediated by a $\lambda_1 \dot{\phi}^3$ interaction while the right is mediated by a $-\frac{\lambda_2^2}{\delta} \dot{\phi}^2$ interaction¹.

The exchange diagram reads:

$$\begin{aligned} \langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{dS}^{CT} &= \sum_{a_{L,R}=\pm} \frac{-a_L a_R i^2 \lambda_1 \lambda_2^2}{8k_1^3 k_2^3 k_3^3 \delta} i \int_{-\infty}^0 \frac{d\tau_L}{\tau_L^{4+\delta}} \frac{d\tau_R}{\tau_R^{4+\delta}} k_1^2 k_2^2 \tau_L^{4+\delta} k_3^2 \tau_R^{2+\frac{\delta}{2}} \\ &\times \tau_L \tau_R \partial_{\tau_L} \partial_{\tau_R} G_{a_L a_R}(p_3; \tau_L, \tau_R). \end{aligned} \quad (3.16)$$

¹Since we are considering an *off-shell* diagram, it makes sense to talk about an exchange diagram where one of the vertex has a quadratic vertex. The *on-shell* counterpart of the story is that figure 3.3 is a contact diagram where an external leg receives loop correction, and this exchange diagram with a quadratic vertex is now a contact diagram removing the divergence from this corrected propagator.

Evaluating this diagram after expanding the mode function and time integration measure in δ gives:

$$\begin{aligned} \langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{dS}^{CT} = & -\lambda_1 \lambda_2^2 \left[\frac{p_3^3 + \omega_T^3 + p_3^2(3\omega_L + \omega_R) + p_3(3\omega_L^2 + 3\omega_L\omega_R + \omega_R^2)}{64k_1k_2k_3\omega_T^3 p_3 E_L^3 E_R} \right] \\ & \times \left(\frac{1}{\delta} - \frac{1}{2} \log\left(\frac{\omega_T}{H}\right) \right). \end{aligned} \quad (3.17)$$

The renormalized correlator reads:

$$\begin{aligned} \langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \rangle_{dS}^{1\text{-loop}} \sim & \lambda_1 \lambda_2^2 \left[-\frac{p_3 \omega_L (3\omega_R^2 + \omega_L^2)}{\omega_T^3 E_R (p_3 - \omega_R) (\omega_R - \omega_L)^3} \log\left(\frac{\omega_R + s}{\omega_T}\right) \right. \\ & - \frac{\omega_R p_3 (p_3^4 (\omega_R^2 + 3\omega_L^2) - p_3^2 (9\omega_L^4 - \omega_R^4) - 9\omega_R^2 \omega_L^4 + 3\omega_R^4 \omega_L^2 + 10\omega_L^6)}{E_1^3 \omega_T^3 (\omega_L - \omega_R)^3 (p_3 - \omega_L)^3} \log\left(\frac{\omega_L + s}{\omega_T}\right) \\ & \left. + \frac{\omega_R (\omega_L^3 + 3p_3^2 \omega_L)}{E_R E_1^3 (p_3 - \omega_R) (p_3 - \omega_L)^3} \log\left(\frac{p_3 + s}{\omega_T}\right) \right]. \end{aligned} \quad (3.18)$$

Notice that the arguments of the logarithmic terms are scale invariant, i.e. if we scale the energies by λ as $\omega \rightarrow \lambda\omega, p_3 \rightarrow \lambda p_3$, the argument remains unchanged.

4-point function at 1-loop with cubic interactions: Once again considering interactions $\lambda_1 \dot{\phi}^3$ and $\lambda_2 \dot{\phi} \sigma^2$ with the conformally coupled scalars running in the loop and massless scalar running in all other legs, we consider the $(+++)$ contribution in Fig. 3.5 to the 4-point function at 1-loop.

Similar to the previous example, it is simple to see that the off-shell correlator in dS is related to the same diagram computed in flat space as the following,

$$\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle_{dS}^{1\text{-loop}} = \left(1 - 2\delta \log\left(\frac{\omega_T}{H}\right) \right) \partial_{\omega_L}^2 \partial_{\omega_R}^2 \left(\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle_{\text{Flat}}^{1\text{-loop}} \right), \quad (3.19)$$

where $\omega_L = \omega_1 + \omega_2$ and $\omega_R = \omega_3 + \omega_4$ are the external energies flowing into the left and right vertices τ_1 and τ_4 respectively. The *unrenormalized* correlator gets an

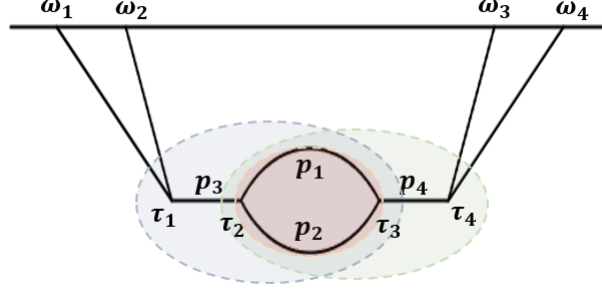


Figure 3.5: 1-loop correction to the 4-point function with cubic interactions $\lambda_1 \dot{\phi}^3$ and $\lambda_2 \dot{\phi} \sigma^2$. The shaded subgraphs involve the loop.

additional logarithmic correction of $-2\delta \log\left(\frac{\omega_T}{H}\right)$ from 4 vertices (each contributing $-\delta \log(-H\tau)$) and 12 modes (each contributes $\frac{\delta}{2} \log(-H\tau)$).

Again, similar to the previous example, renormalizing this loop diagram would give contributions from 4-point exchange diagram, with the loop collapsing to a vertex mediated by a dimension-4 quadratic counter term, e.g. $\frac{\lambda_2^2}{\delta} \dot{\phi}^2$.

After a straightforward computation, the final renormalized 4-point correlator reads,

$$\begin{aligned}
 \langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle_{dS}^{1\text{-loop}} &\sim \lambda_1^2 \lambda_2^2 \left[\frac{-2 \log\left(\frac{\omega_L + s}{\omega_T}\right) Q_1(\omega_L, \omega_R, p_3, p_4)}{\omega_T^5 (p_3 - \omega_L)^3 (\omega_L + p_3)^3 (\omega_L + p_4)^3 (p_4 + \omega_R)^3} \right. \\
 &+ \frac{\log\left(\frac{p_4 + s}{\omega_T}\right)}{p_{34} (\omega_L + p_3)^3 (\omega_L + p_4)^3 (p_4 - \omega_R)^3} \left(2p_3^2 (3\omega_L + p_4) + 2p_3 (3p_4 \omega_L + 3\omega_L^2 + p_4^2) \right. \\
 &+ (\omega_L + p_4)^3 + 2p_3^3 \left. \right) - \frac{2 \log\left(\frac{s + \omega_R}{\omega_T}\right)}{\omega_T^5 (\omega_L + p_3)^3 (p_3 + \omega_R)^3 (p_4^2 - \omega_R^2)^3} Q_2(\omega_L, \omega_R, p_3, p_4) \\
 &+ \left. \frac{\log\left(\frac{p_3 + s}{\omega_T}\right)}{(p_3 - p_4) (p_3 - \omega_L)^3} \left(\frac{1}{(p_4 + \omega_R)^3} - \frac{2p_4}{(p_3 + p_4) (p_3 + \omega_R)^3} \right) \right] \quad (3.20)
 \end{aligned}$$

where Q_1 and Q_2 are polynomials of external and exchange energies, given by,

$$\begin{aligned}
Q_1 = & p_3 \left((2p_4\omega_T^3 (p_3^2 (p_4^2 - 3\omega_L^2) + \omega_L^2 (8p_4\omega_L + 6\omega_L^2 + 3p_4^2) + p_3^4) (p_4 + \omega_R) \right. \\
& + 2p_4\omega_T^2 (p_3^2 (p_4^2 - 3\omega_L^2) + \omega_L^2 (8p_4\omega_L + 6\omega_L^2 + 3p_4^2) + p_3^4) (p_4 + \omega_R)^2 \\
& + 2p_4\omega_T (\omega_L + p_4) (p_3^2 - \omega_L^2) (p_3^2 - \omega_L (3\omega_L + 2p_4)) (2p_4 (\omega_L + 4\omega_R) \\
& + 4\omega_L\omega_R + \omega_L^2 + 3p_4^2 + 6\omega_R^2) + 2p_4 (\omega_L + p_4)^2 (p_3^2 - \omega_L^2)^2 (3p_4 (\omega_L + 5\omega_R) \\
& + 5\omega_L\omega_R + \omega_L^2 + 6p_4^2 + 10\omega_R^2) + \omega_T^4 (p_3^2 (p_4^2 - 3\omega_L^2) + \omega_L^2 (8p_4\omega_L + 6\omega_L^2 + 3p_4^2) \\
& \left. + p_3^4) (2p_4 + \omega_T) \right), \\
Q_2 = & p_4 \left(6p_3^4\omega_T (p_4^2 (-4\omega_L\omega_R + \omega_L^2 - 17\omega_R^2) + 3\omega_R^2 (4\omega_L\omega_R + \omega_L^2 + 5\omega_R^2) + 6p_4^4) \right. \\
& + p_3^2\omega_T (18p_4^4 (4\omega_L\omega_R + \omega_L^2 + \omega_R^2) + p_4^2 (4\omega_L^3\omega_R - 84\omega_L^2\omega_R^2 - 212\omega_L\omega_R^3 + \omega_L^4 - 53\omega_R^4) \\
& + 3\omega_R^2 (20\omega_L^3\omega_R + 70\omega_L^2\omega_R^2 + 84\omega_L\omega_R^3 + \omega_L^4 + 21\omega_R^4)) + 2p_3^5 (p_4^2 (-4\omega_L\omega_R + \omega_L^2 \\
& - 17\omega_R^2) + 3\omega_R^2 (4\omega_L\omega_R + \omega_L^2 + 5\omega_R^2) + 6p_4^4) + 2p_3^3 (p_4^4 (50\omega_L\omega_R + 19\omega_L^2 + 19\omega_R^2) + 3p_4^2 \\
& \times (\omega_L + 2\omega_R) (\omega_L + 3\omega_R) (-6\omega_L\omega_R + \omega_L^2 - 3\omega_R^2) + 3\omega_R^2 (21\omega_L^3\omega_R + 54\omega_L^2\omega_R^2 \\
& + 57\omega_L\omega_R^3 + 3\omega_L^4 + 17\omega_R^4)) + 2p_3 (3p_4^4 (5\omega_L^3\omega_R + 10\omega_L^2\omega_R^2 + 5\omega_L\omega_R^3 + \omega_L^4 + \omega_R^4) \\
& - 3p_4^2\omega_R^2 (17\omega_L^3\omega_R + 30\omega_L^2\omega_R^2 + 15\omega_L\omega_R^3 + 3\omega_L^4 + 3\omega_R^4) + 2\omega_R^3 (19\omega_L^4\omega_R + 56\omega_L^3\omega_R^2 \\
& \left. + 70\omega_L^2\omega_R^3 + 35\omega_L\omega_R^4 + 2\omega_L^5 + 7\omega_R^5)) + \omega_T^5 (-3p_4^2\omega_R^2 + p_4^4 + 6\omega_R^4) \right) \quad (3.21)
\end{aligned}$$

Once again notice that the logarithmic terms are scale invariant, and like the flat space examples all apparent folded poles are spurious.

3.2 Singularity Structure of Loop Corrections

The singularity structure for tree level correlators has been well studied in the literature [96, 20, 78, 97, 79]. It is well known that correlation functions of massless fields at tree level have singularities of total and partial (energy flowing into any subgraph) energy pole. The residue at a partial energy pole is given by the product of a lower-point correlator and a lower-point scattering amplitude [20, 78, 97, 79, 33], whereas the residue at the total energy singularity is related to the scattering amplitude for the same process [76, 77].

The singularity structure of correlators at loops is comparatively much more complicated. Some recent progress has been made in studying the types of singularities arising in wavefunction coefficients (see [101, 102, 103, 104]). However, the singularity structure of correlators can be very different [105, 106, 112, 113] due to different boundary conditions in the past, and as a result different bulk-bulk propagators. Thus correlators at loop level might have additional singularities or some singularities might cancel.

Now, in dimensional regularisation it is convenient to perform the time integrals first, which produces total and partial energy poles. Some of these partial energy poles will involve loop momenta. It is interesting to investigate what kind of singularities such poles produce once momentum integrals are performed. Importantly, there is a way to know the singularity structure of these diagrams without any explicit calculation. We will discuss the diagrammatic rules to extract singularities (the poles and branch cuts) in a correlator at 1-loop in Sec. 3.2.1, and in Sec. 3.2.2 we will demonstrate how these rules follow from the general structure of the integrals in these correlators.

3.2.1 Rules to extract Singularity Structure at 1-loop

There are a few lessons to be learnt from the results of Sec. 3.1. We notice that the singularity structure of the correlators at 1-loop feature poles and logarithms. These poles in total and partial energies (ofcourse, independent of loop momenta) entering vertices and subgraphs of the diagram are produced from time integrals, and also arise in tree level correlations of massless scalars. The branch cuts however are produced from momentum integrals (in dS one also gets logarithms of total energy from time integrals, see Sec. 3.3). Regarding the branch cuts produced, we see a trend emerging, as follows,

- First, we notice the obvious fact that all diagrams have at least one subgraph, trivial or otherwise, which involves only the loop. We call these subgraphs as **Loop-subgraphs**. While the *loop subgraph* is trivial for Figs. 3.1 and 3.2, it has been shaded in red in Figs. 3.3, 3.4 and 3.5. These subgraphs have two vertices of *external* energy injection, through which the energy flows *into* the subgraph. These vertices are of course the loop sites. For example the energy flowing into the *loop-subgraph* through the left loop site in the diagrams of Figs. 3.1, 3.2 is $\mathcal{S}_L = \omega_1 + \omega_2$, in Figs. 3.3, 3.5 is $\mathcal{S}_L = p_3$ and in Fig. 3.4 is $\mathcal{S}_L = p_3 + \omega_3$. The energy flowing in through the right loop site in Figs. 3.1, 3.3 is $\mathcal{S}_R = \omega_3$, in Fig. 3.2 is $\mathcal{S}_R = \omega_3 + \omega_4$, and in Figs. 3.4, 3.5 is $\mathcal{S}_R = p_4$.

Importantly, note that all these diagrams have a branch cut at $\mathcal{S}_L + s$ and $\mathcal{S}_R + s$.

- Apart from *loop-subgraphs*, we considered diagrams in Figs. 3.3, 3.4 and 3.5 which have additional subgraphs involving the full loop. For Fig. 3.3 this subgraph is a trivial one and shaded in blue. For Figs. 3.4 and 3.5 there

are 3 such subgraphs and the non-trivial ones are shaded in green and blue. Notice that atleast one of the *endpoints* of these subgraphs is a loop site, which means that atleast one of the loop sites is not connected to any other vertex of the subgraph, except the other loop site (via, ofcourse the loop arms). Any subgraph which does not involve the full loop, or which does not have a loop site as an endpoint will *not* be considered for determining the logarithmic contributions to the singularity structure of loop diagrams. Note that the trivial subgraph in Figs. 3.4 and 3.5 are such examples, and as mentioned we will not be considering this subgraph.

Now, the loop divides the entire diagram into two parts, one to the right of the loop and the other to the left. We call the subgraphs to the left and to the right of the loop, with one loop site as an *endpoint*, as **left-subgraphs** and **right-subgraphs** respectively. Fig. 3.3 has one *left-subgraph* (shaded in blue). Let us call the external energy injection into this subgraph through all the vertices except the right loop site as $\mathcal{S}_{L_1} = \omega_1 + \omega_2$. Figs. 3.4 and 3.5 have one *left-subgraph* (shaded in blue) and one *right-subgraph* (shaded in green). The external energy injection into the *left- (right-) subgraph* through all the vertices except the right (left) loop site is given by $\mathcal{S}_{L_1} = \omega_1 + \omega_2 + \omega_3$ and $\mathcal{S}_{L_1} = \omega_1 + \omega_2 \left(\mathcal{S}_{R_1} = \omega_4 + \omega_5 \text{ and } \mathcal{S}_{R_1} = \omega_3 + \omega_4 \right)$ for Figs. 3.4 and 3.5 respectively.

Once again, note that Fig. 3.3 has a branch cut at $\mathcal{S}_{L_1} + s$ and Figs. 3.4 and 3.5 have branch cuts at $\mathcal{S}_{L_1} + s$ and $\mathcal{S}_{R_1} + s$.

This leads us to one of the main results of this work : **Rules to extract the singularity structure of any 1-loop diagram at 2-sites**, given below,

- The correlator will feature poles in total and partial energies, entering vertices and subgraphs of the diagram, which are independent of loop momenta p_1, p_2 . These poles are produced from time integrals.
- Consider the energy flowing into the *loop-subgraphs* through the left and right loop sites. We call this energy as $\mathcal{S}_L$ and $\mathcal{S}_R$ respectively. The loop correction coming from the diagram will feature a logarithm of $\mathcal{S}_L + s$ and $\mathcal{S}_R + s$.
- For every *left-* and *right-* *subgraph* consider the energy flowing into the subgraph through all vertices except the right and the left loop site respectively. Let us call these energies as $\mathcal{S}_{L_i}$ and $\mathcal{S}_{R_j}$ respectively. The loop correction coming from the diagram will feature a logarithm of $\mathcal{S}_{L_i} + s$ and $\mathcal{S}_{R_j} + s$.

The number of logarithms hence produced matches with the number of partial energies depending on loop momenta produced after time integrals, which should turn into branch cuts once loop integrals are performed. As we shall see in the following, these rules will apply for derivative as well as polynomial interactions in flat space. In de Sitter one requires derivative interactions if only massless scalars are involved, however, if conformally coupled scalars run in the loop then the rules are valid for both derivative as well as polynomial interactions.

Note that in all the diagrams considered above, the vertices lie on the same side of $\tau = 0$. However, a correlator will have contributions coming from diagrams where the vertices will come from the time ordered as well as the anti-time ordered parts of the in-in expression. We discuss in Sec. 3.2.3 that the rules mentioned above still apply on such diagrams, with an additional constraint on the *left-* and *right-* *subgraphs*.

3.2.2 Working Principle

In the following sections, we will study the general structure of correlators and the singularities arising from these integrals, and we will see how the diagrammatic rules follow.

n -point functions in Flat Space

Consider a general 1-loop diagram with quartic and cubic polynomial interactions in Fig. 3.6. Loop momenta p_1 and p_2 flow between loop sites τ_R and τ_L . Each loop site contracts with multiple legs ($p_3, \dots, p_6$), which may lead to further tree level sub-diagrams denoted by the blobs. The blobs to the left of the loop are labelled as L_1 and L_2 and the blobs to the right of the loop are labelled as R_1 and R_2 . We refer to the energies flowing through the left blobs into the vertices τ_1 and τ_2 as $\omega_{L_{1i}}$ and $\omega_{L_{2j}}$ respectively, where i, j run over partial energies flowing through all possible subgraphs of the blob, including the total energy flowing through the blob. Similarly the energies flowing through the right blobs into the vertices τ_3 and τ_4 are $\omega_{R_{1k}}$ and $\omega_{R_{2l}}$ respectively, with k, l running over all possible partial energies.

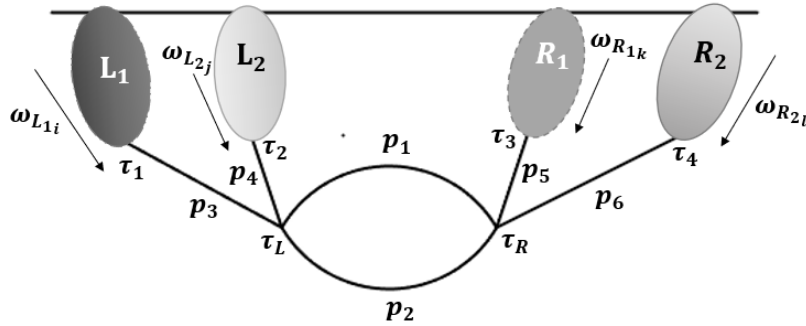


Figure 3.6: A general 1-loop correction diagram. The blobs denote tree-level sub-diagrams with p_3, p_4, p_5 and p_6 as momentum exchanges with the loop sites. The different color gradients of the blobs represent the fact that these subdiagrams may in general be different from each other.

Evaluation of time integrals in this diagram will lead to simple poles in partial energies, and the ones involving p_1 and p_2 are given by,

$$\begin{aligned}
E_{L_{1i}} &= \omega_{L_{1i}} + p_4 + p_+, & E_{L_{2j}} &= \omega_{L_{2j}} + p_3 + p_+, & E_{L_{ij}} &= \omega_{L_{1i}} + \omega_{L_{2j}} + p_+, \\
E_{R_{1k}} &= \omega_{R_{1k}} + p_6 + p_+, & E_{R_{2l}} &= \omega_{R_{2l}} + p_5 + p_+, & E_{R_{kl}} &= \omega_{R_{1k}} + \omega_{R_{2l}} + p_+, \\
E_1 &= p_3 + p_4 + p_+, & E_2 &= p_+ + p_5 + p_6,
\end{aligned} \tag{3.22}$$

where $p_+ = p_1 + p_2$. After performing the time integrals, the loop correction has the following explicit dependence on loop momenta p_1 and p_2 ,

$$\begin{aligned}
& \int d^d \vec{p}_1 d^d \vec{p}_2 \frac{Q(p_i, \omega_j)}{p_1 p_2 E_1 E_2 \prod_{ijkl} (E_{L_{1i}} E_{L_{2j}} E_{L_{ij}} E_{R_{1k}} E_{R_{2l}} E_{R_{kl}})} \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{s}) \\
&= \frac{S_{d-2}}{2} \int_s^\infty dp_+ \int_{-s}^s dp_- \frac{p_1^{d-2} p_2}{s} \sin^{d-3} \theta_1 \frac{Q(p_i, \omega_j)}{p_1 p_2 E_1 E_2 \prod_{ijkl} (E_{L_{1i}} E_{L_{2j}} E_{L_{ij}} E_{R_{1k}} E_{R_{2l}} E_{R_{kl}})},
\end{aligned} \tag{3.23}$$

where $\sin \theta_1 = \sqrt{1 - \left(\frac{s^2 + p_1^2 - p_2^2}{2s p_1} \right)^2}$ and $p_- = p_1 - p_2$. s is the magnitude of the momentum exchanged between the loop sites, and S_{d-2} is surface area of a $(d-2)$ -unit sphere. Q is a function of exchange momenta p_i (excluding the loop arms) and ω_j (energy of the external legs). The equality in Eqn. 3.23 follows from an identity (discussed in the Appendix of [3]). Note that Q may have simple poles in partial energies of subgraphs that do not depend on loop momenta (e.g. $\omega_{L_{1i}} + p_3$, $\omega_{R_{1k}} + p_5$ and so on).

Now, for the computation to be tractable, the integrand is scaled by s . With a slight abuse of notation by suppressing the product over i, j, k, l , we have the

following dependence on p_- and p_+ ,

$$\begin{aligned}
&= s^{m+\delta} 2^{-\delta-1} S_{d-2} \int_1^\infty d\hat{p}_+ \int_{-1}^1 d\hat{p}_- \frac{Q(\hat{p}_i, \hat{\omega}_j)}{\hat{E}_{L_1} \hat{E}_{L_2} \hat{E}_L \hat{E}_{R_1} \hat{E}_{R_2} \hat{E}_R \hat{E}_1 \hat{E}_2} \\
&\times \left((1 - \hat{p}_-^2) (\hat{p}_+^2 - 1) \right)^{\delta/2}, \tag{3.24}
\end{aligned}$$

where m is an integer. The hat denotes scaled energies, i.e. $\hat{\omega}_j = \omega_j/s$ and so on. Since the only p_- dependence in the integrand is the factor of $(1 - \hat{p}_-^2)^{\delta/2}$, using $\int_{-1}^1 dp_- (1 - \hat{p}_-^2)^{\delta/2} = \frac{\sqrt{\pi} \Gamma(\frac{\delta+1}{2})}{\Gamma(\frac{\delta+3}{2})}$, the above expression evaluates to the following,

$$\begin{aligned}
&= s^{m+\delta} 2^{-\delta-1} S_{d-2} \frac{\sqrt{\pi} \Gamma(\frac{\delta}{2} + 1)}{\Gamma(\frac{\delta+3}{2})} \int_1^\infty d\hat{p}_+ \frac{Q(\hat{p}_i, \hat{\omega}_j)}{\hat{E}_{L_1} \hat{E}_{L_2} \hat{E}_L \hat{E}_{R_1} \hat{E}_{R_2} \hat{E}_R \hat{E}_1 \hat{E}_2} (\hat{p}_+^2 - 1)^{\delta/2}. \tag{3.25}
\end{aligned}$$

Recall that the poles in the integrand are all simple, hence using partial fraction decomposition of the integrand in p_+ gives the following structure,

$$s^\delta \int_1^\infty d\hat{p}_+ \frac{(\hat{p}_+^2 - 1)^{\delta/2}}{\hat{p}_+ + x} = -\frac{1}{\delta} - \log(1+x) - \log(s), \tag{3.26}$$

where the $\hat{p}_+ + x$ in the denominator represents the various partial energies in Eqn. 3.22 scaled by s . For example, upon partial fraction decomposition for the term with $\hat{E}_{L_{2j}}$ in the denominator we have $x = \hat{\omega}_{L_{2j}} + \hat{p}_3$ and the RHS of Eqn. 3.26 gives $-\frac{1}{\delta} - \log(s + \omega_{L_{2j}} + p_3)$. Similarly for the term with $\hat{E}_1$ in the denominator we have $x = \hat{p}_3 + \hat{p}_4$ and the RHS of Eqn. 3.26 gives $-\frac{1}{\delta} - \log(s + p_3 + p_4)$, and so on. From Eqn. 3.26 we see that every partial energy pole turns into a logarithm, and the argument of the logarithm is precisely the partial energy pole at $p_+ \rightarrow s$. The loga-

rithms arising from the energies flowing through the loop sites E_1 and E_2 correspond to contributions from what we refer to as *loop-subgraphs*. The logarithms arising from integrating the pole at partial energies flowing through blobs to the left (right) of the loop, i.e. E_{L_1}, E_{L_2} and E_L (E_{R_1}, E_{R_2} and E_R) correspond to contributions from *left- (right-) subgraphs*. To be precise,

$$\begin{aligned} E_1|_{p_+ \rightarrow s} &= \mathcal{S}_L + s, & E_2|_{p_+ \rightarrow s} &= \mathcal{S}_R + s, \\ E_{L_{(1i, 2j, ij)}}|_{p_+ \rightarrow s} &= \mathcal{S}_{L_{(1i, 2j, ij)}} + s, & E_{R_{(1k, 2l, kl)}}|_{p_+ \rightarrow s} &= \mathcal{S}_{R_{(1k, 2l, kl)}} + s, \end{aligned} \quad (3.27)$$

where $\mathcal{S}_L, \mathcal{S}_R$ are the energies flowing into the *loop-subgraph* through the left and right loop sites respectively. $\mathcal{S}_{L_{(1i, 2j, ij)}}$ is the energy flowing into the *left-subgraphs* (involving the blob L_1, L_2 and both respectively) through all vertices except the right loop site, and so on. This demonstrates that the rules mentioned in Sec. 3.2.2 are indeed the correct diagrammatic way of extracting the singularities.

To demonstrate how powerful this analysis is in extracting the singularity structure, we consider an explicit example of the previous arguments in the following. Consider the diagram in Fig. 3.7. Comparing with Fig. 3.6 we see that there is only one left and right blob attached to the loop site. The energy flowing out of the left blob through the vertex τ_1 is $\omega_L = \omega_1 + \omega_2$. There are four partial energies (corresponding to the subgraphs in the grey-shaded regions in Fig. 3.7) flowing out of the right blob through the vertex τ_2 , given by $\omega_{R_1} = \omega_6 + \omega_7 + p_5$, $\omega_{R_2} = \omega_4 + \omega_5 + p_6$, $\omega_{R_3} = p_5 + p_6$ and $\omega_R = \omega_4 + \omega_5 + \omega_6 + \omega_7$.

Now to evaluate this diagram explicitly would require extremely lengthy time integrals (i.e. $6! = 720$ time orderings) and complicated loop integrals. Evaluation of time integrals will lead to partial and total energy poles. The partial energy poles

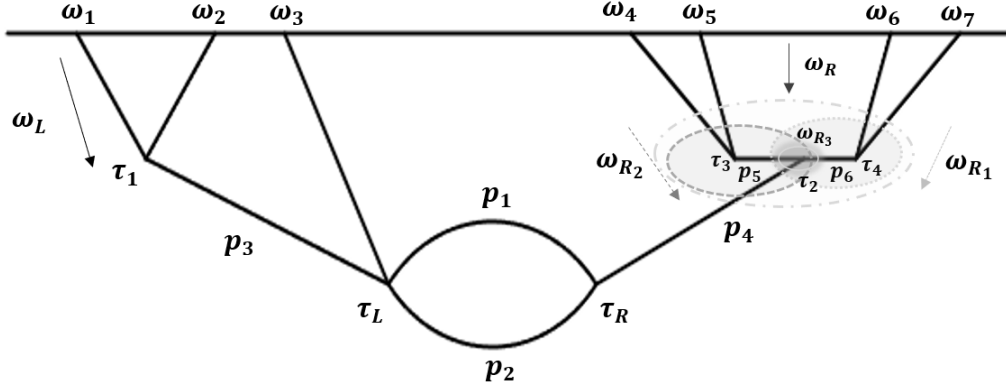


Figure 3.7: A 1-loop correction diagram with cubic and quartic vertices. The grey blobs denotes various subgraphs.

involving loop momenta are,

$$\begin{aligned}
 E_L &= \omega_L + \omega_3 + p_+, & E_1 &= p_3 + \omega_3 + p_+, & E_2 &= p_+ + p_4 \\
 E_{R_1} &= \omega_{R_1} + p_+, & E_{R_2} &= \omega_{R_2} + p_+, & E_{R_3} &= \omega_{R_3} + p_+, & E_R &= \omega_R + p_+. \quad (3.28)
 \end{aligned}$$

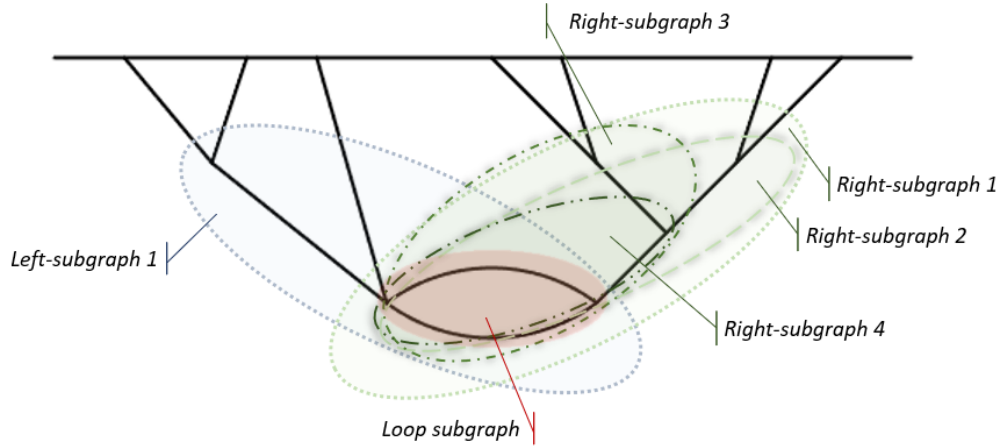


Figure 3.8: The 1-loop correction diagram of Fig. 3.7 with the *loop-subgraph* shaded in red, *left-* and *right-* subgraphs shaded in blue and green respectively. The momentum labels on the arms as well as vertices have been removed for the diagram to remain legible. The reader is requested to refer to Fig. 3.7 for these labels.

Following the rules mentioned in Sec. 3.2.1 we will immediately know the singularity structure without any computations, as follows,

- For the *loop-subgraph* (shaded in red in Fig. 3.8), we have $\log(\mathcal{S}_L + s) = \log(p_3 + \omega_3 + s)$ from the left loop site and $\log(\mathcal{S}_R + s) = \log(p_4 + s)$ from the right loop site.

Note that this is precisely what we get from Eqn. 3.26 considering the E_1 and E_2 pole respectively in the partial fraction decomposition.

- From the *left-subgraph* (shaded in blue in Fig. 3.8), we have $\log(\mathcal{S}_{L_1} + s) = \log(\omega_1 + \omega_2 + \omega_3 + s)$.

Again note this is precisely what we get from Eqn. 3.26 considering the E_L pole in the partial fraction decomposition.

- From the four *right-subgraphs* (shaded in green in Fig. 3.8), we have $\log(\mathcal{S}_{R_1} + s) = \log(\omega_4 + \omega_5 + \omega_6 + \omega_7 + s)$, $\log(\mathcal{S}_{R_2} + s) = \log(\omega_6 + \omega_7 + p_5 + s)$, $\log(\mathcal{S}_{R_3} + s) = \log(\omega_4 + \omega_5 + p_6 + s)$ and $\log(\mathcal{S}_{R_4} + s) = \log(p_5 + p_6 + s)$.²

Once again note this is precisely what we get from Eqn. 3.26 considering the E_R , E_{R_1} , E_{R_2} and E_{R_3} poles respectively in the partial fraction decomposition.

To conclude, this shows how the diagrammatic rules follow from the general structure of a correlator in flat space with polynomial interactions. Notice that with interactions where derivative acts on the fields, such as $\dot{\phi}^3$ or $(\partial_i \phi)^2 \phi^2$ the general structure of the integral is still going to be as in Eqn (3.23), except now the factor of $p_1 p_2$ will feature in the numerator. In that case our arguments still go through and the integral looks like Eqn. (3.33) with $\alpha = 1$ (The value of y obviously depends on the number of derivatives).

In the following we consider a general n -point function in de-Sitter and show how the diagrammatic rules apply to de-Sitter.

²To avoid confusion one should note that here the subscript R_i on $\mathcal{S}$ refers to i^{th} *right-subgraph* of Fig. 3.8. This does not correspond to the subscripts of energies in Eqn. (3.28).

n -point functions in de-Sitter

Unlike flat space, evaluation of time integrals in de-Sitter can produce higher order poles. Additionally the mode functions and measure are dependent on dimensions (Eqns. (1.77), (1.80)), but can be written as a sum of contribution from a three dimensional part, and a $\mathcal{O}(\delta)$ part. Thus the entire correlator may be expressed as a sum of contribution coming from the former and the latter, which we refer to as D_1 and D_2 respectively. In this section, we compute the general structure of D_1 to see what kind of singularities may arise. We will see why the rules to extract the singularities in flat space very much apply to de-Sitter as well.

Also, note that the integral of D_2 is the same as that of D_1 with an extra factor of $\log(-H\tau_i)$. Consequently, the result of performing the time integrals in D_2 is the same as that of D_1 , with an extra factor of $\log(H/\omega_T)$, with some rational functions of momenta [3], i.e.,

$$D_2(\omega_i) = \delta D_1(\omega_i) \sum_i n_i \log(H/\omega_T) + \text{Nlf}, \quad (3.29)$$

where $n_i = -1, 1/2$ for contribution from measure and modes respectively. This is a finite, non- vanishing contribution when D_1 has a $1/\delta$ divergence, thus giving a logarithm of total energy. This logarithm is an interesting feature of de-Sitter, which we discuss in greater detail in Sec. 3.3.

Consider the D_1 contribution to the 1-loop diagram in Fig. 3.6 with quartic ($\dot{\pi}^4$) and cubic ($\dot{\pi}^3$) derivative interactions in de-Sitter. Similar to Sec. 3.2.2, we have loop momenta p_1 and p_2 flowing between the loop sites τ_R and τ_L , and each loop site contracts with multiple legs ($p_3, \dots, p_6$) leading to further tree level sub-diagrams. We refer to the energies flowing through the left blobs into the vertices τ_1 and τ_2 as

$\omega_{L_{1i}}$ and $\omega_{L_{2j}}$ respectively, and the energies flowing through the right blobs into the vertices τ_3 and τ_4 as $\omega_{R_{1k}}$ and $\omega_{R_{2l}}$ respectively.

The partial energies involving the loop momenta are the same as in Eqn. (3.22). Evaluation of time integrals in this diagram will lead to all simple poles in partial energies as in Eqn (3.23), as well as higher order poles due to additional factors of τ in the numerator coming from modes and measure. The correlator thus gets contributions from terms such as,

$$\begin{aligned} & \int d^d \vec{p}_1 d^d \vec{p}_2 \frac{p_1 p_2 Q(p_i, \omega_j)}{E_1^{\alpha_1} E_2^{\alpha_2} \prod_{ijkl} \left(E_{L_{1i}}^{\alpha_3} E_{L_{2j}}^{\alpha_4} E_{L_{ij}}^{\alpha_5} E_{R_{1k}}^{\alpha_6} E_{R_{2l}}^{\alpha_7} E_{R_{kl}}^{\alpha_8} \right)} \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{s}) \\ &= \frac{S_{d-2}}{2} \int_s^\infty dp_+ \int_{-s}^s dp_- \frac{p_1^{d-2} p_2}{s} \sin^{d-3} \theta_1 \frac{p_1 p_2 Q(p_i, \omega_j)}{E_1^{\alpha_1} E_2^{\alpha_2} \prod_{ijkl} \left(E_{L_{1i}}^{\alpha_3} E_{L_{2j}}^{\alpha_4} E_{L_{ij}}^{\alpha_5} E_{R_{1k}}^{\alpha_6} E_{R_{2l}}^{\alpha_7} E_{R_{kl}}^{\alpha_8} \right)}, \end{aligned} \quad (3.30)$$

where α_i 's are integers denoting the order of the poles. The factor of $p_1 p_2$ is now in the numerator due to the derivative acting on the modes, i.e. $\dot{f}_k(\tau) \sim \sqrt{k} \tau \exp(ik\tau)$. Note that the limits of the final time integral $\left(\int_{-\infty}^0 d\tau \right)$ ensures that any additional polynomials in p_1 and p_2 are not produced, and the only dependence on loop energies are of the form in the previous equation.

Once again Q is a function of exchange momenta p_i (excluding p_1 and p_2) and ω_j (energy of the external legs). Scaling the integrand by s and suppressing the labels i, j, k, l , we get the following dependence on p_- and p_+ ,

$$= s^{m+\delta} 2^{-\delta-1} S_{d-2} \int_1^\infty d\hat{p}_+ \int_{-1}^1 d\hat{p}_- \frac{Q(\hat{p}_i, \hat{\omega}_j) (\hat{p}_-^2 - \hat{p}_+^2)^2}{\hat{E}_1^{\alpha_1} \hat{E}_2^{\alpha_2} \hat{E}_{L_1}^{\alpha_3} \hat{E}_{L_2}^{\alpha_4} \hat{E}_L^{\alpha_5} \hat{E}_{R_1}^{\alpha_6} \hat{E}_{R_2}^{\alpha_7} \hat{E}_R^{\alpha_8}} \left((1 - \hat{p}_-^2) (\hat{p}_+^2 - 1) \right)^{\delta/2}, \quad (3.31)$$

where m is an integer and the hat denotes scaled momenta.

Comparing the equation above with the flat space result (Eqn. (3.24)) we see

that in de-Sitter we have higher order poles (the α_i 's), as well as an extra factor of $(\hat{p}_-^2 - \hat{p}_+^2)^2$, due to $p_1 p_2$ being in the numerator as opposed to the denominator. Computing p_- integral, we get,

$$\int_{-1}^1 d\hat{p}_- (\hat{p}_-^2 - \hat{p}_+^2)^2 ((1 - \hat{p}_-^2))^{\delta/2} = \frac{\sqrt{\pi} \Gamma(\frac{\delta}{2} + 1) ((\delta + 5)\hat{p}_+^2 ((\delta + 3)\hat{p}_+^2 - 2) + 3)}{4\Gamma(\frac{\delta+7}{2})}. \quad (3.32)$$

Crucially, the p_- integral above evaluates to polynomials in p_+ because of the fact that the loop propagator does not decay faster than $1/p$, where p is the energy running in the loop. The loop propagator will have this behaviour as long as we are working with interactions featuring derivatives on massless scalars. For conformally coupled scalars running in the loop, polynomial interactions are fine as well. However, if a massless scalar runs in the loop leg without a derivative acting on the field, then certain terms in the loop propagator go as $\sim 1/p^3$, and the p_- integral evaluates to hypergeometric functions of p_+ , which results in the final integral in p_+ turning intractable. Also, the equality in Eqn (3.30) can get messed up by complicated interactions, such as inverse laplacians acting on spatial derivatives of the fields (i.e. $\phi^2 \partial^{-2} \partial_i (\phi \partial_i \phi)$), which will pull out complicated trigonometric functions. It will be worthwhile to investigate the correlator structure for such interactions, which we leave out for future work.

Using the result of p_- integral and performing partial fraction decomposition, we see that the final integral in $\hat{p}_+$ will be a sum of terms having the following structure,

$$s^\delta \int_1^\infty d\hat{p}_+ (\hat{p}_+^2 - 1)^{\delta/2} \frac{\hat{p}_+^y}{(\hat{p}_+ + x)^\alpha}, \quad (3.33)$$

where the $\hat{p}_+ + x$ in the denominator represents the various partial energies in Eqn. 3.22 and α is an integer denoting the order of the pole. The values α can

take depends on the interacting theory as well as the diagram we are computing, whereas y only depends on the interactions in the theory, e.g. for the derivative interactions we considered in this section y takes values 0, 2 and 4. Similarly for interactions other than polynomials in flat space, the poles in Eqn. (3.26) would still be simple, however one would get additional powers of p_+ in the numerator, i.e. the loop integral gets the structure of Eqn. (3.33) with $\alpha = 1$.

Notice that Eq. (3.33) is finite (and hence no logarithms) unless $y \geq \alpha - 1$, and for y, α satisfying this inequality with integer $\alpha \geq 1$, Eq. (3.33) evaluates to the following,

$$= (-1)^{y+\alpha} {}^y C_{\alpha-1} x^{y\alpha+1} \left[\frac{1}{\delta} + \log(1+x) + \log s \right] + \text{NLf}. \quad (3.34)$$

Once again we see that every partial energy pole turns into a logarithm, with the argument of the logarithm being precisely the partial energy pole at $p_+ \rightarrow s$, exactly as in Eqn. (3.27). When $\hat{p}_+ + x = \hat{E}_1, \hat{E}_2$, the contributions correspond to *loop-subgraphs*, when $\hat{p}_+ + x = \hat{E}_{L_1}, \hat{E}_{L_2}, \hat{E}_L$ the contributions correspond to *left-subgraphs*, and finally when $\hat{p}_+ + x = \hat{E}_{R_1}, \hat{E}_{R_2}, \hat{E}_R$ the contributions correspond to *right-subgraphs*.

To conclude this section, we have shown that the rules of Sec. 3.2.1 work for flat-space as well as de-Sitter. Correlators in de-Sitter also have a contribution D_2 coming from $\mathcal{O}(\delta)$ contribution of modes and measure. This produces branch cuts of total energy $\log \omega_T$, which we discuss in greater detail in Sec. 3.3.

3.2.3 Diagrams with vertices above and below $\tau = 0$

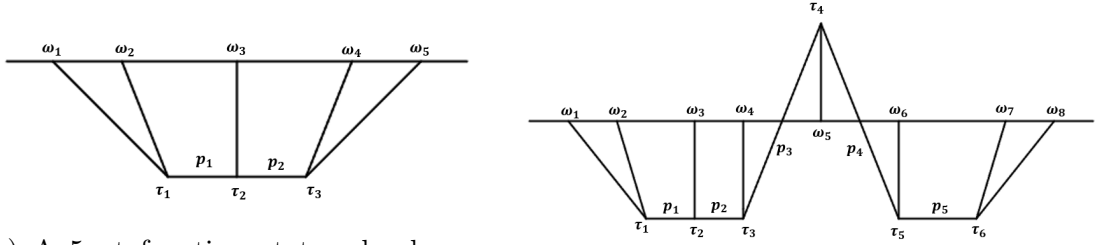
Until now we have been discussing the singularity structure of diagrams with all vertices on the same side of $\tau = 0$. However, the correlator will also receive contributions from diagrams in which vertices lie on both sides of $\tau = 0$. In this section, we discuss why the rules to extract singularity structure will continue to hold in these diagrams, with an additional constraint on the subgraphs considered.

In diagrams where all vertices are above or below $\tau = 0$, every vertex is connected to its adjacent ones by a bulk-bulk propagator. The Heaviside step function in the propagator causes mixing of energies entering adjacent vertices in the exponential functions, which in turn result in partial energy poles once time integrals are evaluated. To see this, consider the tree diagram in Fig. 3.9a, and let us study the time integral corresponding to one of the time orderings $\Theta(\tau_1 - \tau_2)\Theta(\tau_2 - \tau_3)$. Considering flat space for simplicity, we have :

$$\begin{aligned} & \int_{-\infty}^0 d\tau_1 \int_{-\infty}^0 d\tau_2 \int_{-\infty}^0 d\tau_3 e^{i(\omega_1+\omega_2-p_1)\tau_1} e^{i(\omega_3+p_1-p_2)\tau_2} e^{i(\omega_4+\omega_5+p_2)\tau_3} \Theta(\tau_1 - \tau_2) \Theta(\tau_2 - \tau_3) \\ &= \int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 e^{i(\omega_1+\omega_2-p_1)\tau_1} e^{i(\omega_3+p_1-p_2)\tau_2} e^{i(\omega_4+\omega_5+p_2)\tau_3} . \end{aligned} \quad (3.35)$$

where the Heaviside functions in the first line translate to the changed limits of integration in the second line. From here it is easy to see that the evaluation of the integral on τ_3 pulls a factor of $1/(\omega_4 + \omega_5 + p_2)$ (which is a pole at the energy entering the vertex τ_3), modifying the integrand to $e^{i(\omega_1+\omega_2-p_1)\tau_1} e^{i(\omega_3+p_1+\omega_4+\omega_5)\tau_2}$. Further evaluation of the τ_2 integral pulls a factor of $1/(\omega_3 + \omega_4 + \omega_5 + p_1)$ (which is a partial energy pole), modifying the integrand to $e^{i\omega_T\tau_1}$. Finally, the τ_1 integral produces a total energy pole. Clearly, other time orderings will generate other partial energy poles.

However, as pointed out earlier, a n -point correlator will also have contributions



(a) A 5 pt function at tree level. ω_i denote external energies, p_j denote exchange legs.

(b) A 8 pt function at tree level. ω_i are external legs, p_j denote exchange momenta.

Figure 3.9: Tree level correlator diagrams with a) All vertices below, and b) Vertices both above and below the $\tau = 0$ line.

from diagrams where not all vertices lie on the same side of the line $\tau = 0$. Before we study such a diagram at 1-loop diagram, consider the tree level 8-pt function in Fig. 3.9b in flat space.

Evaluation of time integrals will of course give poles at energies entering the vertices $\tau_1, \dots, \tau_6$. However, we will not get partial energy poles entering all possible subgraphs. This is simply because τ_3, τ_4 and τ_5 are connected by Wightman propagators, and hence there are no Heaviside step functions mixing the energies entering these vertices. This implies we will only get partial energy poles for subgraphs with all vertices above *or* below $\tau = 0$, i.e. subgraphs where all the vertices are connected by bulk-bulk propagators.

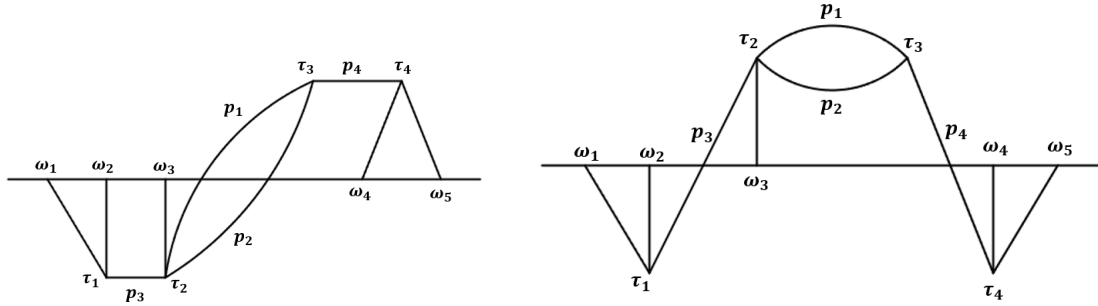
For example in Fig. 3.9b, for the time ordering $\Theta(\tau_2 - \tau_1)\Theta(\tau_2 - \tau_3)\Theta(\tau_5 - \tau_6)$, we get the following pole structure upon computing time integrals,

$$\sim \frac{1}{(\omega_1 + \omega_2 + p_1)} \frac{1}{(\omega_4 + p_2 + p_3)} \frac{1}{(\omega_5 + p_3 + p_4)} \frac{1}{(\omega_7 + \omega_8 + p_5)} \\ \times \frac{1}{(\omega_1 + \omega_2 + \omega_3 + \omega_4 + p_3)} \frac{1}{(\omega_6 + \omega_7 + \omega_8 + p_4)},$$

where the factors in the first line are poles in energies entering the vertices τ_1, τ_3, τ_4 and τ_6 respectively, and the factors in the second line are energies entering subgraphs

where all vertices are connected by bulk-bulk propagators.

Similarly, for 1-loop diagrams with vertices on both sides of $\tau = 0$, evaluation of time integrals will produce poles at the energies entering only those subgraphs where all vertices are connected by bulk-bulk propagators. Furthermore, following the arguments in Sec. 3.2.2 and 3.2.2, the partial energy poles involving loop momentum will turn into branch cuts. Consider Fig. 3.10 as an example. These diagrams, alongwith the one in Fig. 3.4, will contribute to the 5-pt function at 1-loop with cubic and quartic interactions.



(a) A 5 pt function at 1-loop. The loop sites are on opposite sides of $\tau = 0$. (b) A 5 pt function at 1-loop. The loop sites lie on the same side of $\tau = 0$.

Figure 3.10: Contributions to 5-point functions at 1-loop with cubic and quartic interactions.

Upon performing time integrals, poles are produced at energies E_1, E_2, E_3 and E_4 , i.e. the energies entering the vertices τ_1, τ_2, τ_3 and τ_4 respectively, given in Eqn. 3.11. Evaluation of loop integrals will turn two of these into branch cuts, i.e. $\log E_{2,3}|_{p_+ \rightarrow s}$. Following the rules mentioned in Sec. 3.2.1, these are the contributions from *loop-subgraphs*. Time integrals also produce poles at partial energies entering those subgraphs in which vertices lie on the same side, i.e. at $E_L = \omega_1 + \omega_2 + \omega_3 + p_+$ and $E_R = p_+ + \omega_4 + \omega_5$ in Fig. 3.10a and at $E_M = p_3 + \omega_3 + p_4$ in Fig. 3.10b. Evaluating the loop integrals will turn the poles at E_L and E_R into branch cuts $\log E_{L,R}|_{p_+ \rightarrow s}$, whereas Fig. 3.10b does not get any additional branch cuts. The exact same results

also follow from the diagrammatic rules, except now while determining the *left-* and *right-subgraphs* there is an additional constraint that all vertices, except the right and the left loop site respectively, should lie on the same side of $\tau = 0$. A straightforward but lengthy computation of the diagrams in Fig. 3.10 verifies this.

To summarize, the diagrammatic rules of extracting singularity structure are also applicable to diagrams with vertices above *and* below $\tau = 0$, with the additional constraint on *left-* and *right-subgraphs* that all vertices are required to lie on the same side of $\tau = 0$, with the constraint relaxed for the right and left loop site respectively. Also note that these loop diagrams may become finite in de-Sitter if the loop sites are connected by Wightman propagators. If these diagrams are divergent in de-Sitter, then $\log \omega_T$ will also be produced from time integrals, as will be discussed in Sec. 3.3.1.

The evaluation of 1-loop diagrams beyond 2-sites becomes significantly complicated. It is worthwhile discussing the case of the next simplest contribution beyond the simple bubble (2-site) topology, which is the triangle diagram. To our knowledge, the 1-loop contribution to the 3-pt correlation function at 3-loop sites has not been computed yet. Progress has been made in [123] towards the computation of the triangle diagram in flat space, using a range of mathematical tools including graphical tubing rules, cosmological Baikov representation, singularity analysis using Landau equations, and finally the explicit evaluation of loop integrals using Whittaker's representation with the integrand containing logarithms and dilogarithms, which requires carefully tracking the branch cuts, their orientations, and the relevant analytic continuations. However, there are several non-trivial challenges in extending this treatment to de-Sitter. Importantly, the flat-space triangle contribution does not have the irreducible genus-one (or higher) obstruction characteristic of a generic elliptic integral arising in de Sitter loop correlators. Additionally, loop computations

in de Sitter are accompanied by corrections from time dependent background (i.e., $\mathcal{O}(\delta)$ corrections in dimensional regularization, time-dependent regulators in cutoff regularization, and so on) which are substantially more complicated to handle than their 3-dimensional counterpart. Extending the flat-space triangle diagram calculation to de-Sitter space requires an appropriate integral representation that makes the underlying (hyper-)elliptic structure of the loop integrals manifest, potentially at the cost of introducing additional auxiliary parameter integrals. However, identifying a suitable representation for the de Sitter case remains a non-trivial open direction. Hence the singularity structure of correlators beyond 2-site diagrams is yet to be studied.

However, in Appendix A.5, we compute this triangle in the soft limit (Fig. A.1), to gain insight on how our results would generalize beyond 2-site diagrams. We find that the singularities produced from explicit computation matches with the predictions of the diagrammatic rules in this work. We believe this is an important non-trivial check of our results.

3.3 Regularization and analytic structure

Almost everything we have derived so far is in line with expectations from the wavefunction perspective: singularities arise when energy entering a subgraph of the Feynman diagram vanishes. However there is one feature in our example presented in Section 3.1.2 which is not accounted for: in Eqns. (3.18) and (3.20) the logarithmic terms have total energies in their argument, implying the existence of a total energy branch point. This goes against the prediction of the cosmological KLN theorem [34], which states: *“There can be no total energy branch points for a cosmological in-in correlator of massless fields, assuming all interactions are IR finite.”*

Here the “total energy branch point” refers to the *off-shell* total energy branch point, which means that given the sum of off-shell energies $\omega_T = \omega_1 + \omega_2 + \cdots + \omega_n$, we cannot have terms like $\log(\omega_T)$ *without* assuming any dispersion relations (such as $\omega_i = |\mathbf{k}_i|$). For flat space this is not hard to see:

- Only simple poles can arise from doing time integration. This is a direct consequence of the integrand being a product of e^{iEt} .
- For a loop diagram, one can write the correlator as a sum of (momentum integrals over) tree level and loop level wavefunction coefficients. The latter can be further decomposed into momentum integrals over discontinuities of tree level wavefunction coefficients by using the Cosmological Tree theorem.
- By adding these discontinuities to the tree level wavefunction coefficients, one finds that it is impossible to obtain denominators of the form $\omega_T + \sum_i a_i p_i$ in the resulting momentum integrand (see Section 3 in [34] for more details). Since logarithmic terms are generated by these denominators on performing the loop integrals, this implies that no total energy logs can appear in the final expression.

For general cosmological spacetime with IR finite interactions, the usual expectation is that the correlator (or the wavefunction coefficients) can be found by taking derivatives of the flat space wavefunction with respect to the off-shell energies[122, 45]. Since no total energy branch point can arise from either time or momentum integration, the correlators can only have poles in ω_T , and taking derivatives of poles only give rise to higher order poles. Therefore, the expectation is that the cosmological KLN theorem holds assuming there are no IR divergent interactions.

3.3.1 Loophole

An assumption in extending the KLN theorem from flat space to general cosmological spacetime is that branch points in energies arise only from momentum integration. However, as we will discuss below, there is a crucial difference between general FLRW spacetimes and flat space.

In de Sitter, total energy branch points in *regularized* correlators can be generated from *time* integrals, rather than momentum integrals.

For this reason, the arguments of the cosmological KLN theorem are evaded in de Sitter.

Dim Reg To see how this happens in dim reg, consider a simple n -point contact diagram for massless scalars in d -dimensional dS, with $\lambda' \phi^n$ interaction. For $2n - d > 0$ this interaction is IR convergent, and usually we write:

$$\langle \phi^n \rangle = \lambda' \int_{-\infty}^0 d\tau \tau^{2n-d-1} e^{i\omega_T \tau} \sim \frac{\lambda' \Gamma(2n-d)}{\omega_T^{2n-d}} = (-i\partial_{\omega_T})^{2n-d-1} \frac{\lambda'}{\omega_T}. \quad (3.36)$$

However there is one important caveat: here we must assume the dimension d is an integer, otherwise we are taking fractional number of derivatives. If instead we work in $d + \delta$ dimensions, then we have (substituting the d dimensional mode function):

$$\langle \phi^n \rangle = (-i\partial_{\omega_T})^{2n-d-1} \lambda' \int_{-\infty}^0 d\tau \tau^{(\frac{n}{2}-1)\delta} e^{i\omega_T \tau}. \quad (3.37)$$

The main difference from flat space is the following: in the expression of Eqn. (3.36) the power of ω_T is not an integer, therefore $\omega_T = 0$ is not a pole (or a higher order pole as one would naively expect), rather it is a branch point (in the same way that

$\sqrt{x}$ has a branch point at $x = 0$). Usually we do not concern ourselves with this issue: since $\delta \rightarrow 0$, any corrections are proportional to δ and it vanishes. However, if there are any divergences, and we work in dim reg, then these correction may give a finite contribution. For example, let $d = 3 + \delta$, and let $\lambda' = -\frac{\lambda^2}{\delta}$ (i.e. this object is a counterterm to some divergent loop integral), then the integral becomes:

$$I_{ct} = -\frac{\lambda^2 \Gamma(2n - 4 - \delta)}{\omega_T^{2n-5}} \left(\frac{1}{\delta} + \left(\frac{n}{2} - 1 \right) \log(\omega_T) + \text{analytic} \right), \quad (3.38)$$

and so we get a $\log(\omega_T)$ contribution.

At this point it is worth asking what the corresponding divergent loop integral is. In the case where n is an even number, the divergence can arise from a two site loop diagram mediated by the interaction $\lambda \phi^{n/2} \sigma^2$, where σ is a conformally coupled scalar. The loop integral is quite straightforward to calculate, and it gives:

$$\begin{aligned} I_L &= \lambda^2 \partial_{\omega_L}^{n-d-1+2} \partial_{\omega_R}^{n-d-1+2} \left(\frac{\frac{1}{\delta} + \frac{1}{2} \log(\omega_L + s) + \frac{1}{2} \log(\omega_R + s)}{\omega_T^{1+(n/2-2)\delta}} \right) \\ &= \frac{\lambda^2 \Gamma(2n - 4 - \delta)}{\omega_T^{2n-5}} \left(\frac{1}{\delta} + \left(\frac{n}{2} - 2 \right) \log(\omega_T) + \frac{1}{2} \log(\omega_L + s) + \frac{1}{2} \log(\omega_R + s) \right. \\ &\quad \left. + \text{analytic} \right). \end{aligned} \quad (3.39)$$

Therefore, the renormalized answer is:

$$I_{ren} = \frac{\lambda^2 \Gamma(2n - 4 - \delta)}{\omega_T^{2n-5}} \left(\frac{1}{2} \log\left(\frac{\omega_L + s}{\omega_T}\right) + \frac{1}{2} \log\left(\frac{\omega_R + s}{\omega_T}\right) \right). \quad (3.40)$$

Notice how the argument of the log conspire to form a scale invariant combination: the argument of the log remains unchanged under the scaling $\omega_{L,R,T} \rightarrow \omega_{L,R,T}/a$ and $s \rightarrow s/a$. Similar cancellations also occur in more complicated dia-

grams, such as the examples provided in Section 3.1.2. Later we discuss in Section 3.3.2 that this is a general feature, and for any n -point function, the branch cuts always combine to form scale invariant arguments.

The appearance of $\log(\omega_T)$ term is a generic feature of dim reg in dS: the measure for the time integration at each vertex depends on dimension d . In addition, since:

$$\int_{-\infty}^{\tau'} d\tau \tau^{n+\delta} e^{is\tau} = (-is)^{-1-n-\delta} \Gamma(1+n+\delta, -is\tau'), \quad (3.41)$$

by expanding the incomplete gamma function as [124]:

$$\Gamma(a, z) = z^{a-1} e^{-z} \sum_{k=0}^{\infty} \frac{(-1)^k (1-a)_k}{z^k}, \quad (3.42)$$

one can see that no partial energy branch points can arise from intermediate time integration (the prefactor $s^{-1-n-\delta}$ in Eqn. (3.41) cancels the factor of s^δ coming from the z^{a-1} term in Eqn. (3.42), so partial energies always have integer power). As a result only branch points in ω_T can arise from time integration, and the power of the singularity ω_T depends on the number of dimension. Since the number of dimension is no longer an integer in dim reg, naturally this means we get total energy branch points³.

Alternatively, one could argue by expanding the mode functions in δ before integrating [2]. As an example consider the D_2 integral which appears in a two site loop diagram :

$$\int_{-\infty}^0 d\tau_L \int_{-\infty}^{\tau_L} d\tau_R \tau_L^{n_L} \tau_R^{n_R} \log(-H\tau_R) e^{i\omega_L \tau_L} e^{i(p_1+p_2)(\tau_R-\tau_L)} e^{i\omega_R \tau_R}. \quad (3.43)$$

³This argument relies on the commutation of summation (in Eqn. 3.42) and integration operations, however this is tricky. To regularize the loop integrals, δ must be continued analytically to some point in the complex plane, and it is not immediately obvious that the sum of integrals converges at this value of δ . We will comment on this later.

It was shown in [3] that for the bispectrum at 1-loop, evaluation of time integrals with the extra factor of $\log(-H\tau)$ results in a contribution which is equivalent to the logs being pulled outside the integrals as $\log(H/\omega_T)$, plus some rational functions of momenta, as follows,

$$= \log\left(\frac{H}{\omega_T}\right) \int_{-\infty}^0 d\tau_L \int_{-\infty}^{\tau_L} d\tau_R \tau_L^{n_L} \tau_R^{n_R} e^{i\omega_L \tau_L} e^{i(p_1+p_2)(\tau_R-\tau_L)} e^{i\omega_R \tau_R} + \text{NLF.} \quad (3.44)$$

It is straightforward to verify this for higher-point diagrams and conclude that the logarithmic terms generated from expanding the d -dimension mode function must have total energy ω_T as its argument⁴.

Cutoff From the argument above, it may seem like these $\log(\omega_T)$ are simply artifacts of working in dim reg, however we they appear in cutoff regularization as well.

Let us again consider the two site bubble example studied previously, and specialize to $n = 4$, i.e. we compute a four point correlator for a massless scalar field ϕ , with the interaction $\dot{\phi}^2 \sigma^2$ (where σ is a conformally coupled field). The one loop bubble diagram is proportional to the following integral:

$$I_L = \partial_{\omega_L}^2 \partial_{\omega_R}^2 \int_{-\infty}^0 d\tau_L \int_{-\infty}^{\tau_L(1+\frac{H}{\Lambda})} d\tau_R e^{i\omega_L \tau_L + i\omega_R \tau_R} \int_s^{a(\tau_R)\Lambda} dp_+ \int_{-s}^s dp_- \frac{e^{ip_+(\tau_L-\tau_R)}}{4s}. \quad (3.45)$$

Crucially, this is not a derivative of the flat space counterpart of the correlator, due to the additional time dependence of the cutoff. This time dependence is needed, as it is the physical momentum on which the cutoff is imposed. If we evaluate the

⁴Similar to the previous footnote, this relies on an asymptotic expansion of the mode function before performing the integrals, and it is not immediately obvious that the two actions should commute.

momentum integral first, we get:

$$I_L = \partial_{\omega_L}^2 \partial_{\omega_R}^2 \int_{-\infty}^0 d\tau_L \int_{-\infty}^{\tau_L(1+\frac{H}{\Lambda})} d\tau_R e^{i\omega_L \tau_L + i\omega_R \tau_R} \frac{1}{4s(\tau_L - \tau_R)} \left[e^{i\frac{\Lambda}{H}(\frac{\tau_L}{\tau_R}-1)} - e^{is(\tau_L - \tau_R)} \right] \quad (3.46)$$

The first term does not give rise to any logarithmic corrections, and for the purposes of studying divergences and logarithmic corrections we may ignore it [2, 3]. To convince ourselves that this is indeed the case, we can look at the special case $\omega_R \rightarrow 0$, where the τ_R integral can be evaluated analytically to give:

$$\int_{-\infty}^{\tau_L(1+\frac{H}{\Lambda})} d\tau_R \frac{1}{4s(\tau_L - \tau_R)} e^{i\frac{\Lambda}{H}(\frac{\tau_L}{\tau_R}-1)} = \frac{1}{4s} \left(e^{\frac{-i\Lambda}{H}} \text{Ei}\left(\frac{i\Lambda^2}{H^2 + H\Lambda}\right) - \text{Ei}\left(\frac{-i\Lambda}{H + \Lambda}\right) \right). \quad (3.47)$$

Further integration in time will not give rise to any additional logarithmic terms in ω_L , so it cannot possibly generate any total energy logarithms (which in this limit would have the form $\log(\omega_L)$).

The second term integrates to⁵:

$$\begin{aligned} I_L &= \partial_{\omega_L}^2 \partial_{\omega_R}^2 \int_{-\infty}^0 d\tau_L e^{i\omega_T \tau_L} \text{Ei}\left(\frac{iH\tau_L(s + \omega_R)}{\Lambda}\right) \\ &= \frac{6}{\omega_T^5} \left(2\log\left(\frac{\Lambda}{H}\right) + \log\left(\frac{\omega_R + s}{\omega_T}\right) + (\text{analytic}) \right). \end{aligned} \quad (3.48)$$

Upon symmetrizing with $\omega_L \leftrightarrow \omega_R$ and renormalizing we obtain the same answer as (3.40) (in the special case $n = 4$), with the total energy logarithm $\log(\omega_T)$ present.

One may argue that the appearance of logarithmic terms here is an artifact of reversing the order of integration: even if the cutoff does not depend on time, we could still do the momentum integral before the time integral. However, there is a

⁵The answer from doing the time integration is an exponential integral, which can be expanded in terms of logarithmic terms and a series. By studying the behavior of the exponential integrals at $\tau_L \rightarrow 0$ and $\tau_L \rightarrow -\infty$ one can obtain the answer here.

subtle cancellation which occurs when the cutoff does not depend on time:

$$\begin{aligned}
& \int_{-\infty}^0 d\tau_L \int_{-\infty}^{\tau_L(1+\frac{H}{\Lambda})} d\tau_R e^{i\omega_L\tau_L + i\omega_R\tau_R} \frac{1}{4s(\tau_L - \tau_R)} [e^{i\Lambda(\tau_L - \tau_R)} - e^{is(\tau_L - \tau_R)}] \\
&= \int_{-\infty}^0 d\tau_L \frac{e^{i(\omega_L + \omega_R)\tau_L}}{4s} \left(\text{Ei}\left(\frac{iH(s + \omega_R)\tau_L}{\Lambda}\right) - \text{Ei}\left(\frac{iH(\Lambda + \omega_R)\tau_L}{\Lambda}\right) \right) \\
&= \int_{-\infty}^0 d\tau_L \frac{e^{i(\omega_L + \omega_R)\tau_L}}{4s} \left(\log\left(\frac{s + \omega_R}{\Lambda + \omega_R}\right) + \text{analytic} \right).
\end{aligned}$$

Clearly, evaluation of the τ_L integration will no longer give any total energy logarithms. Hence, we have shown that the total energy logarithm arises from time integrals in de Sitter correlators, irrespective of the regularization method.

As it is cumbersome to work with cutoff regularisation in higher loops (or more complicated diagrams) we will not try to generalize this result to general loop diagrams.

3.3.2 Does the presence of total energy logarithm break scaling

So far we have shown that logarithmic corrections can appear either as $\log\left(\frac{\omega_p}{\mu}\right)$ or $\log\left(\frac{\omega_T}{H}\right)$, where ω_p is some linear functional of energy, and we have restored the factors of H and μ to ensure that the arguments in the logarithm are dimensionless. In the *on-shell* limit of the correlators, these logarithms are $\log\left(\frac{k_p}{\mu}\right)$ or $\log\left(\frac{k_T}{H}\right)$. However, as mentioned in the Introduction, the argument of these logarithmic terms are ratios of comoving and physical scales, and when written in terms of physical scales, they turn into $\log\left(\frac{k}{\mu}\right) = \log\left(\frac{a}{\mu} \frac{k_{\text{phys}}}{\mu}\right)$. This introduces an explicit dependence on the scale factor for observables, even though the scale factor, by definition, is unobservable.

However, it is not the *unrenormalized* correlators with all its divergences that are relevant to observations. It is the *full, renormalised* correlator that corresponds to physical observables. In the examples we have seen so far (examples computed in Sec. 3.1.2, as well as in the computations for the Bispectrum presented in [3]), the arguments of the logarithmic terms are always ratios of comoving scales, i.e.

$$\log\left(\frac{k_p}{k_{p'}}\right), \quad (3.49)$$

where k_p, k'_p are linear functionals of energy (for example, it can be $\omega_L + s, \omega_R + s$, where s denotes the magnitude of the exchange momentum. It can also be the total energy k_T), which is perfectly fine from physical arguments⁶. Here we will present a heuristic argument showing why this should always be the case.

First let us note that in a generic loop diagram with massless or conformally coupled scalars, the divergent terms should have this form:

$$\begin{aligned} I_{\text{div}} &= \sum_i \frac{D_1^{(i)}(k)}{\delta^{l_i}} \left(\frac{k_T}{H}\right)^{-n_i\delta} \prod_{p_i} \left(\frac{k_{p_i}}{\mu}\right)^{m_{p_i}\delta} \\ &= \frac{D_1^{(i)}(k)}{\delta^{l_i}} \left(\frac{k_T}{H}\right)^{-(n_i - \sum_{p_i} m_{p_i})\delta} \prod_{p_i} \left(\frac{k_{p_i} H}{k_T \mu}\right)^{m_{p_i}\delta}, \end{aligned} \quad (3.50)$$

where k_p is some linear functional of external energies labelled by p_i , $D_1^{(i)}$ is some scale invariant function in external energies, l_i is an integer which denotes how quickly the term diverges. This form is written with the following consideration:

- Since branch cuts at partial energies only arise from momentum integration, factors of k_p^δ come from the momentum integral measure $\int d^d p$. $\sum_{p_i} m_{p_i}$ is equal to the dimension of the momentum integrals that produced the branch

⁶One way to say this is that the argument of the logarithmic terms are scale invariant, but there are some subtleties about this: see appendix A.4 for more details.

cuts.

- Since branch cuts in total energies are produced only from time integration, thus factors of $k_T^{-\delta}$ come from either the time integration measure ($\sim \int d^\delta \tau$, hence giving a contribution of $\tau^{-\delta}$) or the mode function (which gives $\tau^{\delta/2}$ thanks to our regularization scheme)⁷. n_i is the sum of these contributions.

We would like to cancel the divergences by counterterms, which generically has the following form:

$$I_{\text{CT}} = \sum_j -\frac{D_1^{(j)}(k)}{\delta^{l_j}} \left(\frac{k_T}{H}\right)^{-(n_j - \sum_{p_j} m_{p_j})\delta} \prod_{p_j} \left(\frac{k_{p_j} H}{k_T \mu}\right)^{m_{p_j} \delta}. \quad (3.51)$$

Note that beyond one loop, we also need to take into account lower order loops as counterterms (for example, at two loops we need to consider one loop diagrams with the one loop counterterms at one of the vertices), which is why generically the counterterms should also depend on partial energies as well.

Properly renormalizing the correlator implies the following:

$$\sum I_{\text{div}} + I_{\text{CT}} = \text{Finite}. \quad (3.52)$$

Now consider the case where $n_i - \sum_{p_i} m_{p_i} = n_j - \sum_{p_j} m_{p_j} = N$ for all i, j . Then this implies,

$$\sum I_{\text{div}} + I_{\text{CT}} = \text{Finite and scale invariant} \times \left(\frac{k_T}{H}\right)^{-N\delta}, \quad (3.53)$$

since the terms in the remaining brackets are scale invariant. Now expand $\left(\frac{k_T}{H}\right)^{-N\delta}$

⁷It is possible to relax this assumption: if we get k_p^δ instead we can simply rewrite this as $(\frac{k_p}{k_T})^\delta k_T^\delta$, and continue with the analysis. Since $(\frac{k_p}{k_T})^\delta$ is scale invariant this does not affect the result.

as a series in δ . Since it is multiplying a finite and scale invariant function, this implies there can be no $\log\left(\frac{k_T}{H}\right)$ contribution on its own as these are all proportional to δ and vanishes as $\delta \rightarrow 0$, and so the finite part of the renormalized correlator is scale invariant. It remains to show that $n_i - \sum_{p_i} m_{p_i} = n_j - \sum_{p_j} m_{p_j}$ for all i, j . By dimensional analysis, momentum and time integration measure both contribute k^δ , while internal propagators contribute $k^{-\delta}$. By Euler's formula it is straightforward to see that:

$$n_i - \sum_{p_i} m_{p_i} = \frac{n_{ext}}{2} - L + I - V = \frac{n_{ext}}{2} - 1, \quad (3.54)$$

where n_{ext} , L , I and V denote external lines, loops, internal propagators and vertices respectively. This completes the proof.

For clarity let us provide some examples: consider the two site one loop bubble diagram in Fig. 3.11a. Counting the number of $\log(k_T)$, we get

$$n_i = \frac{1}{2}(n_{ext} + 4) - 2 = \frac{n_{ext}}{2}. \quad (3.55)$$

where n_{ext} denotes the number of external legs. This is because we have 2 internal propagators (each contributing $\tau^{\delta/2 \times 2} \propto k_T^{-\delta}$) and two time integrals (each time integral measure contributes $\tau^{-\delta} \propto k_T^\delta$ from the measure).

The momentum integral measure is $\int d^d p$, and by dimensional analysis after doing the momentum integral the measure contributes a factor of k_s^δ , where k_s is the partial energy of one of the vertex. Therefore for the two site one loop bubble diagram, $m_i = 1$, and

$$n_i - m_i = \frac{n_{ext}}{2} - 1 \quad (3.56)$$

Now let us renormalise the bubble diagram by adding a counterterm. Here the

counter term is just a contact term, which obviously implies $m_i = 0$. Since we need to do exactly one time integration, we have:

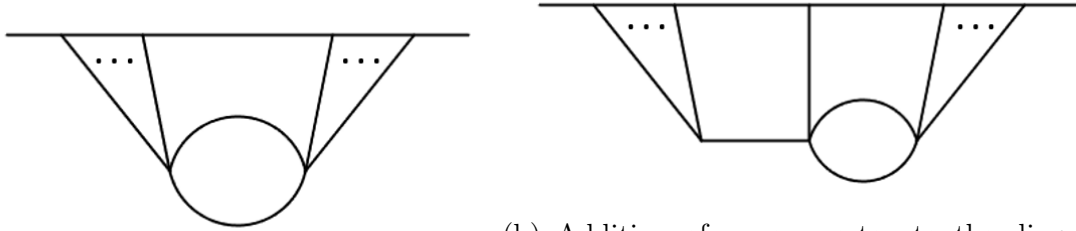
$$n_j - m_j = \left(\frac{n_{ext}}{2} - 1 \right) - 0 = n_i - m_i, \quad (3.57)$$

and from the discussion above, this result implies the one loop bubble is indeed scale invariant.

Starting from this, it is easy to see why we always have $n_i - \sum_{p_i} m_{p_i} = n_j - \sum_{p_j} m_{p_j}$.

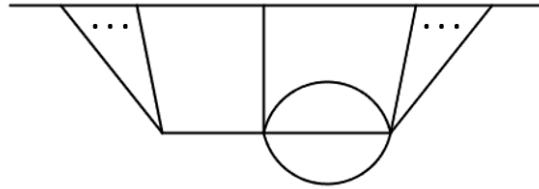
- If we add external legs, i.e. we change n_{ext} , this changes both n_i and n_j in the same way, so this identity still holds.
- Adding a new site in the diagram while remaining at 1-loop does not change n_i , since in doing so the new site must be connected with a new propagator (See Fig. 3.11b). While every propagator contributes $k_T^{-\delta}$, every new site introduces a new time integral, and its dimension dependent measure contributes an additional $\tau^{-\delta} \propto k_T^{\delta}$, and so these contributions cancel.
- To move to higher loops as in Fig. 3.11c, connecting two existing sites in a diagram with an additional propagator increases n_i by one (since propagator contributes $k_T^{-\delta}$). However since we have one more loop, there is another loop momentum integration measure $\int d^d p$. By dimensional analysis, after integration this contributes an additional k_p^{δ} , which increases $\sum_{p_i} m_{p_i}$ by one, and so $n_i - \sum_{p_i} m_{p_i}$ remains unchanged.
- The same arguments carry over to the counterterms, so the number $n_j - \sum_{p_j} m_{p_j}$ remains unchanged and is equal to $n_i - \sum_{p_i} m_{p_i}$.

So order by order in perturbation, for any arbitrary correlator, scale invariance should be preserved in dim reg.



(a) n - point diagram at 1-loop with 2-sites. Changing the number of external legs in this diagram would change n_i and n_j in the same way.

(b) Addition of a new vertex to the diagram in Fig. 3.11a results in an additional time integral which contributes k_T^δ , however this cancels with the contribution coming from the additional propagator.



(c) Connecting two sites in Fig. 3.11b increases number of propagators by one, contributing k_T^δ . However this cancels from the contribution arising from the additional momentum integration measure, since this diagram is now a 2-loop diagram.

Figure 3.11: Various n point diagrams at 1- and 2-loops.

Chapter 4

Singularities in Cosmological Loop Correlators II : Non Local Interactions and Flat Space limits

The material presented in this chapter is based on the author's publication [5].

In Chapter 3 we presented a set of diagrammatic rules to efficiently extract the singularity structure of 1-loop inflationary correlators. These rules make it possible to identify poles and branch cuts in a correlator by examining sub-diagrams and energy flows, without evaluating the full loop integrals. However, those results — and indeed all known results on loop correlator analytic structures so far — were derived for local interactions, i.e. polynomial or derivative couplings of the form $(\partial\phi)^n$ or ϕ^n . Extending beyond simple local interactions is challenging. Generalizing the diagrammatic rules beyond 1-loop diagrams with two interaction vertices (the “bubble” topologies) leads to significantly more complex analytic structures. In fact, the only analytic result for the next-nontrivial one-loop topology (the triangle diagram) has been obtained in the context of the cosmological wavefunction [101].

No intuition or results existed for how more general, non-local interaction terms affect the singularity structure of loop corrections. This is an important gap, because realistic inflationary effective actions naturally contain non-local interactions. For example, when using the ADM formalism to enforce the Hamiltonian and momentum constraints, one finds inverse-Laplacian operators appearing in the cubic [15] and quartic [74, 75] Lagrangians. An illustrative term is a cubic interaction of the form $S_3 \sim \int \dot{\zeta}^2 \partial^{-2} \dot{\zeta}$ (involving an inverse spatial Laplacian). Such non-local vertices introduce non-trivial angular dependencies in loop integrals and can significantly complicate their evaluation. It is therefore both technically interesting and phenomenologically important to understand how these non-local interactions influence the analytic structure of cosmological loop correlators.

In this chapter, we address this question by extending the analysis of Ref. [4] to include non-local inflationary interactions and studying their impact on the 1-loop bispectrum. We focus on a representative set of non-local terms from the cubic and quartic action in the spatially flat gauge (arising from solving the ADM constraints) and compute the corresponding 1-loop corrections to the scalar bispectrum. Our calculation is performed using two complementary regularization schemes – dimensional regularization and a physical momentum cutoff – to cross-check results and handle the various divergences. The inverse-Laplacian factors in the interaction vertices lead to highly non-trivial momentum dependence (for instance, momentum dot products in the numerator of interaction factors), which renders some loop integrals analytically intractable in general d dimensions. In those cases we evaluate the problematic contributions using the cutoff regulator. To our knowledge, this work provides the first analysis of the analytic structure of loop-level cosmological correlators including interactions with inverse-Laplacian (non-local) operators. Rather than attempt an exhaustive computation of all possible interaction terms, our goal is

to isolate and explore the singularity signatures produced by a few salient non-local interactions from the ADM action. This approach allows us to clearly identify which features in the 1-loop bispectrum arise from the non-local nature of the vertices.

Another key aspect of our study is the investigation of the flat-space limit for the 1-loop bispectrum. It is well-known that for tree-level cosmological correlators, taking the total energy ω_T of the external fields to zero relates the leading singularity of the correlator to a flat-space scattering amplitude describing the same process [76, 77, 78, 18, 79, 20]. In particular, the flat-space scattering amplitude can be recovered from the highest-order pole (as $\omega_T \rightarrow 0$) of the tree-level inflationary correlator [33], and this correspondence extends to the integrands of loop diagrams. However, the flat-space limit of the full loop-corrected correlator (after performing the loop integration) has not been derived before. We fill this gap by explicitly deriving the flat-space limit of the complete 1-loop bispectrum (with non-local interactions included) and verifying it in concrete examples. We find that, as expected, the leading singular part of the 1-loop correlator reproduces the corresponding flat-space scattering amplitude. At the same time, the expanding de Sitter background introduces additional features in the loop correlator — notably total-energy branch cuts — which have no direct analogue in flat space. We show that in the renormalized 1-loop bispectrum, all such branch-cut contributions organize into dilatation-invariant logarithms of momentum ratios, similar to what was observed in Ref. [4] for loops with local interactions. Interestingly, we also find that these extra de Sitter features manifest differently depending on the regularization scheme. In both dimensional and cutoff regularization, the same on-shell scattering amplitude is obtained in the $\omega_T \rightarrow 0$ limit, but the two schemes yield slightly different residual logarithmic terms coming from the total-energy branch cuts. We discuss these scheme-dependent differences and their interpretation in Section 4.4.

We summarize the main results of this chapter as follows:

- **Singularities with non-local interactions:** The diagrammatic rules from Ref. [4] for predicting singularity structure at one-loop remain valid even in the presence of non-local (inverse-Laplacian) interaction vertices. We explicitly verify that the pole and branch cut positions inferred by these rules match the results of our direct 1-loop calculations with non-local terms.
- **Branch cuts and logarithmic terms:** Loop corrections involving non-local interactions exhibit branch cuts in the total-energy variable. In the renormalized correlator, all such branch cuts repackage into logarithmic terms that respect dilatation (scale) invariance. This behavior is analogous to what was found in the simpler case of local interactions [4], indicating a degree of universality in how de Sitter symmetry constrains loop correlators.
- **Flat-space limit at 1-loop:** In the limit $\omega_T \rightarrow 0$, the leading singular part of the 1-loop bispectrum with non-local interactions reduces to the corresponding flat-space scattering amplitude, in both dimensional and cutoff regularization schemes. However, due to the time-dependent de Sitter background, the 1-loop correlator also contains features with no flat-space analogue (arising from total-energy branch cuts). These extra contributions manifest differently in the two regularization schemes, resulting in slight scheme-dependent differences in the exact flat-space limit for the branch-cut pieces. Importantly, the physical scattering amplitude itself is consistent between schemes – the scheme differences only affect the additional dS-specific terms (See Sec. 4.4).

Throughout this chapter, we set $M_{pl}^2 = 1$. The rest of the chapter is organized as follows. In Section 4.1 we outline our computational approach for evaluating 1-loop bispectra with both cutoff and dimensional regularization. Section 4.2 presents the

explicit 1-loop bispectrum results obtained from various non-local interactions, and Section 4.3 discusses additional contributions and cross-checks (including interactions omitted from the main text). In Section 4.4, we examine the flat-space limit of the 1-loop correlator in detail and derive the relationship between the singularities of the de Sitter correlators and the flat-space scattering amplitude.

4.1 Details of Computational Scheme

Unlike collider experiments, where initial and final states are well-defined, cosmological observations involve measurements at a single final time slice. To compute expectation values in this context, we employ the *in-in* formalism. The expectation value of an operator $\mathcal{O}(\tau_0)$ at time τ_0 is given by,

$$\langle \mathcal{O}(\tau_0) \rangle = \left\langle 0 \left| \bar{T} e^{i \int_{-\infty(1+i\epsilon)}^{\tau_0} H_I d\tau} \mathcal{O}(\tau_0) T e^{-i \int_{-\infty(1-i\epsilon)}^{\tau_0} H_I d\tau} \right| 0 \right\rangle. \quad (4.1)$$

Here, T and $\bar{T}$ denote time and anti-time ordering respectively, H_I is the interaction Hamiltonian and $|0\rangle$ is the free vacuum state. We will compute the correlators at future boundary of dS, i.e. $\tau_0 \rightarrow 0$. The structure of this expression gives rise to diagrams, often called *Feynman-Witten diagrams* [89], with vertices drawn below (called “right” or “+”) or above (called “left” or “-”) $\tau = 0$ line, depending on whether they come from the time ordered or the anti-time ordered part of the correlator respectively (following notation of [1, 89]). Not all of these diagrams need to be computed since half of these contractions are obtained from complex conjugation of the other half (provided the Hamiltonian is hermitian). The propagators joining

two bulk points are given by,

$$G_{++}(k; \tau_1, \tau_2) = f_k^*(\tau_1)f_k(\tau_2)\Theta(\tau_1 - \tau_2) + f_k(\tau_1)f_k^*(\tau_2)\Theta(\tau_2 - \tau_1), \quad (4.2)$$

$$G_{+-}(k; \tau_1, \tau_2) = f_k(\tau_1)f_k^*(\tau_2), \quad (4.3)$$

$$G_{--}(k; \tau_1, \tau_2) = G_{++}^*(k; \tau_1, \tau_2), \quad (4.4)$$

$$G_{-+}(k; \tau_1, \tau_2) = G_{+-}^*(k; \tau_1, \tau_2), \quad (4.5)$$

where $+$ and $-$ in the subscripts denote whether the interaction vertices τ_1, τ_2 lie below or above $\tau = 0$ line respectively. Depending on the interactions of the theory, the modes in the propagators will be acted upon by operators (spatial/time derivatives or inverse laplacians).

We will compute 1-loop diagrams with two regularization schemes (dimensional regularization and cutoff regularization) wherever possible. For certain contractions, the integral in dim reg becomes intractable due to complicated vertex factors in d -dimensions coming from the non local interactions; for such cases we give the answer from cutoff regularization only.

Dimensional Regularization : Mode functions in de-Sitter are dimension-dependent unlike in flat space. This requires considering full $d = 3 + \delta$ dimensional modes when computing loop integrals in dim reg, with $\mathcal{O}(\delta)$ and higher order corrections from the modes contributing crucially to the structure of the correlator. This is well known from previous 1-loop computations [1, 125, 108, 107, 110], where the unphysical logarithms were tamed by properly regulating the mode functions [2] and renormalising the correlator [3].

In de Sitter, the mode function in d -dimensions for a scalar field with mass m is

given by:

$$f_k(\tau) = H^{-\frac{1}{2}}(-H\tau)^{d/2} H_{i\nu}^{(2)}(-k\tau), \quad (4.6)$$

where $\nu = \sqrt{\frac{m^2}{H^2} - \frac{d^2}{4}}$. However in arbitrary d -dimension, integrals involving these Hankel functions are complicated. Therefore, we analytically continue the mass as [36, 45, 126],

$$\frac{m^2}{H^2} \rightarrow \left(\frac{d^2}{4} - \frac{9}{4} \right), \quad (4.7)$$

which gives back simple massless modes with $\nu = 3/2$. The time integral measure also receives correction in dimensional regularization. Thus the d -dimensional modes and measure simplify as,

$$\begin{aligned} f_k(\tau) &= (-H\tau)^{\delta/2} \frac{H}{\sqrt{2k^3}} (1 - ik\tau) e^{ik\tau} = (-H\tau)^{\delta/2} f_k^{3d}(\tau), \\ a^\delta(\tau) &= (-H\tau)^{-\delta}, \end{aligned} \quad (4.8)$$

where $f_k^{3d}(\tau)$ are 3-dimensional dS modes. In practice, the above is expanded about $\delta = 0$ till $\mathcal{O}(\delta)$ (since 1-loop corrections are at most $\frac{1}{\delta}$ divergent). This gives,

$$\begin{aligned} f_k(\tau) &= \left(1 + \frac{\delta}{2} \log(-H\tau) \right) f_k^{3d}(\tau) + \mathcal{O}(\delta^2) \\ a^\delta(\tau) &= 1 - \delta \log(-H\tau) + \mathcal{O}(\delta^2). \end{aligned} \quad (4.9)$$

We will now sketch the calculational scheme in dim reg. In our calculations, we treat the external mode functions at $\tau = 0$ and the momentum-conserving delta function $\delta^d\left(\sum_i \vec{k}_i\right)$ as three-dimensional, since the counterterms precisely cancel the $\mathcal{O}(\delta)$ corrections arising from these terms [2].

The general expression for one loop correction to the *off-shell* bispectrum (Fig. 4.1)

$D(\vec{k}_i, \omega_i)$ has the general form,

$$D(\vec{k}_i, \omega_i) = \delta^3 \left(\sum_i \vec{k}_i \right) \mu^{-3\delta/2} \int d^d \vec{p}_1 d^d \vec{p}_2 Q(p_1, p_2, \{\omega_i\}, \{\vec{k}_j \cdot \vec{p}_1\}, \{\tau_l\}) \\ \times \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{s}) \prod_l [(-H\tau_l)^{n_l\delta} d\tau_l] , \quad (4.10)$$

where the factor of the renormalisation scale μ has been introduced to account for the δ dependence in the mass dimensions of the couplings in d -dimensions. $\vec{s}(= \vec{k}_3 = -\vec{k}_1 - \vec{k}_2)$ is the momentum exchanged between the loop sites. τ_i are bulk vertices which are integrated over, p_1, p_2 are loop momenta, and Q is a rational function of τ_i 's, external and loop momenta. Contributions from vertex factors involving dot products (arising from spatial derivatives in the interaction) or inverse square norms (from non local operators) of momenta have been absorbed in Q . Also note that Q consists only of the 3-d contribution from modes and measure in Eqn. (4.8). The $\mathcal{O}(\delta)$ contribution from modes and measure has been extracted in the second line in the above equation, with $n_l = \frac{1}{2}, -1$ when considering contribution from modes and measure respectively. Note $\sum_l n_l$ equals the power of μ for any diagrams, this is by construction due to how powers of μ are introduced in the correlator, and hence we get terms like $\log\left(\frac{H}{\mu}\right)$ at 1-loop.

Expanding the last term using Eqn. (4.9) till $\mathcal{O}(\delta)$ terms, we get,

$$D(\vec{k}_i, \omega_i) = \delta^3 \left(\sum_i \vec{k}_i \right) \mu^{-3\delta/2} \int d^d \vec{p}_1 d^d \vec{p}_2 Q(p_1, p_2, \{\omega_i\}, \{\vec{k}_j \cdot \vec{p}_1\}, \{\tau_l\}) \\ \times \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{s}) \left(1 + \sum_l n_l \delta \log(-H\tau_l) \right) \prod_l d\tau_l , \quad (4.11)$$

where the first term in the second line can be identified as D_1 - the d -dimensional

loop integrals with 3-dimensional modes, given by

$$D_1(\vec{k}_i, \omega_i) = \delta^3 \left(\sum_i \vec{k}_i \right) \mu^{-3\delta/2} \int d^d \vec{p}_1 d^d \vec{p}_2 Q(p_1, p_2, \{\omega_i\}, \{\vec{k}_j \cdot \vec{p}_1\}, \{\tau_l\}) \\ \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{s}) \prod_l d\tau_l, \quad (4.12)$$

and the second term, which we call D_2 , is the contribution to the loop diagram coming from $\mathcal{O}(\delta)$ corrections from modes and measure. Clearly the only difference in the computations of D_2 and D_1 is that the former has additional factors of $\log(-H\tau_l)$ in time integrals. For local interactions, it was shown that this factor can be simply pulled out of the time integrals by substituting $\log(-\tau) \rightarrow \log(1/\omega_T)$ [2, 3] (upto some non-logarithmic finite functions of ω_i). In Appendix. A.6, we show that the same relation,

$$D_2(\vec{k}_i, \omega_i) = \delta D_1 \log(H/\omega_T) \sum_l n_l + \text{finite non-log}, \quad (4.13)$$

also holds for the computations in this work involving inverse laplacian interactions.

To evaluate D_1 , it is convenient to compute the time integrals first. With increasing number of interaction vertices, one needs to compute more and more time orderings, hence this calculation is long but straightforward. Next, the loop integrals must be evaluated. In the presence of non-local operators, this can be tricky. These loop integrals are evaluated using an identity which convert $\int d^d \vec{p}_1 d^d \vec{p}_2 \rightarrow \int dp_1 dp_2$ (see Appendix. A.1 for loop diagrams involving local interactions). To briefly sketch the idea- the d -dimensional integral over $\vec{p}_2$ is carried out using the momentum conserving delta function $\delta^d(\vec{p}_1 + \vec{p}_2 - \vec{s})$. Next, the d -dimensional integral over p_1 is

split into norm and angular variables as,

$$\int d^d \vec{p}_1 \rightarrow \int p_1^{d-1} \sin^{d-2} \theta_1 \sin^{d-3} \theta_2 \dots \sin \theta_{d-2} dp_1 d\theta_1 d\theta_2 d\theta_{d-2} d\phi, \quad (4.14)$$

where ϕ is the azimuthal angle and θ_i 's denote polar angles. With $\hat{s}$ chosen along the axis with respect to which θ_1 is measured, we have,

$$|\vec{s} - \vec{p}_1| = \sqrt{s^2 + p_1^2 - 2sp_1 \cos \theta_1} = p_2. \quad (4.15)$$

Now the rest of the derivation proceeds by performing a variable change from θ_1 to p_2 and integrating the chain of sin factors $\sin^{d-3} \theta_2 \dots \sin \theta_{d-2}$ over $d\theta_2 \dots d\theta_{d-2} d\phi$ as $\int d\Omega_{d-1}$. This variable change and integral is trivial if the interactions involve only spatial derivatives, where the vertex contribution involves factors such as,

$$\vec{k}_1 \cdot \vec{p}_1 = k_1 p_1 [\cos \theta_1 \cos \theta_k + \sin \theta_1 \sin \theta_k \cos \theta_2], \quad (4.16)$$

where $\vec{k}_1$ is chosen to lie in the plane of $\hat{s}$ and the axis with respect to which θ_2 is measured, θ_k is angle between $\vec{k}_1$ and $\vec{s}$, given by $\cos \theta_k = \frac{k_2^2 - k_1^2 - s^2}{2k_1 s}$. Importantly, we only need to keep the first term, since the second term in the integrand is odd over $\theta_2 \in (0, \pi)$ due to $\cos \theta_2$.

In this chapter, however, we also have inverse laplacian operators alongwith spatial derivatives in the interactions. This produces, for example in Diagram 31 (see Section. 4.2) a vertex contribution of,

$$\frac{\vec{p}_1 \cdot (\vec{k}_2 + \vec{p}_1)}{|\vec{k}_2 + \vec{p}_1|^2} \xrightarrow{\text{off-shell}} \frac{\omega_2 p_1 (\cos \theta_1 \cos \theta_k + \sin \theta_1 \sin \theta_k \cos \theta_2) + p_1^2}{\omega_2^2 + p_1^2 + 2\omega_2 p_1 (\cos \theta_1 \cos \theta_k + \sin \theta_1 \sin \theta_k \cos \theta_2)}, \quad (4.17)$$

where k_i 's are replaced with ω_i in the offshell limit. θ_k is angle between $\vec{k}_2$ and $\vec{s}$.

Now the integral over angular variables is no longer tractable due to terms such as $\sin^{d-3} \theta_2$. Fortunately, this calculation can still be carried out, atleast partially, in $d = 3$, thus such contractions are to be handled in cutoff regularization only. We discuss the computation in cutoff regularization in the following sub-section.

In Section. 4.2, we notice that the angular dependence can be trivialized in the contractions where both the fields, on which inverse laplacian operator acts, contract to form internal legs, or both contract to form external legs. For e.g. in Diagram 12, the vertex contribution is given by,

$$\frac{\vec{p}_1 \cdot (\vec{p}_1 + \vec{p}_2)}{|\vec{p}_1 + \vec{p}_2|^2} = \frac{\vec{p}_1 \cdot \vec{s}}{s^2} = \frac{s^2 + p_1^2 - p_2^2}{s^2}, \quad (4.18)$$

which introduces no dependence on θ_i ($i = 1, \dots, d-2$) or ϕ . For such contractions, we compute the diagram in both- dim reg as well as cutoff regularization. In such cases, using the loop integral identity we find Eqn. (4.12) gives,

$$D_1(\vec{k}_i, \omega_i) = \delta^3\left(\sum_i \vec{k}_i\right) \mu^{-\frac{3\delta}{2}} \frac{S_{d-2}}{2} \int_s^\infty dp_+ \int_{-s}^s dp_- \frac{p_1^{d-2} p_2}{s} P(p_\pm, \{\omega_i\}, s) \sin^\delta \theta_1. \quad (4.19)$$

where S_{d-2} is surface area of a $(d-2)$ - unit sphere, P is a rational function of $p_\pm, s$ and external energies, and we have defined $p_\pm = p_1 \pm p_2$. Note that although the factor of $\sin^\delta \theta_1$ does not contribute to the $\mathcal{O}(\frac{1}{\delta})$ divergent terms [36] in the correlator, it does give finite contributions and hence is important in determining the structure of the correlator.

Cutoff Regularization: As previously explained, in the presence of non-local interactions, the vertex factors for some contractions become too complicated to be handled in d -dimensions. In such a scenario, we compute the integral in cutoff

regularisation only. In $d = 3$, all θ_i 's drop out of Eqn. (4.14) for $i > 1$, and the integrand simplifies enormously, making the calculation tractable even in the presence of complicated vertex factors such as Eqn. (4.17).

There is another reason for performing computations in cutoff, even for cases when calculation in dim reg is possible. Note that the computations performed in dimensional regularization involved commutation of a summation operator (coming from the Taylor series expansion of the modes about $\delta = 0$) across the loop integrals. This commutation is subtle - for the integral to converge, δ must be analytically continued to some point in the complex plane, and arguing the convergence of the sum of integrals is not straightforward at this value of δ . Thus, a cross check from cutoff regularization demonstrates that answers from dimensional regularization still reliably compute the logarithmic terms, which are the relevant branch cuts we aim to explore in these correlators.

In cutoff regularization, we regulate the momentum integrals by placing a cutoff at $\Lambda a(\tau_f)$, where Λ is the physical cutoff and τ_f is the time being integrated first, e.g. for the $\Theta(\tau_1 - \tau_2)$ part of the integrand $\tau_f = \tau_2$. We integrate the loop integrals first, since the cutoff is now time dependent. The loop momentum identity now becomes :

$$\begin{aligned} & \int d^3\vec{p}_1 d^3\vec{p}_2 Q(p_1, p_2, \{\omega_i\}, \{\tau_l\}) \delta^3(\vec{p}_1 + \vec{p}_2 + \vec{k}_3) \mathcal{Q}(p_1, \{\omega_i\}, \{\vec{k}_j \cdot \vec{p}_1\}) \\ &= \int_0^{\Lambda a(\tau_f)} dp_1 p_1^2 Q(p_1, p_2, \{\omega_i\}, \{\tau_l\}) \int d\theta d\phi \sin\theta \mathcal{Q}(p_1, \{\omega_i\}, \theta, \phi), \end{aligned} \quad (4.20)$$

where we have now separated the vertex factor $\mathcal{Q}$ from the rest of the integrand Q . For inverse laplacian operators acting on fields (φ_1, φ_2) , the ϕ dependence of $\mathcal{Q}$ remains trivial as long as both the modes of φ_1, φ_2 contract in external or internal legs (see Eqn. (4.18)). In this case changing variables from θ to p_2 is straightforward.

However the structure of $\mathcal{Q}$ gets significantly more complicated when one of φ_1 or φ_2 contracts in the loop leg while the other contracts in an external leg. The angular integrals are no longer trivial in this case, and this computation becomes intractable in dim reg.

Let us consider $\mathcal{Q}$ for such cases. For e.g., Eqn. (4.17) in cutoff becomes,

$$\mathcal{Q} = \frac{\omega_2 p_1 (\cos \theta \cos \theta_k + \sin \theta \sin \theta_k \cos \phi) + p_1^2}{\omega_2^2 + p_1^2 + 2\omega_2 p_1 (\cos \theta \cos \theta_k + \sin \theta \sin \theta_k \cos \phi)}. \quad (4.21)$$

Importantly, one can always express other vertex factors in this form by considering the vector $\vec{k}_i$ in the $\hat{x} - \hat{s}$ plane, where $\hat{x}$ is the axis with respect to which ϕ is measured. This leads us to the following integral over ϕ ,

$$\int_0^{2\pi} d\phi \frac{\alpha + \beta \cos(\phi)}{\gamma + \kappa \cos(\phi)} = \left[\frac{\beta\phi}{\kappa} + \frac{2(\beta\gamma - \alpha\kappa) \tanh^{-1} \left(\frac{(\gamma - \kappa) \tan(\frac{\phi}{2})}{\sqrt{\kappa^2 - \gamma^2}} \right)}{\kappa \sqrt{\kappa^2 - \gamma^2}} \right]_0^{2\pi}, \quad (4.22)$$

where we have $\alpha = p_1^2 + \omega_2 p_1 \cos \theta \cos \theta_k$, $\kappa = 2\omega_2 p_1 \sin \theta \sin \theta_k = 2\beta$ and $\gamma = \omega_2^2 + p_1^2 + 2\omega_2 p_1 \cos \theta \cos \theta_k$. Now, the second term on the RHS in Eqn. (4.22) vanishes on the boundary of the integral. However it is discontinuous across $\phi = \pi$, and the integral picks up this jump, yielding,

$$\int_0^{2\pi} d\phi \frac{\alpha + \beta \cos(\phi)}{\gamma + \kappa \cos(\phi)} = \pi + \frac{i\pi(2\alpha - \gamma)}{\sqrt{4\beta^2 - \gamma^2}}, \quad (4.23)$$

where the first and second terms on the RHS of Eqn. (4.23) corresponds to the first and second terms on the RHS of Eqn. (4.22) respectively. The second term introduces extremely complicated dependence on momenta p_i, k_j as well as θ . Infact, the θ integral becomes intractable with this contribution. In the rest of this chapter, we will focus solely on the first term in Eqn. (4.23), and leave any comments on the structure of the correlators coming from the second piece for future work. We will see

in Sec. 4.2, that our results for four of the diagrams (referred to as Diagram 11, 12, 21, 22) are complete, while for the other four case (referred to as Diagram 31, 32, 41, 42) will receive additional contributions from the second term of Eqn. (4.23) which we ignore in this work. Evaluating ϕ integral with the first term in Eqn. (4.23), we get from Eqn. (4.20),

$$\begin{aligned}
&= \int_0^{\Lambda a(\tau_f)} dp_1 p_1^2 Q(p_1, p_2, \{\omega_i\}, \{\tau_l\}) \int d\theta \sin \theta \times \pi \\
&= \pi \int_0^{\Lambda a(\tau_f)} dp_1 \int_{|p_1-s|}^{p_1+s} dp_2 \frac{p_1 p_2}{s} Q(p_1, p_2, \{\omega_i\}, \{\tau_l\}), \tag{4.24}
\end{aligned}$$

and we see that the complicated looking ϕ integral in Eqn. (4.22) does not produce any θ dependence. The equality in second line follows from changing variables $\theta \rightarrow p_2$. Now, some of the terms produced in the integral regime $\int_s^{\Lambda a(\tau_f)} \int_{p_1-s}^{p_1+s} dp_1$ are proportional to $\exp\left(2\frac{\Lambda}{H}\left(\frac{\tau_f}{\tau_l} - 1\right)\right)$ (where $\tau_f > \tau_l$). These terms are dropped when computing the time integrals, since these are exponentially suppressed throughout the integration regime and hence will only produce non-log finite terms [2]. Finally, evaluation of the momentum integrals produces factors of $\tau_1 - \tau_2$ in the denominator. Hence the time integrals are also regulated with the same regulator Λ as,

$$\int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \rightarrow \int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1(1+\frac{H}{\Lambda})} d\tau_2. \tag{4.25}$$

4.2 Results at 1-loop

In this section, we follow the scheme outlined in Sec. 4.1 to compute the bispectrum at 1-loop (Fig. 4.1) with cubic and quartic interactions given by $ig_3 \dot{\varphi}^2 \varphi$ and $ig_4 \dot{\varphi}^2 \partial^{-2} \partial_i (\dot{\varphi} \partial_i \varphi)$, taken from the cubic and quartic action Eqns. (1.85) and (1.87) respectively.

4.2.1 List of all contractions

Let us refer to the fields in the interactions as φ_i , where $i = 1, \dots, 7$. Hence, the interactions are $ig_4 \dot{\varphi}_1 \dot{\varphi}_2 \partial^{-2} \partial_i (\dot{\varphi}_3 \partial_i \varphi_4)$ and $ig_3 \dot{\varphi}_5 \dot{\varphi}_6 \varphi_7$. The possible contractions for the quartic coupling are given as follows,

1. When both the fields being acted upon by the inverse laplacian operator, i.e. φ_3 and φ_4 , contract in the loop legs, with φ_1 and φ_2 contracts in the external legs.
2. When the fields not being acted upon by the inverse laplacian operator, i.e. φ_1 and φ_2 , contract in the loop legs.
3. When φ_3 contracts in an external leg, and φ_4 contracts in a loop leg.
4. When φ_4 contracts in an external leg, and φ_3 contracts in a loop leg.

For each of the contractions mentioned above, we can have the following two cases with the cubic interaction,

1. The field with no time derivative, i.e. φ_7 , contracts in the external leg.
2. φ_7 contracts in one of the internal legs.

We will refer to a diagram with Contraction (i) of quartic coupling with Contraction (j) of cubic coupling as Diagram ij . For example, Diagram 12 refers to the diagram in Fig. 4.1 where fields being acted upon by inverse laplacian operator in the quartic interaction contract in the loop legs (thus we have $\varphi_1(\omega_1, \vec{k}_1)$, $\varphi_2(\omega_2, \vec{k}_2)$, $\varphi_3(p_1)$ and $\varphi_4(p_2)$), while the field without a time derivative in the cubic interaction contracts in one of the loop legs (hence, we have $\varphi_5(\omega_3, \vec{k}_3)$, $\varphi_6(p_1)$ and $\varphi_7(p_1)$).

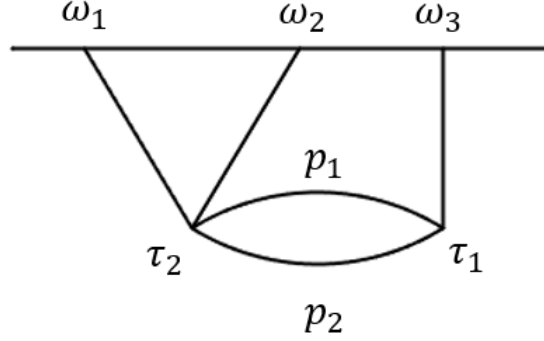


Figure 4.1: Bispectrum at 1-loop computed with one insertion of a cubic and a quartic vertex each.

4.2.2 Results of explicit computations

Below we give the renormalised bispectrum for the Diagrams listed above. We denote the magnitude of the momentum exchanged between the loop sites as $s = |\vec{p}_1 + \vec{p}_2|$, which is equal to k_3 for this diagram once onshell condition is imposed. Note that we ignore some overall factors of Hubble and external momenta, as well as numerical constants, since we are interested in probing the analytic structure of these correlators.

Diagram 11 : The vertex factor produced by inverse laplacian operators for this contraction is given by,

$$\frac{\vec{p}_2 \cdot (\vec{p}_1 + \vec{p}_2)}{|\vec{p}_1 + \vec{p}_2|^2} = \frac{\vec{p}_2 \cdot \vec{s}}{s^2} = \frac{s^2 - p_1^2 + p_2^2}{s^2}, \quad (4.26)$$

which has trivial angular dependencies, and hence can be computed both in cutoff as well as dimensional regularization.

In dimensional regularization, considering 3-d modes and measure, we get,

$$D_1 \sim \frac{-8\omega_3^2 (s^2 + 5\omega_3^2) + 25\omega_{12}\omega_3 (\omega_3^2 - s^2) + 5\omega_{12}^2 (\omega_3^2 - s^2)}{\omega_T^5} \left(\frac{1}{\delta} - \frac{3}{2} \log \mu \right. \\ \left. + \frac{1}{2} \log [(s + \omega_3)(s + \omega_{12})] \right), \quad (4.27)$$

And using Eqn. (4.13), the $\mathcal{O}(\delta)$ correction from modes and measure is,

$$D_2 = \frac{3}{2} \delta D_1 \log \left(\frac{H}{\omega_T} \right) + \text{NLf}, \quad (4.28)$$

since there are 7 internal modes and 2 vertex factors, hence $\sum_i n_i = \frac{7}{2} - 2 = \frac{3}{2}$. Adding D_1 and D_2 , we find “*unphysical*” logarithm of ω_T/μ , which is taken care of once the contact-contribution from counter term is added. This can be seen without explicitly renormalizing the theory. Generally, the counter-terms are local operators of dimension 8 (since $[g_3] = -1$ and $[g_4] = -3$), having the following structure,

$$\mathcal{O}^{\text{CT}} \sim \frac{1}{\delta} \int \mu^{-\delta/2} a^{-3+\delta} \partial^5 (\varphi^3), \quad (4.29)$$

where ∂^5 are 5 space/time derivatives respecting rotational invariance¹. The contact 3-pt diagram computed with d -dimensional modes, renormalizing the divergence in the 1-loop correlator, has 3 internal modes and 1 vertex. Thus counter term contribution $D_{\text{c.t.}}$ is given by,

$$D_{\text{c.t.}} = D_{1,\text{c.t.}} + D_{2,\text{c.t.}} \propto - \left(\frac{1}{\delta} - \frac{1}{2} \log \mu + \frac{1}{2} \log \left(\frac{H}{\omega_T} \right) \right), \quad (4.30)$$

where $D_{(1,2),\text{c.t.}}$ denote contact diagram contributions coming from 3-d and $\mathcal{O}(\delta)$

¹Since the kinematic dependence in the coefficient of the divergent term of the loop correlator comes from inverse laplacian operators, it is interesting to ask whether the counter terms might require some non local operators to produce the correct kinematics.

parts of the modes/measure. The first two terms in Eqn. (4.30) are from $D_{1,\text{c.t.}}$, while the last term is from $D_{2,\text{c.t.}}$ ($\sum_i n_i = \frac{3}{2} - 1 = \frac{1}{2}$ coming from 3 internal modes and 1 vertex). Adding $D_{\text{c.t.}}$ to $D_1 + D_2$, we get $D_{\text{renorm.}} = D_{\text{c.t.}} + D_1 + D_2$ as,

$$D_{\text{renorm.}} \sim \frac{1}{\omega_T^5} \left(-8\omega_3^2 (s^2 + 5\omega_3^2) + 25\omega_{12}\omega_3 (\omega_3^2 - s^2) + 5\omega_{12}^2 (\omega_3^2 - s^2) \right) \\ \left(\log \frac{H}{\mu} + \frac{1}{2} \log \left[\frac{(s + \omega_3)}{\omega_T} \right] + \frac{1}{2} \log \left[\frac{(s + \omega_{12})}{\omega_T} \right] \right) + \text{NLf}, \quad (4.31)$$

As a cross check, we calculate this diagram in cutoff regularisation to find that the unrenormalised² correlator features $\log(\Lambda/H)$ divergence, whose coefficient matches the coefficient of $\log(\mu/H)$ term in dim reg. The other logarithms involving ratios of comoving momenta also match with the above equation.

Diagram 12 The vertex factor produced by inverse laplacian operators in this contraction is the same as Eqn. (4.26). Thus we can once again compute this contraction both in cutoff as well as dimensional regularization. Considering 3-d modes and measure, we get,

$$D_1 \sim \frac{1}{\omega_T^5} \left(8\omega_{12}^2 + 5\omega_3^2 + 31\omega_{12}\omega_3 + 6s^2 \right) \left(\frac{1}{\delta} + \frac{1}{2} \log \left[(s + \omega_3)(s + \omega_{12}) \right] - \frac{3}{2} \log \mu \right), \quad (4.32)$$

Following the arguments outlined for Diagram 11, the renormalized answer $D_{\text{renorm.}} = D_{\text{c.t.}} + D_1 + D_2$ turns out to be,

$$D_{\text{renorm.}} \sim \frac{1}{\omega_T^5} \left(8\omega_{12}^2 + 5\omega_3^2 + 31\omega_{12}\omega_3 + 6s^2 \right) \left(2 \log \left(\frac{H}{\mu} \right) + \log \left(\frac{\omega_{12} + s}{\omega_T} \right) \right. \\ \left. + \log \left(\frac{\omega_3 + s}{\omega_T} \right) \right) + \text{NLf}. \quad (4.33)$$

²The counter term contributions in cutoff regularization are tree level diagrams computed with 3-d massless mode functions, hence no branch cuts are produced.

Performing this computation in cutoff regularisation, the unrenormalised correlator features $\log(\Lambda/H)$ divergence, whose coefficient matches the coefficient of $\log(\mu/H)$ in dim reg. The other logarithms, involving comoving momenta, produced in cutoff regularisation match with the above equation.

Diagram 21 The vertex factor produced by inverse laplacian operators for this contraction is given by,

$$\frac{\vec{k}_2 \cdot (\vec{k}_1 + \vec{k}_2)}{|\vec{k} + \vec{k}_2|^2} = \frac{\vec{k}_2 \cdot \vec{k}_3}{s^2}, \quad (4.34)$$

which has trivial angular dependencies, and hence is computable both in cutoff as well as dimensional regularization. Computing $D_{\text{renorm.}}$, we get,

$$\begin{aligned} D_{\text{renorm.}} = & \frac{\vec{k}_2 \cdot \vec{k}_3}{s^2 \omega_T^5} \left\{ -\log\left(\frac{H}{\mu}\right) \left(8s^2 \omega_3 (5\omega_2 + \omega_3) (3s^2 - 5\omega_3^2) + 8\omega_2 \omega_{12} (10s^2 \omega_3^2 + 3s^4 \right. \right. \\ & - 45\omega_3^4) + \omega_1 (\omega_3^3 (450\omega_{12}^2 - 20s^2) + 10\omega_3^2 (2s^2 \omega_{12} + 45\omega_{12}^3) + 15\omega_3 (2s^4 + 15\omega_{12}^4) + 6s^4 \omega_{12} \\ & \left. \left. - 45\omega_3^5 + 135\omega_{12} \omega_3^4 + 45\omega_{12}^5) \right) + \log\left(\frac{s + \omega_3}{\omega_T}\right) \left(\omega_3 (4s^2 (5\omega_2 + \omega_3) (5\omega_3^2 - 3s^2) \right. \right. \\ & + 5\omega_1 (2s^2 \omega_3^2 - 3s^4 + 9\omega_3^4) + (\omega_1 + 4\omega_2) \omega_{12} (-10s^2 \omega_3^2 - 3s^4 + 45\omega_3^4) \left. \right) + \log\left(\frac{s + \omega_{12}}{\omega_T}\right) \\ & \left(-\omega_1^2 (10\omega_3^2 (s^2 + 135\omega_2^2) + 3(s^4 + 75\omega_2^4) + 180\omega_3^4 + 900\omega_2 \omega_3^3 + 900\omega_2^3 \omega_3) \right. \\ & - 5\omega_1 (-2\omega_3^3 (s^2 - 45\omega_2^2) + 10\omega_2 \omega_3^2 (s^2 + 9\omega_2^2) + 3\omega_3 (s^4 + 15\omega_2^4) + 3\omega_2 (s^4 + 3\omega_2^4)) \\ & + 4(-3s^4 \omega_3^2 + 5s^2 \omega_3^4 + 5s^2 \omega_2 \omega_3 (5\omega_3^2 - 3s^2) + \omega_2^2 (-10s^2 \omega_3^2 - 3s^4 + 45\omega_3^4)) - 45\omega_1^6 \\ & \left. \left. - 225(\omega_2 + \omega_3) \omega_1^5 - 450(\omega_2 + \omega_3)^2 \omega_1^4 - 450(\omega_2 + \omega_3)^3 \omega_1^3 \right) \right\} + \text{NLf}, \quad (4.35) \end{aligned}$$

Performing the same computation in cutoff regularisation, we find that the coefficients of $\log(\frac{\omega_3+s}{\omega_T})$ and $\log(\frac{\omega_{12}+s}{\omega_T})$ match. The same is also true for coefficient of $\log(\Lambda/H)$ (in cutoff) and that of $\log(\mu/H)$ term (in dim reg).

In cutoff, we further find power law divergences in Λ . Some of these terms involve late time log divergences as follows,

$$\sim \frac{3\Lambda (3H^2\Lambda + 2H^3 + 2H\Lambda^2 + \Lambda^3) \log(\tau_0\omega_T)}{128H^4\omega_1\omega_2^3\omega_3^3}. \quad (4.36)$$

It is interesting to ask whether this divergence is a *true* secular divergence of the theory, by which we mean whether this divergence persists once one renormalizes the correlator and all power law divergences cancel. Naively it does not seem to be true, since the $\log(\tau_0)$ term does not arise in the dimensional regularization scheme, and in the cutoff regularization it arises only as a coefficient in the power law divergence in Λ . A true secular divergence in the theory should show up regardless of the UV-regularization scheme. Nevertheless it is important to explicitly demonstrate the existence of the counter terms absorbing the divergence in Eqn. (4.36). We leave the explicit renormalization of the theory for a future work.

Diagram 22 The vertex factor produced by inverse laplacian operators in this contraction is the same as Eqn. (4.34) and remains computable in cutoff as well as dimensional regularization. Computing $D_{\text{renorm.}}$, we get,

$$\begin{aligned} D_{\text{renorm.}} \sim & \frac{\vec{k}_2 \cdot \vec{k}_3}{s^2\omega_T^5} \left(\frac{1}{\delta} + \frac{1}{2} \log \left[\frac{(s + \omega_{12})(s + \omega_3)}{\omega_T^2} \right] \right) \left(\omega_3 (-2(5\omega_2 + \omega_3)(2s^2 + 3\omega_3^2) \right. \\ & \left. - \omega_1(5s^2 + 21\omega_3^2)) - (\omega_1 + 4\omega_2)\omega_{12}(s^2 + 15\omega_3^2) \right) + \text{NLF}. \end{aligned} \quad (4.37)$$

Performing this computation in cutoff regularisation gives the same logarithms of comoving momenta, and the coefficient of $\log(\Lambda/H)$ matches the coefficient of $\log(\mu/H)$ in the previous equation.

Diagram 31 The phase space integral becomes too complicated to be evaluated in d -dimensions, with the vertex factor given by Eqn. (4.17). Hence we evaluate this diagram in cutoff regularization only. The vertex factor in cutoff regularization is given by Eqn. (4.21). Computing the integrals we get,

$$D_{\text{unrenorm.}} \sim \frac{1}{\omega_T^5} \left\{ 2 \log \left(\frac{\Lambda}{H} \right) - \log \left(\frac{\omega_{12} + s}{\omega_T} \right) - \log \left(\frac{\omega_3 + s}{\omega_T} \right) \right\} \left\{ -\omega_3^2 (3\omega_{12}^2 + 2s^2) + 5\omega_{12}\omega_3 s^2 + \omega_{12}^2 s^2 + 15\omega_3^4 - 6\omega_{12}\omega_3^3 \right\} + \text{NLF}. \quad (4.38)$$

Diagram 32 Once again we will compute this diagram in cutoff regularization only due to the phase space integral being too complicated in d - dimensions. There are two cases for this diagram : when φ_7 contracts with φ_4 , and when φ_7 contracts with φ_1 (or φ_2).

We find that in the former case, the loop integral is divergent both in UV as well as IR. For diagrams that are divergent only in the UV, it was noticed that without proper regularization, one finds unphysical features in the loop answer [2, 3, 4]. In dS, the correct way to implement UV-cutoff regularization is to consider a fixed physical cutoff Λ , which turns into a time dependent comoving cutoff $a(\tau_e)\Lambda$ (τ_e is the time being integrated earliest) in the loop integrals. However, it is not very clear what the analogous appropriate regularization is in the IR (for related discussions, see [127, 128, 129, 60, 37, 49] and references therein). A detailed analysis involving regularization of IR divergences, such as the one arising in the above case of Diagram 32, is left for a future work.

For the second case, i.e. when φ_7 contracts with φ_1 or φ_2 , the integrals evaluate to,

$$D_{\text{unrenorm.}} \sim \frac{1}{\omega_T^5} \left\{ \log \left(\frac{\Lambda}{H} \right) - \frac{1}{2} \log \left(\frac{\omega_3 + s}{\omega_T} \right) - \frac{1}{2} \log \left(\frac{\omega_{12} + s}{\omega_T} \right) \right\} (3\omega_{12}(\omega_{12} + 3\omega_3) + 2s^2) + \text{NLF}. \quad (4.39)$$

Diagram 41 The vertex factor produced by inverse laplacian operators in this contraction is given by,

$$\frac{\vec{k}_2 \cdot (\vec{k}_2 + \vec{p}_1)}{|\vec{k}_2 + \vec{p}_1|^2} \xrightarrow{\text{off-shell}} \frac{\omega_2 p_1 (\cos \theta \cos \theta_k + \sin \theta \sin \theta_k \cos \phi) + \omega_2^2}{\omega_2^2 + p_1^2 + 2\omega_2 p_1 (\cos \theta \cos \theta_k + \sin \theta \sin \theta_k \cos \phi)}. \quad (4.40)$$

This involves non-trivial angular dependencies, hence is only computed in cutoff regularization. This dependence is handled exactly as in Eqn. (4.21), and evaluating the integrals we get,

$$D_{\text{unrenorm.}} \sim \frac{1}{\omega_T^5} \left\{ \log \left(\frac{H(s + \omega_3)}{\Lambda \omega_T} \right) \left(\omega_3 (4s^2 (5\omega_2 + \omega_3) (3s^2 - 5\omega_3^2) + 5\omega_1 (-2s^2 \omega_3^2 + 3s^4 - 9\omega_3^4)) + (\omega_1 + 4\omega_2) \omega_{12} (10s^2 \omega_3^2 + 3s^4 - 45\omega_3^4) \right) + \log \left(\frac{H(s + \omega_{12})}{\Lambda \omega_T} \right) \left(\omega_1^2 (10\omega_3^2 (s^2 + 135\omega_2^2) + 3(s^4 + 75\omega_2^4) + 180\omega_3^4 + 900\omega_2 \omega_3^3 + 900\omega_2^3 \omega_3) + 5\omega_1 (-2\omega_3^3 (s^2 - 45\omega_2^2) + 10\omega_2 \omega_3^2 (s^2 + 9\omega_2^2) + 3\omega_3 (s^4 + 15\omega_2^4) + 3\omega_2 (s^4 + 3\omega_2^4)) + 4(3s^4 \omega_3^2 - 5s^2 \omega_3^4 + 5\omega_2 (3s^4 \omega_3 - 5s^2 \omega_3^3) + \omega_2^2 (10s^2 \omega_3^2 + 3s^4 - 45\omega_3^4)) + 45\omega_1^6 + 225\omega_{23} \omega_1^5 + 450\omega_{23}^2 \omega_1^4 + 450\omega_{23}^3 \omega_1^3) \right) \right\} + \text{NLF}. \quad (4.41)$$

Diagram 42 : The vertex factor for this contraction is the same as in Eqn. (4.40).

Computing the diagram in cutoff regularization, we get,

$$\begin{aligned}
D_{\text{unrenorm.}} \sim \frac{1}{\omega_T^5} & \left\{ 2 \log \left(\frac{\Lambda}{H} \right) - \log \left(\frac{\omega_{12} + s}{\omega_T} \right) - \log \left(\frac{\omega_3 + s}{\omega_T} \right) \right\} \left(\omega_3^2 (15 \omega_{12} (\omega_1 + 4 \omega_2) \right. \\
& \left. + 4 s^2) + 5 (\omega_1 + 4 \omega_2) \omega_3 s^2 + \omega_{12} (\omega_1 + 4 \omega_2) s^2 + 6 \omega_3^4 + 3 (7 \omega_1 + 10 \omega_2) \omega_3^3 \right) + \text{NLf.}
\end{aligned}
\tag{4.42}$$

As previously stated, the contributions from Diagram 31 in Eqn. (4.38), from Diagram 32 in Eqn. (4.39), from Diagram 41 in Eqn. (4.41), and from Diagram 42 in Eqn. (4.42) are missing contribution from the second term in Eqn. (4.23), which are analytically intractable. We postpone the analysis of the singularity structure from such contributions to a future work.

Singularities of the bispectrum with non-local interactions : The bispectrum in Fig. 4.1 has no *left-, right-subgraphs* (as defined and discussed in Chapter 3). The *loop-subgraph* of this diagram has external energy injection of $\mathcal{S}_L = \omega_{12}$ and $\mathcal{S}_R = \omega_3$ through the left and right loop site respectively. The diagrammatic rules tell us that evaluation of this diagram should yield $\log(s + \omega_{12})$ and $\log(s + \omega_3)$.

In all the previously computed results we find,

- Logarithms of $s + \omega_3$ and $s + \omega_{12}$. This was precisely expected from the diagrammatic rules. This verifies that the diagrammatic rules hold for the bispectrum computed with the non local interactions in this work.
- Dilatation invariant logarithms, where the denominator of the log argument is the total energy ω_T . It has been previously discussed that this total energy branch cut is a feature of dS correlators (computed with polynomial or derivative interactions), and we find this remains true for non-local interactions as well.

The underlying theory considered in this work is local, and only after eliminating the constraints do these non-local terms appear. It may seem that there is a connection between locality of the theory and the diagrammatic rules. The true signature of locality is reflected when every interaction term is included (i.e. locality = $\sum$ non-local terms), however we find the diagrammatic rules hold in every single contraction for a given interaction term. If additional branch cuts were generated as a consequence of non-locality, their cancellation would require contributions from the other interaction terms. However, no such additional branch cut is observed in any of the computations. Consequently, we find no conclusive evidence for connection between the diagrammatic rules and locality of the theory.

Comments on choice of non-local operators considered : In this chapter, we demonstrate the validity of the diagrammatic rules for identifying the poles and branch cuts of cosmological loop correlators for a representative set of non-local interactions involving inverse Laplacian operators. Unlike the analysis of local interactions in Chapter 3, we do not explicitly derive the diagrammatic rules for general theories with non-local interactions. Such a derivation is more involved because inverse Laplacian operators introduce additional angular dependence into the loop phase-space integrals, which renders part of the loop integral analytically intractable for some of the contractions (see Eqn. (4.23)).

However, we emphasize that the operator combinations considered here have been chosen deliberately to probe a broad range of complicated contributions. In particular, we consider cubic and quartic interactions containing the maximum number of inverse-Laplacian operators in the spatially flat ADM action. This choice allows us to investigate the analytic structure in cases with more intricate vertex factors and phase-space integrals, while the effects of purely local interactions have already been studied extensively in the previous chapters. Although this analysis

does not establish the diagrammatic rules for arbitrary non-local operators, the examples provide non-trivial tests of their validity in the presence of mixed local - non local interactions, and multiple inverse Laplacian operators.

To be precise, the cubic action contains local interaction terms ($S_{3,\text{local}}$), as well as operators involving a single inverse Laplacian ($S_{3,\partial^{-2}}$). The quartic action contains interactions involving one and two inverse-Laplacian operators ($S_{4,\partial^{-2}}$ and $S_{4,\partial^{-2},\partial^{-2}}$ respectively). Every inverse Laplacian operator produces angular dependence into the vertex factors, through terms such as $\vec{k}_i \cdot \vec{p}_j$ and $\vec{p}_i \cdot \vec{p}_j$, further complicating the phase space integral. In this chapter, we have considered the quartic-cubic interaction combinations given by $(S_{4,\partial^{-2}}, S_{3,\text{local}})$ and $(S_{4,\partial^{-2},\partial^{-2}}, S_{3,\partial^{-2}})$ (see Sec. 4.3) for the diagram in Fig. 4.1. These examples probe both the mixed local - non local interactions and contributions in which the quartic and cubic vertices contain two and one inverse-Laplacian operators, respectively. Additionally, for the three vertex contributions of Fig. 4.2 computed in the next section, we consider the interactions $(S_{3,\partial^{-2}}, S_{3,\partial^{-2}}, S_{3,\partial^{-2}})$, thereby examining contributions in which all three vertices involve inverse-Laplacian operators.

Taken together, these examples probe several nontrivial configurations: mixed local-non-local vertices, vertices containing multiple inverse-Laplacian operators, and loop diagrams in which every interaction vertex is non-local. The resulting calculations therefore test the diagrammatic rules across a range of complicated vertex structures and associated phase-space integrals, and we find that the analytic structure agrees with the rules established for local interactions. We thus emphasize that these results constitute a systematic set of illustrative checks of the diagrammatic rules for arbitrary non-local interactions.

4.3 Other contributions to the Bispectrum with non-local interactions

In Sec. 4.2, we computed the bispectrum with interactions $ig_3 \dot{\varphi}^2 \varphi$ and $ig_4 \dot{\varphi}^2 \partial^{-2} \partial_i (\dot{\varphi} \partial_i \varphi)$.

In this section, we inspect the effect of a few other interaction terms, taken from the cubic (Eqn. (1.85)) and quartic action (Eqn. (1.87)), on the structure of the bispectrum.

4.3.1 Additional inverse laplacian operators in quartic and cubic interactions

Let us consider $\partial^{-2} \partial_i (\dot{\varphi} \partial_i \varphi) \partial^{-2} \partial_j (\dot{\varphi} \partial_j \varphi)$ from Eqn. (1.87) and $\dot{\varphi} \partial_i \varphi \partial^{-2} \partial_i \dot{\varphi}$ from Eqn. (1.85) as the quartic and cubic interaction respectively, for the bispectrum in Fig. 4.1. We will only compute one contraction, since the purpose of this computation is not to express the full bispectrum with all interactions at quartic and cubic order, but rather to investigate what happens to the analytic structure in the presence of inverse laplacian operators, which is also why we choose to work with the interactions having the largest number of such operators acting on the fields.

The contraction we compute is described by the following :

1. Let us denote the fields in the quartic vertex as,

$$\partial^{-2} \partial_i \left(\dot{\varphi}(\vec{k}_1) \partial_i \varphi(\vec{k}_2) \right) \partial^{-2} \partial_j \left(\dot{\varphi}(\vec{p}_1) \partial_j \varphi(\vec{p}_2) \right).$$

The first inverse laplacian operator generates a vertex factor of $\frac{\vec{k}_2 \cdot (\vec{k}_1 + \vec{k}_2)}{|\vec{k}_1 + \vec{k}_2|^2}$, and

the second gives a factor of $\frac{\vec{p}_2 \cdot (\vec{p}_1 + \vec{p}_2)}{|\vec{p}_1 + \vec{p}_2|^2} = \frac{s^2 + p_2^2 - p_1^2}{2s^2}$.

2. We denote the fields in the cubic vertex as $\dot{\varphi}(\vec{p}_2) \partial_i \varphi(\vec{k}_3) \partial^{-2} \partial_i \dot{\varphi}(\vec{p}_1)$. This

inverse laplacian gives a vertex contribution of $\frac{\vec{k}_3 \cdot \vec{p}_1}{p_1^2} = \frac{s^2 + p_1^2 - p_2^2}{2p_1^2}$.

We evaluate this diagram in dimensional regularization following the scheme outlined in Sec. 4.1. As usual, we find that the unrenormalized correlator $D = D_1 + D_2$ features unphysical $\log(\frac{\omega_T}{\mu})$, which are removed once contribution from counter terms are added, provided the counter terms absorb the $1/\delta$ divergence of the correlator. The full renormalized correlator $D_{\text{renorm.}} = D_1 + D_2 + D_{\text{c.t.}}$ is then obtained as,

$$\begin{aligned}
D_{\text{renorm.}} \sim & \frac{\vec{k}_2 \cdot \vec{k}_3}{s^2 \omega_T^4} \left[\log \left(\frac{H}{\mu} \right) \left\{ \omega_3^2 (\omega_1 (18s^2 - \omega_2^2 + \omega_3^2) + 36s^2 \omega_2 + 9s^2 \omega_3 - 5\omega_1^3 \right. \right. \\
& - 8\omega_2 \omega_1^2 + 2\omega_2^3) - \omega_{12} \omega_3 (4 (\omega_1 (\omega_2^2 - 3s^2) - 6s^2 \omega_2 + \omega_1^3 + 2\omega_2 \omega_1^2) + (\omega_1 - 5\omega_2) \omega_3^2) \\
& - \omega_{12}^2 (\omega_1 (\omega_2^2 - 3s^2) - 6s^2 \omega_2 + \omega_1^3 + 2\omega_2 \omega_1^2) \left. \right\} + \frac{1}{2} \log \left(\frac{s + \omega_{12}}{\omega_T} \right) \left\{ \omega_3^2 (\omega_1 (18s^2 - 7\omega_2^2) \right. \\
& + 36s^2 \omega_2 + 9s^2 \omega_3 - 11\omega_1^3 - 5(4\omega_2 + \omega_3) \omega_1^2 + 2\omega_2^3 + 5\omega_2^2 \omega_3) + 4\omega_{12} \omega_3 (\omega_1 (3s^2 - 2\omega_2^2) \\
& + 6s^2 \omega_2 - 2\omega_1^3 - 4\omega_2 \omega_1^2) + \omega_{12}^2 (\omega_1 (3s^2 - 2\omega_2^2) + 6s^2 \omega_2 - 2\omega_1^3 - 4\omega_2 \omega_1^2) \left. \right\} \\
& + \frac{1}{2} \log \left(\frac{s + \omega_3}{\omega_T} \right) \left\{ \omega_3^2 (9s^2 (4\omega_2 + \omega_3) + 2\omega_1 (9s^2 + \omega_3^2)) + \omega_{12} \omega_3 (12s^2 (\omega_1 + 2\omega_2) \right. \\
& + (3\omega_1 + 5\omega_2) \omega_3^2) + (\omega_1 + 2\omega_2) \omega_{12}^2 (3s^2 + \omega_3^2) \left. \right\} \left. \right] + \text{NLF}. \tag{4.43}
\end{aligned}$$

Interestingly, the singularity structure of the bispectrum computed with non-local interactions once again follows the diagrammatic rules. The arguments in the logarithms are $(s + \omega_{12})$ and $(s + \omega_3)$, which are exactly what one expects from *loop-subgraph* contributions, with the total energy branch cut combining to give scale invariant, dimensionless logarithms. No additional logarithms are produced.

4.3.2 3 vertex contribution with inverse laplacian operators

There is yet another contribution to the 1-loop bispectrum, obtained by three insertions of cubic operator. This contribution has one *left*- and one *loop-subgraph*, shaded in blue and red in Fig. 4.2 respectively. According to the diagrammatic rules, the contribution expected from this subgraph is $\log(\mathcal{S}_L + s) = \log(p_3 + s)$ and $\log(\mathcal{S}_R + s) = \log(\omega_3 + s)$, where $\mathcal{S}_L = p_3$ and $\mathcal{S}_R = \omega_3$ are the external energy injection into the *loop* subgraph through the left and right loop sites. The *left*-subgraph should contribute a $\log(\mathcal{S}_{L_1} + s) = \log(\omega_{12} + s)$, where $\mathcal{S}_{L_1}$ is the external energy injection into the *left*-subgraph through all vertices except the right loop site. This diagram provides a good premise to investigate what happens to the branch cuts beyond simple graphs such as Fig. 4.1 when non-local interactions are involved.

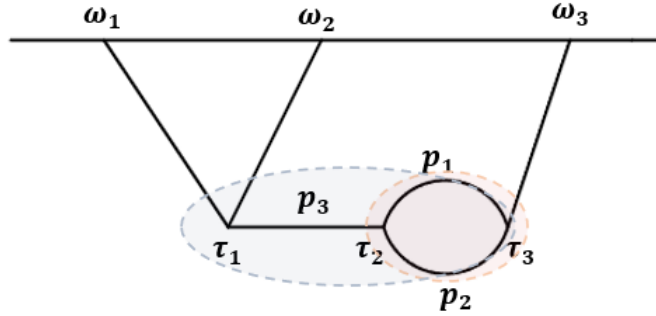


Figure 4.2: The bispectrum at 1-loop computed with three insertions of cubic vertex.

We consider all three interaction vertices of this diagram to be the cubic interaction $\dot{\varphi}\partial_i\varphi\partial^{-2}\partial_i\dot{\varphi}$ taken from Eqn. (1.85). With this, we compute the following contraction :

1. At τ_1 , the cubic vertex is $\dot{\varphi}(\vec{k}_2)\partial_i\varphi(\vec{p}_3)\partial^{-2}\partial_i\dot{\varphi}(\vec{k}_1)$, which generates a vertex factor of $\frac{\vec{p}_3 \cdot \vec{k}_1}{|k_1|^2}$.
2. At τ_2 , the cubic vertex is $\dot{\varphi}(\vec{p}_2)\partial_i\varphi(\vec{p}_3)\partial^{-2}\partial_i\dot{\varphi}(\vec{p}_1)$, which generates a vertex factor of $\frac{\vec{p}_3 \cdot \vec{p}_1}{|\vec{p}_1|^2} = \frac{s^2 - p_2^2 + p_1^2}{2p_1^2}$.

3. At τ_3 , the cubic vertex is $\dot{\varphi}(\vec{p}_1)\partial_i\varphi(\vec{k}_3)\partial^{-2}\partial_i\dot{\varphi}(\vec{p}_2)$, which generates a vertex factor of $\frac{\vec{k}_3\cdot\vec{p}_2}{|\vec{p}_2|^2} = \frac{s^2+p_2^2-p_1^2}{2p_2^2}$.

Upon evaluating the integrals, we once again find the unphysical $\log(\frac{\omega_T}{\mu})$ leftover from $D_1 + D_2$ are removed once contribution from counter terms $D_{\text{c.t.}}$ is added (if the counter terms cancel the $1/\delta$ divergence of the loop), and the final renormalized correlator is given by,

$$\begin{aligned}
D_{\text{renorm.}} \sim & \frac{s \vec{k}_1 \cdot \vec{k}_3}{\omega_T^4 (p_3 + \omega_3)^3 (p_3 + \omega_{12})^2} \left[\left(\log\left(\frac{H}{\mu}\right) + \frac{1}{2} \log\left(\frac{s + \omega_3}{\omega_T}\right) \right) \left\{ -p_3^2 (18s^2 \omega_3 \right. \right. \\
& + \omega_{12} (3s^2 + \omega_3^2) \omega_T^4 - 3p_3 s^2 \omega_3 (2\omega_3 + 3\omega_{12}) \omega_T^4 + 2p_3^6 (\omega_{12} (3s^2 + \omega_3^2) - 2\omega_3 (\omega_3^2 - 6s^2)) \\
& + 2p_3^5 (3\omega_3 + 2\omega_{12}) (\omega_{12} (3s^2 + \omega_3^2) - 2\omega_3 (\omega_3^2 - 6s^2)) - 6p_3^4 \omega_3 (2\omega_3 + 3\omega_{12}) \\
& (-5s^2 \omega_3 - 2s^2 \omega_{12} + \omega_3^3) - 2p_3^3 (3s^2 \omega_3^4 + \omega_{12} \omega_3^3 (7\omega_3^2 - 3s^2) + 6\omega_{12}^2 \omega_3^2 (2s^2 + \omega_3^2) \\
& + 4\omega_{12}^3 \omega_3 (3s^2 + \omega_3^2) + \omega_{12}^4 (3s^2 + \omega_3^2) + \omega_3^6) - 3s^2 \omega_3^2 \omega_{12} \omega_T^4 \left. \right\} + \log\left(\frac{s + \omega_{12}}{\omega_T}\right) \frac{1}{(p_3 - \omega_{12})^2} \\
& \left\{ p_3^3 (p_3 + \omega_3)^3 (-2\omega_3^3 (p_3^2 + 3s^2) + \omega_{12} \omega_3^2 (p_3^2 - 3(8s^2 + \omega_{12}^2))) + 12s^2 \omega_3 (p_3^2 - 3\omega_{12}^2) \right. \\
& + 3s^2 \omega_{12} (p_3^2 - 3\omega_{12}^2) \left. \right\} - \log\left(\frac{p_3 + s}{\omega_T}\right) \frac{1}{2(p_3 - \omega_{12})^2} \left\{ \omega_T^4 (\omega_{12} - 2p_3) (p_3 + \omega_{12})^2 \right. \\
& (9p_3 s^2 \omega_3 + p_3^2 (3s^2 + \omega_3^2) + 3s^2 \omega_3^2) \left. \right\} \Big] + \text{Nlf}, \tag{4.44}
\end{aligned}$$

Once again, the logarithms are correctly predicted by the diagrammatic rules, which combines with the total energy branch cuts to give a scale invariant correlator. The apparent folded poles at $p_3 = \omega_{12}$ cancels from the non-log finite terms (Nlf).

4.3.3 Diagrams with vertices above and below $\tau = 0$

The diagrammatic rules to extract singularity structure also apply to contributions with interaction vertices lying both above and below $\tau = 0$. Hence, let us now consider the bispectrum in Fig. 4.3 with cubic and quartic interactions coming from

the anti-time ordered and time-ordered part of the in-in correlator respectively.

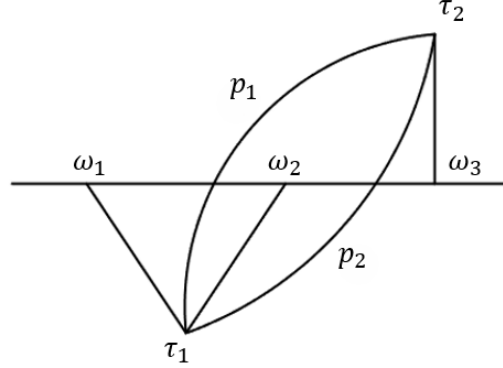


Figure 4.3: Diagram I : The unordered loop contribution to the bispectrum from one insertion of the cubic and a quartic operator. ω_i, p_j denote external and loop momenta respectively.

This diagram was computed in [3] with simple $\dot{\pi}^4$ and $\dot{\pi}^3$ interactions from the EFT of Inflation, and the answer was found to be UV finite. In particular, one gets the following answer in the *off-shell* limit of the correlator,

$$\begin{aligned}
 D_1 \sim & -\frac{1}{2(\omega_1 + \omega_2 - \omega_3)^7} \log\left(\frac{s + \omega_1 + \omega_2}{s + \omega_3}\right) \left(-2\omega_1^2(s^2 - 9\omega_3^2) - 4\omega_1(4s^2\omega_3 \right. \\
 & + \omega_2(s^2 - 9\omega_3^2) - 6\omega_3^3) - 2\omega_2^2(s^2 - 9\omega_3^2) - 8\omega_2(2s^2\omega_3 - 3\omega_3^3) + 3(-4s^2\omega_3^2 \\
 & \left. + 3s^4 + \omega_3^4) \right) + \text{NLf}, \tag{4.45}
 \end{aligned}$$

and the apparent folded poles cancel from NLf. Clearly, the log structure of the computation agrees with the diagrammatic rules. Importantly, no unphysical logarithms (log arguments involving ratios of comoving by physical scales) are found. This is expected from UV-finiteness of the answer : unphysical logarithms require renormalization to get cancelled, hence the unrenormalized UV-finite unordered correlator must feature no unphysical logarithm.

The loop integrands of the unordered pieces have a lower scaling with p_+ as com-

pared to their ordered counterparts, owing to the nested time integrals in the latter, and hence are *less* divergent. However the unordered pieces can be UV-divergent, as we will soon see. This is because, with non-derivative interactions or inverse laplacian operators, scale invariance implies lower powers of τ in the integrand, which leads to lower powers of p_+ in the denominator of the loop integrand, and hence less regulation in the limit $p_+ \rightarrow \infty$ when time integrals are performed. Moreover, with the type of non-local interactions we work with in this manuscript, one can have additional powers of p_+ from vertex factors, which will contribute to UV divergence of the loop integral.

As a concrete example, let us now compute Fig. 4.3 with non local interactions from Eqns. (1.85) and (1.87). Consider the contraction $ig_4 \dot{\varphi}(\vec{p}_1)\dot{\varphi}(\vec{p}_2)\partial^{-2}\partial_i \left(\dot{\varphi}(\vec{k}_1)\partial_i\varphi(\vec{k}_2) \right)$ and $ig_3 \dot{\varphi}(\vec{p}_1)\dot{\varphi}(\vec{p}_1)\varphi(\vec{k}_3)$. Modulo the vertex factor contribution dependent on external momenta, we find the following UV-divergent, unrenormalized correlator $D_{\text{unrenorm.}} = D_1 + D_2$,

$$\begin{aligned}
D_{\text{unrenorm.}} \sim & -\frac{3}{128\omega_2^3\omega_3^3} \left(\frac{1}{\delta} - \frac{1}{2} \log(\omega_T) \right) + \frac{1}{1920\omega_1\omega_2^3(\omega_{12} - \omega_3)^5\omega_3^3} \left\{ \log \left(\frac{s + \omega_3}{\omega_T} \right) \right. \\
& \left(\omega_1^2 (10s^2\omega_3^2 + 3s^4 - 45\omega_3^4) + 5\omega_1 (2s^2\omega_3^3 - 3s^4\omega_3 + \omega_2 (10s^2\omega_3^2 + 3s^4 - 45\omega_3^4) + 9\omega_3^5) \right. \\
& \left. + 4 (3s^4\omega_3^2 - 5s^2\omega_3^4 - 5\omega_2 (3s^4\omega_3 - 5s^2\omega_3^3) + \omega_2^2 (10s^2\omega_3^2 + 3s^4 - 45\omega_3^4)) \right) \\
& - \log \left(\frac{s + \omega_{12}}{\omega_T} \right) \left(\omega_1^2 (10s^2\omega_3^2 + 225\omega_2^4 + 180\omega_3^4 - 900\omega_2\omega_3^3 + 1350\omega_2^2\omega_3^2 - 900\omega_2^3\omega_3 \right. \\
& \left. + 3s^4) + 5\omega_1 (\omega_2 (10s^2\omega_3^2 + 3s^4) + 2s^2\omega_3^3 - 3s^4\omega_3 + 9\omega_2^5 - 45\omega_3\omega_2^4 + 90\omega_3^2\omega_2^3 - 90\omega_3^3\omega_2^2) \right. \\
& \left. + 4 (3s^4\omega_3^2 - 5s^2\omega_3^4 - 5\omega_2 (3s^4\omega_3 - 5s^2\omega_3^3) + \omega_2^2 (10s^2\omega_3^2 + 3s^4 - 45\omega_3^4)) + 45\omega_1^6 \right. \\
& \left. + 225 (\omega_2 - \omega_3) \omega_1^5 + 450 (\omega_2 - \omega_3)^2 \omega_1^4 + 450 (\omega_2 - \omega_3)^3 \omega_1^3 \right) \left. \right\} + \text{NLf}. \quad (4.46)
\end{aligned}$$

The folded pole in the expressions above cancel from the non-log finite part of the answer (NLf). We make two observations. First, in this computation the diagram-

matic rules are verified for UV-divergent unordered correlators with non-local interactions. Second, the unrenormalized correlator features an unphysical logarithm. This is consistent with the fact that this correlator requires renormalization, and the tree level contributions from the counter terms computed with d -dimensional modes must be proportional to $\propto (\frac{1}{\delta} - \frac{1}{2} \log \omega_T)$, and since renormalization absorbs the UV-divergence, it must cancel the unphysical log.

4.4 Polology

In this section, we study the connection between the flat space scattering amplitudes and the cosmological correlators corresponding to the same process ³. It was previously noticed that the leading singularity of cosmological correlators in the limit $k_T \rightarrow 0$ is related to the flat space scattering amplitude corresponding to the same process [76, 77, 78, 18, 79, 20, 33]. The intuitive way of understanding why such a relation should exist is to note that the exponential $i\epsilon$ suppression in the Bunch Davies modes at early time goes away when $k_T \rightarrow 0$, hence this limit probes the early time era when the modes are well approximated by the flat space limit. This was proved at tree level (and for loop integrands) for massless scalars in [33], where the authors developed Feynman rules analogous to those of cosmological correlators, equipped with different propagators for bulk-bulk (internal lines) and bulk-boundary (external legs), as well as vertex factors coming from general (non-Lorentz invariant) interactions involving fields with time and/or spatial derivatives.

³We will often refer to this correspondence as the *flat space limit* of the dS correlators. However, we remark for readers who may not be familiar with this topic, that the flat space limit here simply refers to the leading singular terms of the correlator when total energy is taken to 0, and has nothing to do with the limit $H \rightarrow 0$.

4.4.1 Polology at 1-loop

Below we establish the connection between total energy pole of cosmological correlators at 1-loop and the corresponding flat space scattering amplitude. We then demonstrate this with explicit examples, computed with derivative as well as non-local interactions.

Consider interactions of the type $\partial_i^{r_1} \partial_\tau^{s_1} \phi_1 \dots \partial_i^{r_n} \partial_\tau^{s_n} \phi_n$. Here, every field ϕ_k is being acted upon by r_k spatial derivatives/inverse laplacians ($r_k = 1$ for ∂_i , $r_k = -2$ for ∂^{-2}) and s_k time derivatives (∂_τ). An important thing to note here is that since the same interactions are being used to compute scattering amplitudes as well as the dS correlators, the spatial derivatives will generate the same factors of dot products, since they act on $e^{i\vec{k}\cdot\vec{x}}$ in both cases. As we argue later, this allows us to extend our arguments to interactions involving inverse laplacian operators. Thus the spatial derivatives will cause no trouble; we will plug back their contribution when explicitly evaluating the correlators and scattering amplitudes, however we can ignore these factors for now.

The time derivatives, on the other hand, need to be considered carefully. They act on the flat space modes $f_k(t) = \frac{1}{\sqrt{2k}} e^{ikt}$ and the (3-dimensional) dS modes $g_k(\tau) = \frac{H}{\sqrt{2k^3}} (1 - ik\tau) e^{ik\tau}$ as,

$$\partial_t^s f_k(t) = F_s(k) f_k(t), \quad \partial_\tau^s g_k(\tau) = F_s(k) \frac{H}{\sqrt{2k^3}} (1 - s - ik\tau) e^{ik\tau}, \quad (4.47)$$

where $F_s(k)$ is a factor of k raised to a power depending on s .

With this, the Feynman rules (re)formulated in [33] are summarized below,

1. For every external particle connected to an interaction vertex at t (τ), in the

scattering amplitudes we have,

$$G_{\text{B,flat}}(k, t) = F_s(k)e^{ikt}, \quad (4.48)$$

and external legs in the dS correlator are given by,

$$G_{B,+}(k, \tau) = \frac{H^2}{2k^3}(-H\tau)^s F_s(k)(1 - s - ik\tau)e^{ik\tau}, \quad (4.49)$$

where the $+$ in the subscript of the propagator denotes the interaction is inserted below the $\tau = 0$ line, coming from the time ordered part of the in-in correlator. Complex conjugation of this gives $G_{B,-}$: the propagator corresponding to the interaction insertion above $\tau = 0$. The factor of $(-H\tau)^s$ comes from demanding scale invariance in the interactions.

2. For every exchange between two interaction vertices at t_1 (τ_1) and t_2 (τ_2), with s_1 and s_2 number of time derivatives acting on the exchange fields at the two vertices respectively, for the flat space scattering amplitudes we have,

$$G_{\text{flat}}(p, t_1, t_2) = F_{s_1}(p)F_{s_2}^*(p)\frac{e^{ip(t_1-t_2)}}{2p}\theta(t_2 - t_1) + \left(t_2 \leftrightarrow t_1, s_1 \leftrightarrow s_2\right), \quad (4.50)$$

and for the dS correlators we have,

$$G_{++}(p, \tau_1, \tau_2) = F_{s_1}(p)F_{s_2}^*(p)\frac{H^2}{2p^3}(-H\tau_1)^{s_1}(-H\tau_2)^{s_2}(1 - s_1 - ip\tau_1) \\ (1 - s_2 + ip\tau_2)e^{ip(\tau_1-\tau_2)}\theta(\tau_2 - \tau_1) + \left(\tau_2 \leftrightarrow \tau_1, s_1 \leftrightarrow s_2\right), \quad (4.51)$$

where the $+$ in the subscript again has the same implication as stated previously. One can similarly obtain G_{+-} , G_{-+} and G_{--} , although we will never use the first two. This is because diagrams which have vertices in both upper

and lower region of $\tau = 0$ will never produce a total energy pole, and hence we will only look at totally (un-)ordered diagrams.

3. Every interaction vertex and undetermined (3-) momenta is integrated over. Ofcourse the time integrals for scattering amplitude computations are evaluated all the way from $-\infty$ to ∞ , whereas for correlators they are evaluated till τ_0 : the time at which these correlators are being computed.

We will only consider interactions which are finite at late-time and plug in $\tau_0 = 0$. It can be shown [33] that the tree level scattering amplitude obtained from the rules (with $s_1 = s_2 = 0$) is just a restructured version of the familiar form,

$$\mathcal{A}_n \sim \prod_V^{\text{vertices}} \left(g_V \mathcal{F}_V(\vec{k}) \right) \prod_j^{\text{int legs}} \left(\frac{1}{K_j^\mu K_{j\mu}} \right), \quad (4.52)$$

where $g_V \mathcal{F}_V(\vec{k})$ is the vertex factor for V , K_j^μ is the exchange (4-) momentum of the j^{th} internal leg.

Considering terms with the highest power of conformal time τ^{max} , we see that the dS propagators satisfy,

$$\begin{aligned} G_{B,+}(k, \tau) &\xrightarrow{\tau^{\text{max}}} iG_{\text{B,flat}}(k, \tau)(-1)^{s+1} \frac{H^{2+s}}{2k^2} \tau^{s+1} e^{ik\tau}, \\ G_{++}(p, \tau_1, \tau_2) &\xrightarrow{\tau^{\text{max}}} G_{\text{flat}}(p, \tau_1, \tau_2)(-1)^{s_1+s_2} H^{2+s_1+s_2} \tau_1^{s_1+1} \tau_2^{s_2+1}. \end{aligned} \quad (4.53)$$

For diagrams involving loops, we must use d -dimensional modes in Eqn. (4.9) and split the computation of the full correlator D into D_1 and D_2 , as outlined in Sec. 4.1. At 1-loop, D_2 is immediately obtained from divergent part of D_1 multiplied to $\log(H/\omega_T)$ (see Appendix A.6). Hence, we will first focus on the flat space limit of D_1 , and comments on relation to D_2 will follow later.

Importantly, the exchange propagators G_{++} and G_{flat} are related by factors inde-

pendent of the exchange momenta p . Crucially, since the propagators in D_1 are given by Eqns. (4.49), (4.51), the terms in the loop integrand of dS correlators involving $\tau^{\max}$, and the corresponding flat space amplitude are related by factors which are independent of loop momenta. Hence these factors do not affect the loop integral in D_1 , and a connection at the level of loop integrand leads us directly to a connection at the level of the full loop.

There is a crucial ingredient in realizing the relationship between flat space scattering amplitude and the leading singular terms of the dS correlator. Consider the form of a nested time integral arising in the correlator,

$$\int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \dots \int_{-\infty}^{\tau_{n-2}} d\tau_{n-1} \int_{-\infty}^{\tau_{n-1}} d\tau_n e^{-a_1 \tau_1} \dots e^{i a_n \tau_n} Q(\omega_i, p_j, \tau), \quad (4.54)$$

where $a_1, \dots, a_n$ are functions of external (ω_i) and exchange (p_j) momenta, adding up to the total energy ω_T . Q is some function of external energies, exchange momenta, and it is analytic in τ 's, i.e. all time arguments appear in the numerator, since we work with IR finite interactions. All intermediate integrals would mix arguments of the exponential, leading to $e^{i\omega_T \tau_1}$ in the final time integral, which would produce total energy pole. If we are interested in the most singular term in the limit $\omega_T \rightarrow 0$, then we must only bother with the highest power of τ_1 - the vertex to be integrated last. This highest power is produced from integrating (over $d\tau_2$) the term which has the largest power of τ_2 , which in turn comes from integrating (over $d\tau_3$) the term having the largest power of τ_3 and so on. This implies, to extract the leading singularity, we only have to keep track of the term with the largest power of τ every step of the way.

Note that we are interested in establishing the relation between the *off-shell* dS correlator and the *off-shell* scattering amplitude. An off-shell correlator, as well as

an off-shell amplitude is obtained when the energy of the external legs ω_k is treated as being independent of the momenta $\vec{k}$. An off-shell correlator has internal legs on-shell ($\omega_{p_i} = p_i = |\vec{p}_i|$) and the loop integrand develops poles when one or more vertices become energy conserving, while an off-shell scattering amplitude has energy conserved and loop integrand develops poles when one or more internal particles goes on-shell.

Let us now consider a 1-loop correlator having 2 loop sites τ_L and τ_R , with all ordered vertices, i.e. all interactions below $\tau = 0$ in the Feynman diagram. We will compute the dS correlator and the flat space scattering amplitude corresponding to the process in Fig. 4.4, using dimensional regularization, hence the time integrals will be computed first. We denote external energy by ω_i and exchange as p_j , with p_{l_1} and p_{l_2} as the loop momenta. In Fig. 4.4 the nodes denote interaction vertices, labeled by τ (t for scattering amplitude), exchanges are marked by lines. The interaction vertices can have any number of external lines, which are not drawn in this representation. Since this diagram allows only one closed loop at $\tau_L - \tau_R$, it is possible to find atleast 1 *endpoint*, i.e. a vertex which is connected to only one other vertex. We denote this vertex by τ_{end} , and we label its neighbor as τ_{adj} . To solve the time integrals, we integrate over τ_{end} , and move on to the next endpoint vertex in the graph.

First let us consider the case where neither loop site is an endpoint. Considering the flat space scattering amplitude, integrating the endpoint vertex V (i.e. t_{end})

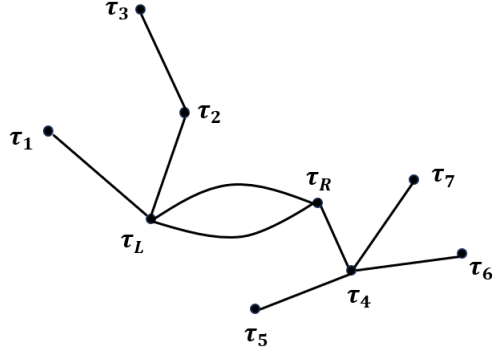


Figure 4.4: A correlator (or scattering amplitude) at 1-loop and 2 loop sites τ_L and τ_R . The nodes denote interaction vertices, labeled by τ_i , the exchange/internal legs are marked by lines, and the external lines connected to the interaction vertices are not drawn.

gives,

$$\begin{aligned}
\mathcal{A}_n : \mathcal{F}_V(\vec{k}, \vec{p}) \int dt_{\text{end}} & \left(\prod_{i \in v} G_{B, \text{flat}}(\omega_i, t_{\text{end}}) \right) G_{\text{flat}}(p, t_{\text{end}}, t_{\text{adj}}) \\
&= \mathcal{F}_V(\vec{k}, \vec{p}) \int dt_{\text{end}} \left(\prod_{i \in v} F_{s_i}(\omega_i) e^{i\omega_i t_{\text{end}}} \right) \left[\frac{F_{s_{t_{\text{end}}}}(p) F_{s_{t_{\text{adj}}}}^*(p)}{2p} e^{-ip(t_{\text{adj}} - t_{\text{end}})} \theta(t_{\text{adj}} - t_{\text{end}}) \right. \\
&\quad \left. + \left(t_{\text{adj}} \leftrightarrow t_{\text{end}} \right) \right], \tag{4.55}
\end{aligned}$$

where v denotes external legs attached to V , and p is exchange of t_{end} with t_{adj} . s_{end} and $s_{t_{\text{adj}}}$ are the number of time derivatives acting on the exchange field at t_{end} and t_{adj} respectively. Although we keep track of the vertex factors $\mathcal{F}_V$ coming from the action of ∂_i and ∂^{-2} on the modes, these operators produce the same vertex factors in flat space scattering amplitudes and dS correlators, and hence we need not worry about them for the purpose of this exercise.

Evaluation of the integral gives [33],

$$\mathcal{A}_n : \mathcal{F}_V(\vec{k}, \vec{p}) \left(\prod_{i \in v} F_{s_i}(\omega_i) \right) \mathcal{G}(p) e^{i(\sum_{i \in v} \omega_i) t_{\text{adj}}}, \tag{4.56}$$

where,

$$\mathcal{G}(p) = \frac{F_{s_{t_{\text{end}}}}(p)F_{s_{t_{\text{adj}}}}^*(p)}{2p(p + \sum_{i \in v} \omega_i)} + \frac{F_{s_{t_{\text{adj}}}}(p)F_{s_{t_{\text{end}}}}^*(p)}{2p(p - \sum_{i \in v} \omega_i)}. \quad (4.57)$$

Similarly, integrating end point vertex V (i.e. τ_{end}) in the D_1 contribution to the dS correlator gives,

$$D_{1_n} : \mathcal{F}_V(\vec{k}, \vec{p})(-H)^{-4+R_V} \int d\tau_{\text{end}} \tau_{\text{end}}^{-4+R_V} \left(\prod_{i \in v} G_{B,+}(\omega_i, \tau_{\text{end}}) \right) \left(G_{++}(p, \tau_{\text{end}}, \tau_{\text{adj}}) \right), \quad (4.58)$$

where $R_V = \left(\sum_{i \in v} r_i \right) + r_{p, \tau_{\text{end}}}$ is the total number of spatial derivatives/inverse laplacians acting on the fields at V . Note $r_i = 1, -2$ for a spatial derivative and an inverse laplacian operator respectively, acting on the i^{th} field. The factors of τ involving powers of R_{V_i} arise due to demanding scale invariant interactions in the presence of (inverse) spatial derivatives. As previously discussed, we will focus on keeping only the terms involving maximum power of τ . Using Eqn. (4.53), we find,

$$\begin{aligned} D_{1_n} : \mathcal{F}_V(\vec{k}, \vec{p})(H)^{-4+R_V} & \left(\prod_{i \in v} \frac{-iH^2}{2\omega_i} (-H)^{s_i} F_{s_i}(\omega_i) \right) (-H)^{s_{p, \tau_{\text{end}}} + s_{p, \tau_{\text{adj}}} + 2} \tau_{\text{adj}}^{s_{p, \tau_{\text{adj}}} + 1} \\ & \int d\tau_{\text{end}} \tau_{\text{end}}^{-4+R_V + v + (\sum_{i \in v} s_i) + s_{p, \tau_{\text{end}}} + 1} e^{i\tau_{\text{end}}(\sum_{i \in v} \omega_i)} \times \\ & \left[\frac{F_{s_{\tau_{\text{end}}}}(p)F_{s_{\tau_{\text{adj}}}}^*(p)}{2p} e^{ip(\tau_{\text{end}} - \tau_{\text{adj}})} \theta(\tau_{\text{adj}} - \tau_{\text{end}}) + \left(\tau_{\text{end}} \leftrightarrow \tau_{\text{adj}} \right) \right] + \dots, \quad (4.59) \end{aligned}$$

where ‘...’ denotes terms with smaller powers of τ_{end} and hence their contributions are subleading in the limit $\omega_T \rightarrow 0$, as previously discussed. We have set $(-1)^{(-4+R_V)} = 1$ since the total number of spatial derivatives in any interaction vertex must be even due to rotational invariance. This integral has the same structure as the flat space case in Eqn. (4.55) modulo some factors, such as extra powers of

τ_{end} , and the upper limit of the time integral in the $\theta(\tau_{\text{end}} - \tau_{\text{adj}})$ piece. The difference in the upper limit is irrelevant here since this limit produces terms subleading in powers of τ_{adj} . Hence, keeping only the highest power of τ_{adj} , evaluation of τ_{adj} gives,

$$D_{1_n} : (H)^{-4+R_V+2v+S_V+2} (-H)^{s_{p,\tau_{\text{adj}}}} (i)^v (-1)^{S_V+v} \left(\prod_{i \in v} \frac{1}{2\omega_i^2} \right) \tau_{\text{adj}}^{-4+2+s_{p,\tau_{\text{adj}}}+R_V+S_V+v} \\ \times \text{flat space expression Eqn. (4.56)}, \quad (4.60)$$

where $S_V = (\sum_{i \in v} s_i) + s_{p,\tau_{\text{end}}}$ is the total number of temporal derivatives acting on all the fields at V (i.e. τ_{end}). Importantly, note that the flat space answer is recovered due to Eqn. (4.53) only when keeping highest power of τ . If one keeps subleading terms as well, then the factors in Eqn. (4.60) become dependent on the exchange p , and evaluation of loop integrals no longer recovers the flat space scattering amplitude.

Going back to the flat space scattering amplitude computation, we see from Eqn. (4.56) that every endpoint vertex integrated out contributes a factor of,

$$\mathcal{F}_V(\vec{k}, \vec{p}) \left(\prod_{i \in v} F_{s_i}(\omega_i) \right) \mathcal{G}(p) e^{it_{\text{adj}}(\sum_{i \in v} \omega_i)}, \quad (4.61)$$

where the exponential modifies the integral structure of the neighboring vertex t_{adj} . Now, t_{adj} may not be an endpoint. For example, in Fig. 4.4, there are multiple choices for endpoint. If we start with $t_{\text{end}} = \tau_3$, then $t_{\text{adj}} = \tau_2$ becomes the next endpoint when t_{end} is integrated over. However, with $t_{\text{end}} = \tau_2$, $t_{\text{adj}} = \tau_L$ does not become an endpoint when τ_2 is integrated. Importantly, considering different endpoints will only affect the order of integration, however the end result remains the same. This is because the integral structure of t_{adj} is simply additive : it does

not matter in which order you consider τ_5, τ_6 and τ_7 as endpoints, finally one ends up with the same factor for integral over $t_{\text{adj}} = \tau_4$,

$$\left(\prod_{i \in v_5, v_6, v_7} \mathcal{F}_i(\vec{k}, \vec{p}) F_{s_i}(\omega_i) \right) \left(\prod_{\beta}^{\text{ex}} \mathcal{G}(p_{\beta}) \right) e^{it_{\text{adj}}(\sum_{i \in v_5, v_6, v_7} \omega_i)}. \quad (4.62)$$

Let us next consider the case when we hit the loop site as the endpoint. Suppose we have $t_{\text{end}} = \tau_L$. The neighboring vertex is $t_{\text{adj}} = \tau_R$ with which it exchanges p_{l_1} and p_{l_2} . So for the integral over τ_L we have,

$$\begin{aligned} \mathcal{A}_n : \mathcal{F}_{\tau_L}(\vec{k}, \vec{p}) \int d\tau_L & \left(\prod_{i \in v_{\tau_L}} G_{B, \text{flat}}(\omega_i, \tau_L) \right) G_{\text{flat}}(p_{l_1}, \tau_L, \tau_R) G_{\text{flat}}(p_{l_2}, \tau_L, \tau_R) \\ & = \mathcal{F}_{\tau_L}(\vec{k}, \vec{p}) \left(\prod_{i \in v_{\tau_L}} F_{s_i}(\omega_i) \right) \left\{ \int_{-\infty}^{\tau_R} d\tau_L e^{i(\sum_{i \in v_{\tau_L}} \omega_i) \tau_L} \frac{e^{i(p_{l_1} + p_{l_2})(\tau_L - \tau_R)}}{4p_{l_1} p_{l_2}} \right. \\ & \quad \left. F_{s_{\tau_L}}(p_{l_1}) F_{s_{\tau_R}}^*(p_{l_1}) F_{s_{\tau_L}}(p_{l_2}) F_{s_{\tau_R}}^*(p_{l_2}) + \int_{\tau_R}^{\infty} d\tau_L e^{i(\sum_{i \in v_{\tau_L}} \omega_i) \tau_L} \frac{e^{i(p_{l_1} + p_{l_2})(\tau_R - \tau_L)}}{4p_{l_1} p_{l_2}} \right. \\ & \quad \left. F_{s_{\tau_L}}^*(p_{l_1}) F_{s_{\tau_R}}(p_{l_1}) F_{s_{\tau_L}}^*(p_{l_2}) F_{s_{\tau_R}}(p_{l_2}) \right\} \\ & = \mathcal{F}_{\tau_L}(\vec{k}, \vec{p}) \left(\prod_{i \in v_{\tau_L}} F_{s_i}(\omega_i) \right) \mathcal{H}(p_{l_1}, p_{l_2}) e^{i(\sum_{i \in v_{\tau_L}} \omega_i) \tau_R}, \end{aligned} \quad (4.63)$$

where,

$$\mathcal{H}(p_1, p_2) = \frac{F_{s_{\tau_L}}(p_1) F_{s_{\tau_R}}^*(p_1) F_{s_{\tau_L}}(p_2) F_{s_{\tau_R}}^*(p_2)}{4p_1 p_2 (p_1 + p_2 + \sum_{i \in v_{\tau_L}} \omega_i)} + \frac{F_{s_{\tau_L}}^*(p_1) F_{s_{\tau_R}}(p_1) F_{s_{\tau_L}}^*(p_2) F_{s_{\tau_R}}(p_2)}{4p_1 p_2 (p_1 + p_2 - \sum_{i \in v_{\tau_L}} \omega_i)}, \quad (4.64)$$

which is exactly what one finds as the integrand when performing integral over the loop (3-)momenta in usual flat space scattering amplitude computations with $s_{\tau_L} = s_{\tau_R} = 0$ (See Appendix. A.7).

Using Eqn. (4.63) we see that the contribution from a loop site endpoint is the same as that of Eqn. (4.61), except instead of $\mathcal{G}(p)$, loop site end points will con-

tribute $\mathcal{H}(p_1, p_2)$. The remaining time integrals are solved with the same algorithm until one is left with the final time integral, which is just a contact diagram giving energy conservation. Hence, the (energy-momentum conserving delta function stripped) flat space correlator is given by the following integral over loop momentum p_l ,

$$\mathcal{A}_n = \int d^3 \vec{p}_l \left(\prod_{\alpha \in \text{vertices}} \mathcal{F}_\alpha(\vec{k}, \vec{p}) \right) \left(\prod_{i \in \text{ext legs}} F_{s_i}(\omega_i) \right) \left(\prod_{\beta \in \text{exchange}} \mathcal{G}(p_\beta) \right) \mathcal{H}(p_{l_1}, p_{l_2}), \quad (4.65)$$

where β runs over non-loop internal lines. Finally, to see how Eqn. (4.60) modifies when considering a loop site in a correlator as an endpoint, we have, considering τ_L as the endpoint and only retaining terms with highest powers of τ_L (and τ_R) as before,

$$\begin{aligned} D_{1_n} : \mathcal{F}_{V_{\tau_L}}(\vec{k}, \vec{p})(H)^{R_V} & \left(\prod_{i \in v} \frac{-iH^2}{2\omega_i^2} (-H)^{s_i} F_{s_i}(\omega_i) \right) (-H)^{s_{p_{l_1}, \tau_L} + s_{p_{l_2}, \tau_L} + s_{p_{l_1}, \tau_R} + s_{p_{l_2}, \tau_R}} \\ & \times (\tau_R)^{s_{p_{l_1}, \tau_R} + s_{p_{l_2}, \tau_R} + 2} \int d\tau_L (\tau_L)^{-4 + R_V + (\sum_{i \in v} s_i) + v + s_{p_{l_1}, \tau_L} + s_{p_{l_2}, \tau_L} + 2} e^{i(\sum_{i \in v} \omega_i) \tau_L} \times \\ & \left[\frac{e^{i(p_{l_1} + p_{l_2})(\tau_L - \tau_R)}}{4p_{l_1} p_{l_2}} F_{s_{\tau_L}}(p_{l_1}) F_{s_{\tau_R}}^*(p_{l_1}) F_{s_{\tau_L}}(p_{l_2}) F_{s_{\tau_R}}^*(p_{l_2}) \theta(\tau_R - \tau_L) + \left(\tau_R \leftrightarrow \tau_L \right) \right]. \end{aligned} \quad (4.66)$$

Substituting $S_V = (\sum_{i \in v} s_i) + s_{p_{l_1}, \tau_L} + s_{p_{l_2}, \tau_L}$ and $s_{p, \tau_{\text{adj}}} = s_{p_{l_1}, \tau_R} + s_{p_{l_2}, \tau_R}$, we see by comparing with Eqn. (4.63) that the above equation is,

$$\begin{aligned} D_{1_n} := & (H)^{-4 + R_V + 2v + S_V + 2 + 2} (-H)^{s_{p, \tau_{\text{adj}}}} (i)^v (-1)^{S_V + v} \left(\prod_{i \in v} \frac{1}{2\omega_i^2} \right)^{\tau_R} \\ & \times \text{flat space expression Eqn. (4.63)}. \end{aligned} \quad (4.67)$$

and once again the flat space integral is recovered due to Eqn. (4.53). This is a

crucial step to recover the flat space amplitude from the correlator, since it leads to these factors being pulled out of the loop integrals.

Similarly, the remaining time integrals in the correlator are integrated until the final (contact) time integral remains. Inspecting Eqns. (4.56), (4.60), (4.63) and (4.67), we see that the integrand of the final time integral will involve,

1. **Factors of H :** Every vertex contributes H^{-4} , every exchange contributes H^2 , thus we have $H^{-4V+N+2n+2I}$, where the total number of vertices is V , $N = \sum_{V_i} (R_{V_i} + S_{V_i})$ is the total number of (inverse) spatial and time derivatives acting on the fields at all interaction vertices, and I is the number of internal lines.
2. **Factors of τ_{fin} :** where τ_{fin} is the final time. Once again, every vertex contributes τ^{-4} and every exchange contributes τ^2 , so we have $\tau_{\text{fin}}^{-4V+2I+N+n}$.

Integrating over the final power of τ_{fin} with $e^{-i\omega_T \tau_{\text{fin}}}$ and keeping the leading singular term in the limit $\omega_T \rightarrow 0$, we find,

$$\lim_{\omega_T \rightarrow 0} D_{1_n} = (H^{\alpha+n}) (-1)^{n+\sum S_{V_i}} i^n \left(\prod_{i \in \text{ext.}} \frac{1}{2\omega_i^2} \right) \frac{(\alpha!) (-1)^\alpha}{i^{\alpha+1}} \frac{\mathcal{A}_n}{\omega_T^{\alpha+1}}, \quad (4.68)$$

where $\alpha = -4V + 2I + N + n$. This is the flat space limit of the D_1 part of the correlator, and it is straightforward to obtain the flat space limit of D_2 from above. Since $\log \omega_T$ terms are produced from time integrals involving $\log(\tau)$, D_1 or $\mathcal{A}_n$ can never have a total energy branch cut [4]. Regardless, due to Eqns. (4.68) and (4.13), the leading singular terms in the limit of $\omega_T \rightarrow 0$ of the (unrenormalized) correlator

are obtained from the flat space scattering amplitude as,

$$\lim_{\omega_T \rightarrow 0} D_n = (H^{\alpha+n})(-1)^{n+\sum S_V} i^n \frac{(\alpha!)(-1)^\alpha}{(i\omega_T)^{\alpha+1}} \left(\prod_{i \in \text{ext.}} \frac{1}{2\omega_i^2} \right) \left(\mathcal{A}_n + \left(\sum_j n_j \right) \log \left(\frac{H}{\omega_T} \right) \mathcal{A}_n^{(\frac{1}{\delta})} \right), \quad (4.69)$$

where $\mathcal{A}_n^{(\frac{1}{\delta})}$ denotes the coefficient of the divergent term in the $\mathcal{O}(1/\delta)$ piece of the flat space scattering amplitude. $n_j = \frac{1}{2}, -1$ for internal modes and measure respectively. Eqn. (4.69) is the flat space limit of the dS correlator at 1-loop computed in dimensional regularization. Note that as far as finite non-log terms (Nlf) are concerned, this limit only retrieves the leading singular terms in D_1 . As pointed out in Appendix. A.6, the contribution from D_2 also produces Nlf, which does not have any flat space counterpart, since this comes from the $\mathcal{O}(\delta)$ correction to modes and measure, which has no analogue in flat space. Also note that in the discussion above, we have suppressed the ‘+’ notation on this correlator : which refers to the fact that D_n^+ (D_n in Eqn. (4.69)) is a completely time-ordered correlator and has all vertices below $\tau = 0$. There is one more diagram (D_n^-) at n -point having the same interactions and with the same order of pole at ω_T , which is completely anti-time ordered, with all vertices lying above $\tau = 0$. D_n^- does not require a separate computation, since it is obtained from D_n^+ by a complex conjugation. Thus the flat space limit of the full correlator ($D_n^+ + D_n^-$) is obtained by taking the Real part of the right hand side in Eqn. (4.69).

It is also worthwhile noting that naively Eqn. (4.53) suggests that an analogous flat space limit should exist for higher loop/loop sites, since the exchange propagators for the dS correlators and scattering amplitudes are related by factors independent of loop momenta. However one should be careful since the discussion above relies heavily on solving the loop integrals by expanding the d -dimensional

modes about $\delta = 0$, and commuting this summation operator with the integral. We have demonstrated that this method computes logarithms reliably by performing a cross check with cutoff regularization. It is not immediately obvious that this method works at higher loop orders, when it would become necessary to consider $\mathcal{O}(\delta^2)$ and higher corrections from modes. Hence it is not clear whether the arguments presented above generalise straightforwardly at higher loops/loop sites. We keep this investigation for a future work.

Also, the flat space limit above is obtained from dimensional regularization. Of course physical answers should not be regularization scheme dependent, however as previously mentioned, the total energy branch cut is unique to dS : there is no analogue of this branch cut in scattering amplitudes. This branch cut is a consequence of the time dependence in the background, however it has different origins in the two schemes: in cutoff regularization, it arises due to time dependence of the cutoff while in dimensional regularisation it arises from $\mathcal{O}(\delta)$ correction of modes and measure. Hence the flat space limit of this branch cut is recovered in slightly different ways for the two schemes. Hence we discuss the flat space limit for correlators being computed in cutoff regularization below. In particular, the power law divergences in Λ arising in cutoff regularized correlators are expected to be absorbed by counter terms, while the $\log \Lambda$ is expected to turn into $\log \mu$ after renormalization. The latter tells us that the coefficient of $\log(H/\Lambda)$ term in cutoff should match the coefficient of the $1/\delta$ divergent term in dim reg, which agrees with our calculations of Sec. 4.2. Also, since the contribution of counter terms renormalizing the 1-loop diagrams are tree level correlators computed with 3-d modes, thus they never produce a $\log \omega_T$, i.e. all total energy branch cuts are produced in the unrenormalized correlator. Hence, instead of the term $\sum_j n_j$ in Eqn. (4.69), the flat space limit in cutoff will take care of all branch cuts, with $\log(H/\omega_T)$ now multiplying the coefficient of term

logarithmically divergent in Λ : $\mathcal{A}_n^{(\log \Lambda)}$. Thus for (unrenormalized) dS correlators and flat space scattering amplitudes computed in cutoff regularization, we have,

$$\lim_{\omega_T \rightarrow 0} D_n = (H^{\alpha+n})(-1)^{n+\sum S_V} i^n \frac{(\alpha!)(-1)^\alpha}{(i\omega_T)^{\alpha+1}} \left(\prod_{i \in \text{ext.}} \frac{1}{2\omega_i^2} \right) \left(\mathcal{A}_n - \log\left(\frac{H}{\omega_T}\right) \mathcal{A}_n^{(\log \Lambda)} \right). \quad (4.70)$$

Note that the flat space limit in cutoff regularization does not recover the finite non-log parts since a finite piece was dropped from the dS calculation in cutoff (See discussion below Eqn. (4.24)).

Before we demonstrate the flat space limit with explicit examples computed with interactions from Eqns. (1.85) and (1.87), we discuss the arguments sketched above with a simple bispectrum (Fig. 4.1) computation using derivative interactions $H_{\text{int}}^4 = \dot{\varphi}^4$ and $H_{\text{int}}^3 = \dot{\varphi}^3$. In dimensional regularization, the loop integrand for the correlator is given by the time integrals,

$$D_3 : \int_{-\infty}^0 \int_{-\infty}^{\tau_1} \tau_1^4 \tau_2^2 e^{i\tau_1(\omega_{12}-p_+) + i\tau_2(p_+ + \omega_3)} d\tau_2 d\tau_1, \quad (4.71)$$

while for the scattering amplitude, the loop integrand is given by,

$$\mathcal{A}_3 : \int_{-\infty}^{\infty} \int_{-\infty}^{\tau_1} e^{i\tau_1(\omega_{12}-p_+) + i\tau_2(p_+ + \omega_3)} d\tau_2 d\tau_1, \quad (4.72)$$

where overall momentum factors/ integrals, as well as a $\Theta(\tau_2 - \tau_1)$ piece, are omitted. We also drop the $\mathcal{O}(\delta)$ contributions involving $\log(-H\tau_i)$ term in the correlator, since it has no flat-space analogue, as discussed in Sec. 4.4.1.

Thus the dS correlator and the corresponding flat-space amplitude differ only by time-dependent prefactors and by the limit of the final time integral. Performing

the τ_2 integral gives,

$$\begin{aligned} D_3 &: \int_{-\infty}^0 \left(\tau_1^6 \frac{e^{i\tau_1 \omega_T}}{i(\omega_3 + p_+)} + (\text{lower powers of } \tau_1) \right) d\tau_1, \\ \mathcal{A}_3 &: \int_{-\infty}^0 \frac{e^{i\tau_1 \omega_T}}{i(\omega_3 + p_+)} d\tau_1. \end{aligned} \quad (4.73)$$

Computing the final time integral gives,

$$D_3 : \frac{7!}{i\omega_T^7(\omega_3 + p_+)} + \dots, \quad \mathcal{A}_3 = \frac{1}{i(\omega_3 + p_+)} \delta(\omega_T). \quad (4.74)$$

where ‘...’ denotes terms in the correlator subleading in the limit of $\omega_T \rightarrow 0$.

The flat space correspondence follows from two observations. First, only the highest power of τ controls the most singular energy behavior; for that term $f_{\text{dS}} \sim \tau f_{\text{flat}}$. Second, the exponential integral preserves its structure:

$$\int_{-\infty}^{\tau} \tau'^m e^{a\tau'} d\tau' = \frac{\tau^m e^{a\tau}}{a} (= \tau^m \times \text{corresponding flat result}) + \text{subleading terms in } \tau, \quad (4.75)$$

so at each time integration step, the dS and flat-space results match up to factors which are time-dependent, but importantly, independent of loop-momenta. After the final time integral, the dS result yields a total-energy pole, while the flat result gives a energy conserving delta function, keeping the correspondence intact.

In the following section, we check these relations explicitly with a few example correlators that have been previously computed in Sections. 4.2 and 4.3.

4.4.2 Examples of flat space limit of dS correlators

Below we consider the leading singular terms of the dS correlator in the $\omega_T \rightarrow 0$ limit, and compute the scattering amplitude corresponding to the same diagram to verify the flat space limit at 1-loop. We will keep explicit all numerical factors, as well as factors of H and external energies ω_i .

Diagram 21 The interaction insertions in this contribution are given by $ig_4 \dot{\varphi}^2 \partial^{-2} \partial_i (\dot{\varphi} \partial_i \varphi)$ and $ig_3 \dot{\varphi}^2 \varphi$, hence the total number of spatial and temporal derivatives are given by $N = 5$. With $V = 2$, $I = 2$ and $n = 3$, we have $\alpha = 4$, and for this diagram involving 7 internal modes and 2 interaction vertices, we have $\sum_j n_j = \frac{7}{2} - 2 = \frac{3}{2}$. Thus, the flat space limit in Eqn. (4.69) gives

$$\lim_{\omega_T \rightarrow 0} D_n = -H^7 \frac{4!}{\omega_T^5} \left(\prod_{i \in 1,2,3} \frac{1}{2\omega_i^2} \right) \left(\mathcal{A}_n + \frac{3}{2} \log \left(\frac{H}{\omega_T} \right) \mathcal{A}_n^{(\frac{1}{\delta})} \right). \quad (4.76)$$

To verify this, we compute the corresponding *off-shell* scattering amplitude in flat space using dimensional regularization, to get (stripping the momentum conserving delta function),

$$\begin{aligned} \mathcal{A}_n &= g_3 g_4 \mu^{-3\delta/2} \frac{\omega_1}{480} \left[\frac{1}{\delta} \left(5 \left(2s^2 \omega_3^2 + 2s^2 \omega_{12}^2 - 3\omega_3^4 - 3\omega_{12}^4 \right) - 6s^4 \right) \right. \\ &\quad \left. + \left(10s^2 \omega_3^2 - 3s^4 - 15\omega_3^4 \right) \log(s + \omega_3) + \left(10s^2 \omega_{12}^2 - 3s^4 - 15\omega_{12}^4 \right) \log(s + \omega_{12}) \right] \\ &\quad + \text{finite}, \end{aligned} \quad (4.77)$$

while the terms of the unrenormalized dS correlator leading in the limit $\omega_T \rightarrow 0$, including $\mathcal{O}(\delta)$ corrections coming from modes and measure, is computed to be,

$$D_n \xrightarrow{\omega_T \rightarrow 0} g_3 g_4 \mu^{-3\delta/2} \frac{H^7}{160 \omega_1 \omega_2^2 \omega_3^2 \omega_T^5} \left[\left\{ \frac{1}{\delta} + \frac{3}{2} \log \frac{H}{\omega_T} \right\} \left(5(-2s^2 \omega_3^2 - 2s^2 \omega_{12}^2 + 3\omega_3^4 + 3\omega_{12}^4) + 6s^4 \right) + \left(-10s^2 \omega_3^2 + 3s^4 + 15\omega_3^4 \right) \log(s + \omega_3) + \left(-10s^2 \omega_{12}^2 + 3s^4 + 15\omega_{12}^4 \right) \log(s + \omega_{12}) \right] + \text{NLf}, \quad (4.78)$$

where NLf denotes non log finite functions from D_1 and D_2 , and the finite contribution from D_1 , given by,

$$\text{NLf} \xrightarrow{D_1} -g_3 g_4 \mu^{-3\delta/2} \frac{H^7}{4800 \omega_1 \omega_2^2 \omega_3^2 \omega_T^5} \left(276s^4 + 50(-8s^2 \omega_3^2 - 3s^3 \omega_3 + \omega_{12}(-8s^2 \omega_{12} - 3s^3 + \omega_{12}^2 \left(3s \left(3 + \frac{4i\pi s}{\omega_{12} - \omega_3} \right) + (9 - 9i\pi)\omega_1 + (9 - 9i\pi)\omega_2 \right) + 9s\omega_3^3 + (9 - 9i\pi)\omega_3^4) \right), \quad (4.79)$$

is related to the finite piece in the scattering amplitude of Eqn. (4.77) via the flat space limit. Note that the unrenormalized dS correlator features unphysical logarithms, which will only be corrected once contribution from counter terms are taken into account, after which the branch cuts are arranged to give $\log(\frac{s+\omega_3}{\omega_T})$, $\log(\frac{s+\omega_{12}}{\omega_T})$ and $\log(\frac{H}{\mu})$, as in Eqn. (4.35) (which includes the subleading pieces).

Clearly, comparing Eqns. (4.77) and (4.78), we see that the flat space limit in Eqn. (4.76) is satisfied.

Diagram 31 This diagram could only be computed in cutoff regularization since the angular dependence coming from the vertex factors becomes intractable in d -dimensions. The values of N, V, I, n, α remain the same as in the previous case

(Diagram 21). Thus the flat space limit in Eqn. (4.70) gives,

$$\lim_{\omega_T \rightarrow 0} D_n = -H^7 \frac{4!}{\omega_T^5} \left(\prod_{i \in 1,2,3} \frac{1}{2\omega_i^2} \right) \left(\mathcal{A}_n - \log\left(\frac{H}{\omega_T}\right) \mathcal{A}_n^{(\log \Lambda)} \right). \quad (4.80)$$

To verify this relation, we compute the flat space scattering amplitude corresponding to this contraction in cutoff regularization. Since the cutoff Λ is now time independent, it is possible to perform the time integrals before the momentum integrals. This is done to avoid taking fourier transforms of complicated functions arising from the loop integrals. The calculation is then straightforward and gives,

$$\begin{aligned} \mathcal{A}_n = g_3 g_4 \frac{\omega_1 \omega_2}{96} & \left[-\log(\Lambda) \left(s^2 \omega_3 - s^2 \omega_{12} - 3\omega_3^3 + 3\omega_{12}^3 \right) + \omega_3 (s^2 - 3\omega_3^2) \log(s + \omega_3) \right. \\ & \left. + \omega_{12} (3\omega_{12}^2 - s^2) \log(s + \omega_{12}) \right] + \text{finite}, \end{aligned} \quad (4.81)$$

while the unrenormalized dS correlator leading in the limit $\omega_T \rightarrow 0$ computed in the cutoff regularization is given by,

$$\begin{aligned} D_n \xrightarrow{\omega_T \rightarrow 0} \frac{g_3 g_4 H^7}{32 \omega_1 \omega_2 \omega_3^2 \omega_T^5} & \left[\log\left(\frac{H}{\Lambda \omega_T}\right) \left(-s^2 \omega_3 + s^2 \omega_{12} + 3\omega_3^3 - 3\omega_{12}^3 \right) \right. \\ & \left. + \omega_{12} (s^2 - 3\omega_{12}^2) \log(s + \omega_{12}) - \omega_3 (s^2 - 3\omega_3^2) \log(s + \omega_3) \right] + \text{NLf}. \end{aligned} \quad (4.82)$$

Once again, comparing Eqns. (4.81) and (4.82), we see that the flat space limit in Eqn. (4.80) is indeed satisfied. Note that the NLf part of the correlator we compute does not contain all finite non log terms of the correlator (since we drop a few terms in the integrals which are not expected to produce logs; see comments below Eqn. (4.24)). However, even with the full NLf explicitly computed, we do not expect any flat space limits for these non-log finite terms to exist. This is because, as previously discussed, there is additional time dependence of the cutoff in dS, which

has no flat space analogue. We leave this explicit check for a future work.

3-vertex Bispectrum at 1-loop We now check the flat space limit of the contribution to the bispectrum in Fig. 4.2, obtained by three interaction insertions of $g_3 \dot{\varphi} \partial_i \varphi \partial^{-2} \partial_i \dot{\varphi}$. This interaction has 2 temporal and spatial derivatives alongwith 1 inverse laplacian operator, thus $N = 6$ including all three vertices. With $V = 3$, $I = 3$, $n = 3$, we have $\alpha = 3$, and with 3 vertices and 9 internal modes, we have $\sum_j n_j = \frac{9}{2} - 3 = \frac{3}{2}$. We will compute this dS correlator (and the corresponding flat space scattering amplitude) in dimensional regularization, thus the flat space limit of Eqn. (4.69) gives,

$$\lim_{\omega_T \rightarrow 0} D_n = -iH^6 \frac{3!}{\omega_T^4} \left(\prod_{i \in 1,2,3} \frac{1}{2\omega_i^2} \right) \left(\mathcal{A}_n + \frac{3}{2} \log \left(\frac{H}{\omega_T} \right) \mathcal{A}_n^{(\frac{1}{\delta})} \right). \quad (4.83)$$

The 3-vertex flat space scattering amplitude computed with these interactions is computed to be (stripped of momentum conserving delta function),

$$\begin{aligned} \mathcal{A}_n = g_3^3 \mu^{-3\delta/2} i \frac{s^3 \omega_2 \cos \theta_k}{48 p_3 (p_3 + \omega_3) (p_3 + \omega_{12})} & \left[\frac{1}{\delta} \left(2p_3 (6s^2 + \omega_3 (\omega_{12} - \omega_3)) + p_3^2 \omega_T \right. \right. \\ & + (6s^2 - \omega_3^2) \omega_T \Big) + \left(\frac{(3s^2 - p_3^2) (p_3 + \omega_{12}) \omega_T}{p_3 - \omega_{12}} \right) \log (p_3 + s) - \left((3s^2 - \omega_3^2) \right. \\ & \left. \left. (2p_3 + \omega_T) \right) \log (s + \omega_3) + \frac{2p_3 (p_3 + \omega_3) (\omega_{12}^2 - 3s^2)}{p_3 - \omega_{12}} \log (s + \omega_{12}) \right] + \text{finite}, \end{aligned} \quad (4.84)$$

where $\cos\theta_k = \hat{k}_1 \cdot \hat{k}_3$, while the leading singular terms of the unrenormalized dS correlator, including $\mathcal{O}(\delta)$ corrections from modes and measure, is computed to be,

$$\begin{aligned}
D_n \xrightarrow{\omega_T \rightarrow 0} g_3^3 \mu^{-3\delta/2} \frac{H^6 s^3 \text{c}\theta_k}{64 p_3 \omega_1^2 \omega_2 \omega_3^2 (p_3 + \omega_3) (p_3 + \omega_{12}) \omega_T^4} & \left[\left(\omega_T (p_3^2 + 6s^2 - \omega_3^2) \right. \right. \\
& + 2p_3 (6s^2 + \omega_3 (\omega_{12} - \omega_3)) \left. \right) \left(\frac{1}{\delta} + \frac{3}{2} \log \frac{H}{\omega_T} \right) + \frac{(p_3^2 - 3s^2) (p_3 + \omega_{12}) \omega_T}{p_3 - \omega_{12}} \log (p_3 + s) \\
& - (\omega_3^2 - 3s^2) (2p_3 + \omega_T) \log (s + \omega_3) + \frac{2p_3 (p_3 + \omega_3) (\omega_{12}^2 - 3s^2) \log (s + \omega_{12})}{\omega_{12} - p_3} \left. \right] \\
& + \text{NLf}, \tag{4.85}
\end{aligned}$$

where NLf denotes non-log finite terms, and the piece coming from D_1 , given by,

$$\begin{aligned}
\text{NLf} \xrightarrow{D_1} -g_3^3 \mu^{-3\delta/2} \frac{H^6 s^3 \text{c}\theta_k}{192 p_3 \omega_1^2 \omega_2 \omega_3^2 (p_3 + \omega_3) (p_3 + \omega_{12}) \omega_T^4} & \left(\omega_T (4p_3^2 + 18s^2 - 3s\omega_3 - 4\omega_3^2) \right. \\
& + p_3 (36s^2 - 3s\omega_{12} + \omega_3 (-3s + 8\omega_1 + 8\omega_2 - 8\omega_3)) \left. \right), \tag{4.86}
\end{aligned}$$

is related to the finite piece in the scattering amplitude of Eqn. (4.84) via the flat space limit. Also, the unrenormalized correlator features unphysical logarithms, which are corrected once the correlator is renormalized, and the branch cuts rearrange to give $\log\left(\frac{s+\omega_3}{\omega_T}\right)$, $\log\left(\frac{s+p_3}{\omega_T}\right)$, $\log\left(\frac{s+\omega_{12}}{\omega_T}\right)$ and $\log\left(\frac{H}{\mu}\right)$, as in Eqn. (4.44).

Finally, Eqns. (4.84) and (4.85) can be compared to see that the flat space limit of Eqn. (4.83) is satisfied.

Chapter 5

Conclusion

In this thesis, we explored the analytic structure of cosmological correlators at 1-loop. We explicitly computed the bispectrum at one loop in the decoupling limit of EFT_{oI}, highlighting the significance of renormalizing loops to observe cancellation of *unphysical* logarithms. We then derived diagrammatic rules to obtain the singularity structure of cosmological correlation function at 1-loop. These rules bypass computation of beastly integrals to obtain the poles and branch cuts produced in a loop diagram, by simple identification of certain subgraphs and the external energy flowing through them. These rules are valid for any n -point correlator at 1-loop with bubble topology (2-loop sites), which is late-time ($\tau_0 \rightarrow 0$) and IR finite. Additionally, we identified the logarithm of total energy is a feature of *off-shell* correlators in de-Sitter, stressing that this branch cut is present regardless of the regularization scheme. Importantly, we find that in a renormalized correlator, the produced branch cuts always repackage themselves into dilatation invariant forms of logarithms at ratios of comoving scales.

We then studied the 1-loop correction to the bispectrum in slow-roll inflation involving non-local interaction terms arising from the ADM formalism. We find that

the analytic structure of the 1-loop inflationary correlator obeys the diagrammatic rules previously derived in [4], exhibiting poles and branch cuts along the negative real axis in the complex external-energy plane. Finally, we examined how flat-space scattering amplitudes are encoded in the total-energy singularities of loop-level de Sitter (dS) correlators, clarifying several subtle aspects of this correspondence. In particular, we observed that each branch cut of the dS correlator arising from the leading $\omega_T \rightarrow 0$ singularity has a counterpart in the flat-space scattering amplitude—except for the total-energy branch cut, which has no flat-space analog, since this branch cut is a consequence of the time-dependent cosmological background.

The following questions remain open for future investigation:

- This thesis only considers one loop bubble diagrams. The natural generalization of this work is to investigate how the rules presented in Sec. 3.2.2 and the arguments of Sec. 3.3.1 generalize to diagrams at higher loop or larger number of loop-sites. The main difficulty in going beyond such 1-loop, 2-site diagrams is that the integral measure for evaluating this type of functional is significantly more complicated than the ones found in Feynman diagrams (see for instance [130]¹). While some progress has been made for the triangle diagram (the simplest extension beyond bubble topology) in flat space [123], the analogous computation in de Sitter is substantially more challenging and no results have been obtained so far. The technical challenges of generalizing beyond bubble topology arises due to a significantly more complicated space of functions involving (integrals of) hyper-elliptic integrals, as well as corrections from time dependent background, such as $\mathcal{O}(\delta)$ corrections in dimensional regularization and time dependent regulators in cutoff regularization. However, as a non-trivial check, we test our rules on the triangle diagram in the soft

¹For progress on the triangle wavefunction, see [101, 131].

limit (See Appendix A.5) and find that our results hold.

- It would be interesting to see if correlators of fields with arbitrary mass and spins obey the same rules presented in our work. The obvious difficulty with computing correlations in these cases is obtaining analytic expressions with Hankel functions, although some progress has been made in this direction [132, 133, 134, 94, 93, 135, 136, 137].
- Our computations for the in-in correlators with dim reg relies heavily on expanding the integrand with an asymptotic series before doing the integrals, computing the integral over each term of the series, and then summing it all up. However, it is not immediately obvious that the series expansion commutes with the integrals. Nevertheless, obtaining analytic expressions for correlators under dimensional regularization without resorting to these asymptotic expansions is very difficult, which is the main reason why this technique is often employed in the first place. It would be nice to understand the subtleties of this procedure: for instance, whether we always get $\log(\omega_T)$ corrections as anticipated from our arguments in section 3.3.1. However it is worthwhile to note that these logarithms arise in cut-off regularisation as well, hence providing a robust cross-check to our results. It is also crucial to investigate the implications of the modifications to the analytic structure introduced by the loop corrections on unitarity and causality, or whether some of these branch cuts are secretly spurious.
- There exists another dimensional regularization scheme where d -dimensional mode functions are used without analytic continuation of the mass (so for instance, the mode function for a massless scalar would be $\tau^{\frac{3+\delta}{2}} H_{3/2+\delta}^{(2)}(k\tau)$). Expanding this mode function in δ gives additional contributions (see Eqn. (46)

in [2] for instance). At one loop, these do not contribute to the logarithmic running and hence the results from this regularization scheme agrees with ours. It would be nice to check what happens at higher loops in this scheme, and whether results from the two schemes agree at higher loops as well.

- Finally, it will be interesting to see if these rules imply a simple way to bootstrap the kinematic dependence of one loop correlators of ζ (curvature perturbation) for full inflationary theory. Using the one loop bubble diagram as an example, we know the type of transcendental functions which can arise from the momentum integration[114, 101]. From our work we understand the location of possible branch points, which in turn fixes the possible arguments of the transcendental functions. It would be interesting to see if we could apply physical constraints such as unitarity [33, 35, 36, 138, 139] and locality [32] to fix all possible forms of one loop correlators in the same way as tree level wavefunction [31, 92].
- In Chapter 4, when regulating loop divergences with cutoff regularization scheme, we encountered late-time logarithmic divergences multiplied by power-law divergences in the cutoff scale Λ . These divergences should cancel upon renormalization—consistent with the fact that the corresponding contractions produce no such divergences in dimensional regularization. It would be interesting to explicitly demonstrate the renormalization that removes these terms. Also, a detailed analysis of handling IR loop divergence in correlators in the context of non-local interactions is left for a future work.
- It would be interesting to determine whether flat-space limits exist for correlators beyond the 1-loop level. Our derivation of the flat-space limit at 1-loop relied on expanding the dS mode functions, which allowed us to relate the

leading terms of the dS propagators to flat-space propagators via factors independent of the loop momentum. However, interchanging this series expansion with the loop integrals in our dimensional regularization scheme is subtle, so a cross-check with a cutoff regularization is needed to verify that this method accurately captures the branch cuts. It also remains an open question whether our dimensional-regularization approach can reliably compute branch cuts at higher loop orders, which would pave the way for a straightforward generalization of flat-space limits to multi-loop correlators.

- In [33], a flat-space limit was formulated at tree level in which a partial energy (external energy injected into a subdiagram) is taken to zero. The resulting correspondence involves more complicated factors than in the $\omega_T \rightarrow 0$ case and depends on the exchange momenta. It would be worthwhile to investigate whether a similar partial-energy flat-space limit exists at loop level.
- The analytic structure of correlator graphs are slowly emerging as simpler than their wavefunction counterparts [105, 137, 140], due to the difference in their integration regimes and boundary conditions. This warrants an investigation into the explicit map between wavefunction coefficients and cosmological correlators, in the theory with derivative and non-local interactions of massless scalars considered in this work. Such a map would make explicit which branch cuts in the correlators arise from *classical/quantum* loops of the wavefunction coefficient, as well as shed light on the simplifications in the correlator due to cancellations when summing up wavefunction contributions.
- The canonical differential equation (DE) framework provides a powerful approach to studying loop corrections to flat-space scattering amplitudes. It is then interesting to investigate whether a similar framework could stream-

line the evaluation of cosmological loop correlators. While such an approach may offer conceptual insight into the analytic structure of cosmological loop diagrams, we expect several obstacles to a direct implementation of this framework in the context of cosmological loops. First, the identification of a *family* of cosmological in-in integrals, that closes under integration-by-parts identities, has not yet been explored, and it is not clear apriori whether such a closed system should exist. Moreover, in flat space loops, Lorentz invariance restricts amplitudes to depend on only a few kinematic variables (the Mandelstam invariants) resulting in a compact DE system. Cosmological loop integrals, by contrast, depend on multiple independent external momenta as well as the structure of the time integrals, so any associated differential-equation system would be considerably larger than in flat space. Furthermore, beyond the simplest “two-site” topology at one-loop, the integrals already cease to be poly-logarithmic, even with simple local (φ^n or $(\partial\varphi)^n$) interactions. With higher loop sites, and at higher loop orders, the complexity of cosmological integral quickly worsens with iterated integral of hyper-elliptic curves. These features are expected to make a systematic construction of a closed DE system, and hence a canonical form, substantially more involved than in flat-space quantum field theory.

- Our analysis is performed in the single-field slow-roll setup, involving massless de Sitter mode functions and late-time finite interactions. In this regime, the diagrammatic rules tested in our work reliably identify the singularity structure of loop correlators, without requiring explicit computations. These rules follow directly from the integral structure of the in-in correlators in a model-independent way. However, in scenarios that significantly depart from slow roll, such as models with a transient ultra slow roll (USR) phase relevant

for primordial black-hole production, the mode functions can be qualitatively more complicated, with interactions that lead to secular growth in these correlators. This makes the question of perturbative control substantially more delicate. Our present work does not address these complications directly, and it remains to be understood how to tame such late time divergences in beyond-slow roll inflationary scenarios. We leave the generalization of the diagrammatic rules framework to such non-attractor or PBH-motivated scenarios for a future work.

Appendix A

Appendix Title

A.1 Loop Momentum Identity

Here, we compute momentum integrals given by

$$D_1(k_i) = \int d^d \vec{p}_1 d^d \vec{p}_2 Q(p_1, p_2, \{k_i\}) \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3). \quad (\text{A.1})$$

To evaluate this, we note that Eq. (A.1) can be rewritten, by carrying out the integral over $\vec{p}_2$ using the delta function, as follows

$$\begin{aligned} &= \int d^d \vec{p}_1 Q(p_1, |\vec{k}_3 - \vec{p}_1|, \{k_i\}) = \int p_1^{d-1} Q(p_1, |\vec{k}_3 - \vec{p}_1|, \{k_i\}) dp_1 d\Omega_d, \\ &= \int p_1^{d-1} Q(p_1, |\vec{k}_3 - \vec{p}_1|, \{k_i\}) dp_1 \sin^{d-2} \theta_1 \sin^{d-3} \theta_2 \dots \sin \theta_{d-2} d\theta_1 d\theta_2 d\theta_{d-2} d\phi, \end{aligned} \quad (\text{A.2})$$

where ϕ is the azimuthal angle and θ_i 's denote polar angles. If $\hat{k}_3$ is chosen along

axis with respect to which θ_1 is measured, then

$$|\vec{k}_3 - \vec{p}_1| = \sqrt{k_3^2 + p_1^2 - 2k_3p_1 \cos \theta_1} = p_2$$

$$\text{or, } \frac{k p_1}{p_2} \sin \theta_1 d\theta_1 = dp_2$$

Thus, a variable change from θ_1 to p_2 in Eq. (A.2) gives us

$$\begin{aligned} &= \int \frac{p_1^{d-2} p_2}{k_3} Q(p_1, p_2, \{k_i\}) \sin^{d-3} \theta_1 dp_1 dp_2 d\Omega_{d-1}, \\ &= S_{d-2} \int \frac{p_1^{d-2} p_2}{k_3} Q(p_1, p_2, \{k_i\}) \sin^{d-3} \theta_1 dp_1 dp_2, \end{aligned} \quad (\text{A.3})$$

where S_{d-2} is surface area of a $(d-2)$ - unit sphere, and $\sin \theta_1 = \sqrt{1 - \left(\frac{k_3^2 + p_1^2 - p_2^2}{2k_3 p_1}\right)^2}$. Defining $p_+ = p_1 + p_2$ and $p_- = p_1 - p_2$, we get

$$\begin{aligned} &\int d^d \vec{p}_1 d^d \vec{p}_2 Q(p_1, p_2, \{k_i\}) \delta^d(\vec{p}_1 + \vec{p}_2 - \vec{k}_3) \\ &= \frac{S_{d-2}}{2} \int_{k_3}^{\infty} dp_+ \int_{-k_3}^{k_3} dp_- \frac{p_1^{d-2} p_2}{k_3} \sin^{d-3} \theta_1 Q(p_1, p_2, \{k_i\}). \end{aligned} \quad (\text{A.4})$$

The factor of $\sin^{d-3} \theta_1$ was dropped in [36]. While it does not contribute to the $\mathcal{O}(\frac{1}{\delta})$ divergent terms in the correlator, it does give finite contributions and hence is important in determining the shape and pole structure of the correlator away from the equilateral limit.

A.2 Computation of D_2 for Diagram I

Here we explicitly compute the $\mathcal{O}(\delta)$ contribution from mode functions and scale factors for the diagram with time ordered vertices in Fig. 4.3. The integral is given

by

$$D_2 = g_3 g_4 \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{1}{8 k_1 k_2 k_3} \int d^d \vec{p}_1 d^d \vec{p}_2 \frac{p_1 p_2}{4} \delta(I_1 + I_2), \quad (\text{A.5})$$

where I_1 and I_2 are the integrals with time orderings $\tau_1 > \tau_2$ and $\tau_2 > \tau_1$ respectively,

$$\begin{aligned} I_1(k_i, p_j) &= \int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 e^{i(k_{12}+p_+)\tau_2} e^{i(k_3-p_+)\tau_1} \left(\frac{1}{2} \log(-H\tau_1) + \log(-H\tau_2) \right) \\ &= I_1^{\tau_1}(k_i, p_j) + I_1^{\tau_2}(k_i, p_j), \\ I_2(k_i, p_j) &= \int_{-\infty}^0 d\tau_2 \int_{-\infty}^{\tau_2} d\tau_1 e^{i(k_{12}-p_+)\tau_2} e^{i(k_3+p_+)\tau_1} \left(\frac{1}{2} \log(-H\tau_1) + \log(-H\tau_2) \right) \\ &= I_2^{\tau_1}(k_i, p_j) + I_2^{\tau_2}(k_i, p_j), \end{aligned} \quad (\text{A.6})$$

where we refer to the terms having log dependence on the time integrated from $-\infty$ to 0 as $I_1^{\tau_1}(k_i, p_j)$ and $I_2^{\tau_2}(k_i, p_j)$, i.e., $\log(-H\tau_1)$ in I_1 and $\log(-H\tau_2)$ in I_2 respectively. Next, we consider terms having log dependence on the time being integrated from $-\infty$ to $\tau_{1(2)}$, and call them $I_1^{\tau_2}(k_i, p_j)$ and $I_2^{\tau_1}(k_i, p_j)$, i.e., $\log(-H\tau_2)$ in I_1 and $\log(-H\tau_1)$ in I_2 respectively.

- From $I_1^{\tau_1}$ and $I_2^{\tau_2}$, we get functions of p_+ , k_{12} and k_3 , plus terms dependent on $\log k_T$. The former gives a $\frac{1}{\delta}$ divergent term after the loop integrals, independent of logarithms, hence giving a finite contribution to D_2 . The log term gives the following finite contribution to D_2 after performing loop integrals,

$$g_3 g_4 \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{k_3(k_1 + k_2)(k_3 - 2k_1 - 2k_2)}{k_1 k_2 k_T^7} \frac{3}{2} \log \left(\frac{H}{k_T} \right). \quad (\text{A.7})$$

- From $I_1^{\tau_2}$ and $I_2^{\tau_1}$, we once again get functions of q_+ , k_{12} and k_3 , and a $\log k_T$ term. Additionally, one also gets terms dependent on $\log(p_+ + k_{12})$ and

$\log(p_+ + k_3)$ for $I_1^{\tau_2}$ and $I_2^{\tau_1}$ respectively. However, upon p_- integration, the p_+ dependence of the result

$$\sim \frac{p_+^4 (p_+^2 - k_{12}^2)^{\delta/2} \log\left(p_+ + \frac{k_{12}}{k_3}\right)}{(p_+ - k_{12})^3 \left(p_+ + \frac{k_{12}}{k_3}\right)^5}, \quad (\text{A.8})$$

vanishes in the limit $p_+ \rightarrow \infty$. Thus the contributions from such terms to D_2 are of $\mathcal{O}(\delta)$, and vanish when $d \rightarrow 3$.

The remaining log independent terms give a finite, log-free contribution to D_2 , and the contribution from $\log k_T$ terms after performing loop integrals is the same as Eq. (A.7)

Hence, summing the contributions, we have

$$D_2 = g_3 g_4 \delta^3 \left(\sum_i \vec{k}_i \right) \frac{H^9}{\left(-\dot{H} M_{pl}^2 \right)^5} \frac{k_3(k_1 + k_2)(k_3 - 2k_1 - 2k_2)}{k_1 k_2 k_T^7} 3 \log\left(\frac{H}{k_T}\right) + \text{finite}. \quad (\text{A.9})$$

Adding this to D_1 gives Eq. (2.19)

A.3 Computation of $\mathcal{O}(\delta)$ terms containing $\log(-H\tau)$

In Appendix A.2, we showed explicitly that D_2 can be computed by pulling the log term out of the integrals and substituting $\tau \rightarrow 1/k_T$, along with some finite terms. In this appendix, we show that this is also true for the D_2 contributions for all diagrams computed in Sec. 2.2

Note that D_2 is given by the same integral as D_1 with an additional factor of $\log \tau$. To compute this contribution, one would need to evaluate up to three time integrals, with logarithms dependent on any of the time vertices. The explicit

computation of D_2 will be lengthy since each such integral can have various time orderings. Instead, let us take a look at the general structure of these integrals.

For diagrams with a cubic and a quartic interaction, the time integrals have the form,

$$\int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \tau_1^n \tau_2^m e^{ia\tau_1} e^{ib\tau_2} \log(-H\tau_{1(2)}) , \quad (\text{A.10})$$

where $a + b = k_T$. In the integrals we compute, (m, n) take the values $(2, 2)$, $(2, 3)$ in Eqns. (2.24), (2.23), (2.26), and $(2, 4)$ in Eqns. (2.17), (2.24), (2.23), (2.26). The log acts on τ_1 or τ_2 . For three cubic diagrams, the time integrals are of the type

$$\int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_3 \tau_1^m \tau_2^n \tau_3^l e^{ia\tau_1} e^{ib\tau_2} e^{ic\tau_3} \log(-H\tau_{1(2,3)}) , \quad (\text{A.11})$$

where $a + b + c = k_T$ and $m, n, l = 2$ for our computations. The log acts on τ_1, τ_2 or τ_3 .

- **Integrals with $\log(-H\tau_1)$:** Performing the integral over τ_2 for Eq. (A.10) (and over τ_3 for Eq. (A.11)), we are left with

$$\int_{-\infty}^0 d\tau_1 \tau_1^m e^{ik_T\tau_1} g(k_i, p_j, \tau_1) \log(-H\tau_1) , \quad (\text{A.12})$$

where g is a polynomial of τ_1 , with coefficients depending on k_i and p_j . Such integrals evaluate to the following,

$$f_1(k_i, p_j) + \log\left(\frac{H}{k_T}\right) f_2(k_i, p_j) , \quad (\text{A.13})$$

where $f_1(k_i, p_j)$ is a finite term and $f_2(k_i, p_j)$ is the result of the time integrals

evaluated without any logs.

- **Integrals with $\log(-\mathbf{H}\tau_2)$:** Evaluation of τ_2 integral in Eq. (A.10) (and τ_3 integral in Eq. (A.11)) gives the following sum of exponential integral functions (Ei) and polynomials in τ_1 ,

$$\begin{aligned} & \int_{-\infty}^0 d\tau_1 \tau_1^m e^{ia\tau_1} \{ \text{Ei}[i(b+c)\tau_1] g_1(k_i, p_j) + e^{i(b+c)\tau_1} g_2(k_i, p_j, \tau_1) \\ & + e^{i(b+c)\tau_1} \log \tau_1 g_3(k_i, p_j, \tau_1) \}, \end{aligned} \quad (\text{A.14})$$

where g_2 and g_3 are polynomials in τ_1 and g_1 is a function of k_i and p_j . Upon evaluation of this integral, the first and second terms give some finite functions of k_i and p_j . The first term also gives a log term of the form $\log\left(\frac{k_T}{p_+ + k_{12}}\right)$. There are however no additional log contributions from this piece since it converges as $p_+ \rightarrow \infty$, which implies this contribution dies at $\delta \rightarrow 0$. This was the case for e.g. in Fig 2.1a, where the functional form of the extra log term in Eqn. A.8 was such that the loop integrals were convergent.

It turns out that the function g_3 is the result of τ_2, τ_3 time integrals performed without logarithms. Thus the third term in Eq. (A.14) has the same structure as Eq. (A.12), and hence the result will have the structure of Eq. (A.13).

- **Integrals with $\log(-\mathbf{H}\tau_3)$:** Evaluation of τ_3 in Eq. (A.11) once again gives a sum of exponential integral functions and polynomials of τ_2 ,

$$\begin{aligned} & \int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \tau_1^m \tau_2^n \{ \text{Ei}[ic\tau_2] h_1(k_i, p_j) + e^{ic\tau_1} h_2(k_i, p_j, \tau_2) \\ & + e^{ic\tau_2} \log \tau_2 h_3(k_i, p_j, \tau_2) \}, \end{aligned} \quad (\text{A.15})$$

where h_2 and h_3 are polynomials of τ_2 and h_1 is a function of k_i and p_j .

The second term gives some finite non-log contribution when the remaining time integrals are carried out. The τ_2 integral on the first term having exponential integral yields some finite answer along with more exponential integral(s), both of which give non-log finite contributions upon performing τ_1 integral. The latter however also gives a $\log\left(\frac{k_T}{p_+ + k_{12}}\right)$, which goes to zero as $\delta \rightarrow 0$ and does not contribute any additional logs, since this term is finite at $p_+ \rightarrow \infty$.

The third term in Eq. (A.15) has the same structure as Eq. (A.14), as the function h_3 is the result τ_3 integral performed without the log. Hence this evaluation will once again give the same structure as Eq. (A.13).

Hence we conclude that in all our computations of $\mathcal{O}(\delta)$ correction terms, D_2 is given by

$$\sim \delta \log\left(\frac{H}{k_T}\right) D_1 + \text{finite}. \quad (\text{A.16})$$

A.4 Scale Invariance

In flat space one of the main features of dim reg is that it preserves the symmetry of the theory (namely Lorentz invariance and gauge invariance for gauge fields). This is one of the main advantages of using dim reg in flat space over other regularization schemes such as cutoff regularization. However from the examples in [3] as well as in section 3.1.2, we noticed the unrenormalised correlator features logarithms of ratios of comoving momenta and physical renormalization scale μ . Aside from issues with secular divergences mentioned in the introduction, this raises another problem: we expect scale invariance to be a symmetry of the theory, and one may worry if dim reg in de Sitter will violate this. In this Appendix we provide clarification on this issue.

“Scale invariance” may refer to a number of things. The first one is the following:

$$\langle \phi(\lambda x_1, \lambda \tau_1) \phi(\lambda x_2, \lambda \tau_2) \dots \phi(\lambda x_n, \lambda \tau_n) \rangle = \langle \phi(x_1, \tau_1) \phi(x_2, \tau_2) \dots \phi(x_n, \tau_n) \rangle. \quad (\text{A.17})$$

This one always holds as long as the spacetime is well-approximated by de Sitter, and we will not discuss this further.

Cosmologists commonly use scale invariance for a stronger condition: for late time correlators, i.e. $\tau_0 \rightarrow 0$, we have

$$\langle \phi(\lambda x_1) \phi(\lambda x_2) \dots \phi(\lambda x_n) \rangle = \langle \phi(x_1) \phi(x_2) \dots \phi(x_n) \rangle. \quad (\text{A.18})$$

This is a special case of the usual notion of scale invariance in CFT. In cosmology we focus on correlators for massless scalars in $d = 3$ at late time, and in this regime massless scalars can usually be decomposed into the following:

$$\phi(x, \tau_0 \rightarrow 0) \sim \mathcal{O}_{\Delta=0}(x) + \tau_0^3 \mathcal{O}_{\Delta=3}(x). \quad (\text{A.19})$$

Since only $\Delta = 0$ survives in the late time limit, demanding scale invariance in the usual CFT notion yields (A.18).

However, these notions do not always coincide, especially if we work in dim reg. In our version of the regularization scheme we modified the mass such that the order of Hankel function is always a half integer¹. For massless scalars this implies:

$$\phi(x, \tau_0 \rightarrow 0; \delta) \sim \tau_0^{\delta/2} \mathcal{O}_{\Delta=\frac{\delta}{2}}(x) + \tau_0^{3+\delta/2} \mathcal{O}_{\Delta=3+\frac{\delta}{2}}(x). \quad (\text{A.20})$$

In our text we are implicitly computing correlators of $\mathcal{O}_{\Delta=\frac{\delta}{2}}$, since we have always

¹If we follow the regularization scheme presented in [2] the CFT notion of scale invariance always coincides with the cosmology one: assuming the mode functions are normalized properly, we always have $\Delta = 0$ in their scheme, and (A.18) always holds.

stripped off factors of τ_0^δ in our computation. This means for our expressions, the correlator scales like the following if we consider the CFT notion of scale invariance:

$$\langle \phi(\lambda x_1) \phi(\lambda x_2) \dots \phi(\lambda x_n) \rangle = \lambda^{-n\delta/2} \langle \phi(x_1) \phi(x_2) \dots \phi(x_n) \rangle. \quad (\text{A.21})$$

The correlators scales as (A.21) instead of (A.18), and this is what people usually mean when they say scale invariance is broken. We refer to the condition (A.21) as “CFT scale invariance”.

Fourier transforming (A.21) gives,

$$\langle \phi(\lambda k_1) \phi(\lambda k_2) \dots \phi(\lambda k_n) \rangle' = \lambda^{-d(n-1)+n\delta/2} \langle \phi(k_1) \phi(k_2) \dots \phi(k_n) \rangle', \quad (\text{A.22})$$

where the prime indicates the momentum conserving delta function have been stripped off.

As an example let us consider the bispectrum from a contact diagram mediated by $g\dot{\phi}^3$ interaction, and we take $d = 3 + \delta$. A straightforward calculation leads to the following:

$$B_3(k_1, k_2, k_3) = \langle \phi(k_1) \phi(k_2) \phi(k_3) \rangle' = \frac{i^{\delta/2} g \Gamma(3 + \frac{\delta}{2})}{8k_1 k_2 k_3 k_T^{3+\delta/2}} \quad (\text{A.23})$$

Under scaling, we have the following:

$$B_3(\lambda k_i) = \lambda^{-6-\delta/2} B_3(k_i), \quad (\text{A.24})$$

which indeed scales as (A.21) and therefore is CFT scale invariant. Importantly this is true *regardless* of the value of δ as well as λ : there is no requirement that $\delta \rightarrow 0$, and λ can be as large as we want.

Should we choose to expand everything as an asymptotic series in δ things become much harder to keep track of. If we expand (A.23) as an asymptotic series in δ , we obtain the following:

$$B_3(k_i) = \frac{g}{4k_1k_2k_3k_T^3} \left[1 - \delta \frac{1}{2} \log(-ik_T) + \dots \right]. \quad (\text{A.25})$$

Under scaling transformation we have:

$$B_3(k_i) = \lambda^{-6} B_3(k_i, \delta = 0) \left(1 - \frac{1}{2} \delta \log(\lambda) + \mathcal{O}(\delta^2) \right). \quad (\text{A.26})$$

The additional $\log(\lambda)$ term, despite appearances, cannot be used as indication of CFT scale invariance being broken, and the reason is as follow: expanding (A.22) as an asymptotic series in δ yields exactly (A.26) at $\mathcal{O}(\delta)$, therefore expression (A.25) is still consistent with scale invariance for $d = 3 + \delta$. Checking full scale invariance requires keeping every single term in the expansion in δ and properly resumming them, and this is not practical in loop calculations.

A.5 Soft limit of the Triangle

The correlator becomes significantly more challenging to compute once we go beyond 2-site case. To probe how our rules would generalize, we inspect the structure of the triangle diagram (Fig.A.1), with one of the legs soft, using $\dot{\phi}^3$ interactions. Naively, the computation looks very similar to the ones performed throughout this work, however the additional time vertex produces significant complications. To start with, handling the d - dimensional phase space integrals is significantly more difficult. Hence we compute this diagram in cutoff regularization. In cutoff, the limits of the loop integrals are time dependent, and there are 6 cases to be computed,

corresponding to the $3!$ time orderings in this diagram. Since these computations are lengthy and tedious, we only compute a few of these cases, and we find that the singularities arising agree precisely with what our diagrammatic rules give, as is shown below. We defer a more rigorous study of the triangle diagram, and 1-loop diagrams involving 3 and more sites, for a later work.

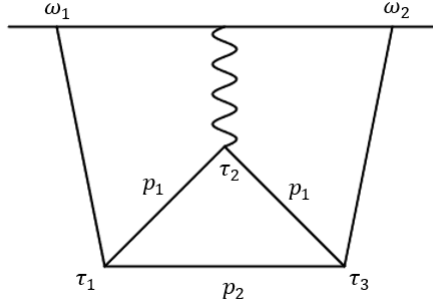


Figure A.1: The triangle diagram with one leg soft (denoted by the squiggly line) and $g_3\dot{\phi}^3$ interaction.

Without loss of generality, we consider the soft leg to contract with τ_2 . Consider the time ordering $\tau_1 > \tau_2 > \tau_3$, which corresponds to the following integral,

$$\sim \int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1(1+\frac{H}{\Lambda})} d\tau_2 \int_{-\infty}^{\tau_2(1+\frac{H}{\Lambda})} d\tau_3 \int_0^{-\frac{\Lambda}{H\tau_3}} dp_1 \int_{|s-p_1|}^{s+p_1} dp_2, \quad (\text{A.27})$$

evaluated over the integrand $\left(\frac{p_1^3 p_2^2 \tau_1^2 \tau_2 \tau_3^2}{s}\right) e^{i\tau_1(\omega_1 - p_1 - p_2)} e^{i\tau_3(\omega_2 + p_1 + p_2)}$, and there is no exponential in τ_2 due to being sandwiched between the other two time vertices. The loop integrals will be performed first since the cutoff is time dependent.

Consider the regime of $p_1 < s$. Evaluation of p_2 integral, due to the exponential terms, would pull out factors of $\tau_1 - \tau_3$ in the denominator,

$$\begin{aligned} \int_{s-p_1}^{s+p_1} dp_2 &\sim \frac{1}{(\tau_1 - \tau_3)^3} \left(f_1(s, p_1, \tau_i, \omega_j) e^{i\tau_1(\omega_1 - s)} e^{i\tau_3(s + \omega_2)} \right. \\ &\quad \left. + f_2(s, p_1, \tau_i, \omega_j) e^{i\tau_1(\omega_1 - s - 2p_1)} e^{i\tau_3(s + \omega_2 + 2p_1)} \right), \end{aligned} \quad (\text{A.28})$$

where f_1 and f_2 are polynomials in their arguments, the exponentials in the first and second lines arise from the lower and upper limits of the integral respectively. The former does not have p_1 dependence, however the latter would pull out additional factors of $\tau_1 - \tau_3$ in the denominator, i.e.,

$$\int_0^s dp_1 \sim \frac{1}{(\tau_1 - \tau_3)^7} \left(f_3(s, \tau_i, \omega_j) e^{i\tau_1(\omega_1 - 3s)} e^{i\tau_3(3s + \omega_2)} + f_4(s, \tau_i, \omega_j) e^{i\tau_1(\omega_1 - s)} e^{i\tau_3(s + \omega_2)} \right), \quad (\text{A.29})$$

where f_3 and f_4 are again polynomials, and the exponential in the first (second) term is produced from the lower (upper) limit of the integral. However the exponential in the first term would cancel from the contribution in $p_1 > s$ regime, since in this regime this same exponential would be produced from the lower limit of the integral, hence leaving us with,

$$\sim \int d\tau_1 d\tau_2 d\tau_3 \frac{f_5(s, \tau_i, \omega_j)}{(\tau_1 - \tau_3)^6} e^{i\tau_1(\omega_1 - s)} e^{i\tau_3(s + \omega_2)}. \quad (\text{A.30})$$

Now, since integrals of the type

$$\int dx \frac{1}{y - x} e^{i(by + ax)} = -e^{iy(a+b)} \text{Ei}[-ia(y - x)], \quad (\text{A.31})$$

produce Exponential Integral functions, it is straightforward to see using integration by parts, that evaluating τ_3 integral in Eqn. A.30 would produce three kinds of terms :

- Polynomials in τ_i and ω_j multiplied to $e^{i\omega_T \tau_1} \text{Ei} \left[i(\omega_2 + s) \left(\tau_2 \left(\frac{H}{\Lambda} + 1 \right) - \tau_1 \right) \right]$.
- Polynomials in τ_i , ω_j and $1 + \frac{H}{\Lambda}$, multiplied to $\exp \left[\frac{i(\tau_2(H + \Lambda)(\omega_2 + s) + \Lambda \tau_1(\omega_1 - s))}{\Lambda} \right]$.

- Polynomials in τ_i , ω_j and $1 + \frac{H}{\Lambda}$, multiplied to,

$$\exp \left[\frac{i (\tau_2 (H + \Lambda) (\omega_2 + s) + \Lambda \tau_1 (\omega_1 - s))}{\Lambda} \right] \frac{1}{\left(\tau_2 \frac{(H + \Lambda)}{\Lambda} - \tau_1 \right)^5}.$$

The second term is a simple exponential, hence upon evaluation of the rest of the time integrals, it produces finite terms alongwith power law divergences in Λ , and no logs. The τ_2 integral on the first term evaluates to exponential and exponential integral functions in τ_1 , however this does not produce any logarithms since the coefficient scales as $1/\Lambda$ for large Λ . Finally, since the coefficient of the last term scales as Λ^0 , using Eqn. (A.31) and the following identity,

$$\int_{-\infty}^0 d\tau_1 e^{ib\tau_1} \text{Ei}[ia\tau_1] = \frac{i \log(1 + b/a)}{b}, \quad (\text{A.32})$$

we see that evaluation of the remaining time integrals will produce,

$$\log \left[\frac{\omega_2 (H + \Lambda)^2 + Hs(H + 2\Lambda) + \omega_1 \Lambda^2}{H(H + 2\Lambda) (\omega_2 + s)} \right],$$

alongwith other non-log finite terms and power law divergences in Λ . In the limit of $\Lambda \rightarrow \infty$, the final answer corresponding to this time ordering is given by,

$$= \frac{(2\omega_1^2 s^2 + 2\omega_2^2 s^2 - 6\omega_1 \omega_2 s^2 + 5\omega_1 \omega_2^3 - 15\omega_1^2 \omega_2^2 + 5\omega_1^3 \omega_2 - s^4)}{\omega_T^6} \log \left[\frac{(\omega_1 + \omega_2) \Lambda}{H (\omega_2 + s)} \right]. \quad (\text{A.33})$$

Since this diagram is identical under switching of τ_1 and τ_3 , the computation corresponding to the time ordering $\tau_3 > \tau_2 > \tau_1$ is exactly same as the integral described above, under $\omega_1 \leftrightarrow \omega_2$, hence producing $\log(\Lambda/H)$, $\log(\omega_1 + \omega_2)$ and $\log(\omega_1 + s)$.

The argument for the computation of the time ordering $\tau_1 > \tau_3 > \tau_2$ is similar, however it is more involved due to the presence of exponentials in τ_2 , which lead to additional poles (i.e. $\sim 1/(\tau_1 - \tau_2)$ and $1/(\tau_2 - \tau_3)$). We skip the details of the evaluation, and state the final result,

$$= \frac{(\omega_2^2 s^2 + \omega_1^2 (7s^2 - 18\omega_2^2) - 3\omega_1 (4\omega_2 s^2 + \omega_2^3) + 15\omega_1^3 \omega_2 - 6s^4)}{8\omega_T^6} \log \left[\frac{(\omega_1 + \omega_2) \Lambda}{H (\omega_2 + s)} \right]. \quad (\text{A.34})$$

Once again we find the answer for the case $\tau_3 > \tau_1 > \tau_2$, under $\tau_1 \leftrightarrow \tau_3$ and $\omega_1 \leftrightarrow \omega_2$ in Eqn. (A.34), to produce $\log(\Lambda/H)$, $\log(\omega_1 + \omega_2)$ and $\log(\omega_1 + s)$.

Thus in the cases we compute, we find $\log \omega_T$, $\log(\omega_1 + s)$ and $\log(\omega_2 + s)$. The first is, as we discuss in detail in Sec. 3.3, a feature of de-Sitter correlators. The second and third logarithms are precisely what *loop-subgraphs* produce from the diagrammatic rules. No other logs are produced, which agrees with the fact that the triangle diagram of Fig. A.1 does not have any *left-* or *right-subgraphs*. Hence we find the triangle contribution in the soft limit to produce branch cuts which agree with our diagrammatic rules.

A.6 Evaluation of integrals involving $\log(-H\tau)$

It was shown in [3], for interactions taken from the Goldstone action of the EFT of inflation, that D_2 , which is the integral involving $\mathcal{O}(\delta)$ corrections from the modes and measure, is simply evaluated by pulling the $\log(-H\tau)$'s out of the time integrals by substituting $\tau \rightarrow \frac{1}{k_T}$, while the rest of the integral is simply D_1 , yielding the

relation in Eqn. (4.13). In particular, integrals of the type,

$$\int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \tau_1^n \tau_2^m e^{ia\tau_1} e^{ib\tau_2} \log(-H\tau_{1(2)}) , \quad (\text{A.35})$$

with $a + b = \omega_T$, involve two cases. The first is when considering $\log(-H\tau_1)$ in the integral; this case is straightforward since the integral over τ_2 is evaluated as in D_1 , and evaluation of τ_1 integral simply gives $(-\delta \log \omega_T)$ multiplied to D_1 , alongwith non-log finite (NLf) polynomial functions. The second case considering $\log(-H\tau_2)$ is slightly more involved. The evaluation of τ_2 integral now yields Exponential integral (Ei) functions of τ_1 , alongwith a $\log \tau_1$ term. The latter is evaluated quite easily following the logic of the previous case, once again producing $(-\delta \log \omega_T)$ multiplied to D_1 and NLf. The Ei functions, however, produce terms of the type $\log\left(\frac{\omega_T}{p_+ + x}\right)$ (where x depends on external energies). However in [3] it was found that these terms are finite for large p_+ , thus they do not contribute to additional branch cuts since they vanish when δ is taken to 0. This was a consequence of the simple structure of the interactions, resulting in the lowest power of τ to be ~ 4 (i.e. $m, n = 2$). Larger powers of τ_i 's in the integrand pull out more factors of p_+ in the denominator when the time integrals are computed. It turns out that $\sim \tau^4$ suffices to produce a finite integrand for large p_+ , despite having an overall $\sim p_+^2$ dependence coming from modes, and another $\sim p_+^2$ coming from the phase space integrals. In particular, it can be shown that a $\sim \tau^3$ or larger power is required for the extra logs to vanish. However, in our case we see that the time integrals fail to pull out enough powers of p_+ in the denominator to counter the factors of p_+ coming from vertex contribution, modes and phase space integrals. This means that the extra logs can produce non vanishing contributions and must be handled with care.

Let us study a concrete example to demonstrate the above. We compute the contribution from D_2 to Diagram 21, whose D_1 contribution was computed in Sec. 4.2. The loop integrand has the structure,

$$\begin{aligned} & \sim (p_+ - p_-)^2 (p_- + p_+)^2 ((p_-^2 - s^2)(s^2 - p_+^2))^{\delta/2} \times \\ & \left[\int_{-\infty}^0 d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 f(\omega_i, \tau_1, \tau_2) e^{i(\omega_L - p_+)\tau_1} e^{i(\omega_R + p_+)\tau_2} \log(-H\tau_{1(2)}) \right. \\ & \left. + \left[\int_{-\infty}^0 d\tau_2 \int_{-\infty}^{\tau_2} d\tau_1 f(\omega_i, \tau_1, \tau_2) e^{i(\omega_L + p_+)\tau_1} e^{i(\omega_R - p_+)\tau_2} \log(-H\tau_{1(2)}) \right] \right], \quad (\text{A.36}) \end{aligned}$$

where f is a function of external energies and τ_i , and the dependence on p_+, p_- in the first line arises from modes ($\sim p_1 p_2$) and phase space integral. ω_L and ω_R is the external energy entering the left and right vertices (τ_1 and τ_2 respectively). Notice that the vertex factor does not contribute additional factors of p_+ in Diagram 21. The terms in $f = \sum_i f^{(i)} \tau_1^{m_i} \tau_2^{n_i}$ are dependent on time as $(m = 2, n = 0)$, $(m = 2, n = 1)$, $(m = 3, n = 0)$, $(m = 3, n = 1)$. We will only focus on the first term, since the others are finite in the large p_+ limit, hence vanishing when $\delta \rightarrow 0$.

For the $\tau_1 > \tau_2$ piece with $\log(-H\tau_1)$, as well as the $\tau_2 > \tau_1$ piece with $\log(-H\tau_2)$, the integral is straightforward, resulting in $(-\delta \log \omega_T)$ multiplied to D_1 alongwith NLf. The $\tau_1 > \tau_2$ piece with $\log(-H\tau_2)$ and the $\tau_2 > \tau_1$ piece with $\log(-H\tau_1)$, however, also produce Exponential integrals (Ei) when the first time integral is carried out,

$$\begin{aligned} \int_{-\infty}^{\tau_1} d\tau_2 e^{i(\omega_L - p_+)\tau_1} e^{i(\omega_R + p_+)\tau_2} \tau_1^2 \log(\tau_2) &= \frac{i\tau_1^2 e^{-i\tau_1(p_+ - \omega_L)}}{p_+ + \omega_R} \text{Ei}[i\tau_1(p_+ + \omega_R)] + \dots, \\ \int_{-\infty}^{\tau_2} d\tau_1 e^{i(\omega_L + p_+)\tau_1} e^{i(\omega_R - p_+)\tau_2} \tau_1^2 \log(\tau_1) &= -\frac{2ie^{-i\tau_2(p_+ - \omega_R)}}{(\omega_L + p_+)^3} \text{Ei}[i\tau_2(p_+ + \omega_L)] + \dots, \end{aligned} \quad (\text{A.37})$$

where ... represents terms involving $\log(\tau)$ multiplied to a polynomial which is

exactly equal to the result of the first time integral performed without the log, thus the result of the final time integral follows the case of $\tau_1 > \tau_2$ with $\log(-H\tau_1)$.

Evaluation of the final time integral over the Ei function gives logarithms with p_+ in the arguments,

$$\begin{aligned} \int_{-\infty}^0 d\tau_1 \frac{i\tau_1^2 e^{-i\tau_1(p_+ - \omega_L)}}{p_+ + \omega_R} \text{Ei}[i\tau_1(p_+ + \omega_R)] &= \frac{2 \log\left(\frac{\omega_L + \omega_R}{p_+ + \omega_R}\right)}{(\omega_L - p_+)^3 (p_+ + \omega_R)} + \dots, \\ \int_{-\infty}^0 d\tau_2 \frac{-2ie^{-i\tau_2(p_+ - \omega_R)}}{(\omega_L + p_+)^3} \text{Ei}[i\tau_2(p_+ + \omega_L)] &= \frac{2 \log\left(\frac{\omega_L + \omega_R}{\omega_L + p_+}\right)}{(\omega_L + p_+)^3 (\omega_R - p_+)}, \end{aligned} \quad (\text{A.38})$$

where ... represents polynomial functions, which contribute to NLf when loop integrals are carried over. The logs, when multiplied to the prefactors involving p_+ in the first line of Eqn. (A.36) and integrated over p_- , yield a loop integrand which is not finite for $p_+ \rightarrow \infty$. This integral must now be computed explicitly since this can give non-vanishing contribution to the singularity structure of the correlator. However, due to the particular pole structure of Eqn. (A.38), these contributions cancel out between the $\tau_1 > \tau_2$ and $\tau_2 > \tau_1$ piece, as we will see below.

Adding the two cases up, the evaluation of p_- integral gives,

$$\begin{aligned} &\sim (-s^2 + p_+^2)^{\frac{\delta}{2}} g(s, \delta, p_+) \left[\log \omega_T \left\{ \frac{2}{(\omega_L + p_+)^3 (\omega_R - p_+)} \right. \right. \\ &\quad \left. \left. - \frac{2}{(p_+ - \omega_L)^3 (p_+ + \omega_R)} \right\} + \frac{2 \log(\omega_L + p_+)}{(\omega_L + p_+)^3 (p_+ - \omega_R)} + \frac{2 \log(p_+ + \omega_R)}{(p_+ - \omega_L)^3 (p_+ + \omega_R)} \right], \end{aligned} \quad (\text{A.39})$$

where g is a function of s, δ and p_+ . To be more precise, g has terms dependent on p_+^0, p_+^2 and p_+^4 . We only focus on the p_+^4 dependence since this is the term that blows up for large p_+ . Also, we can simplify the above integral by writing, $(-s^2 + p_+^2)^{\frac{\delta}{2}} = p_+^\delta (1 - s^2/p_+^2)^{\frac{\delta}{2}}$. Expanding $(1 - s^2/p_+^2)^{\frac{\delta}{2}}$ about $\delta = 0$, we can discard all $\mathcal{O}(\delta^1)$ and

higher terms since the integrand vanishes for $p_+ \rightarrow 0$.

Now naively, Eqn. (A.39) seems like bad news. Integrating over p_+ can produce non-vanishing and “*unphysical*” pieces of $\log(\omega_T/\mu)$ from the first line. However notice that,

$$\begin{aligned} \int dp_+ p_+^\delta \frac{p_+^4}{(p_+ + a)^3(p_+ - b)} &= \frac{3a - b}{\delta} + \mathcal{O}(\delta^0), \\ \int dp_+ p_+^\delta \frac{p_+^4}{(p_+ - a)^3(p_+ + b)} &= \frac{-3a + b}{\delta} + \mathcal{O}(\delta^0), \end{aligned} \quad (\text{A.40})$$

where we keep terms till $\mathcal{O}(1/\delta)$ since there is an overall δ multiplying Eqn. (A.39). Hence the $\log(\omega_T)$ terms cancel from the $\tau_1 > \tau_2$ and $\tau_2 > \tau_1$ pieces. Similarly, $\log(p_+ + \dots)$ pieces will generate dilogs once the integral over p_+ is carried out (however these dilogs vanish since they multiply $\mathcal{O}(\delta^0)$ coefficients, and hence are overall of $\mathcal{O}(\delta)$). They can also generate $\frac{1}{\delta^2}$ terms, which will contribute to the overall $1/\delta$ divergence of the correlator. Interestingly, these terms also cancel since,

$$\begin{aligned} \int dp_+ p_+^{4+\delta} \frac{\log(p_+ + a)}{(p_+ + a)^3(p_+ - b)} &= \frac{-3a + b}{\delta^2} - \frac{a}{\delta} + \mathcal{O}(\delta^0), \\ \int dp_+ p_+^{4+\delta} \frac{\log(p_+ + b)}{(p_+ - a)^3(p_+ + b)} &= \frac{3a - b}{\delta^2} - \frac{b}{\delta} + \mathcal{O}(\delta^0), \end{aligned} \quad (\text{A.41})$$

which ultimately contributes to NLF. Thus the contribution froms the $\tau_1 > \tau_2$ and $\tau_2 > \tau_1$ pieces once again add up to cancel the additional logarithm contributions, resulting in Eqn. (4.13) to hold in our case.

A.7 $2 \rightarrow 2$ scalar scattering at 1-loop in flat space

Here we express the flat space scattering process in Fig. A.2 as an integral over the loop 3-momentum. Consider massless ϕ^4 theory. The external particles have

(4-)momenta $p_i = (\vec{p}_i, E_i)$, $i = (1, 2, 3, 4)$, and the loop momenta are labeled by q_1, q_2 . Due to conservation of momenta, we have, $q_{\text{net}} = q_1 + q_2 = p_1 + p_2 = p_3 + p_4$.

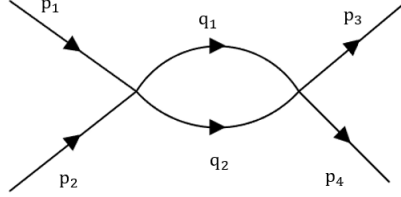


Figure A.2: $2 \rightarrow 2$ scattering of massless scalars with ϕ^4 interaction.

The flat space rules are framed as integral over 4-momentum and the Feynman rules of [33] specify loops as integrals over 3-momentum. We must integrate the former over dq_0 to check whether the Feynman rules used in this work reproduce the correct flat space amplitude. Following the usual flat space Feynman rules, we get the scattering process to be,

$$\begin{aligned} I &= \int d^4 q_1 \frac{1}{q_1^2 + i\epsilon} \frac{1}{q_2^2 + i\epsilon} \\ &= \int d^3 \vec{q}_1 dE_{q_1} \frac{1}{-|\vec{q}_1|^2 + E_{q_1}^2 + i\epsilon} \frac{1}{-|\vec{q}_2|^2 + (E_1 + E_2 - E_{q_1})^2 + i\epsilon}, \end{aligned} \quad (\text{A.42})$$

where $\vec{q}_2 = \vec{p}_1 + \vec{p}_2 - \vec{q}_1$. The integrand has simple poles in the upper half plane at $E_{q_1} = -\sqrt{q_1^2 - i\epsilon} = E_A$, $E_{q_1} = E_1 + E_2 - \sqrt{|\vec{q}_2|^2 - i\epsilon} = E_B$ and in the lower half plane at $E_{q_1} = \sqrt{q_1^2 - i\epsilon} = E_C$, $E_{q_1} = E_1 + E_2 + \sqrt{|\vec{q}_2|^2 - i\epsilon} = E_D$. The integrand is finite at large E_{q_1} , so we can close the contour in the upper half plane. The residue at these two poles are,

$$\begin{aligned} \text{Res}_{(E_{q_1}=E_A)} I &= -\frac{1}{2|\vec{q}_1|} \frac{1}{(E_1 + E_2 + |\vec{q}_1|)^2 - |\vec{q}_2|^2}, \\ \text{Res}_{(E_{q_1}=E_B)} I &= -\frac{1}{2|\vec{q}_2|} \frac{1}{-|\vec{q}_1|^2 + (E_1 + E_2 - |\vec{q}_2|)^2}. \end{aligned} \quad (\text{A.43})$$

Hence, we have,

$$\begin{aligned}
I &= \int d^3\vec{q}_1 \left[-\frac{1}{2|\vec{q}_1|} \frac{1}{(E_1 + E_2 + |\vec{q}_1|)^2 - |\vec{q}_2|^2} - \frac{1}{2|\vec{q}_2|} \frac{1}{(E_1 + E_2 - |\vec{q}_2|)^2 - |\vec{q}_1|^2} \right] \\
&= \int d^3\vec{q}_1 \left(\frac{1}{E_1 + E_2 + |\vec{q}_1| + |\vec{q}_2|} + \frac{1}{|\vec{q}_1| + |\vec{q}_2| - E_1 - E_2} \right) \frac{1}{4|\vec{q}_1||\vec{q}_2|}, \quad (\text{A.44})
\end{aligned}$$

which is exactly what we get in Eqn. (4.64).

A.8 Hamiltonian for $f(\dot{\varphi})$ interactions

Let us consider the Lagrangian for an interacting scalar field theory involving time derivatives,

$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 + \frac{g_3}{2}\dot{\varphi}^2\varphi. \quad (\text{A.45})$$

Comparing with the ADM action Eqn. (1.85) at cubic order, we have $g_3 \sim \sqrt{\epsilon}$.

The canonical conjugate is obtained by,

$$\pi = \dot{\varphi}(1 + g_3\dot{\varphi}\varphi). \quad (\text{A.46})$$

To obtain the interaction Hamiltonian, we must invert the above equation to trade $\dot{\varphi}$ for π , and clearly this inversion will generate the interaction $\dot{\varphi}^2\varphi$, as well as additional higher dimensional operators suppressed by the coupling. We want to investigate whether a quartic coupling will contribute to leading order. However, the quartic action in the spatially flat gauge is $\mathcal{O}(\epsilon^0)$ suppressed, and clearly at $\mathcal{O}(g_3^0)$ there are no quartic operators generated from the cubic interactions. To see

this, the Hamiltonian at $\mathcal{O}(g_3^2)$,

$$\mathcal{H} = \frac{\pi^2}{2} + \frac{(\partial_i \varphi)^2}{2} - \frac{1}{2} g_3 \pi^2 \varphi + \frac{1}{2} g_3^2 \pi^2 \varphi^2. \quad (\text{A.47})$$

Now to extract the interaction Hamiltonian, we separate this into a quadratic $\mathcal{H}_0$ and an interaction part $\mathcal{H}_{\text{int}}$, and set $\frac{\partial \mathcal{H}_0}{\partial \pi} = \dot{\varphi}$. This gives, at $\mathcal{O}(g_3)$,

$$\mathcal{H} = \frac{\dot{\varphi}^2}{2} + \frac{(\partial_i \varphi)^2}{2} - \frac{1}{2} g_3 \dot{\varphi}^2 \varphi, \quad (\text{A.48})$$

hence generating no additional quartic terms at leading order in slow roll, and we recover $\mathcal{H}_{\text{int}} = -\mathcal{L}_{\text{int}}$

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